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Instability in Shear of Simply Supported Square Plates With Reinforced Hole

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The critical shear stresses were determined for five elastic, simply supported square plates with central circular holes of diameter one-eight and one-fourth the length of a side. Comparison of the numerical results obtained with those computed for plates without holes shows that, although an unreinforced hole may cause a large reduction in the critical shear stress of the plate, reinforcement of the hole by a circular doubler plate causes a substantial increase in the shear buckling load.

I. Introduction

Holes must frequently be cut in the stressed skin surfaces of airplane wings or fuselages to provide access to the interior of the wing or fuselage. To prevent weakening of the entire structure thereby, the hole is usually reinforced by a circular doubler plate riveted to the inner surface of the sheet.

The present report is one of a series of studies undertaken for the Bureau of Aeronautics, Navy Department, in order to understand more fully the effect of reinforcements in strengthening and stabilizing the structure around a hole. In reference [1],[1] a theoretical investigation was presented of the effect of various reinforcements on the stresses and displacements near a small circular hole in a plane sheet under uniform tension in all directions. Numerical values of stress concentration and of radial displacement at the edge of the hole for various doubler plates indicated that the greatest reduction in stress concentration for a given volume of reinforcing material occurs when the outer radius of the reinforcement is smallest and that the reduction in distortion of the hole depends principally upon the volume of the reinforcement.

Tests were made on plates with reinforced circular holes, under uniform tension in one direction only, reference [1], to check the theoretical analysis, and it was found that, if the reinforcements were attached to the sheet with two concentric rows of rivets or bonded to the sheet, the theoretical analysis gave a sufficiently accurate description of the stresses and displacements in the neighborhood of the hole.

In reference [2], results of compressive tests of curved panels with circular holes unreinforced or reinforced with doubler plates are given, which show that curvature has no appreciable effect on the median fiber strain distribution around a hole and that the plane stress theory can be used to reliably predict the stress distribution in the unreinforced regions of curved panels with holes.

A theoretical analysis for the stress distribution in a flat plate near a reinforced circular hole loaded by a pin was presented in reference [3]. Results obtained in the theoretical analysis and those from a test of a plate of sandwich construction showed good agreement.

A method is presented in reference [4] for determining the instability under edge compression of a rectangular plate with a reinforced circular hole. The values of critical compressive stresses of square plates obtained by this method indicated that, although the buckling stress of a plate is reduced only a small amount by the presence of an unreinforced hole, reinforcement of the hole causes a substantial increase in the buckling stress. Reinforcement of a hole, therefore, can be expected to stabilize as well as strengthen the structure in the neighborhood of a hole when the structure is under compressive load.

[1] Figures in brackets indicate the literature references at the end of this paper.

Using the method of reference [4], a study is made in this report of the effect of the reinforcement of a circular hole in a square plate on the stability of the plate under shearing loads uniformly distributed along the edges of the plate.

II. Method of Analysis

In reference [4], an energy method was presented for computing the compressive buckling load of a simply supported elastic rectangular plate having a central circular hole reinforced by a circular doubler plate. This method can also be used to determine the critical shearing stress for simply supported rectangular plates having a central circular hole.

The integrals I_1 and I_2, which must be evaluated for the plate are:

$$\left.\begin{aligned} I_1 &= \iint_{\text{surface}} D\left\{\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\mu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\right\} dx\, dy \\ I_2 &= \iint_{\text{surface}} h\left[\frac{\sigma_x}{\tau_\infty}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\sigma_y}{\tau_\infty}\left(\frac{\partial w}{\partial y}\right)^2 + 2\frac{\tau_{xy}}{\tau_\infty}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right] dx\, dy \end{aligned}\right\} \quad (1)$$

where h = plate thickness, function of x and y;
x, y = rectangular coordinates with origin at center of plate;
w = lateral deflection of plate;
$D = Eh^3/12(1-\mu^2)$, flexural rigidity of plate, function of x and y;
μ = Poisson's ratio, 0.3;
σ_x = tensile stress in x direction;
σ_y = tensile stress in y direction;
τ_{xy} = shear stress in x and y directions;
τ_∞ = shear stress in x and y directions far from the hole.

The stress ratios σ_x/τ_∞, σ_y/τ_∞ and τ_{xy}/τ_∞ just prior to buckling may be obtained from reference [4] in the following way. If the origin of coordinates is taken as shown in figure 1, shear stresses along the edges of the plate will be equivalent to tensile stresses acting at the edge of the plate at $\theta = 45°$ and equal compressive stresses acting at the edge of the plate at $\theta = -45°$. The constants F and K of reference [4] are zero in the case of shear loading. The stresses σ_r/τ_∞, $\sigma_\theta/\tau_\infty$, and $\tau_{r\theta}/\tau_\infty$ referred to polar coordinates r, θ are then given as

$$\left.\begin{aligned} \sigma_r/\tau_\infty &= -(A+3CR^4/r^4+2DR^2/r^2)\cos 2\left(\theta+\frac{\pi}{4}\right) \\ \sigma_\theta/\tau_\infty &= (A+6Br^2/R^2+3CR^4/r^4)\cos 2\left(\theta+\frac{\pi}{4}\right) \\ \tau_{r\theta}/\tau_\infty &= (A+3Br^2/R^2-3CR^4/r^4-DR^2/r^2)\sin 2\left(\theta+\frac{\pi}{4}\right) \end{aligned}\right\} \quad (2)$$

where r, θ = polar coordinates with origin at center of hole (see fig. 1);
θ = angle between radius r to point and positive horizontal axis. Positive θ measured in counter clockwise direction;
R = radius of hole;
A, B, C, D = coefficients with different values in sheet and reinforced region.

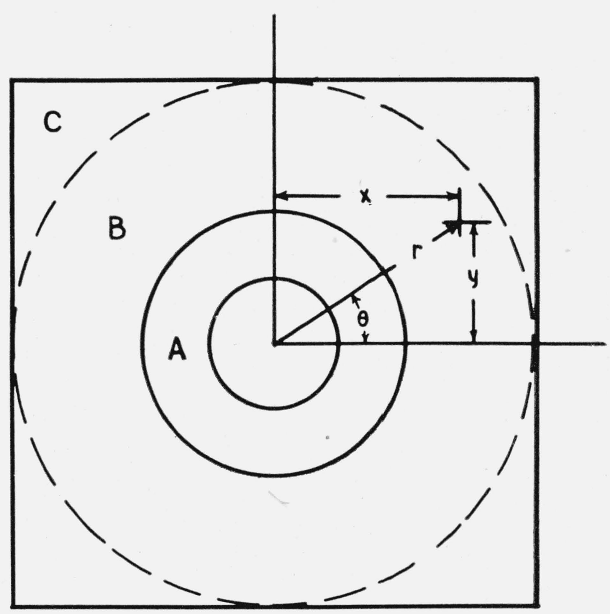

FIGURE 1. *Subdivision of plate into three portions A, B, and C, in evaluating I_2*

The values of A, B, C, and D may be determined as outlined in reference [4]. The stresses in rectangular coordinates may be computed from the stresses in polar coordinates, eq 2, by use of the conversion formulas:

$$\left.\begin{aligned}\sigma_x&=\sigma_r\cos^2\theta+\sigma_\theta\sin^2\theta-\tau_{r\theta}\sin 2\theta\\ \sigma_y&=\sigma_r\sin^2\theta+\sigma_\theta\cos^2\theta+\tau_{r\theta}\sin 2\theta\\ \tau_{xy}&=\frac{\sigma_r-\sigma_\theta}{2}\sin 2\theta+\tau_{r\theta}\cos 2\theta\end{aligned}\right\}\qquad(3)$$

The lateral deflection, w, of a simply supported rectangular plate of length a and width b can be approximated by the first terms in the trigonometric series

$$\begin{aligned}w=&a_{11}\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}+a_{22}\sin\frac{2\pi x}{a}\sin\frac{2\pi y}{b}+\\ &a_{13}\cos\frac{\pi x}{a}\cos\frac{3\pi y}{b}+a_{31}\cos\frac{3\pi x}{a}\cos\frac{\pi y}{b}+\\ &a_{24}\sin\frac{2\pi x}{a}\sin\frac{4\pi y}{b}+\ldots\end{aligned}\qquad(4)$$

After substituting eq 4 into eq 1 and performing the indicated integration by a numerical procedure, the integrals I_1 and I_2 become quadratic expressions in terms of the coefficients a_{11}, a_{22}, a_{13}, ... The critical value of τ_∞, the shear stress far from the hole, at which buckling of the plate occurs is that value of τ_∞ which reduces to zero the determinant of the coefficients of a_{11}, a_{22}, a_{13}, etc. in the set of simultaneous equations

$$\left.\begin{aligned}&\frac{\partial I_1}{\partial a_{11}}+\tau_\infty\frac{\partial I_2}{\partial a_{11}}=0\\ &\frac{\partial I_1}{\partial a_{22}}+\tau_\infty\frac{\partial I_2}{\partial a_{22}}=0\\ &\frac{\partial I_1}{\partial a_{13}}+\tau_\infty\frac{\partial I_2}{\partial a_{13}}=0\\ &\ldots\ldots\ldots\ldots\end{aligned}\right\}\qquad(5)$$

III. Numerical Integration

The evaluation of the integrals in eq 1 over the surface of the plate could not be done directly, since the plates have a rectangular outer boundary and a circular inner boundary and involve a stress that is a complicated function when expressed in rectangular coordinates.

The integral I_1, eq 1, was taken as

$$I_1=I_{1a}+I_{1b}-I_{1c},\qquad(6)$$

where I_{1a} = the integral I_1 for the surface enclosed by the outer rectangular boundary of the plate, taking the flexural rigidity $D=D_1$, the value of D in the unreinforced portion of the plate;

I_{1b} = the integral I_1 for the surface enclosed by the outer circular boundary of the reinforcement taking $D=D_2-D_1$, where D_2 is the value of D in the reinforcement;

I_{1c} = the integral I_1 for the surface enclosed by the circular boundary of the hole taking $D=D_2$.

The double integration necessary to evaluate I_{1a} could always be done directly, but this was not possible in evaluating I_{1b} and I_{1c}. The integrals I_{1b} and I_{1c} were obtained by integrating directly with respect to x and using Gauss' method of numerical integration, reference [5], for integrating with respect to y. In all cases except case 2, five Gauss points were used in this numerical integration as computations showed that, in increasing the number of points used from three to five, the value of the critical stress was changed only 2 percent. If more than five points had been used, the change in the value of the critical stress would have been even less. For case 2, three Gauss points were used in evaluating I_{1c} because the circular area over which I_{1c} was evaluated had a much smaller radius for case 2 than for the other cases.

The integral I_2 was evaluated as the sum of three integrals

$$I_2=I_{2a}+I_{2b}+I_{2c},\qquad(7)$$

where I_{2a} = the integral I_2 for the circular disk of the reinforced area (A, fig. 1);

I_{2b} = the integral I_2 for the circular disk between the outer boundary of the reinforcement and the largest inscribed circle in the plate (B, fig. 1;

I_{2c} = the integral I_2 for the remainder of the plate (C, fig. 1).

The integral for each circular portion was determined, using Gauss' method of numerical integration, by first integrating numerically in a circumferential direction and then in a radial direction. The integral for the remainder of the plate was obtained also by using Gauss' method, first integrating numerically in the x-direction and then in the y-direction.

A typical distribution of Gauss points for computing I_2 is shown in figure 2 for one-eighth of a square plate with a central circular hole. For this distribution, the integral of a function F over the portion of the plate surface enclosed by heavy lines in figure 2 is

$$\left.\begin{aligned}\int\int F dA=\frac{a^2}{8}[&0.0481444F_1+0.0770310F_2+0.0481444F_3+0.0380048F_4+0.0571868F_5+\\&0.0914988F_6+0.0571868F_7+0.0727220F_8+0.116355F_9+0.0727220F_{10}+\\&0.0337158F_{11}+0.0539452F_{12}+0.0337158F_{13}+0.0142967F_{14}+0.0228747F_{15}+\\&0.0142967F_{16}+0.0181805F_{17}+0.0290888F_{18}+0.0181805F_{19}+0.00842899F_{20}+\\&0.0134864F_{21}+0.00842899F_{22}]\end{aligned}\right\} \quad (8)$$

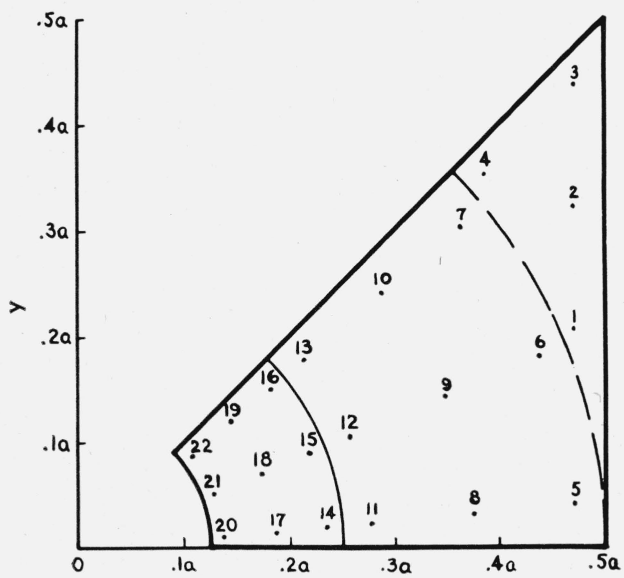

FIGURE 2. *Distribution of Gauss points for computing I_2 for case 3.*

Coordinates of points

Point	x/a	y/a	Point	x/a	y/a
1	0.469052	0.206523	12	0.257000	0.106453
2	.469052	.321115	13	.213318	.178541
3	.469052	.435706	14	.234988	.020855
4	.384501	.352062	15	.217954	.090280
5	.469978	.041709	16	.180908	.151415
6	.435910	.180559	17	.186766	.016575
7	.361818	.302831	18	.173228	.071753
8	.373532	.033150	19	.143784	.120343
9	.346455	.143506	20	.138543	.012295
10	.287568	.240686	21	.128501	.053227
11	.277086	.024591	22	.106659	.089271

where $F_1, F_2, F_3, \ldots F_{22}$ are values of the function F at the positions shown in Figure 2.

In each case a sufficient number of Gauss points was used to reduce the estimated error to less than 5 percent. Twenty-two points for each octant of the plate were used for cases 1, 3, and 4; twenty-eight points for case 2; and twenty-five points for case 5.

As an indication of the adequacy of the numerical integration methods used, the critical shear stress of a square plate without a hole was determined by approximating the deflection by

$$w=a_{11}\cos\frac{\pi x}{a}\cos\frac{\pi y}{a}+a_{22}\sin\frac{2\pi x}{a}\sin\frac{2\pi y}{a},$$

integrating for I_1, eq 1, exactly and integrating for I_2, eq 1, both exactly and by numerical integration with 22 points in one-eighth of the plate as shown in figure 3. The resulting critical stresses differed by 0.8 percent. In the case of a plate with a reinforced hole, it is probable that the more complicated stress distribution and the use and prominence of higher order terms in the series used for the deflection will cause the error to be somewhat higher.

IV. Convergence of Deflection Function

The correct value of τ_∞ for buckling of the plate is that value of τ_∞ which reduces the determinant of the coefficients of $a_{11}, a_{22}, a_{13}, \ldots$, in the infinite set of eq 5 to zero. In order to limit the work of computing τ_∞ to a finite amount, preliminary computations were made to determine which terms in the deflection function, eq 4, were most important in evaluating τ_∞. The computations were made for the plate of case 3, table 1.

TABLE 1. *Dimensions for square plates investigated*

Case	Length of side	Radius of hole, R	Outer radius of reinforcement	Thickness	
				Reinforced region	Unreinforced region
1	a	0.125 a	([a])	([a])	h
2	a	.0625 a	([a])	([a])	h
3	a	.125 a	0.25 a	2.0 h	h
4	a	.125 a	.25 a	1.5 h	h
5	a	.125 a	.1875 a	3.4 h	h

[a] Hole in plate not reinforced.

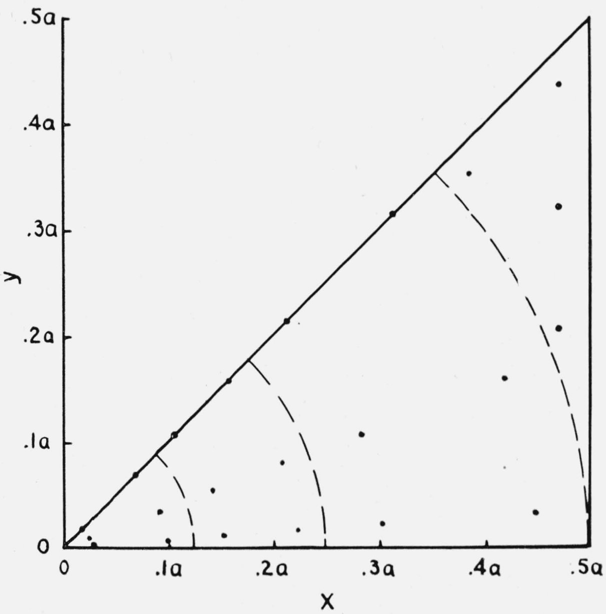

FIGURE 3. *Distribution of Gauss points for computing I_2 for case of plate without a hole.*

Using the distribution of points shown in figure 4 to carry out the numerical integration for I_2, the critical shear stress τ_∞ was 18.78 Eh^2/a^2 using only the first two terms of eq 4,

$$w=a_{11}\cos\frac{\pi x}{a}\cos\frac{\pi y}{a}+a_{22}\cos\frac{2\pi x}{a}\cos\frac{2\pi y}{a}.$$

Using as the deflection function

$$w=a_{11}\cos\frac{\pi x}{a}\cos\frac{\pi y}{a}+a_{22}\sin\frac{2\pi x}{a}\sin\frac{2\pi y}{a}+$$

$$a_{13}\cos\frac{\pi x}{a}\cos\frac{3\pi y}{a}+a_{31}\cos\frac{3\pi x}{a}\cos\frac{\pi y}{a},$$

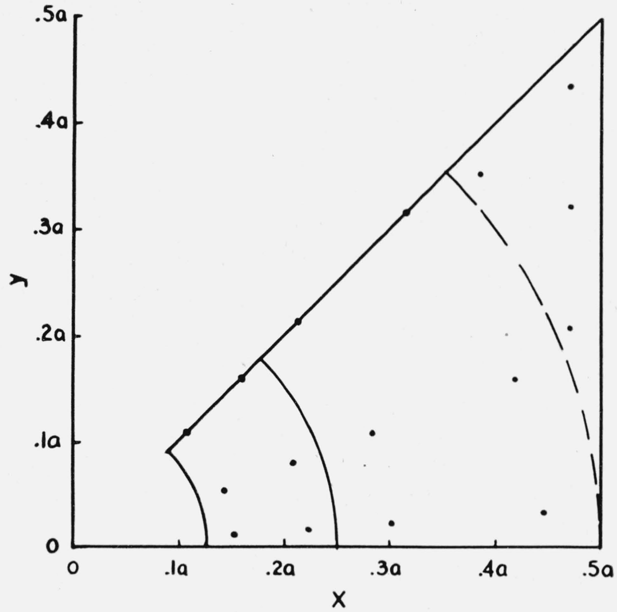

FIGURE 4. *Distribution of Gauss points for evaluating I_2 in study of convergence of deflection function.*

τ_∞ was 18.10 Eh^2/a^2, a decrease of 3.6 percent. Using

$$w=a_{11}\cos\frac{\pi x}{a}\cos\frac{\pi y}{a}+a_{22}\sin\frac{2\pi x}{a}\sin\frac{2\pi y}{a}+$$

$$a_{33}\cos\frac{3\pi x}{a}\cos\frac{3\pi y}{a},$$

τ_∞ was 17.46 Eh^2/a^2, a decrease of 7.0 percent. Using

$$w=a_{11}\cos\frac{\pi x}{a}\cos\frac{\pi y}{a}+a_{22}\sin\frac{2\pi x}{a}\sin\frac{2\pi y}{a}+$$

$$a_{24}\sin\frac{2\pi x}{a}\sin\frac{4\pi y}{a}+a_{42}\sin\frac{4\pi x}{a}\sin\frac{2\pi y}{a},$$

τ_∞ was 17.37 Eh^2/a^2, a decrease of 7.5 percent. Using

$$w=a_{11}\cos\frac{\pi x}{a}\cos\frac{\pi y}{a}+a_{22}\sin\frac{2\pi x}{a}\sin\frac{2\pi y}{a}+$$

$$a_{35}\cos\frac{3\pi x}{a}\cos\frac{5\pi y}{a}+a_{53}\cos\frac{5\pi x}{a}\cos\frac{3\pi y}{a},$$

τ_∞ was 18.50 Eh^2/a^2, a decrease of 1.5 percent. Using

$$w=a_{11}\cos\frac{\pi x}{a}\cos\frac{\pi y}{a}+a_{22}\sin\frac{2\pi x}{a}\sin\frac{2\pi y}{a}+$$

$$a_{44}\sin\frac{4\pi x}{a}\sin\frac{4\pi y}{a},$$

τ_∞ was 18.58 Eh^2/a^2, a decrease of 1.1 percent.

The use of additional terms of the trigonometric series with the first two terms probably would have shown a proportionately smaller decrease in the buckling load. Therefore, it is believed that the function

$$w=a_{11}\cos\frac{\pi x}{a}\cos\frac{\pi y}{a}+a_{22}\sin\frac{2\pi x}{a}\sin\frac{2\pi y}{a}+$$
$$a_{33}\cos\frac{3\pi x}{a}\cos\frac{3\pi y}{a}+a_{24}\sin\frac{2\pi x}{a}\sin\frac{4\pi y}{a}+$$
$$a_{42}\sin\frac{4\pi x}{a}\sin\frac{2\pi y}{a},$$

approximated the lateral deflection of a square plate with simply supported edges under shearing loads with sufficient accuracy. This deflection function was used in the computations of the critical shearing stresses.

After the critical shearing stresses had been computed for the five square plates considered in this report, the relative importance of the various terms in the deflection function was determined. Values of a_{22} were 25 to 42 percent of a_{11}; coefficients a_{33}, a_{24}, and a_{42} were 0.4 to 5.0 percent of a_{11}. It seems reasonable to assume that higher order terms would have been even smaller as compared to a_{11} and that the most important terms were used for the deflection in all cases.

The number of points used in the numerical integration for I_2 has an effect on the accuracy of the determination of the critical stress. To carry out a complete investigation of convergence, as more points are used, was not possible because of the amount of work involved. To obtain a partial indication of the accuracy to be expected, however, the critical stress for case 3, table 1, was computed using various deflection functions and using both the distribution of points in figure 2 and that in figure 4 for computing I_2. The results are given in table 2.

TABLE 2. *Values of critical stress obtained from the spacings of Gauss points investigated*

Terms in deflection function eq 4	Critical stress using points in figure 4 for I_2 τ_{cr}	Critical stress using points in figure 2 for I_2 τ_{cr}	Difference
			Percent
a_{11},a_{22}	18.78 Eh^2/a^2	20.05 Eh^2/a^2	7
a_{11},a_{22},a_{33}	17.46 Eh^2/a^2	18.56 Eh^2/a^2	6
$a_{11},a_{22},a_{24},a_{42}$	17.37 Eh^2/a^2	18.21 Eh^2/a^2	5
$a_{11},a_{22},a_{35},a_{53}$	18.50 Eh^2/a^2	19.58 Eh^2/a^2	6

It was decided on the basis of these computations that the use of the point distribution in figure 2 would be acceptable.

The over-all accuracy of the computations of critical stress are estimated to be such that the critical stresses are within 7 percent of the exact values for the cases investigated.

V. Results and Discussion

The critical shear stress was determined for five square plates with reinforced and unreinforced holes, figure 5. The dimensions for the plates are given in table 1.

The analysis gave the critical shear stress far from the hole τ_∞, corresponding to the stress distribution for a plate under uniform shear at an infinite distance from the hole. The shear stress τ_∞ is larger than the average shear stress on the

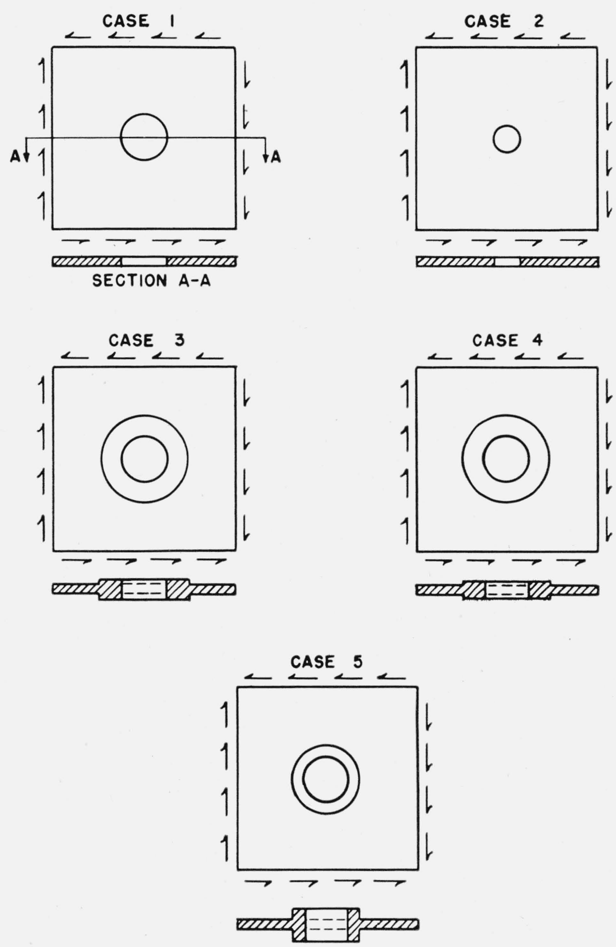

FIGURE 5. *Loading of square plates with simply supported edges. (Dimensions in table 1.)*

center cross section obtained by dividing the load on that section by ah. A better value of the average buckling stress τ_{cr} of the plate was obtained from

$$\tau_{cr}=\frac{1}{2}\frac{P_1+P_2}{ah}, \tag{9}$$

where P_1=shearing force acting on edge of plate;
P_2=shearing force transmitted across center section, $y=0$.

The value of P_1 was obtained by numerical integration and the value of P_2 by direct integration of the stresses τ_{xy} given by eq 3. Values of τ_∞, $\tau_1=P_1/ah$, $\tau_2=P_2/ah$, and τ_{cr} are given in table 3.

TABLE 3. *Critical shear stress for plates investigated*

Case	τ_∞	τ_1	τ_2	τ_{cr}
0	----------	----------	----------	$8.44\frac{Eh^2}{a^2}$
1	$6.95\frac{Eh^2}{a^2}$	$6.94\frac{Eh^2}{a^2}$	$6.11\frac{Eh^2}{a^2}$	$6.53\frac{Eh^2}{a^2}$
2	$8.55\frac{Eh^2}{a^2}$	$8.55\frac{Eh^2}{a^2}$	$8.29\frac{Eh^2}{a^2}$	$8.42\frac{Eh^2}{a^2}$
3	$17.47\frac{Eh^2}{a^2}$	$17.52\frac{Eh^2}{a^2}$	$17.01\frac{Eh^2}{a^2}$	$17.27\frac{Eh^2}{a^2}$
4	$10.45\frac{Eh^2}{a^2}$	$10.46\frac{Eh^2}{a^2}$	$9.78\frac{Eh^2}{a^2}$	$10.12\frac{Eh^2}{a^2}$
5	$28.83\frac{Eh^2}{a^2}$	$28.86\frac{Eh^2}{a^2}$	$27.80\frac{Eh^2}{a^2}$	$28.33\frac{Eh^2}{a^2}$

It was assumed that the boundaries of the square plates were sufficiently far from the hole so that only a uniform shear was acting on the boundaries. Numerical integration of the stresses σ_x as given by eq 3 on the boundary of the plate showed that there was an average compressive stress on half the side of the plate and a tensile stress on the other half equal to 3 to 12 percent of the shear stress. This accounts for the small differences in the values of τ_1 and τ_2.

The shear buckling stress of a simply supported square plate of constant thickness is given on page 360 of reference [6] (taking Poisson's ratio as 0.3) as

$$\tau_{cr}=8.44\ Eh^2/a^2. \tag{10}$$

Comparison of this value, case 0, table 3, with the values of the critical stresses for cases 1 and 2 shows that the unreinforced holes of diameter one-fourth and one-eighth the length of a side reduced the buckling stresses by 22.6 and 0.2 percent, respectively. Comparison of the value for case 0 with the values for cases 3 to 5 of table 3 shows that reinforcement of the hole increased the shear buckling stress of the plate over that for a plate without a hole from 20 to 236 percent. These percentages are to be taken only as indications of the order of magnitude since the accuracy of the present determination of critical stresses is estimated to be 7 percent.

The effect of thickness of the reinforcement on the buckling load is seen by comparing cases 3 and 4, table 3. In these cases, the hole size and area of the reinforcing material is the same, but the thickness of the reinforcement of case 4 is only one-half the thickness of the reinforcement of case 3. The buckling load for case 3 is 71 percent higher than that for case 4.

The effect of shape of reinforcement on the critical shear stress is indicated by comparing cases 3 and 5, table 3. The volume of material reinforcing the holes in these two cases is the same, but the reinforcing material is concentrated nearer the edge of the hole in case 5 than it is in case 3. The buckling stress is 64 percent greater for case 5 than for case 3.

Comparison of the values for case 1 and cases 3 to 5 shows that reinforcement of the hole raises the critical shear stress from 55 to 334 percent, depending on the thickness and shape of the reinforcement.

VI. Conclusions

Using a numerical procedure for evaluating the integrals for the energy stored in a plate, the critical shear stress can be estimated for plates with circular holes and doubler plate reinforcement.

The critical shearing stress of plates may be reduced considerably by the presence of unreinforced holes. A square plate with a hole diameter one-eighth the length of a side showed practically no reduction in critical shearing stress, but a square plate with a hole diameter twice as long showed a reduction of 23 percent.

Reinforcement of the doubler plate type causes marked increases in the critical shear stress. The critical shearing stresses were 55 to 334 percent higher than that for a plate with the same size unreinforced hole.

The volume and shape of the reinforcement has a marked effect on the critical shearing stress. The critical stresses for two plates with the same size hole and same area of reinforcement, but different thickness, were compared. The critical stress increased from 10.12 Eh^2/a^2 to 17.27 Eh^2/a^2, an increase of 71 percent, when the thickness of the reinforced region was increased from 1.5 h to 2.0 h. The critical stresses for two plates with the same size hole and same volume of reinforcing material were compared. The plate with the material concentrated closer to the edge of the hole had a 64 percent larger critical shearing stress.

VII. References

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