Precise Topography of Optical Surfaces[1]

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The unit of length usually used in measuring optical surface features, by interference of light, is one-half the wavelength of the monochromatic light that is used. A method is described in which the unit of length is a much smaller fraction of the wavelength. Topographic maps of optical surfaces are made in which the contour interval is less than one-thirtieth the wavelength. The smallness of the unit depends upon the quality of the surfaces. Irregularities, too small to be detected with the Fizeau or two-beam fringes, are made to appear very prominent. Surface markings, caused by the final polishing actions, can be made to stand out in bold relief.

The accuracy of results that may be obtained with any type of interferometer depends upon the accuracy with which one can locate a line in the fringe field, along which the order of interference is constant. The intensity of light in a direction normal to these lines, in a set of two-beam or Fizeau [2] type of interference fringes, is a smooth mathematical function without abrupt changes, whereas for multiple beam or Fabry-Perot [3] type fringes it has sharp maxima and minima for transmitted and reflected light, respectively. The accuracy attainable with the two types of fringes is roughly proportional to the ratio of fringe separation to fringe width, the fringe width being defined here as the width at one-half maximum intensity.

Accurate contour maps of one surface relative to a standard surface of known shape are obtained by photographing the interference fringes, produced by these two surfaces, with monochromatic light. The contour intervals are one-half wavelength. As in geodesy, the smaller the contour interval the more detail is revealed in the contour map and the narrower the lines the more detail is revealed along the line. With broad fringes, of the two-beam type, irregularities of several hundredths of a wavelength can be completely hidden in or between the dark broad lines. An irregularity of several tenths of a wavelength can be completely invisible if it falls within the area between two adjacent Fabry-Perot fringes that appears uniformly illuminated. The one-half wavelength interval is, therefore, inadequate for representing local irrregularities on high-quality optical surfaces.

This paper describes a procedure whereby very narrow fringes are used for producing contour maps of optical surfaces with contour intervals of a small fraction of a wavelength. These intervals can be made as small as the narrowness of the fringes will permit. If m resolved fringes can be equally spaced between two of the original fringes, the separation of which corresponds to one-half wavelength or to one order of interference, then the contour intervals will be $[1/2(m+1)]\lambda$. Values of m as large as 19 have been used, for which the intervals are $\lambda/40$.

The usual procedure in precision measurements on glass optical surfaces is to place the unknown close to and almost parallel to a standard surface of known shape, with a thin air wedge between, and to observe the fringes produced by the interference of monochromatic light reflected normally from these two surfaces. Figure 1 shows a set of two-beam interference fringes, observed by reflection with a Fizeau viewing instrument, which one might have under observation for the purpose of measuring the deviation of an unknown surface from a standard plane surface. The orders of interference corresponding to each fringe, although not usually known, are indicated by the numerals. The exact position of the lines that represent the loci of integral orders of interference, are only known to lie somewhere within the darker portion of the fringes. This locus for fringe number 3 is indicated by the narrow solid line. Usually, as will be shown later, these lines are not as smooth [4] as the much broader fringes appear to be. The assumption that this line lies at the center of the dark fringe is usually false and, consequently, results based on this assumption are in error by the amount of displacement of the line from the center; which can amount to over a tenth of the separation between adjacent fringes. Even if one could locate the lines of integral orders, he would still be unable to measure the change in order of interference along a straight line such as the one shown dotted in figure 1, for the purpose of surface measurement. The change in order of interference along this straight line is an accurate measure of the deviation of the unknown surface from a plane, but if the shape of the surface between the fringes is unknown one has no way of determining the fractional orders of interference at points on this straight

[1] This work was done as part of a research project sponsored by the Air Force. The paper was presented at the March 1950 meeting of the Optical Society of America (Abstract, J. Opt. Soc. Am. **40**, 258 (Apr. 1950)). Since then the author's attention has been called to a paper by Takemara Sakurai and Kôrô Shishido at the Research Institute of Scientific Measurements, on "Test of an optical flat by the Fabry-Perot etalon" and published in the Scientific Report of the Research Institute, Tôhoku University, [A], **I**, No. 1 (May 1949).

[2] Fizeau, Compt. rend. **54**, 1237 (1862).

[3] C. Fabry and A. Perot, Ann. Chim. Phys. **12**, 459 (1897).

[4] The irregular serrations along the edges of the fringes are a good indication of the shape of the lines of constant orders. In fact, the location of lines of constant order (although not integral) are most accurately locatable along the margin of the fringes where the density gradients are a maximum. This is the most accurate means, known to the author for locating lines of integral differences in order of interference for surface measurements.

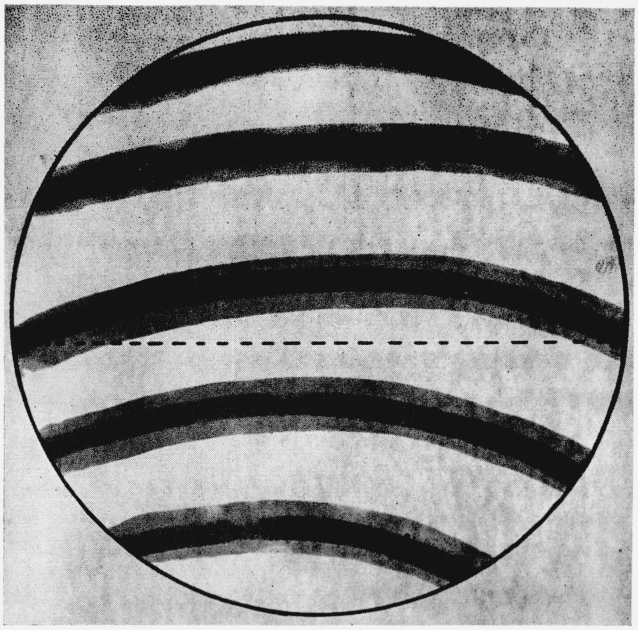

FIGURE 1. *Two-beam interference fringes.*

line that are not also on the lines of integral orders. Linear interpolation, for points not on a fringe, can be in error by several tenths of an order. The measurement of deviations of a surface from a plane, by measuring the change in order of interference along this line, is in error both from the inability to locate the lines of integral orders of interference and also from interpolation based on the false assumption that the order of interference varies linearly with the displacement of the point from the adjacent fringes. Unless a fringe passes through the center of a surface, the deviation of the surface at this point, relative to others, cannot be accurately measured.

The accuracy of measurements on the deviations of the unknown surface from an arbitrarily chosen reference plane, along any one of the lines of integral orders such as the narrow solid line in fringe 3, is limited only by the accuracy with which the line can be located. The deviation of the surface at any point on this line, from a plane through the indicated straight line (which is conincident with neither of the reflecting surfaces but intersecting the unknown surface at the two ends of fringe 3) and making an angle θ with the standard reference plane, is equal to the product of θ and the perpendicular distance of the point to the straight line where θ equals one-half the wavelength of the light used divided by the fringe separation. The fringe separation is defined as the average distance between the centers of adjacent fringes and is the separation that would be obtained if the unknown was replaced by a true plane, placed in the position of the arbitrarily chosen reference plane. The value for θ should be constant for all computations, although the observed fringes may neither be parallel nor equally spaced. The use of different values for fringe separation is equivalent to measuring deviations from different planes at various angles to the standard.

If both the standard and the unknown are coated with a uniformly thin coat of highly reflecting metal, such as silver or aluminum, the fringes in which the lines of integral orders of interference are located become narrower. The narrowness of these fringes, relative to their separation, increases with the reflectance of the surfaces and also with decreasing angle of incidence. The decrease in width of these narrow line fringes, relative to their separation, increases the accuracy of locating the lines of integral order of interference. With modern methods of metal deposition and with smooth surfaces that are almost parallel to each other, fringes may be obtained whose width, at one-half the maximum intensity, are only one-fiftieth part of their separation.[5] With such narrow fringes, deviations can be measured to better than one-hundredth of a wavelength. These narrow-line or Fabry-Perot type fringes are dark by reflected light and bright with a dark background, with transmitted light. Figure 2, A, shows a set of two-beam fringes produced by reflected light, and B shows a graph of multiple reflection fringes, formed by the same surfaces by transmitted light after applying the thin coat of aluminum. Note the increased accentuation of detail along these narrow fringes over that shown by the broad ones.

When measuring surface features to one-hundredth of a fringe, the mechanical errors in the micrometer (assuming a micrometer eyepiece to be used) and personal errors of settings become large unless the changes in order of interference with movement of the cross hair, are relatively small. This requires that the fringes have considerable separations. At least two fringes must appear on the surface in order to determine the unit of deviation of a fringe from a straight line corresponding to a known unit of deviation of the surface from a plane. This unit is usually one-half the wavelength of the monochromatic light being used. Figure 3, A, shows such a set of fringes, formed by the same surface that was used for figure 2, B. Further enhancement of detail along the fringes in figure 3, A, over those of figure 2, B, is obvious. This is partially due to the decrease in width of the fringes relative to their separation, caused by making the wedge between the plates smaller and, consequently, reducing the angle of incidence that favors the narrowing of these fringes.

In order to further enhance irregularities along a fringe and still be able to measure the magnitude of the corresponding deviations in the surface, a multiple spectral line source, such as the mercury arc for instance, may be used. Each spectral line produces its own set of fringes. If the absolute orders of interference of each and all of the fringes are known, the separation of the surfaces along the fringes is known and, consequently, the change in separation of the surfaces between two adjacent fringes of different colors is also known, although this difference in separation may be only a small

[5] S. Tolansky, Multiple-beam interferometry of surfaces and films (Oxford at the Clarendon Press, 1948).

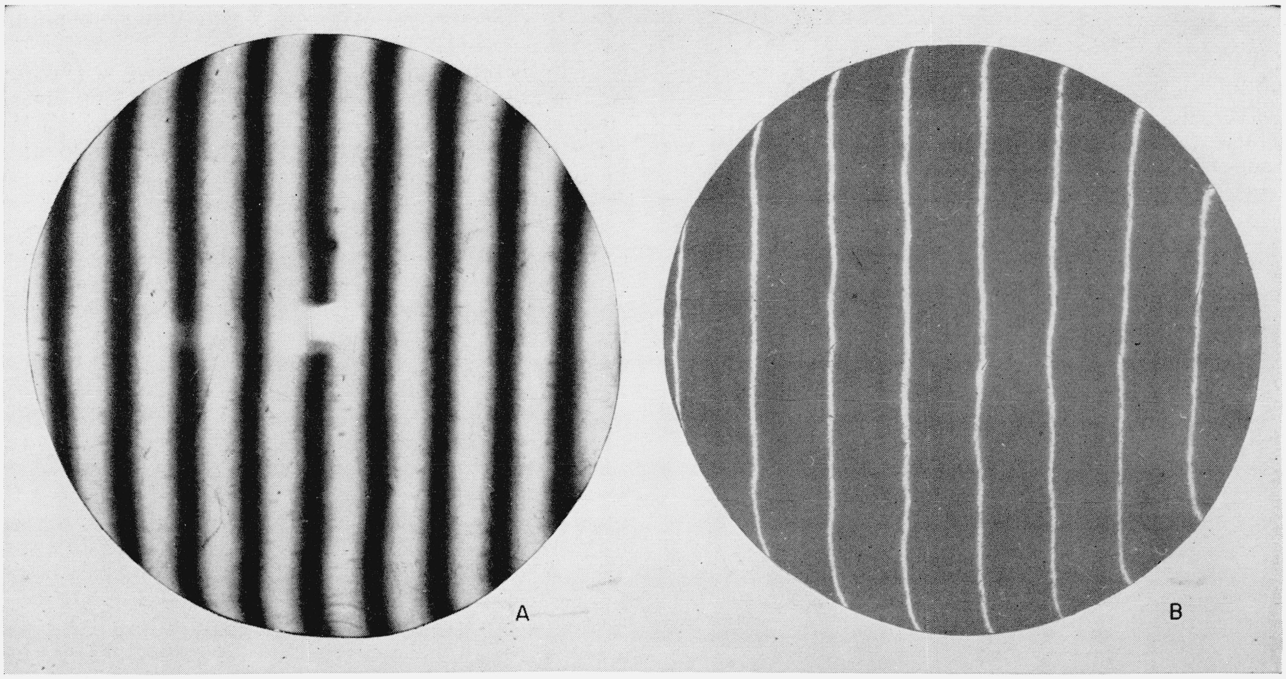

FIGURE 2. *Equal-thickness interference fringes.*

A, Two-beam fringes produced by reflected light; B, Multiple-beam fringes produced by transmitted light.

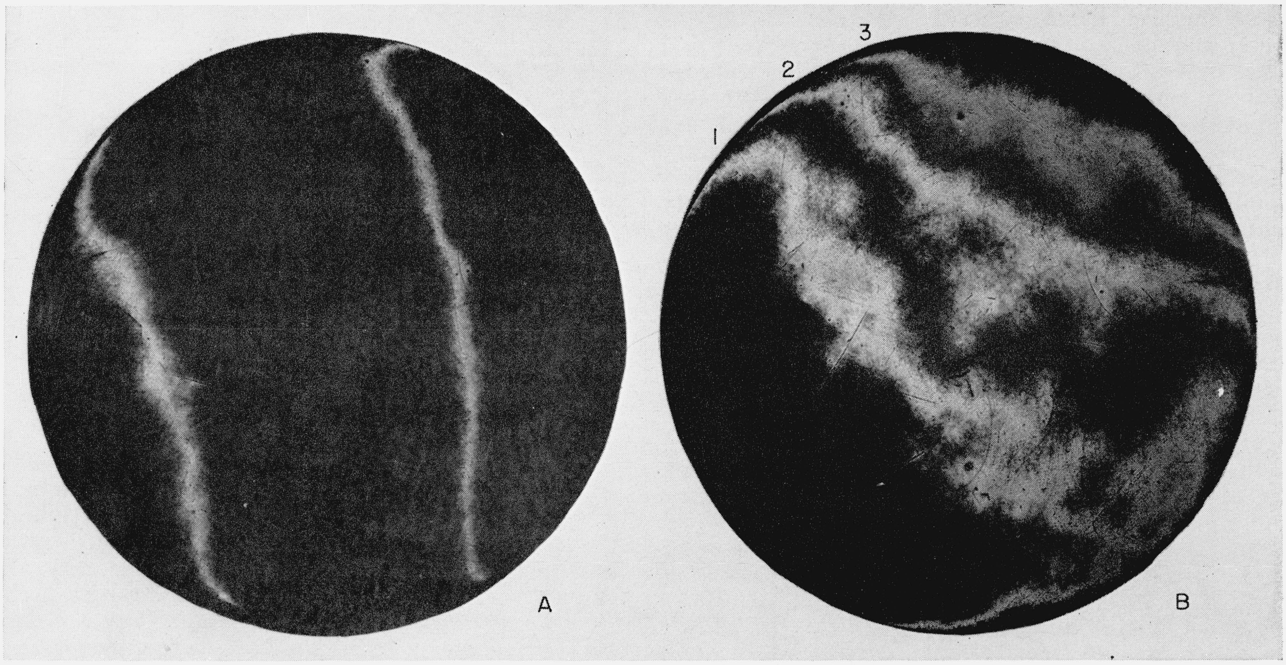

FIGURE 3. *Multiple-beam interference fringes.*

A, Single spectral line. Note increase enhancement of surface detail. B, Multiple spectral line. One order of each of three lines shown on plate. Wedge too small to include other lines. Note further enhancement of surface detail.

fraction of the unit of length, one-half wavelength. The thickness and angle of the air wedge between the surfaces may now be adjusted so that one of two or more closely spaced fringes of different color pass through the center of the surface and one or more of the others just inside the edge. The surfaces are then as near parallel as is practicable, thus favoring the attainment of the narrowest fringe possible, relative to fringe separation, and measurements may be made at all radial distances. Still greater enhancement of the irregularities is thereby attained, and two or more fringes are available for quantitative evaluation of these irregularities. Figure 3, B, shows a set of fringes, produced by a mercury source, in which the total change in order of interference across the plate is less than three-tenths of a unit ($\lambda/2=0.27\mu$).

Maximum enhancement of surface irregularities is obtained by adjusting the surfaces to as near parallelism as is possible and the separation near to some integral number of half wavelengths. If one had two plane reflecting surfaces (the existence of which is doubted by the author) these could be placed parallel to each other, which is the condition for fringes of infinite width. If the separation is now adjusted to some value in the neighborhood of where the resultant interference produces maximum variation of intensity with change in separation, then small deviations in phase, caused by irregularities of only a few angstroms, such as may be produced by the individual particles of the polishing compound, become quite visible. Figure 4 shows the abovementioned surfaces (actually both surfaces contribute equally) in which the parallelism and separation are most favorable to the revelation of small surface detail. Note the fiber-like appearance in the darker regions where maximum enhancement of detail occurs. Quantitative measurements of these finer irregularities have not yet been attempted.

The manner of determining absolute orders of interference may be described by means of figure 5 The large circularly enclosed fringe system represents a set that might be under observation. The colors of the fringes are indicated by R for red, Y for yellow, G for green, B for blue, and V for violet. The wavelengths of the light, forming the fringes, are identified usually by the color of the corresponding fringes. The small circularly enclosed fringe system represents the same fringes as those shown in the large circle. They are straight as if formed by parallel surfaces and are inserted to show the relationship between the fringes in the large circle and the lines, which correspond to fringes, in the chart above. If we assume the surfaces that form this fringe system to be extended in the direction of their decreasing separation to their line of intersection, and one could observe the fringe system in this extended area, one would find and could identify the zero order of interference for each set (color) of fringes, all of which coincide. Extending from the zero order fringe, one would observe equally spaced red fringes (assuming plane surfaces), the order of which could be identified by counting from the zero order.

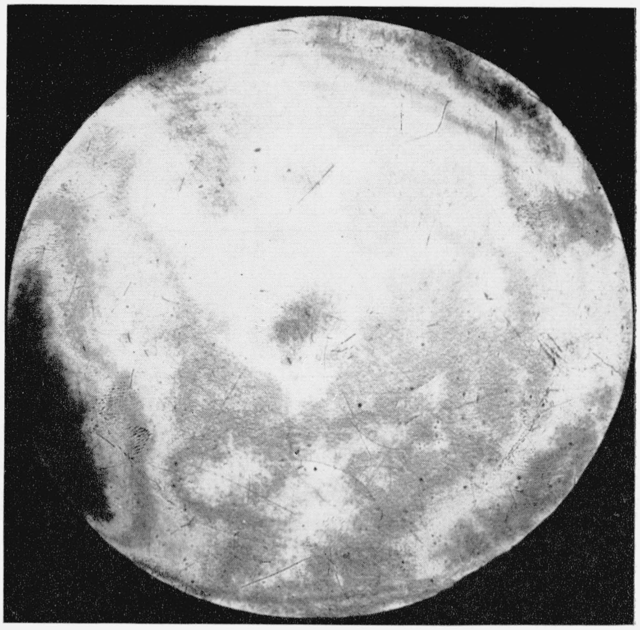

FIGURE 4. *Multiple-beam interference fringes.*

Surfaces are as near parallel as the surfaces will permit (maximum enhancement of detail). Note fabric-like marks; believed to be due to final polishing action.

Thus the numbers associated with the vertical lines indicates the order of interference for all the visible colors in the source, and their positions correspond to the positions of the fringes. One sees on this chart, in the neighborhood of 3.3-μ separation of the plates, that several fringes fall in a space corresponding to 0.12-μ increase in separation of the plates. This space corresponds to a change of only 0.36 in the order of interference for this red spectral line. If the maximum and minimum separations of the plates are adjusted to 3.2 and 3.4 μ, respectively, the particular set of fringes that correspond to the lines falling in this range on the chart can be seen on the plates. We therefore have a fringe system in which the increase in separation of the surfaces from one fringe of a given color to another fringe of a different color is a small fraction of the conventional unit, ½λ, that is usually used for this type of measurement. Furthermore, the width of the bright line fringes, relative to separation of adjacent orders, is made smaller by the two surfaces being made nearer parallel than is practical when using monochromatic light. The narrower the fringes, relative to separation of adjacent orders, the more concentrated a cluster or grouping of fringes can be used, without overlapping. Figure 3, B, shows a constellation or group of fringes whose separations correspond to an increase in separation of the surfaces of 270 A from fringe 1 to 2, and 225 A from 2 to 3.

When an optical flat is placed on an unknown surface with only an air film separator, the weight of the top plate forces the air film to become relatively thin, but the plates will not come into absolute contact at any point. Usually, small particles of dust will prevent the air film from becoming as

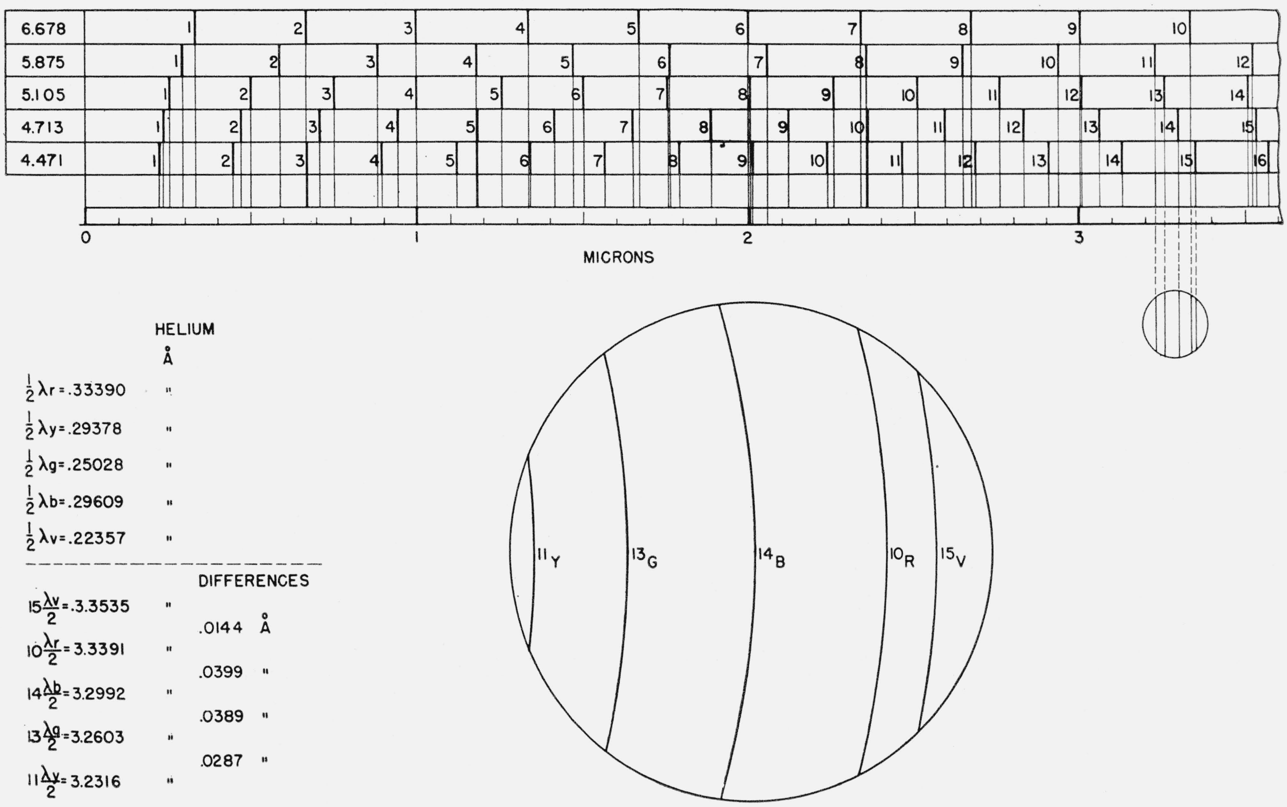

FIGURE 5. *Chart showing how the absolute order of interference is identified from the relative distribution of colored fringes produced by a multiple line source.*

thin as would be attained in their absence The usual range in thickness, for circular surfaces of 4-in. diameter and free from dust, is from 8 to 9μ. For larger surfaces this range may extend to 10μ. The range, 0 to 3.6μ, shown in figure 5 is not practical but was used for explanatory purposes only. The practical range is from 6 to 9μ and is shown in figure 6 for several different light sources. If one has access to different sources he may choose a constellation most suitable for a given film-thickness range.

A more elegant topographic map of an optical surface, relative to a standard, can be obtained by using monochromatic light and photographing the image of a given fringe in several different positions on a single film; the shift in the fringe system being accomplished by changing the optical separation of the surfaces. There are several different ways of changing the optical separation, only one of which will be described in detail. The interferometer plates are enclosed in an airtight chamber (see fig. 7) in which the pressure of the enclosed gas may be controlled at will. The collimator and collector lenses form the windows to the chamber. A gas line, connected to a manometer through a two-way stop cock, permits either compression by a compressor, or evacuation by a vacuum pump. A needle valve in the line, not shown, facilitates control of the pressure in the chamber.

According to the law of Gladstone and Dale, the index of refraction of a gas varies linearly with the density. For a constant temperature of 25° C, which was maintained during this experiment, the density is directly proportional to pressure, to a close approximation. The relation between refractive index and pressure, is $n-1=P(n_0-1)/760$ where n is the refractive index of the gas, P is the pressure in millimeters of mercury, and n_0 is the refractive index of the gas at 760 mm pressure and at 25° C.

If Δn is the change in n for a given change of ΔP in P, it may be shown that the corresponding change in order of interference, ΔN, at a point where the separation of the surfaces is L, is

$$\Delta N=\frac{2L\Delta n}{\lambda}=\frac{2L}{\lambda}\frac{(n_0-1)}{760}\Delta P.$$

From this relation one may compute the separation, L, that is necessary in order to get a chosen change in order of interference for any desired pressure range. For instance, suppose the available pressure range to be 1 atm or 760 mm of mercury and the desired change in order of interference to be one (the maximum practical change for this work), then $\Delta P=760$, and $\Delta N=1$. Consequently,

$$L=\frac{\lambda}{2(n_0-1)},$$

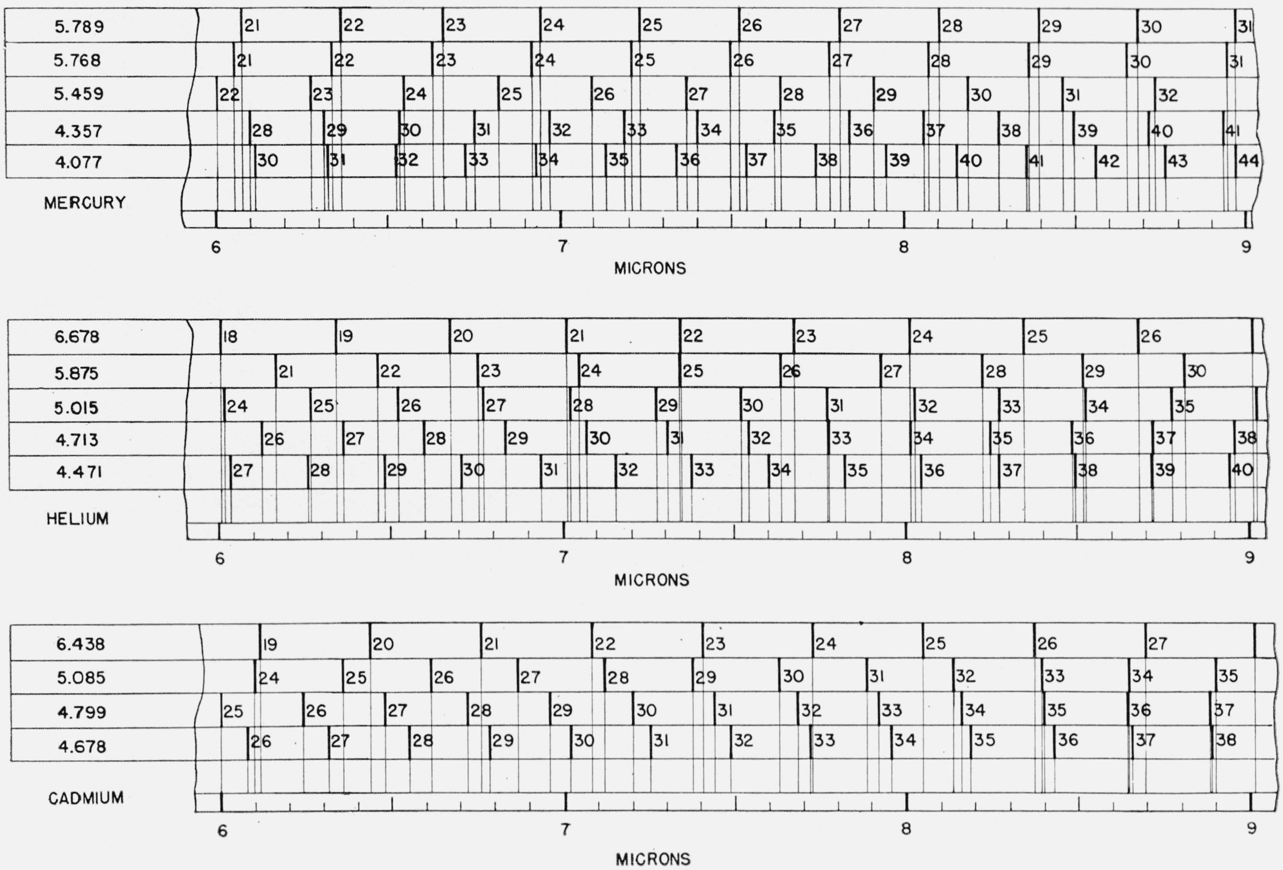

FIGURE 6. *Charts showing relative distribution of colored fringes for normal separation of plates when placed one on the other for test.*

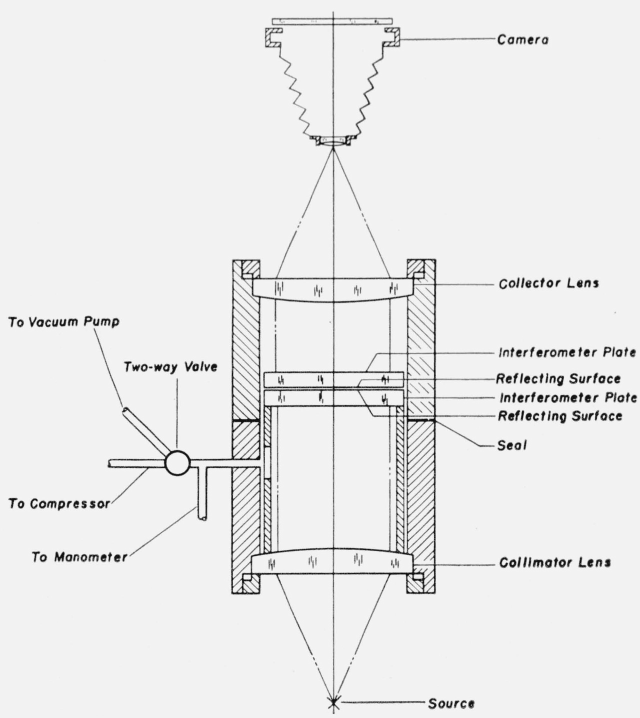

FIGURE 7. *Vacuum chamber used to change optical separation without changing the geometrical separation of two interferometer plates.*

FIGURE 8. *Topographic map of one optical surface, relative to a standard, produced by photographing a given fringe (order of interference) successively, as it was moved across the plate, by controlled evacuation of air wedge between the plates.*

Total movement corresponds to a change of one-half order of interference.

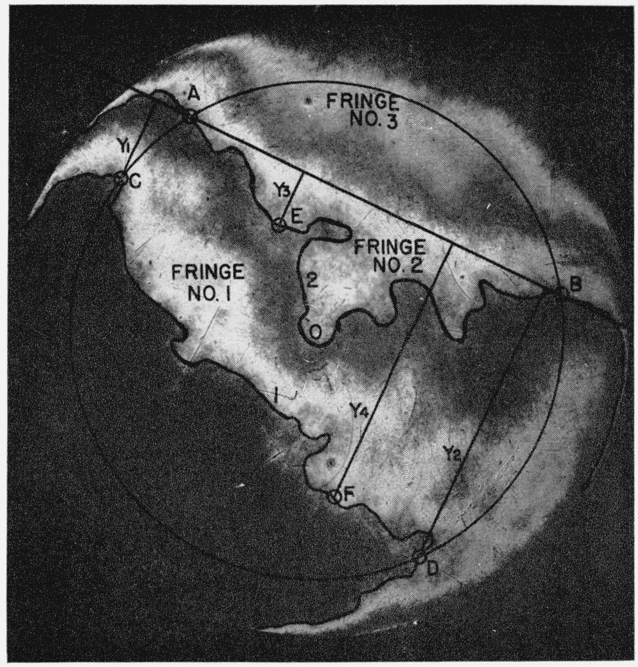

FIGURE 9. *Same photograph as in figure 3, B, with inserted drawing, for surface measurement.*

If, for instance, the light to be used is the green line of mercury, for which $\frac{1}{2}\lambda = 2.7 \times 10^{-5}$ cm and $n_0 - 1$ is 2.8×10^{-4}, then L equal 0.96 mm. Consequently, with this separation, pressure range and light source, a fringe can be made to take on any position between the positions of this fringe and that of one of its adjacent neighbors at the extremes of the pressure range. If there is more than one fringe on the plate and one wishes to interpolate nine additional, equally spaced fringes, between each of the adjacent original fringes, thus completely filling the photographed field with fringes whose separations are one-tenth that of the original set, then it will be necessary to change the pressure in steps of 76 mm for nine times, or a total of nine-tenths of an atmosphere. If the wedge between the surfaces is such as to give a maximum change in order of interference of one-half, then one-half-atm change in pressure is sufficient to move the center of a fringe completely across the surface. If, for this wedge, one wishes to photograph 10 fringes on the plate, the necessary change in pressure will be nine-twentieths of an atmosphere. It is, of course, essential that this pressure range be within that available, which in this particular case is 0 to 760 mm.

Figure 8 represents a topographic map with contour intervals of approximately 170 A, or one-thirtieth of a wavelength of cadmium green light. The separation of the two surfaces was 0.82 mm, and the change in separation, in terms of interference fringes, was approximately one-half fringe (1,300 A).

To illustrate the application of these smaller-than-one-half-wavelength-unit topographic maps to quantitative determination of surface features, measurements have been made on the deviations of one of the surfaces used to produce figure 3, B, relative to the other. For this discussion one surface will be assumed a true plane (called the reference surface) and the other (called the unknown) to have all the irregularities corresponding to the irregularities shown in this fringe pattern. Lines and points have been inserted in figure 3, B, and reproduced as figure 9.

The deviations of the unknown from a true reference plane, for several points along the lines AEB and CFD have been computed and are plotted in figure 10. The chosen reference plane is an imaginary plane that passes through points A and B (consequently including all points on the straight line AB) and making an angle θ with the reference surface approximating the average angle between the two reflecting surfaces. The unknown surface and the chosen reference plane are made as near parallel to each other as the irregularities in the unknown will permit. The value of θ is obtained from circle $ABDC$, which is concentric with respect to the center of the plates, the straight line AB, and the perpendicular projections of points C and D on line AB. Circle $ABDC$ must intersect at least two locatable lines, such as AEB and CFD, along each of which the order of interference is constant, and between which the change in separation of the reflecting surfaces is known. The line AEB is the lower edge of fringe 2, and line CFD is the lower edge of fringe 1. The change in separation of the plates between these two lines is approximately equal to that between the centers of the corresponding fringes and, as was given previously, equal to 270 A. The magnitude of θ is taken to be $2d/(Y_1+Y_2)$, where d equals the change in separation of the plates between fringes 1 and 2, Y_1 equals the normal distance of the point C from the straight line AB, and Y_2 equals the normal distance of point D from AB. This choice for θ is entirely arbitrary and in no way affects the computed shape of the unknown surface.

To correlate the data shown in figure 10 with the topographic map (fig. 9) from which it was derived, the designated points in figure 9 are labeled likewise in the graph. Thus, the ordinate of point E in figure 10 is the deviation of the unknown at point E of figure 9. The lower branch of solid curve $2R$, figure 10, represents deviations along the right-hand portion of line 2 figure 9, and the upper branch, $2L$ having the circled points, corresponds to the left-hand portion of this line.

These measurements are not limited to points along a single fringe or line, but may be made at any point along any fringe that appears on the photograph. The deviation of the unknown from the chosen reference plane, at a point on any other line, such as point F on line 1, is equal to $Y_4\theta - Nd$, where Y_4 is the normal distance between point F and line AB, and N is the difference in order of interference between F and any point on line 2. Thus, measurements may be made at all points along each and every contour line shown.

The plot shown in figure 10 does not give any better idea of the shape of the unknown surface than the contour map from which it was derived. The values are, however, better defined numerically.

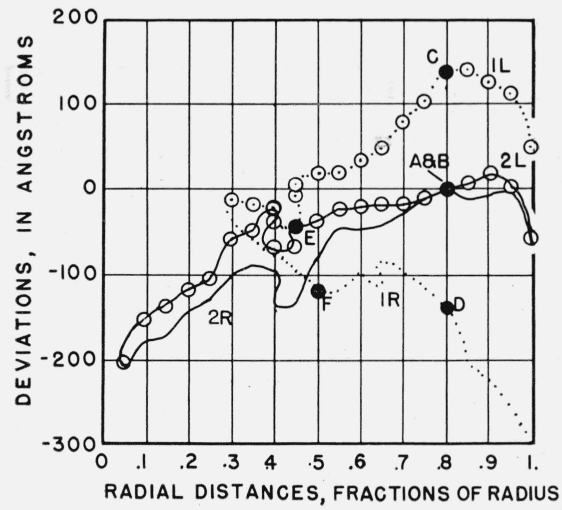

FIGURE 10. *Radial distances, fractions of radius.*
Radius equals one-half diameter of plates.

The failure of all computed points to fall on a single smooth curve indicates that the unknown surface is not a figure of revolution about its center. The appearance of more than two values of a given radial distance and on a single line is due to local depressions or elevations. The irregular shape of the curves also indicates the degree of local irregularities in the surface. These local irregularities represent deviations approximating 50 A, in some cases.

WASHINGTON, January 2, 1951.