U. S. Department of Commerce
National Bureau of Standards

Research Paper RP2089
Volume 44, April 1950

Part of the Journal of Research of the National Bureau of Standards

Interference Methods for Producing and Calibrating End Standards

By Chauncey G. Peters and Walter B. Emerson

An interferometric apparatus developed at the National Bureau of Standards for ruling meter scales in terms of wavelengths of light requires accurate determination of the length of a decimeter etalon. To compare the results of the interferometric methods of measurement used at this Bureau with those of other national laboratories, etalons of fused quartz having highly polished plane and parallel end surfaces were made and calibrated at this Bureau. They were then sent to the Bureau International des Poids et Mesures, Physikalish-Technische Reichsanstalt, and The National Physical Laboratory for calibration. The lengths of the etalons as determined by the various laboratories are:

Etalon No.	By other Laboratory	By NBS	Difference
	mm	*mm*	*mm*
11	BIPM 99.999 848	99.999 855	−0.000 007
2	PTR 99.999 934	99.999 907	+.000 027
15	PTR 99.999 899	99.999 919	−.000 020
13	NPL 99.999 862 (initial measurements)	99.999 872	−.000 010
14	NPL 99.999 906 (initial measurements)	99.999 888	+.000 018

These results indicate that diverse interferential methods used by the different laboratories give the same lengths to an average deviation of 0.016 μ (1 part in 6,000,000) and to a probable uncertainty much less.

Besides furnishing a direct comparison of the results obtained by different interferential methods, the investigation gives values for the index of refraction of air and also wavelength values for several radiations of krypton and cadmium. These are included in the paper.

Because of inherent advantages of fused quartz etalons as separators for Fabry-Perot interferometers, the methods used in making and calibrating these etalons are given in detail.

I. Introduction

1. Light Waves as Length Standards

Impetus for the use of light waves as standards of length developed from the classical measurements of Michelson and Benoît [1],[1] who in 1893 determined the number of waves of red, green, and blue radiations of cadmium in the standard meter; and those of Benoît, Fabry, and Perot [2], who repeated these measurements in 1905–06 with modified apparatus.

In 1925 the International Astronomical Union provisionally adopted [3] the red radiation of cadmium as the standard for wavelength measurement, assigning as its wavelength for specified conditions of gravity, air temperature and pressure, and operation of the source the value determined by Benoît, Fabry, and Perot; namely $\lambda_R=6438.4696\times10^{-10}$ m. In 1927 the International Conference on Weights and Measures likewise adopted [4] the red radiation of cadmium as the primary wavelength standard with the same value, $\lambda_R=6438.4696\times10^{-10}$ m, but with a specification for the source that differed in some details

[1] Figures in brackets indicate the literature references at the end of this paper.

from that adopted by the IAU. Further changes in the specification of the cadmium source were adopted by the International Committee on Weights and Measures in 1935 [5]. The selection of this particular radiation appears to have been happy in that no definitely superior line has since been discovered in nature. Recently, however, artificial Hg^{198} has been produced, which gives superior radiations. Furthermore, the original value of Benoît, Fabry, and Perot that was adopted for its wavelength is confirmed within the uncertainties of measurements by later determinations, as seen in table 1, compiled by Barrell [6].

TABLE 1. *Determinations of λ_R by various observers*

Date of determination	Observer	Original values	Values corrected and adjusted for normal air	Difference from the mean
		A	*A*	*A*
1892–93	Michelson & Benoît (BIPM) [1]	6438.4722	6438.4691	−0.0005
1905–06	Benoît, Fabry, & Perot (BIPM) [2]	6438.4696	6438.4703	+.0007
1927	Watanabe & Imaizumi (Japan) [7]	6438.4685	6438.4682	−.0014
1933	Sears & Barrell (NPL) [8]	6438.4711	6438.4713	+.0017
1933	Kösters & Lampe (PTR) [9, 10]	6438.4672	6438.4689	−.0007
1934–35	Sears & Barrell (NPL) [11]	6438.4709	6438.4709	+.0013
1934–35	Kösters & Lampe (PTR) [11]	6438.4685	6438.4690	−.0006
1937	Kösters & Lampe (PTR) [12]	6438.4700	6438.4700	+.0004
1940	Romanova, Varlich, Kartashev, & Batarchukova (USSR) [13]	6438.4677	6438.4687	−.0009
	Mean		6438.4696	±0.0009

Although individual determinations of λ_R differ by more than one part in 2 million, the mean of all nine corrected determinations is identical with the adopted value. The wavelengths of certain neon and krypton lines have been determined relative to λ_R with high precision, thus making length measurements with these lines equivalent to direct comparisons with the cadmium standard. Repeated intercomparisons of lines from numerous sources over long periods of time show no definite variations in their wavelengths greater than the uncertainty of measurement. It is therefore evident that suitable lines can be confidently adopted as standards of length.

It is a well-known fact that length standards of alloys or other materials have shown considerable change in length with time. The results in table 1 show that the meter bars have, however, during the interval 1892 to 1940, remained constant in length within the limits of error imposed by intercomparison of ruled scales and light waves.

For the past 30 years interferential methods using light waves as length standards have been employed by this Bureau to calibrate standard and commercial end gages. The original methods of making these tests have been adequately described by Peters and Boyd [14]. During the same period other national standardizing laboratories likewise developed methods and apparatus for calibrating gages by means of light waves.

To ascertain the degree of uniformity of interferential calibrations by different laboratories, this Bureau in 1927 submitted four 1-in. Johansson gages, designated: $.B_1$, $:B_1$, $\therefore B_1$, $::B_1$, to the National Physical Laboratory (NPL), the Physikalisch-Technische Reichsanstalt (PTR), and the Bureau International des Poids et Mesures (BIPM) for calibration. A summary of these measurements is given in table 2. The individual measurements differed from the mean by 0.00 to 0.03μ.

TABLE 2. *Calibration of 1.0-in. gages by different laboratories*

Measured by—	Length at 20° C			
	$.B_1$	$:B_1$	$\therefore B_1$	$::B_1$
	mm	*mm*	*mm*	*mm*
NPL	25.400 054	25.400 059		
PTR			25.400 08	25.400 09
BIPM			25.400 10	25.400 12
NBS	25.400 11	25.400 06	25.400 09	25.400 10

In view of the availability, reproducibility, permanency, and adaptability of light waves for precise measurements of end gages, their use is favored for calibrations up to the maximum length to which they may be directly applied with accuracy. One further step in their evaluation as length standards remains; i. e.: to determine the precision with which the prototype meter line scale itself may be reproduced by means of light waves. A meter so reproduced will give a direct check, within the errors of ruling and measuring, of the accuracy of the lengths that have been assigned to the waves employed.

2. Purpose of this Investigation

This Bureau has developed apparatus for ruling meters in terms of light waves, reversing the procedure of Michelson and Benoît. The method selected requires the stepping-off of a meter by using a decimeter end gage that has been measured in terms of standard waves. An accurate direct interferometric calibration of this decimeter gage is imperative, because any uncertainty in its length is multiplied by a factor of 10 in the meter ruled with it. Although the 1927 comparisons gave good agreement among observers for 1-in. gages, no international comparisons were then available for gages of 1-decimeter length for which the path difference approaches the limits of satisfactory interference set by the inherent characteristics of the radiations.

The purpose of the present investigation was to to determine the accuracy of this Bureau's interferential measurements of decimeter lengths. To do this, decimeter end gages of fused quartz were calibrated at the NBS, then two of these were sent to the NPL, two to the PTR, and two to the BIPM. Although decimenter gages had been prepared from the same material (a special steel) as that to be used in ruling meters, it seemed advisable in this investigation to use gages that were capable of more precise measurement than were steel gages. From considerations of permanency, low thermal expansivity, and the perfection with which optical surfaces may be polished plane and parallel, fused quartz glass was selected as the most suitable material. Although this is too fragile for commercial gages, its fragility proved no real handicap in the present investigation. The results obtained by the use of these fused quartz gages give a critical test of the uniformity of measurements by interferential methods at the various standardizing laboratories.

3. Data

In order to meet the more exacting requirements of present-day gage calibrations, interferential methods employed at the NBS have been modified somewhat since the paper of Peters and Boyd was published. Detailed information is therefore given of these present methods, together with the results obtained at this Bureau for 12 fused quartz decimeter end gages, including 5 that were also measured by standardizing laboratories that use other interferential methods.

As an auxiliary investigation, gages of fused quartz, combined with fused quartz plates to form a Fabry-Perot interferometer, proved adaptable to the determination of the refractive index of air and to the determination of other useful wavelengths by comparison with the standard cadmium wavelength. The results of preliminary measurements of the refractive index of air and also the wavelength values that were obtained for several radiations of cadmium and krypton are included. Because of the proved value of these fused quartz gages and consequently their possible value to other investigators, the procedure in making them is included in this paper.

II. Procedure in Making the Gages

1. Preparation of Blanks

Fused quartz was obtained commercially in the form of cylindrical slugs (diameter 8 cm, length 11 cm). These slugs were clear except for scattered minute bubbles, which did not affect their use for end gages. Each slug was sawed into blanks 2 cm square and slightly over 10 cm long, which were afterward heated in an electric furnace to 1,145° C, held at this temperature 35 min and cooled in the furnace to 900° C at the rate of 1° C per minute. The heating current was then cut off, increasing the cooling rate somewhat. This heat-treatment left the quartz in about the same strain-free condition when examined in polarized light as that of well-annealed optical glass. After cooling, the blanks were machine-ground on all faces and finished with 303 emery to within 15 fringes (4.5μ) of the correct length.

2. Finishing of the Gages

First an attempt was made to work the gages in multiple. Four over-length blanks were cemented together and fused quartz blocks 1 by 2 by 5 cm were cemented at the ends of the blanks, as shown in figure 1, to prevent rounding of the edges during polishing. The surfaces were then ground and polished to correct form and length. Multiple working of the gages proved unsatisfactory however, for after the pressure on the blanks (which resulted from the cementing operation) was released the gage surfaces were found to be distorted and their parallelism destroyed. Final procedure was to work each blocked-up end surface of each gage separately. Blocks of fused quartz 1 by 2 by 3 cm were cemented to one end of a blank,

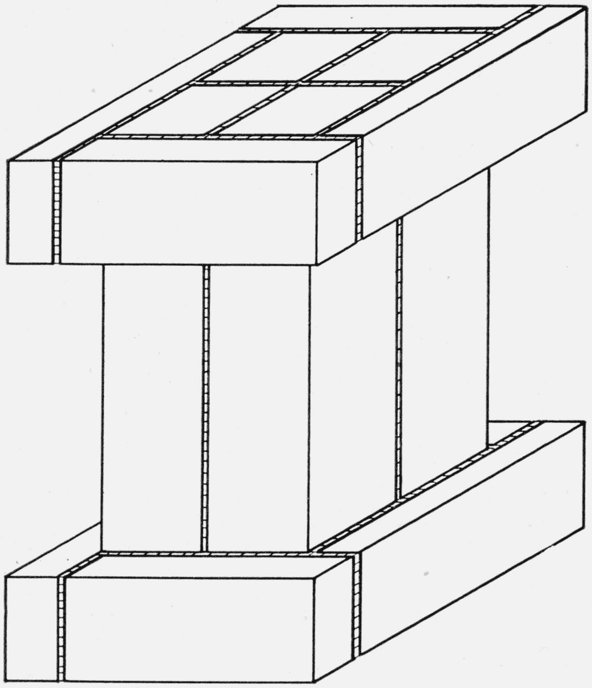

FIGURE 1. *Gage blanks mounted for multiple working of their end surfaces.*

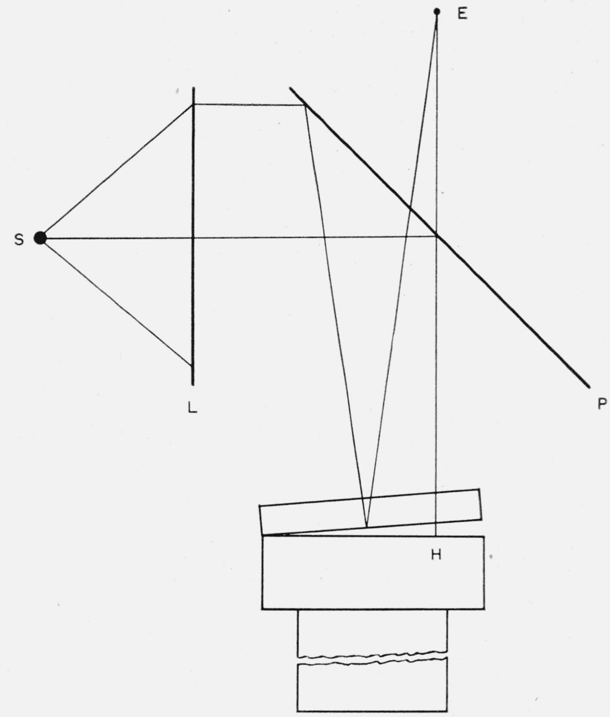

FIGURE 2. *Apparatus for inspecting the gage surfaces.*

after which that end surface was fine-ground with 304 emery on a cast iron lap and pitch-polished plane, using opticians lump rouge. During the early polishing operation, surfaces were inspected with the simple interferometric testing device of Peters and Boyd shown in figure 2 consisting of source, S, of monochromatic light, diffusing glass plate, L, and a thin glass plate, P, set at 45 deg to perpendicular EH from the eye to the surface. As a surface approached its final form, more precise measurements were made with apparatus described below.

After one end of a gage had been finished, the blocks were removed and applied to the unfinished end, which was then worked to make it plane, parallel to and 100 mm distant from the first surface. Length measurements were first made with a Zeiss optimeter until the gage was within 6 or 7 fringes (2μ) of the required length, and errors in parallelism were down to 1 fringe. At this stage more sensitive methods of measuring both parallelism and length were substituted. It was found necessary to continue to completion work once started on a surface because of slippage of the cemented blocks with time and, for this reason, rapidity was an added requirement of the test methods.

3. Method of Checking Parallelism and Planeness of the Ends

The apparatus for testing parallelism and planeness of the ends is illustrated in figure 3. Three quartz end gages (1, 2, 3), each 2 by 2 by 10.03 cm, made accurately to the same length, were wrung to quartz flat B. To the other end of each of these, quartz flat A was also wrung. The adjacent surfaces of A and B were thus plane and parallel (to 0.01λ), and 10.03 cm apart. S is a thin wooden strip that was attached to the rear of supports 1 and 2 by means of wax. Gage G is in position for test. The assembled apparatus is shown in figure 4.

The procedure for testing parallelism of surfaces was to insert a gage between the plane surfaces of A and B and to incline it slightly to the vertical by so resting it against the strip S as to give a spacing of about 2 mm to the straight fringes that were formed by interference of helium light reflected from the lower surface of A and from the top of the gage. These fringes were viewed normally by means of a Pulfrich instrument [15], which is a self-contained device for illuminating and viewing the surfaces and for measuring deviations and displacements of the fringes. The

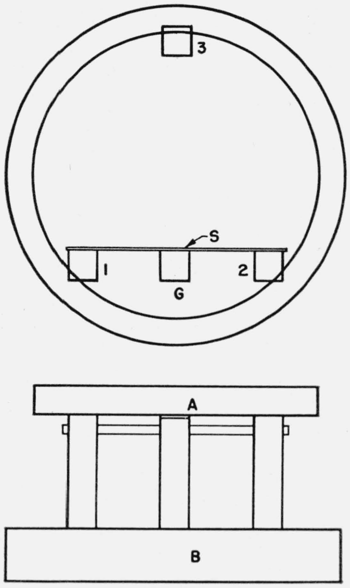

FIGURE 3. *Apparatus for testing parallelism and planeness of the gage surfaces.*

cross wires of this instrument were made parallel to the fringes in direction *ab*, figure 5. Next, the instrument was refocused upon a lower set of fringes formed by interference at the upper surface of *B* and the bottom of the gage. The condition of parallelism required that the latter set of fringes should also be parallel to the cross wires, i. e., the upper and lower sets of fringes should be mutually parallel. The deviation from this condition was a measure of lack of parallelism. For example: if distance *e* from the fringe to the cross wire differs from *f* by one-third the distance between two fringes, then the gage lengths at edges *a* and *b* differ by ⅓ fringe (0.1μ). This difference in length at opposite edges, or lack of parallelism, will be termed the "slope" of the gage. In order to compensate for possible slope in the apparatus, the gage was measured in one position, rotated 180 deg. and remeasured; the recorded slope of a gage is the mean of these measurements. The slope was determined at *a*,

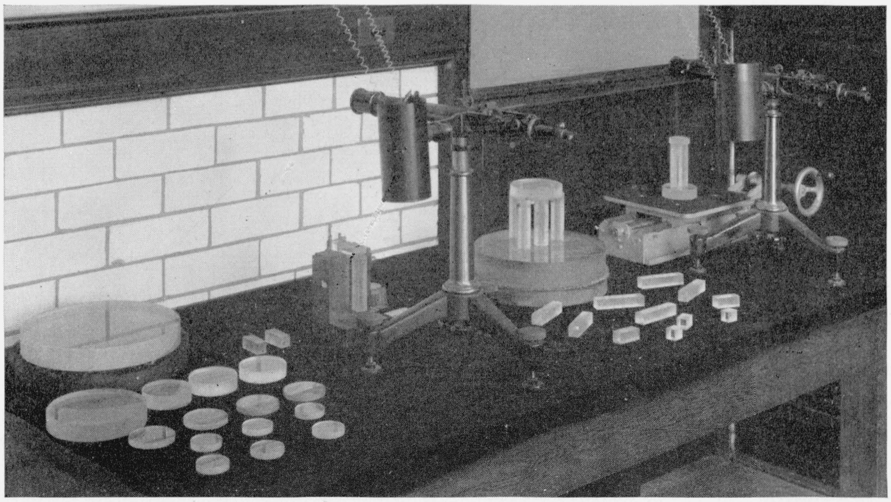

FIGURE 4. *Assembled apparatus for testing the gages.*

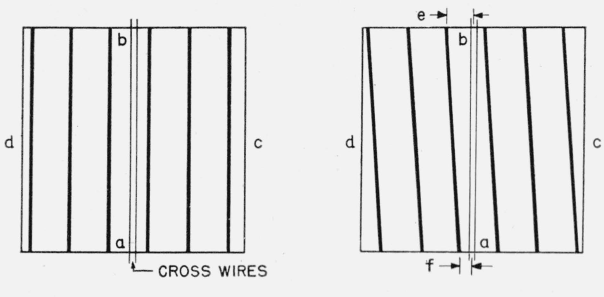

FIGURE 5. *Upper and lower sets of interference fringes.*

b and *c*, *d* for each gage. These tests could be made rapidly and consequently served as a convenient basis for correcting errors in parallelism. The accuracy of the method was verified by length measurements at opposite edges of the gages.

For making accurate measurements of planeness with the same apparatus, the procedure was to place the tested surface beneath plate *A* as shown by *G* in figure 3. Its curvature was determined by placing straight edge *E*, figure 6, above the fringe pattern and measuring the departure of a fringe from a straight line. If a surface was higher at its center than at its edges by ⅓ fringe, then the end of the gage was considered ⅓ fringe (0.1μ) convex. In general, departures from planeness assumed the form of regular curvatures.

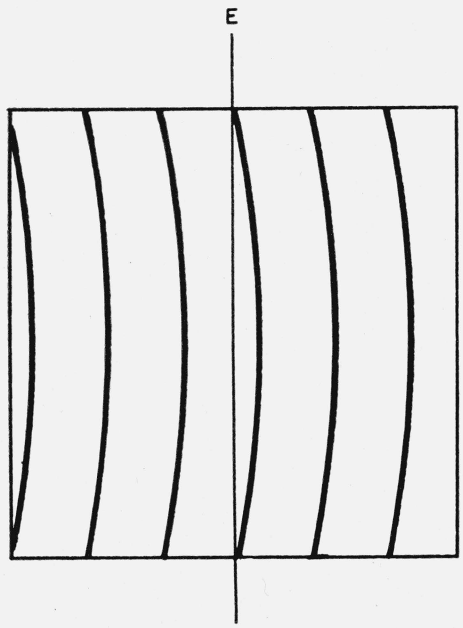

FIGURE 6. *Method of determining surface curvature.*

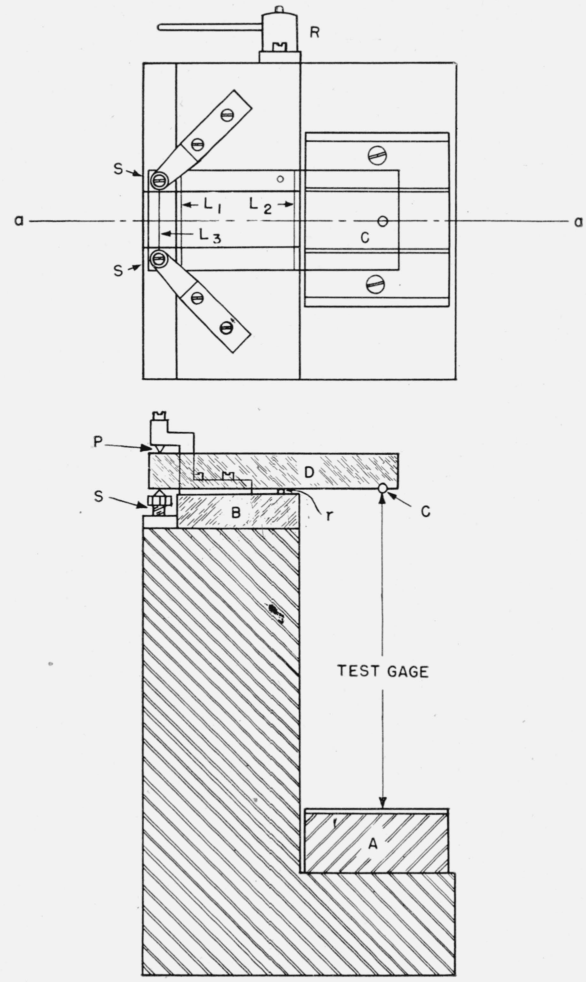

FIGURE 7. *Interferometric length comparator.*

4. Rapid Method for Checking Lengths

In order to expedite length measurements during the making of the gages, a comparator that is an adaptation of the interferometer described by Priest [16] was used. This is shown in figure 7 and also in figure 4 (slightly to the left of center). For temperature stability, the base and the interferometer support were machined from a heavy steel casting. *A*, figure 7, is a stainless steel anvil with two parallel strips, each 0.5 mm wide, which supports alternately a standard quartz gage and the gage to be tested. The bottom of anvil *A* and the top of the base on which it rests were lapped plane, wrung together, and then held by means of screws. *B* is a glass flat that is cemented in a recess in the top of the block. *D* is another glass flat that rests upon adjustable screw points *S*, *S* and is held in place by spring plungers *P* above *S*, *S* which press down lightly in depressions in the top of plate *D*, above and in line (L_3) with the supporting screws. Contact *C* is a small ball that is cemented into a depression in *D*. Lines L_1 and L_2 are ruled transversely on plate *B*; their separation equals one-half the distance between L_3 and *C*. Screws *S*, *S* are adjusted to place the surface of *D* parallel to and slightly above *B*. A device *R*, operates a small rod *r*, which permits the making or breaking of contact at *C*. If the inclination of *D* with respect to *B* is relatively small, straight fringes are seen when the surfaces are illuminated and viewed normally. The number of fringes between L_1 and L_2 depends upon the inclination of the adjacent surfaces of *B* and *D* and the length of the light wave.

In testing, a standard gage is first inserted between *C* and *A* and the fringes between L_1 and L_2 counted. The count is repeated after substituting the unknown gage for the standard. The length of the tested gage will be

$$l = l_s + (n - n_s)\lambda, \text{ in which}$$

l=the length of the unknown gage

l_s=the length of the standard gage

n=the number of fringes between L_1 and L_2 with the unknown gage

n_s=the number of fringes between L_1 and L_2 with the standard gage

λ=the wavelength of the source

n and n_s may be positive or negative with respect to each other, and their signs must be considered in the equation. If the fringes are viewed with a Pulfrich instrument, n and n_s may be determined to 0.1 fringe by inspection; thus, the difference in length of the two gages can be quickly measured to 0.1 wavelength (0.06μ).[2]

Measurements were made immediately after each attempted correction (in which great care had to be taken to avoid over-correction) when the gage was at a nonuniform and indeterminate temperature that was higher than that of the standard gage as a result of the polishing operation. Consequently, the end point of each attempt depended upon the judgment of the operator in allowing for contraction of a gage upon cooling.[3]

5. Gages Produced

Following the above procedure, fifteen decimeter gages were made and later, using modified testing apparatus, six 50-mm gages, one 35-mm gage, and two 25-mm gages were produced.

After removal of the end blocks, the edges of the gages were bevelled slightly. The gages were then carefully measured again for planeness and parallelism by the methods as described, and twelve of the decimeter gages were found satisfactory for use as standards. Table 3 gives the planeness and parallelism of these gages, and figure 8 will aid in its interpretation. Let a, b, c, d be the medial points of the edges of one face of a gage and e, f, g, h be the corresponding points of the opposite face. The headings of columns 2 to 6 refer to the directions in which the planeness measurements were made. (It will be noted that one face—always the surface first brought to final correction—is more nearly plane than the other face). The edge containing a was taken as the standard edge in subsequent calibrations. Column 6 gives the combined curvature of the gage surfaces in plane *abfe*. Subscripts A, B, C, D, later associated with a gage number, refer to lengths at a, b, c, and d, respectively.

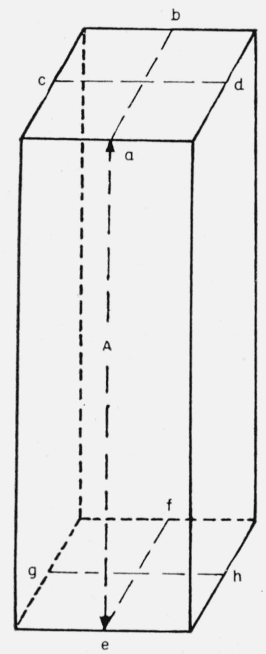

FIGURE 8. *Lettering assigned to a gage to aid in the interpretation of the measurements.*

TABLE 3. *Planeness and parallelism of twelve fused quartz decimeter gages*

Gage	Planeness					Parallelism	
	a, b	c, d	e, f	g, h	a,b+e,f	A>B	C>D
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	μ	μ	μ	μ	μ	μ	μ
3	[a] 0.005c	0.005c	0.011c	0.004c	0.016c	−0.004	+0.057
5	.002c	.000	.004v	.000	.002v	−.006	+.003
6	.006c	.004v	.025c	.025c	.019c	−.011	+.006
7	.005c	.004c	.035c	.007c	.040c	+.002	+.035
8	.010c	.001v	.015c	.010c	.025c	−.007	+.012
9	.006c	.001v	.001c	.001c	.007c	−.002	+.002
10	.018c	.001v	.006c	.003c	.024c	+.007	−.041
11	.000	.002v	.012c	.003c	.012c	+.006	−.016
12	.001c	.004c	.013c	.027c	.014c	+.002	−.033
15	.002v	.002v	.009v	.011v	.011v	−.015	+.021
16	.000	.001c	.032c	.032c	.032c	−.032	−.004
17	.003v	.013c	.005v	.003v	.008v	+.007	−.034

[a] c=concave; v=convex.

[2] In making the fused quartz gages, the total length could not be controlled to this accuracy.

[3] Subsequent measurements using more precise methods show the importance of having the surfaces plane and parallel for accurate calibrations; and, in view of the difficulty experienced in meeting simultaneous requirements of planeness, parallelism, and exact length, the attainment of exact length should have been subordinated to the first two requirements. The making of fused quartz gages to the accuracy of those here produced requires a high degree of skill in optical polishing. The present gages resulted from the exceptional capability of E. L. Robinson of this Bureau who applied to the task the knowledge, skill, and technique acquired over many years in producing optical parts of the highest refinement of workmanship.

III. Calibrating the Gages

1. General Methods

The length of a gage may be determined by: (a) Interferential comparison with a standard gage of known length. (b) Direct interferential measurement of the distance between its faces.

(a) Comparative Method

Method (a) is a relatively simple and very accurate means for calibrating a gage when the length of a like standard is known to the required accuracy. If the gages are of the same material, shape, and finish and are placed side-by-side for comparison, departures from standard temperature conditions tend to affect the gages alike. Only the differential effect on the gages will in this case be included in the measurements and, consequently, the requirements for temperature control and measurement are not specially exacting. Furthermore, the difference in length of the standard and tested gage is usually small and may be measured with great accuracy by means of simple interferometric apparatus. These considerations reduce the necessary equipment to simple temperature apparatus, two interferometer plates, a source of monochromatic radiations, and an instrument for viewing and measuring fringe displacements.

(b) Direct Method

If a standard gage of the required accuracy is not available or if the length of a standard itself is required, accurate calibrations may be made by direct measurement with light waves for gage lengths less than 5 in. The measuring requirements of this method are more exacting than for method (a) and therefore, more elaborate and sensitive apparatus is needed. Accurate knowledge of the temperature and thermal expansivity of a gage is also necessary, because the full difference in length at standard temperature (20° C in US) and at the existing temperature must be taken into account. This is of major importance in calibrating steel gages but less so with fused quartz, which has a thermal expansivity one-thirtieth that of steel. Furthermore, since the order of interference (number of waves) to be determined is large, the lengths of the waves must be accurately known. This requires knowledge of the density and refractive index of the medium surrounding the etalon in order that wavelengths at standard conditions may be converted to wavelengths at existing conditions.

Measurements are sometimes made in vacuum and, when so made, require exact values of refractive indices and dispersions in order to correct air wavelengths to vacuum wavelengths; and require also a correction for the expansion of the gage with reduced pressure (compressibility correction). These corrections must be applied for the total existing atmospheric pressure.

2. Adaptations of the General Methods to the Calibration of Decimeter Gages

Adaptations of these general methods were used in calibrating the fused quartz gages. Because of the simplicity and accuracy of the comparative method, the following procedure given in greater detail was used:

Two of the 12 gages were selected as reference standards. These were carefully compared with each other, and relative lengths of the remaining gages were determined by comparison with each standard. For computational convenience, one standard was assumed to have a mid-side length of 100.000 000 mm.

Actual lengths of the two reference standards were next determined by direct measurement with wavelengths of the standard radiation of cadmium and of two homogeneous radiations of krypton. Later, the actual lengths of the other gages were likewise determined.

From these measurements a corrected value was obtained for the length of each reference standard, by means of which the previously determined relative lengths were converted to actual lengths.

Thus, two values resulted for each of the rest of the 12 gages—one, by comparison with the two reference standards; and the other, by direct measurement with light waves. Although lengths so determined were not independent values, they gave a basis for appraising the measurements and eliminating any unduly large difference in the two values.

IV. Relative Lengths

1. Description of the Comparative Method

Comparative lengths of the fused quartz gages were determined by the method illustrated in figure 9 and in figure 4 at extreme right.

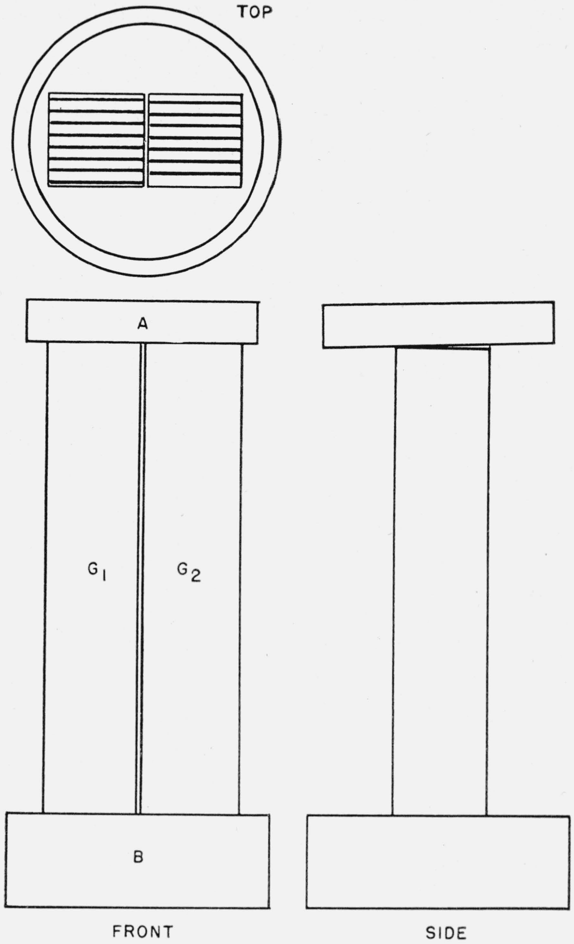

FIGURE 9. *Method for comparing two gages.*

Gages G_1 and G_2 were wrung [4] side-by-side on the fused quartz base B (diameter, 57 mm, thickness, 20 mm) having the top surface plane to 0.01λ. Upper quartz plate A (diameter 50 mm, thickness, 9 mm) having its lower surface plane to 0.01λ was placed on top of the gages forming a wedge that produced about 10 fringes across the surfaces.

The combination was placed in a chamber regulated at $20° \pm 0.01°$ C by means of apparatus described by Souder and Peters [18,19]. Although this accurate temperature control was unnecessary for comparing fused quartz gages, it was essential when for a special test a quartz and a steel gage were compared.

The source of illumination was a helium tube incorporated in a Pulfrich viewing instrument, which disperses the light. The relative displacement of the two sets of fringes formed by reflection from the lower surface of A and the upper surfaces of the two gages was measured at adjacent edges of the gages for the red, yellow, and green lines of helium. Difference in length of the gages was calculated from the measured displacements of the fringes and the wavelengths of the light.

2. Comparative Measurements: Relative Lengths of the Gages

Gages 5_A and 9_A were selected as reference standards.[5] Gage 9_A was 0.085μ longer than 5_A as found by the above method of comparison. Assuming 5_A to be 100.000 000 mm and 9_A to be 100.000 085 mm, the relative lengths of the remaining gages were determined by comparison with each of these as standard. Table 4 shows that the lengths of a gage obtained by comparison with 5_A and 9_A agree within from 0.001 to 0.022μ, thus assuring the reliability of the comparative measurements. The "average difference" of -0.001μ confirms the relative values that were determined for 5_A and 9_A. Relative midside lengths given in table 4 were later used (section VI) to obtain lengths at the center of the gages.

Additional measurements included intercomparisons of six 50-mm fused quartz gages and comparisons of the 100-mm reference standards with combinations of these 50-mm gages and, also, with 100-mm and 4-in. steel gages.

[4] The method of wringing is as follows:

The surfaces are first cleaned with alcohol and absorbent cotton and then covered with a thin film of grease by thoroughly rubbing with an otherwise clean cloth freshly impregnated with petroleum jelly. This film is reduced in thickness by vigorous polishing with clean surgical cotton until no trace of film is visible in reflected light. Although the surfaces are apparently free from lint and dust, tiny particles sometimes remain, or are attracted to the surfaces by the electric charge produced by polishing. A camel's-hair brush is helpful in removing these particles. Next, the gage, slightly tilted, is held with a bottom edge resting lightly on the plate surface near an edge. The gage is then slid across the plate—sweeping ahead any particles that may be present as it goes—to the desired place and moved simultaneously to vertical position in actual contact. The surfaces are next brought intimately together by pressing lightly on the gage. Contact is indicated by a black appearance at the contacted edge of the gage, which gradually spreads over the entire surface provided particles have been successfully removed. The presence of particles is indicated by a colored or grayish appearance around them in the field of black contact. When particles are in evidence, the surfaces must be separated and the process of wringing repeated. Separation is effected by several hours immersion in water. Our experience in comparing standard end gages indicates that the separation of the surfaces when properly wrung into contact is less than 0.005 μ. Separations of this magnitude have been found by others [17].

[5] Subscript A refers to the length of a gage at the center of edge a as noted in section II.

TABLE 4. *Relative mid-side length of the gages by comparison with 5_A and 9_A*

Gage	Length at mid-side by comparison— With 5_A (assumed length=100.000 000 mm)	With 9_A (length by comparison with 5_A=100.000 085 mm)	Difference in length by comparison with 5_A and 9_A
	mm	*mm*	μ
3_A	99.999 926	99.999 909	+0.017
5_A	100.000 000	(100.000 000)	---------
6_A	99.999 781	99.999 796	−.015
7_A	100.000 017	100.000 032	−.015
8_A	100.000 025	100.000 010	+.015
9_A	100.000 085	100.000 085	---------
10_A	100.000 104	100.000 039	−.002
11_A	99.999 994	99.999 972	+.022
12_A	100.000 025	100.000 093	−.014
15_A	100.000 018	100.000 031	−.013
16_A	100.000 021	100.000 030	−.009
17_A	99.999 958	99.999 957	+.001
Average difference by comparison with 5_A and 9_A			−0.001

V. Lengths by Direct Interferential Measurement

1. Description of the Direct Method

(a) General Plan

The Fabry-Perot method was selected for direct measurements of length because of the accuracy and convenience with which temperatures may be controlled and because of the sharply defined fringes it furnishes. In its application to gage measurements, the gage itself, G, serves as a separator for two Fabry-Perot interferometer plates.

The arrangement of the apparatus is shown in figures 10 and 11. Interferometer P_1P_2 is placed within constant temperature chamber, A. The transmitted waves, after multiple reflections, produce a system of interference rings that is focused by lens L_2 upon slit F of the spectrometer. The images of the slit corresponding to different wavelengths from the source, S, are separated by the spectrometer. Each image will be crossed by arcs of the ring system formed by that particular radiation, and the diameters of these circular fringes are measured either directly by means of a micrometer eyepiece at E, or the fringes are photographed and then measured. The optical distance between the interferometer plates may be determined from the measured diameters of the rings and the wavelengths of the source.

(b) Interferometer

The interferometer was formed by wringing interferometer plates to the ends of the gage. Plates of optical glass (thermal expansivity 9×10^{-6}) proved unsatisfactory for combining with fused quartz separators (expansivity 4×10^{-7}). Because of the differential expansion, distortion of the plates resulted after wringing unless gage and plates were at the same temperature when brought together, and distortion likewise resulted whenever the temperature of the combination was

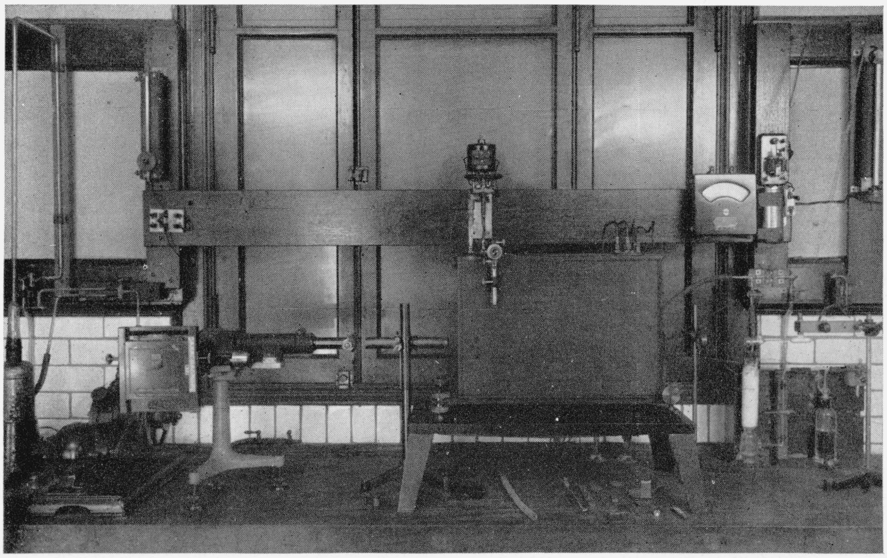

FIGURE 10. *Apparatus for direct interferential measurement of length.*

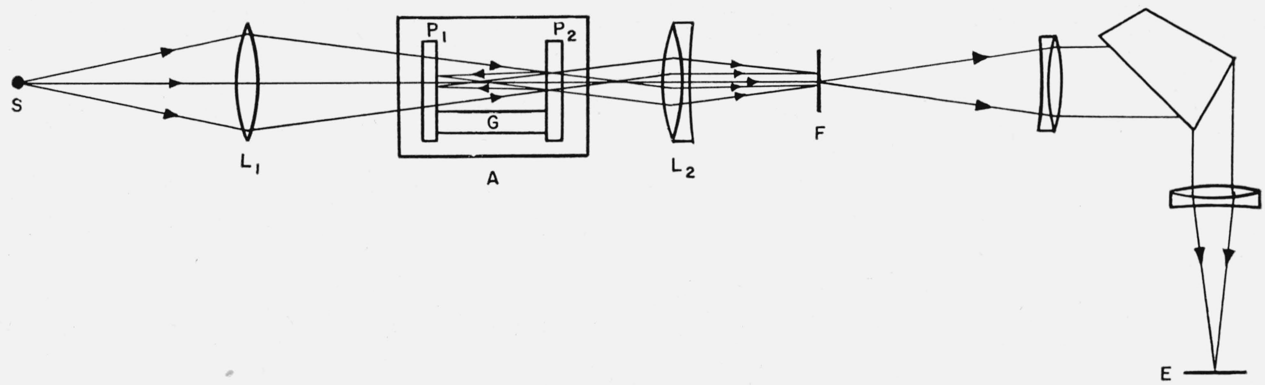

FIGURE 11. *Assembled apparatus for direct interferential measurement of length.*

different from the wringing temperature. Distortion was avoided by using plates of fused quartz selected for its optical homogeneity. These were plane to 0.01μ, and one surface of each was coated with a thin silver film by sputtering from a silver electrode in vacuum. The silver coating was removed from sufficient area to permit the gage to be wrung to the quartz glass surface. Two pairs of plates (diameter 56 mm, thickness 9.5 mm) and (diameter 68 mm, thickness 8.5 mm) respectively, resilvered as required, were used during the measurements. Interferometers thus constructed were sufficiently plane and parallel to give measurable fringes with several different radiations for a 250-mm path.

To avoid errors caused by bending of the gage, consideration was given to the method of supporting the interferometer in its horizontal position. Measured lengths differing by 0.15μ were obtained when the combination was supported at its ends and middle, respectively. Any error attributable to bending was finally reduced to the required limit by supporting the interferometer at points 10 mm from the ends of the gage.

(c) Constant Temperature Apparatus

During the measurements the interferometer I, figure 12, was maintained at 20° $\pm$0.01°C in a water-jacketed tubular brass container. The ends of this tube were closed with optically flat glass plates, one of which was permanently sealed in a recess at one end of the tube, whereas the other was removable to permit access to the interferometer chamber. Slidable metal baffles B_1, B_2 provided with windows isolated the central part of the chamber where the temperature was most uniform. Light from the source was diaphragmed by means of cardboard disks placed in B_1 and B_2. A copper tube led from the inteferometer chamber to the outside and, through this, air could be either admitted through a drying train or the container evacuated. Calcium chloride was used as a drying agent within the chamber.

For proper regulation of the temperature, water was circulated (by means of a motor-driven propeller) between the tube containing the interferometer and a surrounding brass tube, and given directional flow around the inner tube by means of vanes and baffles (not shown) soldered to the tubes. The bath liquid was water-cooled by coil C and electrically heated by coil H. A relay operated by thermostat J regulated the amount of heat supplied. Temperature of the bath was measured by means of an exceptionally rapid mercury thermometer calibrated at frequent intervals by the Bureau's Thermometry Section. This constant temperature apparatus was encased in a wooden box and insulated with mineral wool.

With the glass plates attached to the tube ends and with baffles and diaphragms in place for calibration of decimeter gages, the interferometer chamber was explored with differential thermocouples, one junction of each couple being in contact with the thermometer bulb. Exploration showed that after the apparatus had been operating 3 or more hr the chamber attained the temperature of the bath to 0.01° C, provided room temperature was within 1.0° C of the bath temperature. Under these conditions no measurable gradient existed in the inner chamber.

(d) Spectrometer

The present method differs from that of Peters and Boyd by using a constant deviation spectrom-

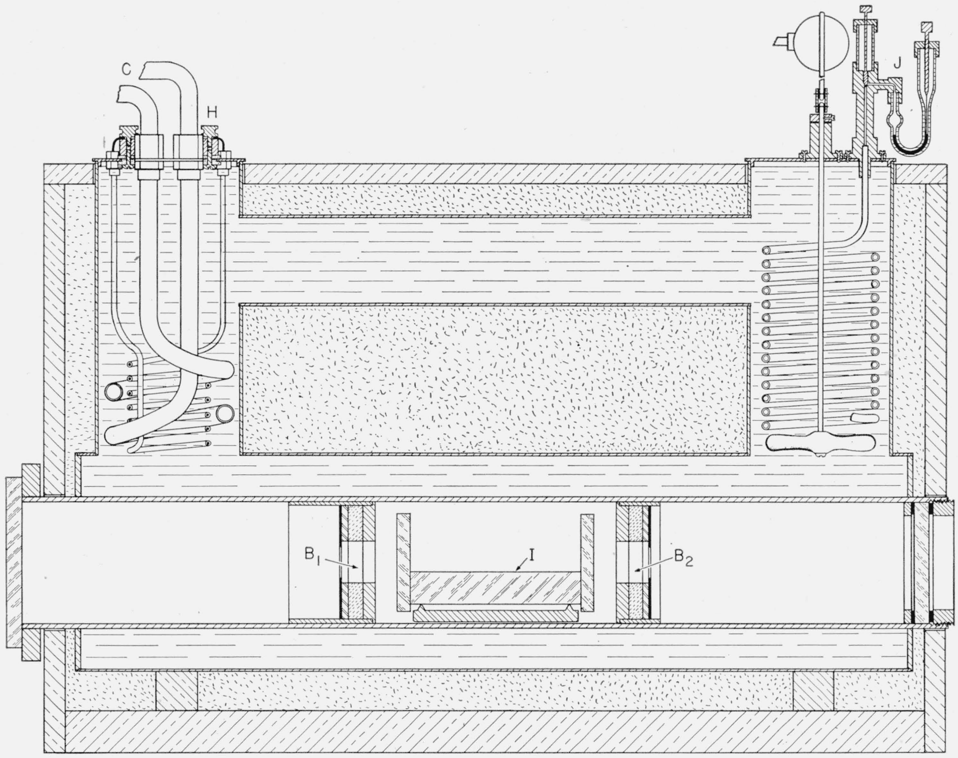

FIGURE 12. *Constant-temperature apparatus.*

eter (Hilger 78: index 1.74 for the *D* line) rather than a diffraction grating for dispersing light.

In the preliminary work ring diameters were usually measured by means of a micrometer eyepiece, but in the final calibrations a camera replaced the eyepiece and the fringes were photographed for permanent record using Eastman Type III—*F* plates. Three or four exposures were made on each plate for a given source, with periods ranging from 30 sec to 12 min—to suit the individual lines. Gage lengths computed from the photographed rings agreed with those obtained by direct visual measurement with the micrometer eyepiece.

(e) Barometer

Atmospheric pressures were measured by means of a standard mercury barometer that had been calibrated by the Aeronautical Instrument Section of this Bureau. Observations were recorded to the nearest 0.05 mm, although the limiting accuracy of the measurements probably did not exceed 0.1 mm.

(f) Sources of Light

Neon, krypton, and cadmium discharge tubes were used during the investigation. The final calibrations were however based solely upon measurements with the red radiation (λ_R= 6438.4696 A) of cadmium as a primary standard and upon two green radiations (λ_{G_1}=5562.2255 A, λ_{G_2}=5570.2892 A) of krypton as secondary standards.

The cadmium tubes were obtained from Meggers and were similar to those used by Meggers and Humphreys [20] for wavelength comparisons. These were operated in a furnace maintained at 320° C.

The krypton tubes were used either "side-on" or "end-on" without affecting the results. Final measurements were made with the tubes end-on because of the greater intensity of the lines. The values given above for the krypton wavelengths were obtained by Meggers and Humphreys by comparison with the standard wavelength λ_R.

(g) Reduction of Observations

(1) *Method of calculation.* The optical separation, *L,* of the interferometer surfaces at the center of the ring system is given by the equation

$$L=\left(\frac{N}{2}\right)\lambda, \tag{1}$$

in which N is the order of interference (the number of waves of length λ in the distance $2L$ over which the light travels).

Equation 1 may also be stated as

$$2L = N\lambda = (n+a)\lambda, \tag{2}$$

in which n is the order of interference at the first ring, and a is the fraction of a wave that N differs from n. Thus N consists of an integral number n plus fraction a.

The fractional difference a for a given radiation may be computed from the measured diameters of two or more rings by the method of Childs [21] and Burns and Kiess [22] in the form given by Humphreys [23]. In the present work, results in general were based on the value of a determined from five measured rings.

In order to determine n, fractional values a_1, a_2 . . . for several radiations of known wavelength λ_1, λ_2 . . . are required, together with an approximate value, l, for L which may usually be obtained micrometrically to within 1 or 2μ. The subsequent computing procedure is given by Meggers [24] and modified by Peters [25].

(2) *Conversion of wavelengths at standard conditions to wavelengths at conditions of measurement.* Since light waves vary in length with air density, their known lengths at standard conditions (15° C; 760 mm) must be converted to lengths at the existing temperatures and pressures. Conversion factor (f) is sufficiently accurate for this purpose:

$$f = \frac{\mu_{15}}{\mu_t} \text{ where } \mu_t = 1 + \frac{(\mu_{15}-1)P288}{760T},$$

in which μ_{15} and μ_t are refractive indices at 15°C and at the observed temperature, T is the observed absolute temperature, and P is the observed pressure.

(3) *Correction for phase change at reflection and film thickness.* Calculated length L is the distance between reflecting surfaces of the silver films, whereas the required length is that between the clear glass surfaces onto which a gage is wrung. Therefore, a small correction that is equal to the thickness of the silver films plus or minus the phase change that occurs on reflection at their surfaces must be applied to L.

To determine the magnitude of this correction, a quartz glass test flat is placed over each interferometer plate so as to cover both clear and silvered parts. Each plate gives two systems of fringes arising from the interference between the front and back surfaces of the separating air films. The system at the silvered area, owing to the combined effect of difference in thickness of the air film and difference in phase change by reflection, is displaced (less than 1 order) relative to the system at the clear portion. The combined displacement for the two plates multiplied by $\lambda/2$ gives the required correction to L.

(4) *Correction for curvature and slope.* The length thus determined is the separation of the glass surfaces of the interferometer plates at the center of the ring system, which lies outside the gage. This will also be the required axial length of the gage (unless account is taken of the very thin wringing films) provided its surfaces are plane and parallel. For gages that are not plane and parallel, the difference in length at the center of the ring system and at the center of the gage may be calculated from the measured planeness and parallelism of the gage surfaces and the approximate distance between the center of the ring system and the gage. In order to make this calculation, it must necessarily be assumed that, upon wringing, the surfaces of the plate conform to the surfaces of the gages. The validity of this assumption depends greatly upon the contour and regularity of curvature of the surfaces and emphasizes the necessity of having plane and parallel gages for precise calibrations.

(5) *Illustration of computations.* Table 5 illustrates the method for making the computations for a 100-mm gage. Column 1 gives the wavelengths used; 2, the number of rings from the center of the ring system; 3, the measured diameter of the rings; 4, the square of these diameters; 5, the difference in squares of the diameters of successive rings; 6, the fractional part a of the order at the center of each ring system; 7, the order of interference N or the number of waves in double the distance between reflecting surfaces of the plates at the center of the ring system; and 8, the distance in millimeters between these reflecting surfaces.

Table 5. *Computations for a 100-mm gage*

1 λ (Angstrom units)	2 Ring	3 D_6; D_5; etc.	4 D_6^2; D_5^2; etc.	5 D_6^2-D_5^2; D_5^2-D_4^2; etc.	6 a	7 N	8 L $\left[\frac{N}{2}\lambda\right]$
							mm
Kr 5562.2255	6th	1026	10527				
				1803	0.839		
	5th	934	8724				
				1852	.711		
	4th	829	6872				
				1845	.725		
	3'd	709	5027				
				1891	.658		
	2'd	560	3136				
				1804	.738		
	1st	365	1332				
					.734	359 563.734	99.998 729
Kr 5570.2892	6th	979	9584				
				1893	.063		
	5th	877	7691				
				1839	.182		
	4th	765	5852				
				1820	.216		
	3'd	635	4032				
				1842	.189		
	2'd	468	2190				
				1770	.237		
	1st	205	420				
					.177	359 043.177	99.998 717
Cd 6438.4696	6th	1123	12611				
				2166	.822		
	5th	1022	10445				
				2109	.953		
	4th	913	8336				
				2142	.892		
	3'd	787	6194				
				2098	.952		
	2'd	640	4096				
				2230	.837		
	1st	432	1866				
					.891	310 628.891	99.998 734

Optical separation of interferometer surfaces, uncorrected for air density	99.998 727
Optical separation corrected to density of air at 760 mm; 15° C, L×(f=1.0000096)	99.999 687
Correction for phase change at reflection and film thickness	+0.000 010
Combined slope and curvature correction to convert the distance between plates at the center of fringe system to the separation at the axis of the gage	−.000 021
Axial length of the gage	99.999 676

2. Measurements

(a) Preliminary Measurements

Before making final direct measurements of the 100-mm gages, steps were taken to assure the accuracy of the integral order of interference.

Direct measurements by the Fabry-Perot method were first made of two 50-mm fused quartz gages with krypton and neon sources. These furnish a large number of spectral lines that are sufficiently homogeneous in structure to stand interference over this shorter path. The following lengths were obtained from visual measurements of the fringe diameters:

Gage	Length at center of fringe system (measured)	Length at A (edge) (converted)
	mm	*mm*
2	50.000 580	50.000 556
4	49.999 897	49.999 890
	Sum 100.000 477 (1)	

Next, gages 2 and 4 were wrung together, and their combined length was determined in the same manner from measurements of photographed interference patterns for three lines of krypton. Lengths thus obtained were:

λ	Measured length of combination at center of fringe system
A	*mm*
5562	100.000 503
5570	100.000 507
5870	100.000 497
	Mean 100.000 502 (2)

The small difference (0.025 μ) between (1) in which gages 2 and 4 were measured individually and (2) in which the gages were measured in combination established the integral order of interference for the decimeter etalon.

Later, 50 mm gages 4 and 5 were measured directly, using the standard radiation of cadmium and the two secondary standard radiations of krypton. Values for gage 4 were computed from diametric measurements of photographed fringes; values for gage 5 (first wringing) were computed from visual measurements of the diameters by two observers. Later, the interferometer plates were removed from gage 5 and rewrung (second wringing) to the gage. The length of this etalon was measured both visually and photographically. The results of the determinations, table 6, show no systematic or appreciable difference for (a)

TABLE 6. *Lengths of two 50-mm fused quartz etalons at the center of the ring system by direct measurement with light waves*

Gage	Date	Observer	Length at center of ring system						
			Using cadmium (λ6438.4696)		Using krypton (λ5562.2255)		Using krypton (λ5570.2892)		Mean of three radiations
			Visually	From photo.	Visually	From photo.	Visually	From photo.	
			mm	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*
4	Feb. 4, '35	1	----	49.999 885	----	49.999 894	----	49.999 886	
			----	49.999 884	----	49.999 885	----	49.999 885	
			----	----	----	----	----	49.999 886	
			----	----	----	----	----	49.999 887	
			M=49.999 885		M=49.999 890		M=49.999 886		49.999 887
	FIRST WRINGING								
5	Dec. 10, '34	1	----	----	49.999 933	----	49.999 940	----	
	Dec. 11, '34	2	----	----	49.999 936	----	49.999 927	----	
			----	----	49.999 933	----	49.999 940	----	
	Dec. 12, '34	2	----	----	49.999 922	----	49.999 931	----	
			----	----	49.999 923	----	49.999 931	----	
	SECOND WRINGING								
	Jan. 31, '35	1	49.999 935	49.999 944	49.999 953	----	49.999 943	49.999 935	
			----	49.999 935	----	49.999 933	----	49.999 933	
			----	----	----	----	----	49.999 937	
	Jan. 31, '35	1	----	49.999 928	----	49.999 930	----	49.999 929	
			----	49.999 930	----	49.999 927	----	49.999 927	
			----	----	----	----	----	49.999 925	
			----	----	----	----	----	49.999 935	
			M=49.999 934		M=49.999 932		M=49.999 933		49.999 933

two observers, (b) visual or photographic measurements, or (c) the three spectral sources. The mean values given in the last column of table 6 are equivalent to the following mid-edge lengths;

$$4_A=49.999\ 880 \text{ mm}$$

$$5_A=49.999\ 932 \text{ mm}$$

This value for 4_A is but 0.01 μ different from that found previously with krypton and neon radiations. Using these values for 4_A and 5_A, the comparative lengths (table 7) of other 50-mm quartz gages were obtained. As a check upon the accuracy of the comparisons, gages 2 and 3 and gages 4 and 7 were wrung together and compared. From table 7, combination $2_A+3_A=$ 99.999 486 mm and combination $4_A+7_A=$ 99.999 144 mm; or a difference in length of +0.342 μ, whereas the difference found by direct comparison of the combinations was +0.358 μ.

TABLE 7. *Lengths of the 50-mm gages by comparison with 4_A and 5_A*

Gage	Length by comparison			Length-preliminary direct measurements
	Compared with 4_A (4_A=49.999 880 mm)	Compared with 5_A (5_A=49.999 932 mm)	Mean	
	mm	*mm*	*mm*	*mm*
2_A	50. 000 563	50. 000 552	50. 000 558	50. 000 556[a]
3_A	49. 999 922	49. 999 933	49. 998 928	----
4_A	49. 999 880	49. 999 900	49. 999 890	49. 999 890[a]
5_A	49. 999 912[a]	49. 999 932[a]	49. 999 922	----
6_A	49. 999 723	49. 999 743	49. 999 733	----
7_A	49. 999 257	49. 999 251	49. 999 254	----

[a] Direct measurements.

Approximate lengths were next established for the 100-mm reference standards by the following procedures:

(a) By comparison with combinations of the 50-mm gages that are summarized in table 7.

(b) By comparison with an NBS standard 4-in.

steel gage. This gage had been measured directly with light waves and had also been compared with combinations of 1-in., 2-in., and 50-mm steel gages, which in turn had been calibrated directly with light waves. Its length was confirmed within 0.000002 in. (0.05 μ) by comparing it with gages that had been certified by other national standardizing laboratories. In making these comparisons, gages were placed in the constant temperature chamber at least 4 hrs before measurements were taken.

The following approximate lengths were thus obtained for the reference standards.

Reference standard	Length by comparison— With combinations of 50-mm quartz gages	With 4-in. steel gage
	mm	*mm*
5_A	99. 999 901	99. 999 890
9_A	99. 999 972	99. 999 981

These measurements established the lengths of the reference gages to well within 1 fringe; and the preliminary measurements *in toto* assured the exact integral orders of interference for decimeter etalons when the red radiation of cadmium and the two green radiations of krypton were used.

(b) Final Measurements

Final direct measurements of reference standards 5 and 9 were made at 20° C with cadmium and krypton sources; and it was considered advisable to measure in the same manner the gages to be calibrated by other laboratories.

Calibrations for most of the gages are based upon two wringings of gages and plates. Three exposures were usually made for each source, with times ranging from 30 sec to 12 min for the individual lines. Although several radiations of cadmium and krypton gave fringes that were measured, only the three standard lines were used for computing gage lengths because of uncertainty as to the homogenity of the other lines. The individual values computed from the three standard radiations usually agreed to 0.01μ, as shown in table 8.

No orderly difference was found in measurements made with large and with small interferometer plates. These plates were resilvered from time to time, and the combined corrections for film thickness and phase change for pairs of plates ranged from 0.000 to 0.041μ, for different coatings.

Measurements of a given etalon when undisturbed in its position within the constant temperature chamber gave the same lengths within 0.01-μ on successive days during which barometric pressure varied from 750 to 761 mm Hg. The same values were, however, not always obtained for successive wringings of gages and plates. The maximum difference observed (0.065μ for gage 16; table 8) is greater than would be expected from wringing or from correcting for curvature and slope, and greatly exceeds that obtained with 50-mm gages. Upon reviewing the measurements, the most likely explanation appeared to be that discordance resulted from the method that was used to adjust the etalon to its correct lateral position in the constant temperature chamber. The adjustment was effected by applying a slight pressure to the rear face of an interferometer plate by means of a rod. This operation could conceivably strain the gage and result in a bending that might not be relieved upon removal of the pressure. This explanation was verified in later work by the excellent agreement obtained for successive

TABLE 8. *Lengths at the center of the ring system by direct measurement with krypton and cadmium wave lengths*

[Two or more wringings were made for nine of the gages]

Gage	Wringing No.	Length at center of ring system — Krypton $\lambda=5562.2255$	Krypton $\lambda=5570.2892$	Cadmium $\lambda=6438.4696$	Mean of Kr and Cd values for each wringing	Mean length for all wringings
		mm	*mm*	*mm*	*mm*	*mm*
3	1	99.999 831	99.999 828	99.999 840	99.999 833	99.999 833
5	1	99.999 878	99.999 881	99.999 891	99.999 883	
	2	99.999 859	99.999 874	----------	99.999 867	
	3	99.999 870	99.999 874	99.999 877	99.999 874	
	4	99.999 864	99.999 853	99.999 856	99.999 858	99.999 871
6	1	99.999 696	99.999 699	99.999 696	99.999 697	99.999 697
7	1	99.999 953	99.999 938	99.999 941	99.999 944	
	2	99.999 906	99.999 902	99.999 911	99.999 906	99.999 925
8	1	99.999 926	99.999 921	99.999 929	99.999 925	
	2	99.999 909	99.999 906	99.999 899	99.999 905	99.999 915
9	1	100.000 001	100.000 003	100.000 004	100.000 003	
	2	99.999 961	99.999 959	99.999 967	99.999 962	99 999 983
10	1	100.000 046	100.000 040	100.000 041	100.000 042	
	2	100.000 026	100.000 027	100.000 029	100.000 027	100.000 035
11	1	99.999 858	99.999 861	99.999 859	99.999 859	
	2	99.999 880	99.999 878	99.999 865	99.999 874	99.999 867
12	1	99.999 920	99.999 925	99.999 924	99.999 923	
	2	99.999 963	99.999 956	99.999 978	99.999 966	99.999 945
15	1	99.999 959	99.999 962	99.999 964	99.999 962	
	2	99.999 896	99.999 897	99.999 923	99.999 905	99.999 934
16	1	99.999 892	99.999 891	99.999 907	99 999 897	
	2	99.999 957	99.999 959	99.999 967	99.999 962	99.999 930
17	1	99.999 862	99.999 859	99.999 870	99.999 864	99.999 864

wringings when other adjusting procedures were used.

By the use of the corrections in table 9, the lengths that were determined at the center of the ring system (located 5 mm from the edge of a gage) by direct measurement with light waves were converted to mid-edge lengths and to axial lengths of the gages. These are given in table 10.

TABLE 9. *Corrections for planeness and parallelism of the gage surfaces*

Gage	Total curvature [a]	Slope ($A>B$)	Correction to center of gage from center of ring system			Total correction to edge, A, from center of fringe system
			Curvature correction	Slope correction	Total correction applied	
	μ	μ	μ	μ	μ	μ
3	−0.016	−0.004	−0.024	+0.003	−0.021	−0.007
5	+.002	−.006	+.003	+.005	+.008	+.003
6	−.019	−.011	−.029	+.008	−.021	−.008
7	−.040	+.002	−.060	−.002	−.062	−.021
8	−.025	−.007	−.038	+.005	−.033	−.011
9	−.007	−.002	−.011	+.002	−.009	−.003
10	−.024	+.007	−.036	−.005	−.041	−.013
11	−.012	+.006	−.018	−.005	−.023	−.008
12	−.014	+.002	−.021	−.002	−.023	−.008
15	+.011	−.015	+.017	+.011	+.028	+.009
16	−.032	−.032	−.048	+.024	−.024	−.008
17	+.008	+.007	+.012	−.005	+.007	+.003

[a] The algebraic sum of the curvatures of the end surfaces in directions a-b and c-f. A (−) value indicates a resulting concavity, a (+) value indicates convexity.

TABLE 10. *Lengths at the center and mid-edge by direct measurement*

Gage	Lengths by direct measurement		
	At center of ring system	At center of gage (axial length)	At mid-edge, A, of gage
	mm	*mm*	*mm*
3	99.999 833	99.999 812	99.999 826
5	99.999 871	99.999 879	99.999 874
6	99.999 697	99.999 676	99.999 689
7	99.999 925	99.999 863	99.999 904
8	99.999 915	99.999 882	99.999 904
9	99.999 983	99.999 974	99.999 980
10	100.000 035	99.999 994	100.000 022
11	99.999 867	99.999 844	99.999 859
12	99.999 945	99.999 922	99.999 937
15	99.999 934	99.999 962	99.999 943
16	99.999 930	99.999 906	99.999 922
17	99.999 864	99.999 871	99.999 867

VI. Summary of Comparative and Direct Measurements of the Gages

Before the comparative calibrations of the decimeter gage can be summarized, the relative mid-side lengths given in table 4 must be converted to actual lengths at the axis. Lengths in table 4 are based upon the assumption that $5_A=100.000\ 000$ mm and by comparison with this, $9_A=100.000\ 085$ mm. The actual lengths of the reference standards as found by direct measurement were $5_A=99.999\ 874$ and $9_A=99.999\ 980$ mm. Instead, however, of using the measured lengths of the reference standards to convert table 4 values to actual values, lengths $5_A=99.999\ 898$ mm; $9_A=99.999\ 982$ mm were used. These corrected values (see table 11) were derived from the directly measured lengths of all gages and the very accurate comparisons of each reference standard with the other gages upon which table 4 is based. These corrected lengths are considered more reliable than those obtained from the relatively few direct measurements of the standards. The corrected lengths of 5_A and 9_A are 0.024 and 0.002μ greater than the directly measured lengths and, since the final comparative length of a third gage was taken as the mean of its comparisons with 5 and 9, its value is increased 0.013μ, using corrected lengths. The difference (0.084μ) in the corrected lengths of 5_A and 9_A concords better with the difference (0.085μ) that was obtained in the original comparisons of 5_A and 9_A than does the difference of 0.106μ resulting from the direct measurements of the two gages. Table 12 shows the close agreement of the pre-

TABLE 11. *Corrected length of 5_A and of 9_A derived from the direct measurement of all gages*

Length of comparison gage by direct measurement	Derived length of reference standard	
	Gage 5_A [a]	Gage 9_A [a]
mm	*mm*	*mm*
3_A=99.999 826	99.999 900	100.000 002
5_A=99.999 874	99.999 874	99.999 959
6_A=99.999 689	99.999 908	99.999 978
7_A=99.999 904	99.999 887	99.999 957
8_A=99.999 904	99.999 879	99.999 979
9_A=99.999 980	99.999 895	99.999 980
10_A=100.000 022	99.999 918	100.000 001
11_A=99.999 859	99.999 865	99.999 972
12_A=99.999 937	99.999 912	99.999 983
15_A=99.999 943	99.999 925	99.999 997
16_A=99.999 922	99.999 901	99.999 977
17_A=99.999 867	99.999 909	99.999 995
Corrected, length	99.999 898	99.999 982

[a] The derived lengths of 5_A and 9_A were obtained by correcting the directly measured lengths of the comparison gages of column 1 by the differences that were obtained by comparing these gages with the standard gages.

liminary calibrations of 5_A and 9_A with the measured and corrected lengths.

TABLE 12. *Lengths of reference standards by various methods*

Method of derivation	5_A	9_A
	mm	*mm*
From direct and comparative measurements of 12 gages (corrected value)	99.999 898	99.999 982
By direct measurement with light waves	99.999 874	99.999 980
By comparison with combinations of 50-mm gages that had been measured directly	99.999 901	99.999 972
By comparison with 4.0 steel gage	99.999 890	99.999 981

The length of each gage at its *A* edge was obtained from the comparative values given in table 4 by using the corrected values of 5_A and 9_A for the conversion. Lengths at the center of the gages (axial lengths) were obtained from the mid-edge lengths by applying the slope and curvature corrections in table 9.

The axial lengths determined by comparing the gages with the reference standards and also those determined by direct measurement with light waves are summarized in table 13. It will be noted that the greatest difference in length (Δ) of any gage by the two methods is 0.024 μ.

TABLE 13. *Summary of axial lengths by direct measurement and by comparison*

Gage	Axial length					
	By direct measurement (1)	By comparison—			Δ (1)−(2)	Final value, Mean of (1) and (2)
		With 5_A	With 9_A	Mean (2)		
	mm	*mm*	*mm*	*mm*	μ	*mm*
3	99.999 812	99.999 810	99.999 792	99.999 801	+0.011	99.999 807
5	99.999 879	[a] 99.999 903	99.999 902	99.999 903	−.024	99.999 891
6	99.999 676	99.999 666	99.999 680	99.999 673	+.003	99.999 675
7	99.999 863	99.999 874	99.999 888	99.999 881	−.018	99.999 872
8	99.999 882	99.999 901	99.999 885	99.999 893	−.011	99.999 888
9	99.999 974	99.999 977	[a] 99.999 976	99.999 977	−.003	99.999 976
10	99.999 994	99.999 974	99.999 975	99.999 975	+.019	99.999 985
11	99.999 844	99.999 877	99.999 854	99.999 866	−.022	99.999 855
12	99.999 922	99.999 908	99.999 921	99.999 915	+.007	99.999 919
15	99.999 962	99.999 935	99.999 947	99.999 941	+.021	99.999 952
16	99.999 906	99.999 903	99.999 911	99.999 907	−.001	99.999 907
17	99.999 871	99.999 860	99.999 858	99.999 859	+.012	99.999 865

[a] From corrected values of 5_A and 9_A.

VII. Comparison of the Calibrations by the Various Standardizing Laboratories

Following completion of the measurements the gages were assigned NBS numbers, and two, together with their calibration certificates, were submitted to each of the following laboratories for calibration: Bureau International des Poids et Mesures, Physikalish-Technische Reichsanstalt, and National Physical Laboratory. Either final or preliminary reports have been received for five of the gages.

1. Methods Used at the Other Laboratories

The following methods were used at the other testing laboratories:

(a) Bureau International des Poids et Mesures

The gage was wrung to the plane surface of a fused quartz base plate and placed in an industrial interferometer,[6] which carried a glass interferometer plate at a short distance from the upper surface of the gage. Lengths were determined from measurements of the Fizeau fringes between the part of the glass plate over the gage and the upper face of the gage, and those between another part of the glass plate and the upper surface of the quartz base plate. The radiations used were two yellow lines of mercury and one yellow and two green lines of krypton.

(b) Physikalish-Technische Reichsanstalt

Gages sent to the PTR were likewise calibrated by the use of Fizeau fringes. At this laboratory

[6] Presumably the industrial interferometer described by Pérard [26]. The general principle of this method is similar to the comparative method used with the decimeter gages. Monochromatic light is obtained, however, by passing radiations from a source through a dispersive system of prisms to a slit, from which it is projected vertically onto the interferometer by means of a total reflecting prism and mirror. Fringes are viewed with a telescope.

a gage was mounted horizontally with one surface wrung to a fused-quartz base plate. The distance from the middle of the other surface of the gage to the base surface was measured interferometrically, statedly by the PTR method,[7] which is different from that employed at the BIPM. The standard radiation of cadmium and five lines of krypton were used—the wavelengths of the krypton lines having been determined by comparison with the standard wavelength of cadmium.

(c) The National Physical Laboratory

One end of a gage was wrung to a crystalline quartz plate. A second crystalline quartz plate was supported about 15 mm above the upper surface of the gage by means of a fused quartz tube that enclosed the gage and had plane and parallel end faces in wringing contact with the upper and lower plates. Light from the source was projected vertically to the interferometer producing, by reflection from the lower surface of the upper plate and the top surface of the gage and likewise from adjacent surfaces of the plates, two systems of Haidinger fringes. These were focused upon the slit of a spectrograph. The calibrations were based upon four wave-lengths of krypton.

[7] The method in common use at the PTR is basically a modification of Michelson's interferometer. A brief description of it is given by Dr. Kösters in a report by the special committee on wavelengths of light. [27].

It will be noted that the interferential methods used by the above laboratories and the NBS differ among themselves. For calibrating fused quartz gages, which do not require the exacting temperature control necessary for the direct measurement of steel gages and for which the Fabry-Perot method is particularly adaptable, some of the other methods have certain advantages, namely: (1) bending of a gage is eliminated at the BIPM and NPL by measuring it in a vertical position; and (2) the errors introduced by wringing together imperfect surfaces are less by the BIPM and PTR methods, which require but one wringing.

2. Summary and Comparison of the Calibrations

A summary of the results at present available is given in table 14, in which the NBS bracketed lengths are those originally given in the gage certificates. These differ less than 0.010μ from the final derived lengths, which are based upon additional planeness and parallelism measurements not included in the original computations.

TABLE 14. *Axial length of fused quartz gages by different laboratories*

Gage No.		Testing laboratory	Measured length			Notes
Original	NBS		Direct method	Comparative method	Mean	
			mm	*mm*	*mm*	
		BIPM	99.999 840			
			99.999 856			
11	11		99.999 848		99.999 848	
		NBS	99.999 844	99.999 866	99.999 855	Final calculated length.
			[99.999 844]	[99.999 857]	[99.999 851]	[Reported length].
		PTR	99.999 934		99.999 934	
16	2	NBS	99.999 906	99.999 907	99.999 907	Final calculated length.
			[99.999 914]	[99.999 906]	[99.999 910]	[Reported length].
		PTR	99.999 899		99.999 899	
12	15	NBS	99.999 922	99.999 915	99.999 919	Final calculated length.
			[99.999 929]	[99.999 910]	[99.999 920]	[Reported length].
		NPL	99.999 862			Before immersion.
			99.999 926			Measured 7 hr after partial immersion for several hr.
7	13		99.999 927			Measured 5 hr after drying 16 to 17 days in a desiccator.
		NBS	99.999 863	99.999 881	99.999 872	Final calculated length.
			[99.999 861]	[99.999 873]	[99.999 867]	[Reported length].
		NPL	99.999 906			Before immersion.
			99.999 969			Measured 16 hr after partial immersion for several hr.
8	14		99.999 982			Measured 5 hr after drying 16 to 17 days in a desiccator.
		NBS	99.999 882	99.999 893	99.999 888	Final calculated length.
			[99.999 882]	[99.999 886]	[99.999 884]	[Reported length].

The agreement of the NBS calibrations with those of the other laboratories may be stated as follows:

Gage NBS 11: BIPM value—NBS value= -0.007μ

Gage NBS 2: PTR value—NBS value= $+0.027\mu$

Gage NBS 15: PTR value—NBS value= -0.020μ

Gage NBS 13: NPL values—NBS value= -0.010μ [1] to $+0.055\mu$ [2]

Gage NBS 14: NPL values—NBS value= $+0.018\mu$ [1] to $+0.094\mu$ [2]

[1] From NPL initial measurements.

[2] From NPL measurements after wetting and drying treatment noted in table 14.

Comparison of the NBS with the NPL calibrations is less definite than with the BIPM and PTR determinations. Apparent increases in length resulted from various wetting and drying treatments of the gages at the NPL when measured interferentially, although purely mechanical methods of measurement gave no change. Extensive interferometric comparisons at the NBS of two gages, one being alternately immersed in water and dried for several days, agreed with the NPL mechanical measurements, giving no change as great as 0.005μ.[8]

3. Discussion and Conclusion

The NBS calibrations differ from the BIPM calibration by $+0.007\mu$; from the PTR calibrations by $+0.020$ and -0.027μ; and from the initial measurements at the NPL by $+0.010$ and -0.018μ. The lack of agreement with the NPL measurements that were made after wetting and drying the gages has been mentioned.

Gages 2 and 15 were returned to this Bureau after calibration by the PTR. Repeated comparisons of these gages (each having one surface plane and the other 0.03μ concave), made with the plane and concave surfaces wrung in different sequence to the base plate, gave values that differ among themselves as much as the PTR calibrations differ from those of the NBS. Later tests give a possible explanation for the differences obtained and show the importance of plane end surfaces for precise calibrations. These tests were made with 50-mm fused quartz gages having one plane and one concave surface, each being wrung alternately to a fused-quartz base plate. A shorter mid-side length was measured when the concave surface was wrung to the plate, thus indicating a "flattening" of the concave surface—the magnitude depending upon the intimacy of contact of the wrung surfaces.

Agreement of the calibrations by the various observers is thus limited by uncertainties of measurement that may be attributed to lack of planeness of the gages. The greatest difference (0.027μ) in the independent calibrations can reasonably be ascribed entirely to lack of planeness of the gages. It may certainly be concluded that the diverse interferential methods used by the BIPM, PTR, NPL, and NBS give the same lengths to a maximum uncertainty of 0.027μ (1 part in 3,700,000) and to a probable uncertainty much less.

The results give assurance that decimeter lengths can be measured by the Fabry-Perot method to the accuracy required for ruling standard meter scales. In the light of the information obtained from this investigation, the surfaces of the steel decimeter gages to be used in stepping-off the meter scales should be reworked to the highest planeness attainable through present technique. Improvement in the gages, in the temperature control, and in the measuring apparatus may reasonably be expected to reduce the uncertainty of calibration to 0.02μ for the steel gages. This is equivalent to 0.2μ in a meter or 1 part in 5 million.

VIII. Application of Fused Quartz Gages to Other Investigations

Fabry-Perot interferometers are commonly used for determining refractive indices of gases and for wavelength comparisons. Fused quartz etalons similar to those used in the gage calibrations are particularly suitable for these measurements because of the rigid construction and low thermal expansivity of the etalons, and the planeness and

[8] It is suggested that distortion in the interferometer system may account for the changes obtained interferentially at the NPL. Distortion of interferometer surfaces has been observed at the NBS when glass plates were combined with a fused quartz gage (section V, 1, (b)) and also when a fused quartz plate was combined with a steel gage of high wringing quality. In the latter case a small change in temperature caused appreciable distortion of the plate, although its thickness was 20 mm. Since differences in thermal expansivity between fused quartz and materials such as glass, steel, and crystalline quartz are approximately the same, distortion may reasonably have been experienced in the NPL combination of fused and crystalline quartz. The NPL lack of fused quartz interferometer plates of the required size was noted in its report. These have since been acquired, and repetition of the measurements with these plates may clarify the present uncertainties.

parallelism of the interferometer surfaces. Indices and wavelengths are calculated directly from the measured orders of interference between plates; consequently, they are not subject to the uncertainties of gage calibrations in which measured optical lengths must be transferred to mechanical lengths at the center of the gages.

1. Refractive Index of Dry Air

The refractive index of air, n, may be defined as

$$N=\frac{\text{Number of waves in thickess } (t) \text{ of air}}{\text{Number of waves in thickness } (t) \text{ of vacuum}},$$

and may be determined by obtaining the order of interference of monochromatic radiations in air and in vacuum.

Preliminary measurements to test the suitability of fused quartz Fabry-Perot etalons for index determinations were made in April, 1934. These consisted of five individual determinations at 15° C of the refractive index of dry air containing a normal carbon dioxide content, and of two similar determinations at 20° C. In this work the orders of interference with a 50-mm etalon were obtained alternately in air and vacuum for 23 radiations of krypton and neon. Corrections were made for the expansion of the etalon in vacuum, using $C=9.9\times10^{7}$ [28] as the coefficient of compressibility. Index values so obtained are compared in a subsequent table with those of Sears and Barrell at the NPL; Pérard at the BIPM; Kösters and Lampe at the PTR; and Meggers and Peters at the NBS, to which reference will first be made.

In 1918 Meggers and Peters [29] determined the refractive index of air at 0°, 15°, and 30° C for more than 200 wavelengths of Cu, Fe, Ne, H, and A between 2,218 and 9,000 A. Their results were presented in tables giving the observed index at each wavelength, and also in more convenient tables giving the indices as calculated from dispersion equations that were derived from the observed data.

In 1934 Pérard [30] determined the indices at five wavelengths in the visible spectrum at temperatures between 0° to 100° C and derived from these the following dispersion equation for CO_2-free dry air:

$$(N-1)10^{6}=\left[288.02+\frac{1.478}{\lambda^{2}_{\text{air}}}+\frac{0.0316}{\lambda^{4}_{\text{air}}}\right]\frac{h(1+\beta h)}{760(1+760\beta)}\,\frac{1}{1+0.0037160\theta}$$

in which λ=wavelength in air (microns),
h=pressure (mm),
θ=temperature (° C),
$\beta=2.4\times10^{-6}$ (which can be taken as zero without appreciable error).

In 1934 Kösters and Lampe [9, 10] gave the following equation for the index of CO_2-free dry air at 20° C

$$(n_{t:B}-1)=\left(268.036+\frac{1.476}{\lambda^{2}_{\text{vac}}}+\frac{0.01803}{\lambda^{4}_{\text{vac}}}\right)\frac{1+\alpha\cdot 20}{1+\alpha\cdot t}\,\frac{B}{760}10^{6},$$

in which λ=wavelength in vacuum (microns),
$\alpha=0.00367$.

In 1937 Sears and Barrell [31, 32] gave the following equations for CO_2-free dry air, which were derived from determinations with eight wavelengths in the visible spectrum:

$$(n_{T:p}-1)=(n_{20:760}-1)\,\frac{p(1+\beta p)}{760(1+760\beta)}\,\frac{1+20\alpha}{1+\alpha T},$$

$$(n_{20:760}-1)10^{6}=267.8725+\frac{1.5189}{\lambda^{2}_{\text{vac}}}+\frac{0.01246}{\lambda^{4}_{\text{vac}}},$$

in which λ=wavelength in vacuum (microns)
$\beta=0.73\times10^{-6}$
$\alpha=0.003674$

Table 15 gives the indices calculated by Sears and Barrell from the various equations at eight wavelengths and converted by them to like conditions of CO_2-free, dry air at 15° C, 760 mm Hg. The values of Pérard, Sears and Barrell, and Kösters and Lampe are appreciably higher then those of Meggers and Peters, which are admittedly low because, for lack of the necessary data, no correction was applied for the increased length of the etalons in vacuum, although they were the first to call attention to this required correction. Furthermore their equation does not fit their observations particularly well in the red end of the spectrum and for this reason their observed values are more reliable than the curve values in this region. For example: (n-1) for the 6,438 Cd line (15° C; 760 mm) is 2758×10^{-7} by their equation, whereas their observed data for this spectral region gives 2761×10^{-7} which, if corrected by 3×10^{-7} for increase in the length of the etalon when changing from atmospheric to vacuum conditions gives $(n-1)=2764\times10^{-7}$ as found by the other observers.

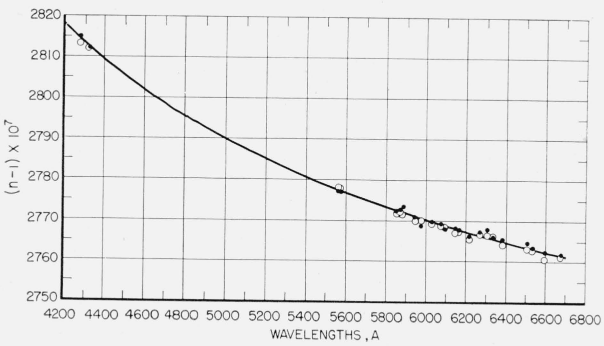

FIGURE 13. $(n-1)\times10^7$ *for dry air; normal* CO_2; *15°C; 760 mm, by various observers.*

———, Pérard, Kösters, and Lampe; Sears & Barrell. ●●●○○○, Peters and Emerson.

The indices determined in the present preliminary study are seen in figure 13: circles are the mean ($n-1$) values of five determinations at 15° C, while dots are the means of two determinations at 20° C, which have been converted to 15° indices using Meggers and Peters' optical coefficients. The

TABLE 15. $(n-1)\times10^7$ *for dry, carbon-dioxide-free air at 15° C; 760 mm by various observers*

Source	Wave-length	$(n-1)\times10^7$ by different observers				
		Meggers and Peters (1)	Pérard (2)	Kösters and Lampe (3)	Sears and Barrell (4)	Mean: Observers (2, 3, and 4)
	A					
Cd(R)	6438.4696	2757.5	2763.7	2764.3	2763.4	2763.8
He(Y)	5875.623	2764.4	2771.2	2772.0	2771.2	2771.5
Hg(G)	5460.7430	2771.1	2778.5	2779.4	2778.6	2778.8
Cd(G)	5085.8212	2778.8	2786.8	2787.7	2787.0	2787.2
Cd(B)	4799.9104	2786.0	2794.6	2795.6	2794.8	2795.0
Cd(V)	4678.1493	2789.6	2798.4	2799.4	2798.6	2798.8
He(V)	4471.477	2796.4	2805.7	2806.6	2805.8	2806.0
Hg(V)	4358.325	2800.6	2810.2	2811.1	2810.2	2810.5

full line in figure 13 represents the average dispersion curve obtained by converting the equation values of Pérard, Sears and Barrell, and Kösters and Lampe for CO_2-free air to values for air having a normal CO_2 content.

In general the present observations lie close to the average curve of the other observers, although a slight difference in the form of that curve and one drawn through the observed points is indicated. Unfortunately, several of the neon lines used are not suitable for a 100-mm path, causing departures of 1 or 2×10^{-7} from a smooth curve. Also the accuracy of determinations with the 4,274 and 4,320 A Kr lines is questionable, because with optical paths of this magnitude the limiting order of interference was approached for these wavelengths causing the fringes to be indistinct.

2. Wavelengths by Comparison With the Standard Wavelength

In calibrating the decimeter gages by direct measurement with light waves, krypton and cadmium sources were alternated, and the order of interference was determined for all lines that gave measureable fringes. From these data the wavelengths of several Kr and Cd radiations were obtained by comparison with wavelength λ_R of the standard radiation of Cd in the following manner:

The optical path was taken as the product of the order of interference and the wavelength employed. The order of interference, as determined, included the phase change at reflection from the silver surfaces. The combined correction for film thickness and phase change evaluated for red, yellow, green, and blue radiations of helium indicated that any difference in optical path attributable to selective phase change within this spectral range was too small in comparison with the total path to introduce appreciable error in the determined wavelengths. Consequently, the optical path obtained for the standard radiation was considered common to all visible radiations of Cd and Kr. This optical path divided by the order for a given line gave, then, the wavelength of that line for the atmospheric conditions at which the measurements were made. The final step was to convert these values to wavelengths at 15° C and 760 mm Hg.

The gage data were sufficient to give the values shown in table 16 for the wavelengths of the 5,562;

5,570; 5,871 Kr and the 4,800; 5,086 Cd radiations with an optical path of 200 mm.[9] Wavelengths determined for these radiations by various investigators are given in table 17.

TABLE 16. *Wavelength determinations with a 200-mm optical path*

Series	Wavelengths (krypton)			Wavelengths (cadmium)	
	A	*A*	*A*	*A*	*A*
1	5562. 2256	5570. 2897	5870. 9166	4799. 9114	5085. 8216
2	55	90	57	14	13
3	42	84	43	----------	22
4	63	98	77	23	14
5	48	92	55	13	13
6	64	94	72	----------	26
7	66	90	58	18	14
8	53	92	55	----------	08
9	41	81	50	----------	09
10	50	92	66	----------	17
11	60	97	----------	----------	08
12	39	80	44	----------	07
13	52	86	57	----------	05
14	57	94	82	----------	19
15	56	. 2900	61	----------	16
16	55	. 2893	70	14	17
17	59	96	72	21	18
18	58	93	59	23	05
19	57	91	55	17	15
20	59	96	60	19	13
21	47	91	58	----------	17
22	56	96	66	11	06
23	50	91	55	11	08
24	52	88	54	11	07
25	55	89	47	22	12
26	62	. 2900	65	15	----------
27	54	. 2891	53	----------	13
28	47	85	44	13	17
29	48	91	62	18	18
30	43	82	47	17	10
31	62	. 2903	68	14	13
32	56	. 2895	61	14	11
33	59	96	59	----------	16
34	58	96	60	----------	15
35	61	98	56	----------	----------
36	51	88	56	----------	02
37	47	91	----------	----------	21
38	55	92	60	16	----------
39	56	92	63	19	16
Mean	5562. 2254	5570. 2892	5870. 9159	4799. 9116	5085. 8213
Avg dev [1]	0. 00052	0. 00039	0. 00066	0. 00031	0. 00042
Avg dev [2]	. 00008	. 00006	. 00011	. 00007	. 00007

[1] Average deviation of the observations from the mean.
[2] Average deviation of the mean.

[9] Three exposures were customarily made with each source for a given series. These usually gave three determinations of the order of interference for the 6,438 standard line and two or three determinations for each of the other lines, except the 4,800 Cd for which a single observation was ordinarily obtained. The tabulated wavelengths are based upon the average values of the order of interference of all determinations in a series with the standard radiation and with a given radiation.

Although the 5,650; 5,672; 5,944 Kr and the 4,678 Cd lines also gave measurable fringes with several etalons, the data are too scanty to include in table 16.

Jackson's values for Kr wavelengths are greater than those given by the other observers and exceed the average values of those observers by 0.0011, 0.006, and 0.0010 A for the 5,562; 5,570; 5,871 radiations. The uniformity of the values, particularly for the 5,562; 5,570 lines, by observers 1, 3, and 5 indicates an incompatability of Jackson's results with theirs. The preponderance of determinations giving shorter values makes the inclusion of Jackson's wavelengths questionable when arriving at the most reliable values for the wavelengths. The following wavelengths based upon the determinations of Meggers and Humphreys, Meggers and Burns, Pérard and Peters and Emerson are considered by the authors to be the most reliable values now available: Kr_1, 5562.2255; Kr_2, 5570.2893; Kr_3, 5870.9157.

TABLE 17. *Wavelength determinations by different observers*

Observer	Wavelength				
	Krypton			Cadmium	
	(1)	(2)	(3)	(4)	(5)
	A	*A*	*A*	*A*	*A*
Meggers and Humphreys [20].	[a] 5562.2255	[a] 5570.2892	[a] 5870.9154	----------	----------
	[b] 5562.2254	[b] 5570.2893	[b] 5870.9153	----------	----------
Meggers and Burns [33]. [c]	----------	----------	----------	4799. 9139	5085. 8230
Sears and Barrell [34] [d]	----------	----------	----------	4799. 9104	5085. 8212
Pérard [30, 35] [e]	5562. 2257	5570. 2894	5870. 9159	4799. 9113	5085. 8224
Jackson [36] [f]	5562. 2266	5570. 2899	5870. 9167	----------	----------
Peters and Emerson [g]	5562. 2254	5570. 2892	5870. 9159	4799. 9116	5085. 8213

[a] By comparison with a cadmium standard, using 25- and 35-mm etalons.
[b] By comparison with neon standards, using 43-mm etalons.
[c] Values are based upon measurements with optical paths up to 80 mm.
[d] Values are based upon measurements with 20- to 240-mm optical paths.
[e] Values are based upon measurements with optical paths of 10 to 260 mm for the krypton lines and of 10 to 190 mm for the cadmium lines.
[f] Values are the means of determinations with optical paths of 20, 40, and 60 mm.
[g] Values obtained with an optical path of 200 mm.

The greater discordance by the various observers for the 4,800 and 5,086 Cd wavelengths may be attributed to nonhomogeneity of these lines; studies by Michelson and Pérard have confirmed the presence of satellites that affect the wavelength measurements and give different wavelength values when paths of different length are used. The hyperfine structure of these and other cadmium lines has been studied in detail by Schüler and Bruck [37] and by Schüler and Keyston [38].

877106—50——7

IX. References

[1] A. A. Michelson and J. R. Benoît, Travaux et Mémoires du Bureau International des Poids et Mesures **11,** 85 (1895).

[2] J. R. Benoît, C. Fabry and A. Perot, Travaux et Mémoires du Bureau International des Poids et Mesures **15,** 131 (1913).

[3] Trans. Intern. Astronomical Union **2,** 232 (1925).

[4] Comptes rendus, Septième Conférence Générale des Poids et Mesures 52 (1927).

[5] Procès-verbaux des Séances, Comité International des Poids et Mesures, Deuxième Série **XVII,** 91 (1935).

[6] H. Barrell, Rev. Métrologie, 2d Série, **VII,** 33, (Feb. 1947).

[7] N. Watanabe and M. Imaizumi, Proc. Imp. Acad. Tokyo **4,** 351 (1928).

[8] J. E. Sears and H. Barrell, Phil. Trans. Roy. Soc. (London) [A], **233,** 143 (1934).

[9] W. Kösters and Lampe, Phys. Zeit. **35,** 224 (1934).

[10] W. Kösters and Lampe, Compt. rend., Huitième Conférence Générale des Poids et Mesures 79 (1933).

[11] Procès-verbaux des Séances, Comité International des Poids et Mesures, 2d Série, **XVII,** 113 (1935). Results are given by Kösters and Sears for repetitions of the 1933 measurements at the PTR and the NPL.

[12] W. Kösters and Lampe, Werkstatt, und Werksleiter **32,** 527 (1938), Rev. Métrologie, 2d Série, **IV,** 147 (1943).

[13] Romanova, Varlich, Kartasher et Batarchukova: Compt. rend. (Doklady) de l'Académie des Sciences de l'URSS **37,** 46 (1942) (In English).

[14] C. G. Peters and H. S. Boyd, BS Sci. Pap. **17,** 677 (1922) S436.

[15] C. Pulfrich, Z. Instr. **18,** 261 (1898).

[16] I. G. Priest, BS Sci. Pap. **15,** 669 (1919–20) S365.

[17] F. H. Rolt, Gauges and fine measurements **1,** 164 (1929).

[18] W. H. Souder and C. G. Peters, Tech. Pap. BS 157 (1920).

[19] W. H. Souder and C. G. Peters, Dental Cosmos. (March 1920).

[20] W. F. Meggers and C. J. Humphreys, J. Research NBS **13,** 293 (1934) RP710.

[21] W. H. J. Childs, J. Sci. Inst. **3,** 99 to 129 (1926).

[22] K. Burns and C. C. Kiess, Pub. Allegheny Obs. **6,** 125 (1927).

[23] C. J. Humphreys, J. Research NBS **5,** 1041 (1930) RP115.

[24] W. F. Meggers, BS Bull. **12,** 203 (1915) S179.

[25] C. G. Peters, BS Sci. Pap. **20,** 635 (1926) S521.

[26] Albert Pérard, Applications Pratiques des Interférences Lumineuses: Travaux et Mémoires du Bureau International des Poids et Mesures **18,** 11 (1930).

[27] W. Kösters, Procès-verbaux des Séances, Comité International des Poids et Mesures, 2d Série, **XVII,** 82 (1935).

[28] W. B. Emerson, J. Research NBS **18,** 683 (1937) RP1003.

[29] W. F. Meggers and C. G. Peters, BS Bull. **14,** 697 (1918) S327.

[30] A. Pérard, Travaux et Mémoires du Bureau International des Poids et Mesures **19,** 78 (1934).

[31] J. E. Sears and H. Barrell, Procès-verbaux des Séances, Comité International des Poids et Mesures, 2d Série, **XVIII,** 94 (1937).

[32] J. E. Sears and H. Barrell, Phil. Trans. Roy. Soc. (London) [A] **238,** 1 to 64 (Feb. 1939).

[33] W. F. Meggers and K. Burns, BS Sci. Pap. **18,** 185 (1922–23) S441.

[34] J. E. Sears and H. Barrell, Proc. Roy. Soc. [A], **139,** 202 (1933).

[35] A. Pérard, Réunions de l'Institut d'Optique 6 Année, 10 (1935).

[36] C. V. Jackson, Proc. Roy. Soc. [A], **138,** 409 (1932).

[37] H. Schüler and H. Bruck, Z. Phys. **56,** 291 (1929).

[38] H. Schüler and J. E. Keyston, Z. Phys. **67,** 433 (1931).

WASHINGTON, October 6, 1949.