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Transmission of Reverberant Sound Through Double Walls

By Albert London

The transmission of reverberant sound through a double wall, which consists of two identical single walls coupled by an airspace, is investigated both theoretically and experimentally. A theory is developed, which gives good agreement with experiment. In order to compute the transmission loss of a double wall, it is necessary to know the impedance Z_w of the single wall. Z_w was determined from experiments conducted on the single wall and includes the effects of mass, dissipation, and flexural motion. The treatment shows that it is impossible to get a large improvement in transmission loss for a double wall relative to a single wall under reverberant-sound field conditions if the single wall is considered to have only mass reactance. In addition, the customary normal incidence theory is totally inadequate in explaining the behavior of a double wall in a reverberant-sound field.

For double walls having air coupling only, very shallow airspaces can produce appreciable increases in transmission loss over a single wall. An absorbent material, when inserted in the airspace, produces large improvements only when the mass of the walls is relatively light and has but little effect for heavy walls. Honeycomb or other nonabsorbent cellular structures having no cell walls in a direction normal to the wall faces do not result in an increase in transmission loss. Air-coupled walls having no solid sound-conducting paths between individual septa are extremely effective sound insulators as compared to conventional double-wall constructions. The theory indicates that a large improvement in the transmission loss of a double wall can be obtained by using as components single walls with high internal dissipation.

I. Introduction

In a previous paper [1] the transmission of reverberant sound through homogeneous single walls was investigated theoretically and experimentally. The attenuation of an obliquely incident plane sound wave upon transmission through a single wall was computed, and using the customary reverberant sound field statistics the attenuation was integrated over all angles of incidence to give the average transmission loss. A similar technique is employed in this paper in studying the transmission of sound through a double wall consisting of two identical single walls. The materials comprising the double walls are the same as were used in the single walls, i. e., aluminum, plywood, and plasterboard. From the experimental results obtained in RP1998, an expression for the wall impedance, Z_w, for each material was determined, this expression containing terms that include the effects of the mass, dissipation or resistance, and flexural motion of the wall. This value of Z_w is used in the double wall theory to compute the transmission loss for a double wall.

[1] A. London, Transmission of reverberant sound through single walls, J. Research NBS **42**, 605 (1949) RP1998.

II. Transmission Through Double Walls

1. Attenuation of an Obliquely Incident Wave

In figure 1, an oblique plane wave is incident at an angle θ on the first partition. As a result there exist in the three airspaces formed by the infinite double partition: an incident and reflected wave in space (1); a standing wave in space (2), consisting of a wave moving to the right

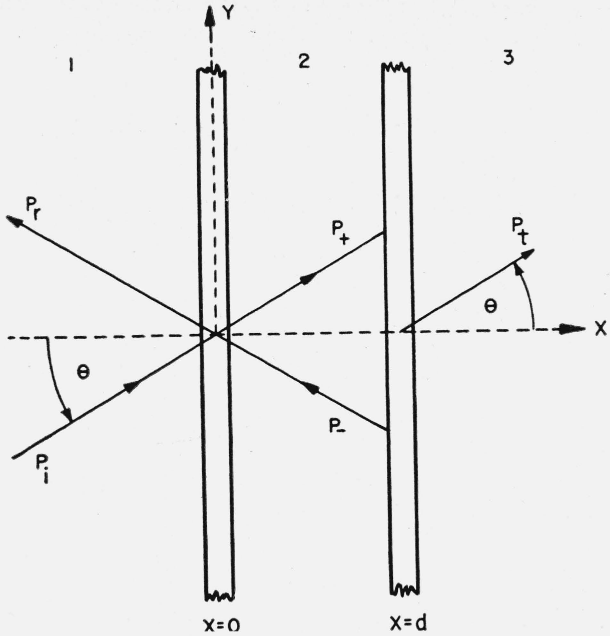

FIGURE 1. *Geometrical relation between incident and reflected wave in space (1); standing wave in airspace of double wall, space (2); and transmitted wave in space (3).*

and one moving to the left; and a transmitted wave in space (3). It is desired to know the ratio of the transmitted pressure wave amplitude P_t to the incident pressure wave amplitude P_i, where the pressures in each airspace are given by eq 1.

$$\begin{aligned} p_1 &= P_i e^{i\omega t - ik(x\cos\theta + y\sin\theta)} + \\ &\quad P_r e^{i\omega t - ik(-x\cos\theta + y\sin\theta)} \qquad x \leq 0 \\ p_2 &= P_+ e^{i\omega t - ik(x\cos\theta + y\sin\theta)} + \\ &\quad P_- e^{i\omega t - ik(-x\cos\theta + y\sin\theta)} \qquad 0 \leq x \leq d \\ p_3 &= P_t e^{i\omega t - ik(x\cos\theta + y\sin\theta)} \qquad x \geq d \end{aligned} \tag{1}$$

where

$\omega = 2\pi \times$ frequency

$k = 2\pi/\lambda = \omega/c$, λ being the wavelength, c the velocity of sound in air.

The four ratios P_r/P_i, P_+/P_i, P_-/P_i, and P_t/P_i may be determined from the two boundary conditions, the continuity of the x-component of velocity at $x=0$, and $x=d$, and the two equations of motion, one for each partition. In deriving the equations of motion it is only necessary to consider a small area of the panel upon which the projection of the wave front has practically constant phase. The justification for such an assumption has been given previously in section 2 of RP1998, titled Basic Assumptions.

The ratio P_t/P_i will be computed for the same value of y, so that this coordinate will not appear in the calculation. Thus, since the particle velocity is proportional to the pressure gradient, there results from the continuity of the x-component of particle velocity at $x=0$

$$\left(\frac{\partial p_1}{\partial x}\right)_{x=0} = \left(\frac{\partial p_2}{\partial x}\right)_{x=0},$$

$$P_i - P_r = +P_+ - P_-, \tag{2}$$

and at $x=d$

$$\left(\frac{\partial p_2}{\partial x}\right)_{x=d} = \left(\frac{\partial p_3}{\partial x}\right)_{x=d},$$

$$P_+ e^{-ikd\cos\theta} - P_- e^{+ikd\cos\theta} = P_t e^{-ikd\cos\theta}. \tag{3}$$

If p_{10} and p_{20} are the pressures acting on the left and right side, respectively, of the panel at $x=0$, p_{2d}, and p_{3d} the pressures acting on the left and right side of the panel at $x=d$, the equations of motion for each panel are

$$p_{10} - p_{20} = Z_w \dot{\eta}_{x=0}, \tag{4}$$

and

$$p_{2d} - p_{3d} = Z_w \dot{\eta}_{x=d}, \tag{5}$$

where Z_w is the mechanical impedance per unit area of the two identical walls, and $\dot{\eta}$ is the velocity of the wall in the x-direction. Furthermore, since the wall velocity must be the same as the x-component of particle velocity of the air at the wall, there results

$$\dot{\eta}_{x=0} = \frac{i}{\rho\omega}\left(\frac{\partial p_1}{\partial x}\right)_{x=0} = \frac{\cos\theta}{\rho c}(P_i - P_r) e^{i\omega t - iky\sin\theta}, \tag{6}$$

$$\dot{\eta}_{x=d} = \frac{i}{\rho\omega}\left(\frac{\partial p_3}{\partial x}\right)_{x=d} = \frac{\cos\theta}{\rho c} P_t e^{i\omega t - ik(d\cos\theta + y\sin\theta)}. \tag{7}$$

Substituting eq 1 and 6 into 4 causes the latter equation to reduce to

$$(P_i + P_r) - (P_+ + P_-) = \frac{Z_w \cos\theta}{\rho c}(P_i - P_r), \tag{8}$$

and similarly eq 5 becomes

$$P_+ e^{-i\beta} + P_- e^{i\beta} - P_t e^{-i\beta} = \frac{Z_w \cos\theta}{\rho c} P_t e^{-i\beta}, \tag{9}$$

where

$$\beta = kd\cos\theta. \tag{10}$$

Let

$$\gamma = \frac{Z_w \cos\theta}{2\rho c}, \tag{11}$$

then solution of the four simultaneous equations (2, 3, 8, and 9) results in the following expressions:

$$A = \frac{P_i}{P_t} = 1 + 2\gamma + \gamma^2(1 - e^{-2i\beta}), \tag{12}$$

and also

$$\alpha = \frac{P_+}{P_i} = 1 + \gamma. \tag{13}$$

It is of interest to observe that eq 13 is precisely the expression for the ratio of incident to transmitted amplitude for a single wall given by eq 1.6 of RP 1998, inasmuch as the boundary conditions, i. e., the existence of an incident, reflected, and transmitted wave, are the same as that for a single wall.

Equation 12, which is of primary interest to this development, can be tested for agreement with the solution, eq 13, for the attenuation of a single septum. Thus, if $d=0$, the double wall becomes a single wall having an impedance $2Z_w$. Of course, this is strictly a mathematical experiment, inasmuch as was shown in the previous paper (see footnote 1), the resistive and reactive components of Z_w are not twice as great when a single wall's thickness is doubled. Setting $d=0$ in eq 12 results in

$$A_{d=0} = 1 + 2\gamma = 1 + \frac{2Z_w \cos\theta}{2\rho c}, \tag{14}$$

in agreement with eq 13.

Also, it is possible to compare eq 12 with the results obtained by previous investigators [2] for the special case when the wave is incident normally, i. e., $\theta=0°$, and the wall impedance, Z_w, is a pure mass reactance only, given by eq 2.1 of RP1998 or

$$Z_w = i\omega m,$$

where m is the mass of the wall per unit area. Now, from eq 12, if the wall acts as a mass reactance only, it is readily shown that

$$|A|^2_m = \left|\frac{P_i}{P_t}\right|^2_{Z_w = i\omega m} = 1 + 4a^2 \cos^2\theta (\cos\beta - a\cos\theta \sin\beta)^2, \tag{15}$$

where $a = \omega m / 2\rho c$. (16)

[2] A. Schoch, Die physikalischen und technischen Grundlagen der Schalldammung im Bauwesen, p. 86 (Hirzel, Leipzig, 1937).

Letting $\theta=0°$, reduces eq 16 to

$$|A|^2_{0,m} = 1 + 4a^2 (\cos b - a \sin b)^2, \tag{17}$$

where

$$b = \frac{\omega d}{c} = \frac{2\pi d}{\lambda}, \tag{18}$$

and eq 17 is identical with the expression given by Schoch [2].

From eq 15 all of the incident energy will be transmitted when

$$(\cos\beta - a\cos\theta \sin\beta) = 0, \tag{19}$$

or when

$$\tan\beta = 1/(a\cos\theta). \tag{20}$$

For cases where β is small ($d \ll \lambda$) $\tan\beta$ may be replaced by β. Using eq 16, there results an expression for the frequency f_θ, for which a wave incident at angle θ, will be perfectly transmitted in the case where each wall acts as a pure mass.

$$f_\theta = \frac{1}{2\pi\cos\theta}\left(\frac{2\rho c^2}{md}\right)^{\frac{1}{2}}. \tag{21}$$

The value of f_θ for normal incidence is f_0, the characteristic frequency for the air-mass sandwich, i. e.

$$f_0 = \frac{1}{2\pi}\left(\frac{2\rho c^2}{md}\right)^{\frac{1}{2}}. \tag{22}$$

f_0 is the frequency for normally incident waves for which the mass reactance of the panel is exactly equal to the stiffness reactance of the air-space. It is also the lowest frequency for which the attenuation of the panel is zero. At frequencies above f_0 there will be some angle of incidence for which zero attenuation will occur. Since in a reverberant sound field, energy is incident from all directions, the attenuation measured in a reverberant field will never reach zero. For frequencies above f_0 there will be some waves that will be totally transmitted, consequently resulting in a diminution of the transmission loss of the panel as compared to that predicted by the normal incidence theory.

Since $1/a$ decreases with increasing frequency, at high enough frequencies eq 20 may be written as

$$\tan\beta = 0,$$

and

$$\beta = n\pi,\ n = 1, 2, 3, \ldots,$$

which results in

$$d \cos \theta = n\lambda/2, \; n=1, 2, 3, \ldots, \qquad (23)$$

as the expression for the frequencies, or wavelengths, at which higher-order minima occur. Here too, for a reverberant field, considerations similar to those discussed in connection with f_0 apply.

Equation 12, which gives the attenuation, A, for a double wall may readily be compared with the attenuation, α, for a single wall given by eq 1.6 of RP1998 or its identity eq 13. Since γ is ordinarily much larger than unity, $\alpha \approx \gamma$, and

$$A \approx 2\alpha + \alpha^2(1-e^{-2i\beta}) \qquad (24)$$

The term containing α^2, multiplied by a factor $(1-e^{-2i\beta})$ which is never greater than 2 in absolute value and which depends on the spacing between the two walls, therefore, represents the chief difference in attenuation caused by a double wall relative to that of a single wall.

As shown in RP1998 the most general expression for the wall impedance is given by

$$Z_w = \frac{2r}{\cos\theta} + i\omega m \left(1 - \frac{f^2}{f_c^2}\sin^4\theta\right), \qquad (25)$$

or

$$\gamma = \frac{Z_w \cos\theta}{2\rho c} = R + ia\cos\theta\left(1 - \frac{f^2}{f_c^2}\sin^4\theta\right), \qquad (26)$$

where $R = r/\rho c$, the resistance of the wall in ρc units, and f_c = the critical frequency above which flexural waves will appear in the wall. The parameters R and f_c for different materials were determined from the experimental observations made in RP1998. Substituting eq 26 into 12 results in

$$A = 1 + 2R(1 - pv\sin 2bv) + (R^2 - p^2v^2)(1-\cos 2bv) + i\{(R^2-p^2v^2)\sin 2bv + 2pv\,[1 + R(1-\cos 2bv)]\}, \qquad (27)$$

where $v = \cos\theta$, $b = kd$, and

$$p = a\left[1 - \frac{f^2}{f_c^2}(1-v^2)^2\right]. \qquad (28)$$

For $|A|^2$ there results

$$|A|^2 = 1 + 4[R(R+1) + p^2v^2] + 4\sin^2 bv\{[R(R+1)+p^2v^2]^2 - p^2v^2\} - 4pv\sin 2bv\{R(R+1)+p^2v^2\}. \qquad (29)$$

When $R=0$, 29 reduces to an equation analagous to 15 with a replaced by p, i.e.,

$$|A|^2_{R=0} = 1 + 4p^2v^2(\cos bv - pv\sin bv)^2. \qquad (30)$$

Utilizing 30, eq 29 may be rewritten as

$$|A|^2 = |A|^2_{R=0} + 4R(R+1)\{1 + [R(R+1) + 2p^2v^2]\sin^2 bv - pv\sin 2bv\}, \qquad (31)$$

or an equivalent form is

$$|A|^2 = |A|^2_{R=0} + 4R(R+1)\{(\cos bv - pv\sin bv)^2 + [p^2v^2 + R(R+1) + 1]\sin^2 bv\}. \qquad (32)$$

Inasmuch as the second member on the right-hand side of eq 32 is always positive, it will be seen that the attenuation of a double wall, each component of which has dissipation or resistance, is always greater than for the case in which each component is dissipationless.

2. Average Attenuation of a Double Wall in a Reverberant Sound Field

In accordance with the reverberant sound field statistics discussed in section 3 of RP1998, if τ_d is the ratio of the total energy transmitted by the double wall to the total energy incident on the wall, we get from eq 3.1 of RP1998 and eq 29

$$\tau_d = 2\int_0^1 \frac{v\,dv}{|A|^2}, \qquad (33)$$

where $v = \cos\theta$.

The integral in eq 33, unfortunately, is highly intractable. It was not possible to evaluate it other than by numerical integration. This has been done for a number of different constructions on which experimental results were obtained and will be discussed in section II, 3. However, for the special case where it is assumed that each single wall has a mass reactance only, the integral has been computed[3] for a wide range of values in a systematic manner. For the mass reactance case we may set $R=0$ and $f/f_c=0$, whence, 29 reduces to 15 and 33 assumes the following form

$$\underset{(R=0,\,f/f_c=0)}{\tau_d} = 2\int_0^1 \frac{v\,dv}{1 + 4a^2v^2(\cos bv - av\sin bv)^2}. \qquad (34)$$

[3] We are indebted to G. Blanch and I. Stegun of the National Bureau of Standards' Computation Laboratory for carrying out these integrations. They used a combination of numerical integration and analytic representations for different regions of a certain parameter to evaluate the integrals.

It is convenient to introduce two nondimensional parameters into eq 34, namely,

$$\mu=\frac{b}{2a}=\frac{\rho d}{m}, \tag{35}$$

and

$$X=\frac{f}{f_0}=\sqrt{ab}, \tag{36}$$

where f_0 is defined by eq 22. Thus, μ is the ratio of the mass of air in the airspace to the mass of one wall, whereas X is the ratio of the frequency of the sound wave to the frequency for which a wave, normally incident on a double wall possessing mass reactance only, will be perfectly transmitted. In addition we let

$$u=2\,av \tag{37}$$

and 34 becomes

$$\underset{(R=0,f/f_c=0)}{\tau_d}=\frac{\mu}{X^2}\int_0^{X\sqrt{2/\mu}}\frac{u\,du}{1+u^2\left(\cos\mu u-\frac{u}{2}\sin\mu u\right)^2}. \tag{38}$$

It is of interest to compare the transmission loss, 10 log $(1/\tau_d)$, computed from eq 38 with that which one obtains for a single wall when it is assumed that the wall has a mass reactance only. An expression for the latter transmission loss is given by eq 3.2 of RP 1998. If we replace a^2 by its equivalent expression in terms of X and μ, i. e.,

$$a^2=\frac{X^2}{2\mu}, \tag{39}$$

eq 3.2 of RP 1998 may be written

$$TL=10\log\left(\frac{1}{\tau}\right)=10\log\frac{X^2}{2\mu}-10\log\left[\ln\left(1+\frac{X^2}{2\mu}\right)\right]. \tag{40}$$

In figure 2, the computed transmission loss for a single and double wall, having mass reactance only, i. e., $Z_w=i\omega m$, have been plotted for three different values of the parameter μ. It will be seen that on this basis the predicted improvement of a double wall over a single wall is small and in fact may actually be negative. This astonishing behavior results from the fact that for a double wall there is some angle of incidence for which the transmission is perfect and in the integrated effect of all angles of incidence, this minimum

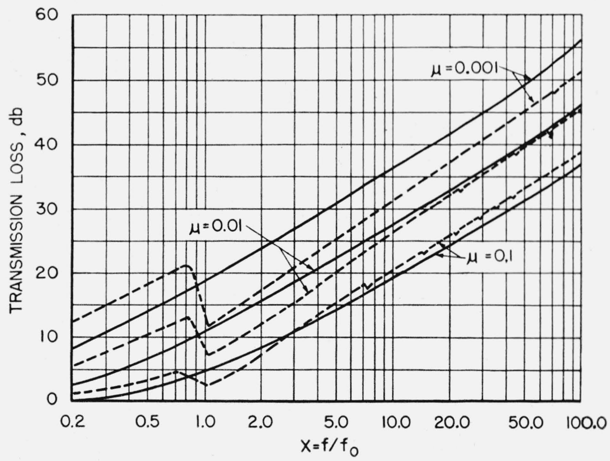

FIGURE 2. *Comparison between theoretical transmission loss of a single wall to that of the corresponding double wall in a reverberant sound field when wall is considered to be a pure mass reactance.*

___, Single wall; ----, double wall.

transmission loss swamps out the effect at other angles of incidence. In the case where a resistive term is included in the impedance, there is no angle for which the transmitted wave is not attenuated. Hence, it is not sufficient to treat each component of the double wall as a pure mass.

With regard to figure 2, it is well to point out that the small maxima and minima indicated in the double wall curves are a result of the higher order minima, which are approximately given by eq 23. Values of the integral (eq. 38) were computed for $X=0.2$, 0.5, 1.0, 2.0, 5.0, 10.0, 20.0, 50.0, and 100.0 for $\mu=0.1$, 0.08, 0.06, 0.04, 0.01, 0.006, 0.004, 0.002, and 0.001. This information has not been reproduced here but is available upon request.

3. Comparison Between Experimental and Computed Results

Figure 3 is a schematic drawing showing the arrangement of the double wall in the sound transmitting opening. Each leaf of the double wall was made separately, a practical procedure inasmuch as the concrete walls of the test chamber are isolated from each other by a 3-in. airspace except for the common foundation of the walls. Thus, there are no solid sound-conducting bridges between the two faces of the double wall, a circumstance that allows a close approximation to the conditions set down in the theory. The experimental method utilized in making the trans-

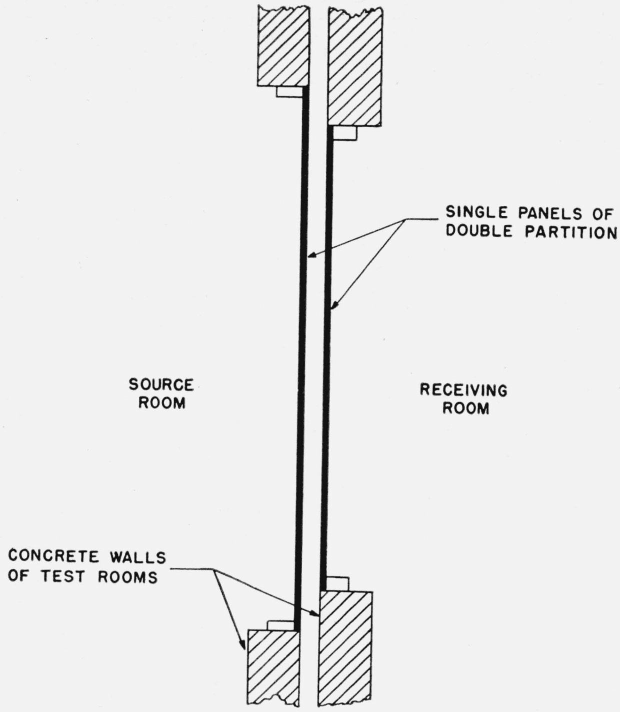

FIGURE 3. *Arrangement of double wall in sound transmitting opening.*

mission loss measurements is that described as the usual method in NBS Research Paper RP1388.[4]

The next figure, figure 4, shows the results obtained on a double wall consisting of single walls of 1/64-in. aluminum separated by a 3-in. airspace. Here $m=0.12$ g/cm^2, and the mass and thickness of the single wall are such that the critical flexural frequency f_c, is approximately 30,000 c/s. Thus, $f/f_c \approx 0$, and no flexural effects will be observed. For reference purposes, the results obtained in the single wall case are shown in the lower part of the figure.

The best fit for the single wall case was obtained when $R=2.16$. The same value of R was used for the double wall calculations, which were carried out by a numerical integration of eq 33. It will be noted that eq 33 as opposed to eq 38 predicts a sizeable improvement in transmission loss of a double wall over a single wall.

According to eq 22 there should be a minimum in the transmission loss curve at $f_0=279$ c/s, corresponding to the frequency for which the mass reactance of the wall is exactly equal to the stiffness reactance of the airspace. However, this minimum is based on the assumption that the wall has zero resistance. As a matter of fact, in this particular case, the value of R is such that no noticeable minimum occurs in the τ_a integral calculations for critical values of v or θ corresponding to eq 19. In particular, from eq 15, 19, and 32 we see that $|A|^2$ for v or $\cos\theta$ satisfying eq 19 becomes

$$|A|^2_{v\theta} = 1+4R(R+1)\sin^2\left(\sqrt{\frac{b}{a}}\right)\left[\frac{a}{b}+R(R+1)+1\right]. \tag{41}$$

Since $|A|^2_{R=0}=1$, $p=a$ (i. e. $f/f_c=0$), and the critical value of v, say v_0, is given by $v_0^2=(ab)^{-1}$ from eq 20. From eq 3.5 $a/b=1/(2\mu)$ and for this wall $\mu=.075$, so that $a/b=6.66$. Since $R=2.16$, we get from eq 41

$$|A|^2_v = 57.4.$$

Thus, the minimum value is much larger than 1, which is the value that would result if $R=0$. Furthermore, it will be noted that eq 41 predicts this same minimum value of $|A|^2$ independent of frequency provided θ is such that eq 19 is satisfied. This same minimum will occur at frequencies above f_0, thus tending to depress the natural increase in transmission loss resulting from mass law behavior.

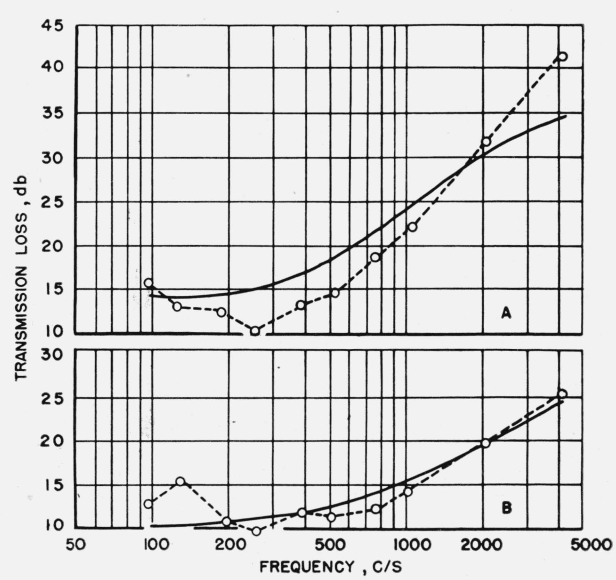

FIGURE 4. *Comparison of computed and experimental transmission losses of 1/64 in. single and double aluminum walls.*

A, Double wall; ——, computed; ----, experimental; $d=3$ in.; $f_0=279$ c/s.
B, Single wall; ——, computed; ----, experimental; $R=2.16$; $m=0.12$ g-cm^{-2}.

[4] A. London, J. Research NBS **36**, 419 (1941) RP1388.

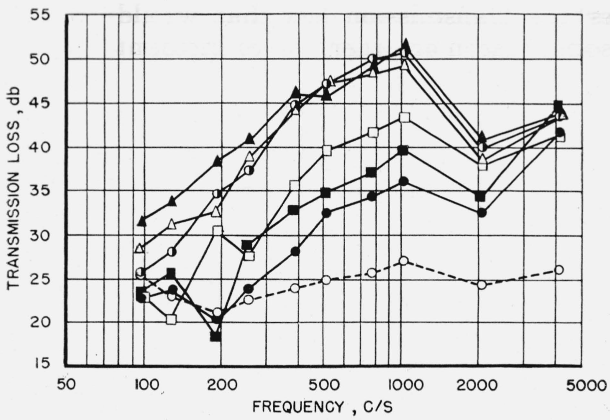

FIGURE 5. *Experimental transmission loss results for a series of double walls of ½-in plywood with airspace varying from ⅜ to 12 in.*

Dotted broken line is experimental transmission loss for corresponding single wall. Air space· ●, ⅜ in.; ■, 1.5 in.; □, 3 in.; ◑, 6 in.; △, 9 in.; ▲, 12 in.

With regard to the reliability of the computed values relative to the experimental values, it is probable that they agree within the accuracy of experimental observations for frequencies below 500 c/s. Above that frequency, it is to be noted that the computed curve deviates from the experimental curve in the same direction for both the double and single walls. In fact, these two curves intersect at about the same frequency for both the double and single walls. Thus, the discrepancy between computed and experimental curves in the double wall case is apparently due to the imperfect fit obtained for single walls and, furthermore, the effect of this imperfect fit seems to be magnified for double walls.

Figure 5 shows the experimental results obtained on a series of ½-in. plywood walls in which the airspace was varied from ⅜ to 12 in., together with the transmission loss obtained on the single wall. Several pertinent observations may be made concerning the general nature of these experimental results. First, it will be seen that even for the ⅜-in. airspace there is a considerable range of frequencies for which there is a significant improvement of the double wall over the single wall. Second, all of the curves have a minimum in the vicinity of 2,000 c/s. As was pointed out in RP1998, the minimum in the single wall *TL*, which also occurs at this frequency, was due to a flexure wave having an f_c=1,885 c/s. Consequently, the effect of flexure shows up in the double wall case at the same frequency. Third, for large airspaces (6 to 12 in.) and for frequencies in the range from 400 to 1,000 c/s the transmission loss of a double wall approaches a value that is twice that of the single wall, showing that the second wall is almost entirely decoupled from the single wall for this frequency range.

In attempting to compute the transmission loss of the double plywood walls, we chose the ⅜-, 3-, and 12-in. airspace cases for detailed analysis. As was pointed out earlier, any error in fit between computed and experimental results for the single walls would result in much larger errors in the double wall case. In figure 6 we reproduce the computed and experimental data for the single wall. Using R=8.3, results in a computed curve that agrees well up to 1,500 c/s but gives larger than experimental values above this frequency. However, if R=5 is used, the computed result will agree with the experimental at f=2,048 c/s, but will still be too high at 4,096 c/s. R=1.8 at 4,096 c/s gives much better agreement, whereas R=1.0 is a perfect fit. These data indicate that R decreases with increasing frequency.

The necessity for using an accurate value of R is illustrated best by figure 7. Here the transmission loss of the single and double ½-in. plywood wall has been computed as a function of R for f=4,096 c/s. A variation of R from 1.0 to 8.0 causes a change in loss of 7 db for the single wall, whereas in the double wall case a 20-db change results. In fact, it would seem to be somewhat easier to determine R from the double wall results than from the single wall measurements. The value of R=1.8 used for further computations at f=4,096 c/s was selected because it gave exact agreement with experiment for a 3-in. airspace

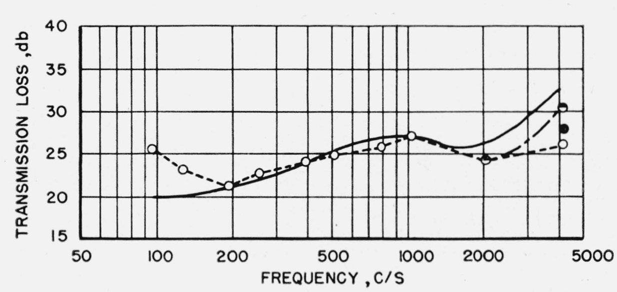

FIGURE 6. *Effect of varying R on computed transmission loss for a ½-in. plywood single wall.*

Dotted broken line corresponds to experimental transmission loss. ○, Experimental; ◒, computed, R=5; ●, computed, R=1.8; at 4,096 c/s R=1.0, computed, coincides with experimental point. ___, Computed R=8.3; f_c=1,885 c/s.

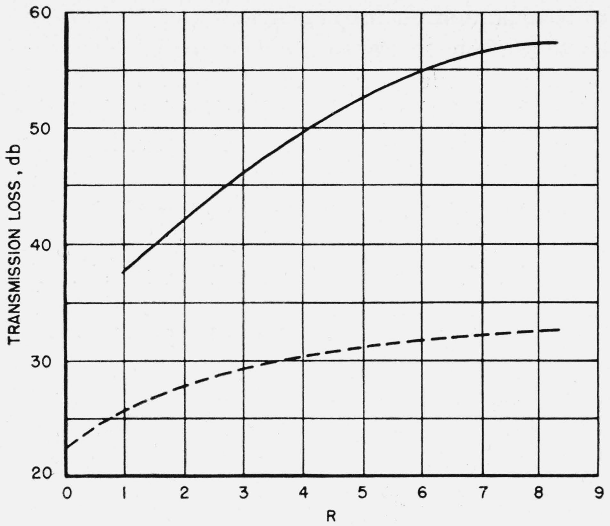

FIGURE 7. *Variation of computed transmission loss with R for double and single wall of ½-in. plywood at a frequency of 4,096 c/s.*

——, Double wall 3-in. airspace; ----, single wall; f=4,096 c/s; f_c=1,885 c/s.

double wall. According to this treatment it should be possible to obtain significant improvement in a double wall by building into each of the single walls a layer of attenuating material.

Figure 8 is a comparison between the computed and experimental transmission losses using $R=8.3$ for frequencies up to 1,024 c/s, $R=5.0$ at 2,048 c/s, and $R=1.8$ at 4,096 c/s. The solid lines are drawn through the computed points, the dotted through the experimental points. It is believed that there is reasonably good agreement considering the complexity of the problem and, in particular, the computations. For example, the point at $f=4{,}096$ c/s for the 12-in. airspace case represents the results of 40 pages of calculations. In figure 9, the integrand of the τ_d integral, eq 33 for this point, is plotted as a function of v. Very sharp half wavelength maxima corresponding to $v=n\pi/b$ and covering a range of variation of several orders of magnitude are evident. In addition, there is a less sharp peak due to flexure. If one compares the area under the peaks, one finds most of the area exists in the neighborhood of the flexure angle thus showing the importance of this effect.

Figure 10 is another representation of the data shown in figure 9. Here, the τ_d integral, eq 33 instead of being integrated from $v=0$ to $v=1.0$, is integrated from a variable lower limit v_l to $v=1.0$. The quantity 10 log $(1/\tau')$ so defined, therefore gives the transmission loss that would result, if, for some reason or other, waves incident at angles greater than θ_l corresponding to v_l were not transmitted. Thus, for $0<v_l<v_1=\pi/b$, grazing incident waves are first excluded, but the angle, or v_1, corresponding to the first maximum nearest grazing incidence, would be allowed. It will be seen that not until the fifth maximum is exceeded is there a change in loss. This is because the first five maxima do not contribute anything to the integral, since they are so sharp. When the flexure angle is excluded, however, there is a large jump in loss because there is a large transmission of sound energy resulting from this cause. As we approach more closely to the conditions where the angles of incidence are restricted to the neighborhood of normal incidence, the transmission loss increases greatly.

At the lower frequencies fewer angles at which maximum transmission occurs are observed. At the lowest frequencies none may occur at all.

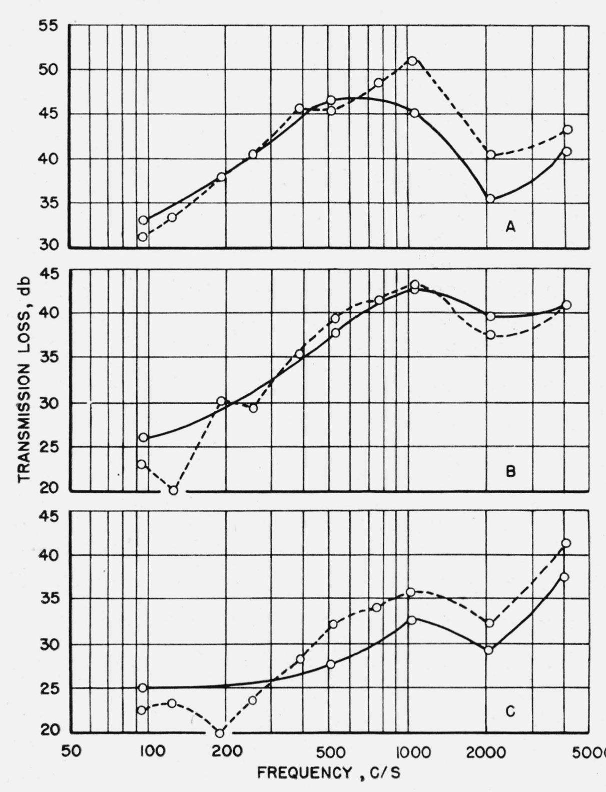

FIGURE 8. *Comparison between computed and experimental results for ½-in. plywood double walls.*

Solid line, computed; broken dotted line, experimental. In the computations, R=8.3 for frequencies up to and including 1,024 c/s, R=5.0 for f=2,048 c/s, and R=1.8 for f=4,096 c/s. A, 12-in. airspace; B, 3-in. airspace; C, ⅜-in. air space.

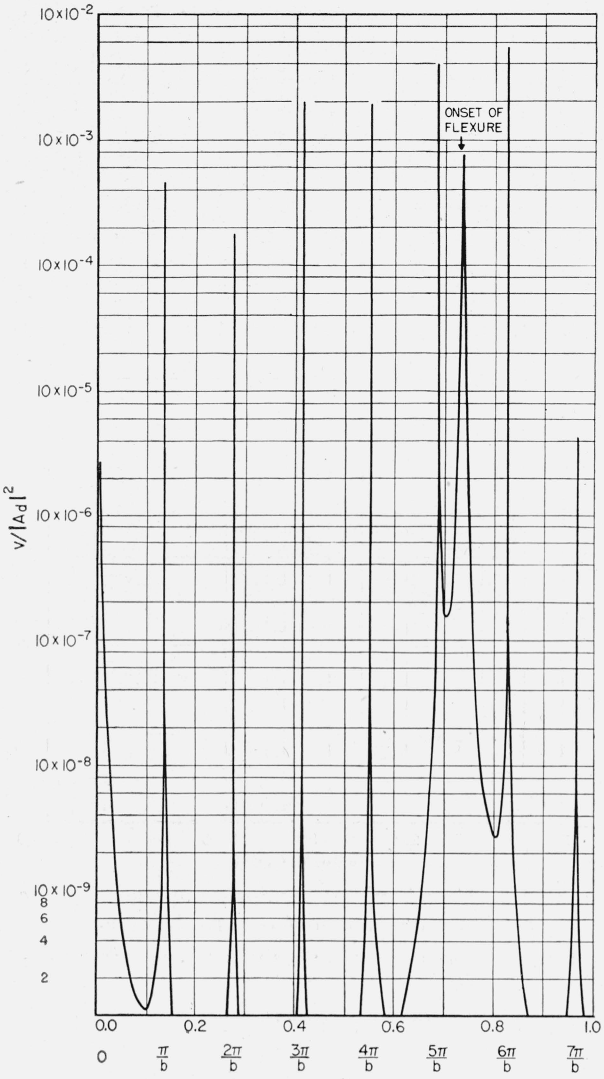

FIGURE 9. *Plot of $v/|A_d|^2$ vs v for a ½-in. plywood wall for $f=4.096$ c/s.*

Here $v=\cos\theta$, where θ is the angle of incidence of the sound wave. Very sharp transmission maxima occur at $v=n\pi/b$ or when $d\cos\theta=n\lambda/2$, where d is airspace thickness. In addition, a less sharp maximum occurs at the angle of incidence corresponding to the occurrence of flexural waves in the wall. Double wall, 12-in. airspace, ½-in. plywood; f=4,096 c/s; f_c=1,885 c/s; R=1.8.

This is shown in figure 11, which is a graph similar to figure 10, but indicates the value of the τ' integral plotted in decibels for other frequencies for the same 12-in. airspace double wall.

4. Additional Experimental Results

In this section we consider some additional experimental results obtained on double walls, for which, however, no analytical computations were carried out, principally because of the tedious nature of such calculations.

Figures 12, 13, and 14 show the experimental results obtained on double walls consisting of ½-, 1-, and 2-in. plasterboard single walls. For comparison purposes the experimental and computed results obtained on the corresponding single walls are also shown on the figures. In the ½-in. plasterboard case it will be seen that the double wall experiments tend to confirm the selection of f_c=4,096 c/s as preferable to f_c=2,048 c/s. The

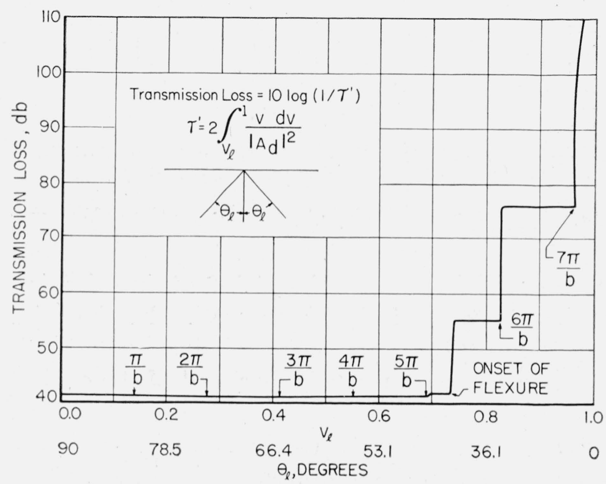

FIGURE 10. *Another representation of curve of figure 9.*

The figure shows the variation of 10 log $(1/\tau')$ with θ, where the integration occurs from a variable lower limit of integration, θ_l, to $\theta=0°$. As the wave packets in the reverberant soundfield are confined to a cone for which θ_l is decreasing, a sudden increase in transmission loss occurs when the angle of incidence corresponding to flexure is excluded, showing that most of the transmission of sound occurs as the result of flexural waves.

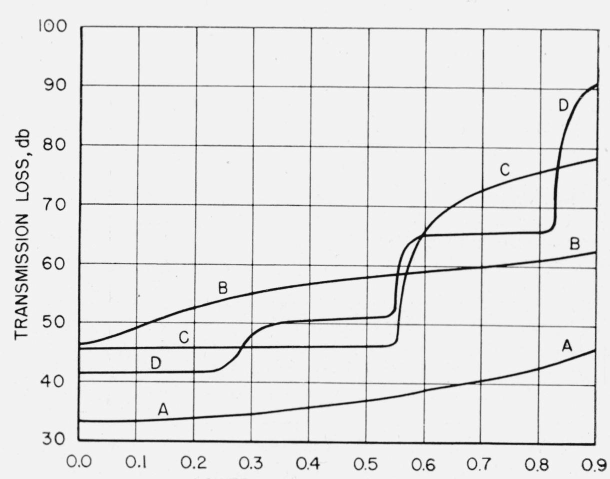

FIGURE 11. *Plot of 10 log $(1/\tau')$, as in figure 10, for a ½-in. plywood double wall for various frequencies.*

A, 96 c/s; B, 512 c/s; C, 1,024 c/s; D, 2,048 c/s.

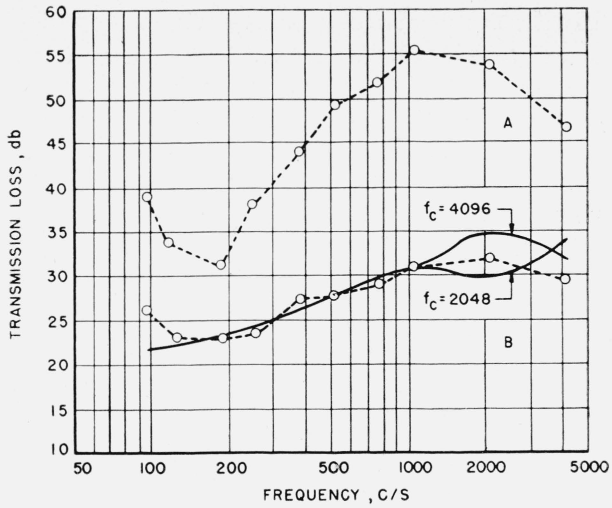

FIGURE 12. *Experimental transmission loss results on a single and double wall of ½-in. plasterboard.*

A, Double wall 3-in. airspace; B, single wall; ----, experimental; ——, theoretical R=10.5.

double wall TL drops off at 4,096 c/s in a fashion similar to that of the single wall. In the 1-in. and 2-in. TL measurements the single wall minimum occurring in the neighborhood of the critical flexural frequency did not appear in the double wall case. It is of interest (table 1) to compare the average TL for the nine frequencies in the range of 128 to 4,096 c/s with that of ordinary plaster and stud walls.[5]

It will be seen from the data in table 1, that if no mechanical ties or sound-conducting bridges exist between the two components of a double wall, very large transmission losses result even with relatively light weight walls. Thus, a 45-db loss may be obtained for a weight of only 4.2 lb/ft². The situation rapidly worsens if solid coupling between each component exists. Comparing the last four entries in the table it will be seen that all have approximately the same weight. The 2-in. air-coupled wall, however, is some 20 db better than the stud-coupled wall; some 10 db better when the studs are staggered so that coupling exists only due to a top and bottom

TABLE 1.

Description	Average TL	Weight
	db	*lb/ft²*
½-in. plasterboard double wall	45.2	4.2
1-in. plasterboard double wall	55.5	8.3
2-in. plasterboard double wall	59.6	16.6
½-in. gypsum plaster on wood lath on 2 by 4 studs.	37.5	17.1
⅞-in. gypsum plaster on metal lath on 2 by 4 staggered wood studs, 4-in. airspace	49.8	19.8
Double wall consisting of two 2-in. solid plaster single walls resting on 1-in. corkpad, 3-in. airspace	54.1	17.2

[5] For data of this kind see: Building Materials and Structures Report BMS17 and two supplements, Sound insulation of wall and floor constructions, available from the Superintendent of Documents, Government Printing Office, Washington 25, D. C. at a total cost of 35¢; also Technical Report on Building Materials, TRBM-44, Fire resistance and sound-insulation ratings for walls, partitions, and floors, free upon request at National Bureau of Standards, Washington 25, D. C.

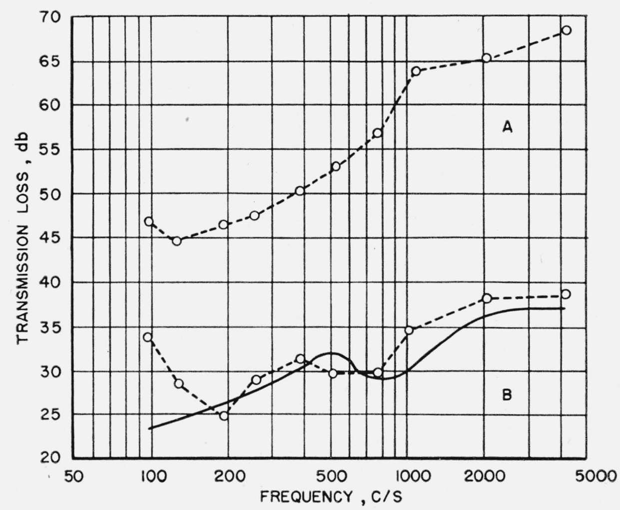

FIGURE 13. *Experimental transmission loss results on a single and double wall of 1-in. plasterboard.*

A, Double wall 3-in. airspace; B, single wall; ----, experimental; —— theoretical R=10.5, f_c=768 c/s.

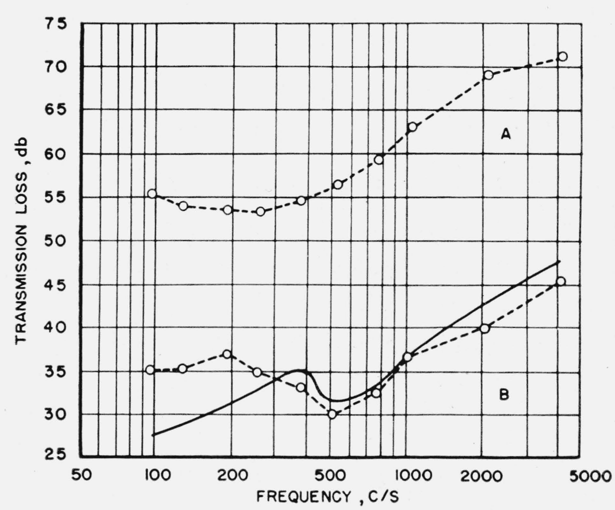

FIGURE 14. *Experimental transmission loss results on a single and double wall of 2-in. plasterboard.*

A, Double wall 3-in. airspace; B, single wall; ----, experimental; ——, theoretical R=5.3, f_c=512 c/s.

plate to which the staggered studs are attached, and some 5 db better when coupling is only due to a cork base.

The question often arises as to the effect of placing an absorbent in the airspace. Accordingly, some measurements were taken with a 3-in. thick fiberglas blanket having a density of about 1.0 lb/ft² inserted in the airspace. Table 2 gives the average improvement in transmission loss for the frequency range of 128 to 4096 c/s over the untreated airspace double wall.

TABLE 2.

Description of double wall	Average improvement
	db
½-in. plaster board	9.6
1-in. plaster board	3.0
2-in. plaster board	3.5

In the previous paper (see footnote 1), the effect of placing this same fiberglas blanket in front of and in juxtaposition to a single wall was discussed. The walls were ½-, 1-, and 2-in. plasterboard single walls. In this case the average *TL* improvement for the frequency range of 128 to 4,096 c/s was 8.2 db and was approximately the same for all three walls. Thus, for the double wall having the lightest weight the improvement using the absorbent was equal to or better than that obtained for the single wall. On the other hand, for the heavier double walls, a relatively small effect is observed. This fact has been observed many times in more conventional construction using wood studs, staggered, or otherwise. For light-weight construction significant increases in the *TL* are measured, whereas for heavy-weight constructions only minor increases result. In conventional construction this is in part due to the existence of sound-conducting paths. This explanation, however, does not hold in these experimental double walls, since the components of the double walls were isolated from each other and the blanket was arranged in the airspace so as not to touch the walls. Evidently, the effect depends on the ratio of the impedance of the airspace material to the impedance of the walls. For the heavy walls the material in the airspace can add little to the already large impedance of the walls.

Meyer [5] has considered the effect of the airspace absorbent material on reducing transverse modes of sound in the airspace, that is, those modes in which the sound travels parallel to the wall surfaces. He pointed out that if these modes are important, it should be possible to absorb them by placing this material only on the boundaries of the airspace. Accordingly, the boundaries of the airspace shown in figure 3 were stuffed with Fiberglas, early in the double wall experiments starting with the double aluminum wall. No significant difference due to the insertion of the boundary absorbent occurred, so that it was concluded that the effect of the transverse modes was negligible.

Additional confirmation of this was obtained by inserting the "strawcomb" shown in figure 11 of RP1998 in the airspace of several double walls. The term strawcomb refers to a honeycomb structure that was made by cutting soda straws into 2⅞-in. lengths. These were placed with their long axis perpendicular to the wall surfaces. Some 150,000 straws were used in the strawcomb used in these experiments. Because of the large number of cell walls that would be intersected by a transverse wave, it is hardly to be expected that they would occur. The average *TL* increase, again for the ½-, 1-, and 2-in. double plasterboard walls, was only 0.7 db, showing that the strawcomb had a negligible effect.

5. Conclusions

A theory of air-coupled double walls has been developed, which gives good agreement with experimental results. In order to apply the theory it is necessary to know the wall impedance, Z_w, of the identical single wall components. This quantity may be determined from the transmission loss results obtained on the single walls. Inasmuch as it is theoretically possible to evaluate the resistance, R, and flexural frequency, f_c, from mechanical impedance measurements on small scale samples, we have here, in principle, a method of computing double wall transmission losses from small scale experiments. The experimental results indicate that both normal incidence theory and the mass-reactance assumption are entirely inadequate for explaining the behavior of single

[5] E. Meyer, Elek. Nachr. Tech **12**, 393 (1935)

and double walls in a reverberant sound field. The importance of including resistance and flexural wave effects has been demonstrated.

For double walls having air-coupling only, very shallow airspaces can produce appreciable increases in transmission loss over a single wall. An absorbent material, when inserted in the airspace, produces large improvements only when the mass of the walls is relatively light and has but little effect for heavy walls. Honeycomb or other nonabsorbent cellular structures having no cell walls in a direction normal to the wall faces do not result in an increase in transmission loss.

The author is indebted to S. Edelman and Henry J. Leinbach, Jr.. for making many of the experimental observations; in addition, the latter carried out most of the required numerical integrations.

WASHINGTON, July 26, 1949.