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Determination of Efficiency of Microwave Bolometer Mounts from Impedance Data

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An impedance method of determining the input-to-bolometer-element power-transfer efficiency of bolometer mounts of the type used in microwave power measurements is formulated, and the theoretical basis of the method is outlined. The method stems from the consideration of the bolometer mount as a transducer. Under certain conditions, which are discussed, this transducer is representable as a two-terminal-pair network, and its parameters can be determined, essentially as in the case of ordinary networks, from observation of input impedance (in waveguide or coaxial line) as a function of load impedance (bolometer resistance). Convenient formulas are given for the calculation of power-transfer efficiency from such impedance data.

I. Introduction

Bolometric methods, employing fine platinum wires or bead thermistors as bolometers, have received wide use in power measurement at microwave and lower frequencies. The bolometer is mounted in a waveguide [1] structure (fig. 1) whose function is to enable the bolometer to absorb radio-frequency power from a source whose output is to be measured. The bolometer is also connected into a direct-current (or low-frequency alternating-current) network whose functions are to detect or measure changes of bolometer resistance, and to supply bias power to the bolometer. Thus radio-frequency power may be measured in terms of the change in bolometer resistance produced by the application of the power to be measured, or it may be measured by the change in bias power required to keep the resistance constant. Both methods are used; [2] the details, however, are not needed for the purpose of this paper, which is concerned only with what may be called the efficiency of bolometer mounts.

[1] Throughout this paper the term *waveguide* is used in a somewhat general sense, in that two-conductor systems, such as coaxial line, are included within the meaning of the term. It is immaterial to the discussion whether the input waveguide is of a type that does or does not support a principal mode.

[2] See, for example, Technique of microwave measurements **11**, chap. 3 (Radiation Laboratory Series, McGraw-Hill Book Co., New York, N. Y., 1947).

Bolometer mounts are ordinarily arranged to present very nearly a reflectionless load to the waveguide to which they are connected. If there is reflection, however, the net radio-frequency input power consists of that corresponding to an incident wave less that corresponding to a reflected wave. Let P_1 denote the net power input to the bolometer mount at a conveniently located terminal surface in the input waveguide. Let P_2 denote the radio-frequency power absorbed by the bolometer element itself. Then the efficiency of the bolometer mount is defined as

$$\eta = P_2/P_1 \tag{1}$$

Since P_2 may be considered to be the useful output, the efficiency is simply the ratio of output power to input power. P_2 is of course less than P_1, the difference being ordinarily accounted for largely by skin-effect losses on the metal surfaces constituting the interior surface of the bolometer mount.

A knowledge of bolometer-mount efficiency is not particularly important if a bolometric instrument is to be used only for relative measurements, or if it is to be used merely as a transfer instrument to be calibrated by some other means. If one is interested in attempting to establish absolute values of power (or of voltage or current in cases

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where these quantities are useful) by means of bolometric measurements, a knowledge of the efficiency of the mount used is important. Nevertheless, it appears that not much work has been done on the determination of the efficiency of bolometer mounts. It has in fact rather generally been assumed that the mount losses could be neglected in routine applications, even when absolute power is at least of nominal interest. But the negligibility of mount losses obviously depends not only on the actual value of the losses, but also upon the accuracy of measurement toward which one is working.

Unfortunately, it is rather difficult to determine the efficiency of a bolometer mount—at least when the determination is based upon power measurements. Comparison of two mounts can at best yield only the ratio of their efficiencies. Direct calorimetric measurements are handicapped by the fact that the powers involved are small, of the order of a few milliwatts, and by the fact that the heating effect of the losses gets distributed throughout a relatively large mass. Calorimetric measurements are somewhat easier to accomplish at higher power levels, but then one has the problem of attenuating the measured level by an accurately known amount down to the level at which bolometers are ordinarily used. The situation is, in short, that accurate measurement of microwave or UHF power at the milliwatt level is a difficult task. Bolometer-mount efficiency, however, is a power ratio—an attenuation, in fact—and does not fundamentally require power measurements for its determination.

The method of determining mount efficiency to be described stems from the consideration of the bolometer mount as a transducer. Under certain conditions, which will be discussed, this transducer becomes a four-pole (representable as a two-terminal-pair network) and its parameters can be determined, essentially as in the case of ordinary networks, from observation of input impedance as a function of load impedance. (In this case the load impedance is varied by varying the bias power supplied to the bolometer.) Once the parameters of the four-pole are found, the power-transfer efficiency can of course be calculated for any desired value of load impedance.

The formulation is based upon field concepts. As is to be expected, the formulas eventually obtained are formally the same as those for the same type of problem in conventional network theory. Thus the more basic portion of the development may be considered as an example of the reduction of what is in detail a problem described by field equations to a problem described by equations of the form of network equations, and the essential concepts and conditions on the basis of which the reduction is accomplished are those that in general underlie the treatment of waveguide and circuit problems by means of network equations. This paper is intended to be reasonably complete and self-contained; however, a much more elaborate discussion of the fundamentals is contained in a previous paper by the author.[3]

II. Formulation of the Method

Consider a bolometer mount, a structure of the general form indicated in figure 1, as a transducer.

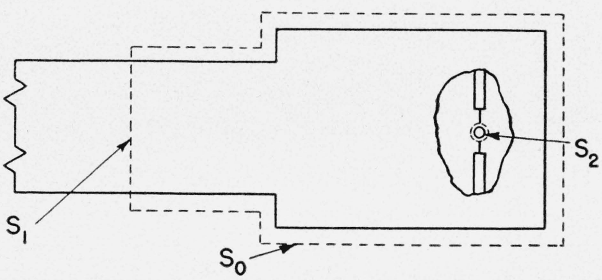

FIGURE 1. *Bolometer mount schematized.*

The terminal surfaces, through which radio-frequency power enters or leaves the system considered, are the surfaces S_1 and S_2. S_1 is a transverse surface in the input waveguide. This waveguide may be of any type or cross section (including, in particular, coaxial line); it is necessary merely that there exist a section that may be considered uniform and lossless. The terminal surface S_2 encloses the region occupied by the bolometer element. This surface is drawn as shown in figure 2, a, or 2, b, for a bolometer consisting of a fine wire or of a bead thermistor, respectively. S_2 is thus pierced by conductors that carry not only radio-frequency current but also bias current. (The bias current comes into consideration only insofar as it serves to establish the operating resistance of the bolometer.) If, as is ordinarily the case, the electromagnetic field

[3] David M. Kerns, Basis of the application of network equations to waveguide problems, J. Research NBS, **42**, 515 (1949) RP1990. (This reference is of a general nature and contains no discussion specifically directed to the bolometer problem).

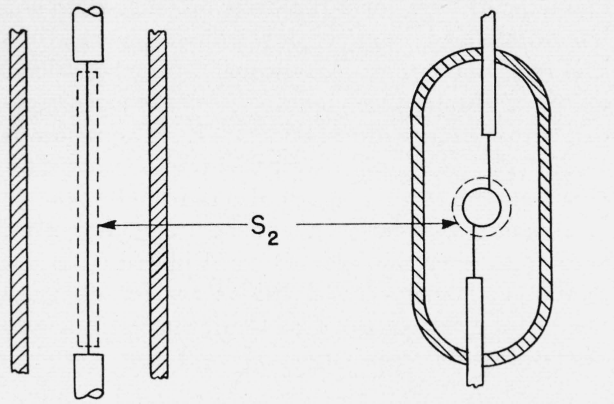

FIGURE 2. *Illustrating terminal surface S_2.*
Cross-hatching indicates protective dielectric enclosure.

of the transducer is effectively limited to a finite region by sufficiently thick metal walls, the interior of the transducer is (by definition) the region R bounded by the surface $S_0+S_1+S_2$, as illustrated in figure 1. S_0 is drawn outside the metallic walls of the structure, and the field may be assumed to be zero on this surface. It is, however, by no means necessary to assume that the transducer field is confined to a finite region. If the bolometer mount does not form a closed metallic structure, so that the electromagnetic field can extend to infinity, the surface S_0 and the region R would be chosen somewhat differently. But the subsequent argument would not be altered, so that it is not necessary to consider this case explicitly.

The whole structure within R, which may include auxiliary tuning devices, joints, bends, etc., may be counted simply as a (decidedly) nonhomogeneous medium. This medium is to be linear, i. e., such that Maxwell's equations become linear equations, and is to contain no sources. The electromagnetic field within R is then uniquely determined by the values of the tangential components of either the electric or the magnetic field on the boundary of R. (It is sufficient to consider only the case in which all field quantities vary harmonically with time at a given frequency, ω. The time dependence will be represented implicitly by a factor $\exp(j\omega t)$.) On the surface S_0, the boundary condition is simply that the field shall vanish; the fields on S_1 and S_2 will be specified in the manner outlined in the following two paragraphs.

In practice, only one waveguide mode need be considered in specifying the field on the waveguide terminal-surface S_1. The transverse components $\boldsymbol{E}_1$, $\boldsymbol{H}_1$ of the field on this surface may then be written

$$\left.\begin{aligned}\boldsymbol{E}_1&=V_1\boldsymbol{E}_{01},\\ \boldsymbol{H}_1&=I_1\boldsymbol{H}_{01},\end{aligned}\right\}\qquad(2)$$

where $\boldsymbol{E}_{01}$, $\boldsymbol{H}_{01}$ are suitable two-component vector functions of coordinates in the transverse plane. V_1, I_1, are numbers, in general complex, which are respectively linear measures of $\boldsymbol{E}_1$, $\boldsymbol{H}_1$. It is convenient to choose $\boldsymbol{E}_{01}$, $\boldsymbol{H}_{01}$ as that field that corresponds to a waveguide field consisting solely of an incident wave, of unit power. The integral of the complex Poynting's vector $\frac{1}{2}(\boldsymbol{E}_{01}\times\boldsymbol{H}^*_{01})$ taken over the surface S_1 becomes [4]

$$\tfrac{1}{2}\int_{S_1}\boldsymbol{E}_{01}\times\boldsymbol{H}^*_{01}\cdot\boldsymbol{n}dS=1\,\text{watt},\qquad(3)$$

where $\boldsymbol{n}$ is the unit normal vector on S_1 directed into the transducer. The complex power input to the transducer at terminal surface S_1 is then, for arbitrary values of V_1, I_1,

$$W_1=\tfrac{1}{2}\int_{S_1}\boldsymbol{E}_1\times\boldsymbol{H}^*_1\cdot\boldsymbol{n}dS=V_1I_1^*.\qquad(4)$$

The looking-in impedance of the field on S_1 is defined as the ratio

$$z=V_1/I_1.\qquad(5)$$

This impedance, which will be called the input impedance, can be determined by means of standing-wave measurements in the input waveguide.

It is assumed that the tangential components of the field on the surface S_2, which encloses the bolometer element, can be represented in the form 2

$$\left.\begin{aligned}\boldsymbol{E}_2&=V_2\boldsymbol{E}_{02},\\ \boldsymbol{H}_2&=I_2\boldsymbol{H}_{02},\end{aligned}\right\}\qquad(6)$$

where $\boldsymbol{E}_{02}$, $\boldsymbol{H}_{02}$ denote suitable vector functions of coordinates designating points on S_2. The assumption 6 is justifiable if, in particular, the condition $kd \ll 1$ is satisfied, where d is a representative linear dimension of the region enclosed by S_2, and k is the wave-number $\omega\sqrt{\mu\epsilon}$ in the

[4] The rationalized meter-kilogram-second system of units is used.

medium (of permeability μ, dielectric constant ϵ) immediately surrounding the bolometer element.[5] For sufficiently small values of kd, equations of the form 6 will hold, with V_2 and I_2 equal (or proportional) to conventional voltage and current, respectively, at the bolometer terminals. Existence of voltage and current, however, is a sufficient but not a necessary condition. Validity of eq 6 insures formal validity of the method being set up; existence of voltage and current on S_2 is not essential in this respect (any more than existence of voltage and current on S_1 is essential). But for application to practical cases, the most favorable condition for knowing that eq 6 will hold to a good approximation occurs when voltage and current do in fact exist to a good approximation.

It is convenient to assume that $\boldsymbol{E}_{02}$, $\boldsymbol{H}_{02}$ satisfy the normalizing condition

$$\tfrac{1}{2}\int_{S_2} \boldsymbol{E}_{02}\times\boldsymbol{H}_{02}^{*}\cdot\boldsymbol{n}dS=1 \text{ watt},$$

where $\boldsymbol{n}$ is here the unit normal on S_2 directed into the transducer. The complex power input to the transducer at terminal surface S_2 is then

$$W_2=\tfrac{1}{2}\int_{S_2} \boldsymbol{E}_{2}\times\boldsymbol{H}_{2}^{*}\cdot\boldsymbol{n}dS=V_2I_2^{*}. \tag{7}$$

The looking-in impedance on S_2 is defined as the ratio V_2/I_2; the load impedance, w, is defined as the negative of the same ratio:

$$w=-V_2/I_2. \tag{8}$$

It is essential for the practical application of the method being formulated that the load impedance be known and variable through known (but not necessarily prescribed) values. The load impedance certainly can be varied by varying the bias power and hence the direct-current resistance of the bolometer; since the load impedance cannot be directly measured (except perhaps at sufficiently low frequencies), the essential problem is to relate it to the direct-current resistance in an adequate fashion. If, in addition to the previous condition $kd\ll 1$, the skin-depth $\sqrt{2/\mu\omega\sigma}$ (permeability μ, conductivity σ) is not less than the diameter of wire for a fine-wire bolometer or not less than bead-diameter for a bead thermistor,[6] it is reasonable to expect the radio-frequency load impedance to be approximately equal to the direct-current resistance of the bolometer. The assumption of this equality is equivalent to the assumption that the current distribution in the bolometer is substantially independent of frequency from zero up to the radio frequency considered.[7] When the above-mentioned conditions are fulfilled, the use of the value of the direct-current resistance as an approximation to the value of the load impedance naturally suggests itself. This approximation is favored by the following facts (to be demonstrated later): (a) When the angle of the load impedance is considered to be a small quantity of the first order, the error in the determination of power-transfer efficiency caused by neglecting the angle and using real values for the load impedance tends to be small of the second order; (b) no error in the determination of efficiency is caused by the use of an arbitrary constant real multiple of the load impedance in place of the load impedance itself—the magnitude of the load impedance may indeed remain undefined to the extent of a constant real factor. This discussion outlines what appear to be the principal factors involved in judging the usefulness of the type of approximation considered.

The bolometer mount, as a transducer, is characterized by the relations that it imposes upon its four terminal variables V_1, I_1, V_2, I_2. Precisely two such relations must exist, since, by the uniqueness theorem mentioned above, the transducer field is determined by the boundary conditions, which in turn are fixed by fixing two of the four terminal variables. Since Maxwell's equations are, by hypothesis, linear and homogeneous in the interior of the transducer, it follows that the relations among V_1, I_1, V_2, I_2 must also be linear and homogeneous. The transducer is thus a linear, source-free four-pole, and its equations may be written

$$\left.\begin{aligned} V_1&=Z_{11}I_1+Z_{12}I_2,\\ V_2&=Z_{21}I_1+Z_{22}I_2.\end{aligned}\right\} \tag{9}$$

[5] Typical dimensions for a platinum-wire bolometer are roughly 0.00015 cm. in diameter by 0.25 cm. in length; a typical bead thermistor has a bead diameter of roughly 0.05 cm. At a frequency of 10,000 Mc/s, the value of k for air is approximately 2.0/cm.

[6] For bolometers of the materials and dimensions mentioned in footnote 5, the skin depth becomes comparable to the respective dimensions in the two cases for frequencies in the neighborhood of 10,000 Mc/s.

[7] A condition of this kind is also important in assuring that radio-frequency power and direct-current power have equivalent heating effects in a bolometer. This question is of course basic in the measurement of absolute values of power by the bolometric method.

The Z's are constants in that they do not depend upon V's and I's, but they of course do depend upon frequency. Equation 9 can be interpreted and could be derived on the basis of the following remarks. If, for example, the boundary conditions

$$\boldsymbol{H}_1=0,\ \boldsymbol{H}_2=I_2\boldsymbol{H}_{02},$$

are prescribed, the corresponding field is determined and is proportional to I_2. Thus, in particular, the corresponding $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ are proportional to I_2, so that V_1 and V_2 are proportional to I_2, the factors of proportionality being Z_{12} and Z_{22}, respectively. The interpretation with $I_1\neq 0$ and $I_2=0$ is similar; eq 9 expresses the general case, with I_1 and I_2 both different from zero.

For the present purpose it is expedient to write equation 9 in a different form, viz,

$$\left.\begin{aligned} V_1&=aV_2+b\bar{I}_2,\\ I_1&=cV_2+d\bar{I}_2,\end{aligned}\right\} \tag{10}$$

where $\bar{I}_2=-I_2$ is used for convenience. The new parameters a, b, c, d are of course related in a simple manner to the Z's appearing in eq 9. These relations will not be written down, since they will not be needed explicity.

Under the assumptions that have been made, the reciprocity condition will apply.[8] Reciprocity must be assumed on theoretical grounds in the present problem, since it is not possible to make radio-frequency measurements at both ends of the transducer. The expression of the reciprocity condition is that $Z_{12}=Z_{21}$, or, equivalently, that the determinant of the coefficients in eq 10 is equal to unity:

$$ad-bc=1. \tag{11}$$

This equation, together with three further equations that are determined by measurement, provides the necessary number of equations to determine the four parameters a, b, c, d.

In accordance with eq 10, input impedance z is given as a function of load impedance w by a so-called linear fractional transformation,

$$z=\frac{aw+b}{cw+d}. \tag{12}$$

[8] A reciprocity theorem in a form adapted to the present application is derived in the reference in footnote 3.

It is noted that this equation may be written in the form

$$wa+b-wzc-zd=0. \tag{12a}$$

Suppose that, for any three distinct values of load impedance w_1, w_2, w_3, the corresponding values of input impedance z_1, z_2, z_3 are measured. Then, apart from a common constant multiplier, the values of a, b, c, d are determined by three simultaneous equations of the type 12, which may be written

$$\left.\begin{aligned} w_1a+b-w_1z_1c-z_1d&=0,\\ w_2a+b-w_2z_2c-z_2d&=0,\\ w_3a+b-w_3z_3c-z_3d&=0.\end{aligned}\right\} \tag{13}$$

If now these equations are solved by Cramer's rule for three of the parameters, say a, b, c, in terms of the remaining one, and if the resulting expressions are used to eliminate a, b, c, d from eq 12a, it may be seen that the final result is expressed by

$$\begin{vmatrix} w & 1 & wz & z\\ w_1 & 1 & w_1z_1 & z_1\\ w_2 & 1 & w_2z_2 & z_2\\ w_3 & 1 & w_3z_3 & z_3\end{vmatrix}=0. \tag{14}$$

(This equation may indeed be written down directly, by recognizing that it is the necessary condition that the system of *four* equations, consisting of eq 13 and 12a, shall form a compatible system for values of a, b, c, d not all zero). Equation 14 gives the relation between general values of z and w, as determined by three pairs of measured values of z and w. The equation may be reduced to

$$\frac{(z-z_3)(z_2-z_1)}{(z-z_1)(z_3-z_2)}=\frac{(w-w_3)(w_2-w_1)}{(w-w_1)(w_3-w_2)}. \tag{15}$$

It should be observed that there exists a transformation of the form 12, determined by eq 14 or 15, which will transform any three given distinct w-values into *any* three given z-values. Thus, in order to obtain a check on the consistency of experimental data, as well as on the validity of the method itself, it is necessary to observe more than three pairs of corresponding values of z and w, and to note whether all the data are represented by a transformation determined by just three pairs of corresponding values. A check of this kind does not directly check the validity of the reciprocity condition, since the transformation of w depends only on the ratios of the values of a, b, c, d. But the reciprocity theorem applies rigorously, provided merely that the form of the description of the terminal fields (eq 2 and 6) is valid. Equation 12 is a rather direct consequence of eq 2 and 6; insofar as a verification of the existence of an equation of the form of eq 12 verifies the basic eq 2 and 6, it at the same time verifies the applicability of the reciprocity theorem.

Explicit formulas for the values of a, b, c, d will now be obtained. As an abbreviation let

$$\gamma=\frac{(w_2-w_1)(z_3-z_2)}{(z_2-z_1)(w_3-w_2)}$$

and solve eq 15 for z:

$$z=\frac{(z_3-\gamma z_1)w+(\gamma z_1 w_3-z_3 w_1)}{(1-\gamma)w+(\gamma w_3-w_1)}.$$

By comparison with eq 12

$$\left.\begin{aligned} a=\alpha(z_3-\gamma z_1),\ b=\alpha(\gamma z_1 w_3-z_3 w_1),\\ c=\alpha(1-\gamma),\quad d=\alpha(\gamma w_3-w_1),\end{aligned}\right\} \tag{16}$$

where α is a constant that must be determined with the aid of the reciprocity condition 11. This condition yields

$$\left.\begin{aligned} ad-bc=\alpha^2\gamma(w_3-w_1)(z_3-z_1)=1,\\ \alpha=[\gamma(w_3-w_1)(z_3-z_1)]^{-\frac{1}{2}}.\end{aligned}\right\} \tag{17}$$

Equations 16 and 17 together determine the values of a, b, c, d corresponding to the measured pairs of values of z and w.

Once a, b, c, d are known for a given bolometer mount, the power-transfer efficiency for a given load impedance can easily be calculated. By using the four-pole equations in the form 10, with load impedance w_0, one finds

$$V_1=(a+b/w_0)V_2,$$

$$I_1=(cw_0+d)\overline{I}_2.$$

Hence the complex power input W_1 and the complex power output $\overline{W}_2(=-W_2)$ are related by

$$W_1=(a+b/w_0)(cw_0+d)^*\overline{W}_2.$$

Let $(a+b/w_0)(cw_0+d)^*=N\ \exp\ (j\theta)$, with N and θ real; and let $\overline{W}_2=|\overline{W}_2|\ \exp\ (j\phi)$ (ϕ is the angle of the load impedance w_0). Then the efficiency η_0 is

$$\eta_0=\frac{Re(\overline{W}_2)}{Re(W_1)}=\frac{\cos\phi}{N\cos(\theta+\phi)}. \tag{18}$$

On the basis of the foregoing formulas, the two supplementary facts invoked in the discussion of load impedance may be demonstrated. The formulas 16, 17, 18 enable one to calculate the parameters a, b, c, d and the efficiency η_0 with load w_0 for a four-pole that transforms w_i into z_i ($i=1, 2, 3$). Suppose that the load impedance is erroneously assumed to be $w'=m^2w$, where m^2 is a constant, and that the parameters a', b', c', d' and the efficiency η_0' with load impedance $w_0'=m^2w_0$ are calculated for a fictitious four-pole that transforms $w_i'=m^2w_i$ into z_i ($i=1, 2, 3$). By using eq 16 and 17, one obtains

$$a'=a/m,\qquad b'=mb,$$

$$c'=c/m,\qquad d'=md.$$

Defining δ by writing the constant m^2 in the form $m^2=|m|^2\ \exp\ (j\delta)$, and using eq 18, one obtains

$$\eta_0'\cos\phi=\eta_0\cos\phi', \tag{19}$$

where $\phi'=\phi/+\delta$ is the angle of w_0'. Since $|m|$ does not enter into eq 19, any constant real multiple of the load impedance may be used in place of the load impedance itself without affecting the calculated efficiency η_0'. If w_0' is real and ϕ is small,

$$\eta_0'=\eta_0/\cos\phi$$

$$\cong\eta_0(1+\phi^2/2).$$

Hence, under the stated conditions, the difference $\eta_0'-\eta_0$ is of the second order (and is positive). Thus the previous statements (b) and (a) are proved.

A bolometer mount (which may in particular include auxiliary tuning equipment in addition to the mount proper) is preferably and frequently arranged so that, with the bolometer at its desired operating resistance, there is substantially no reflected wave at the waveguide input. The corresponding value of input impedance is unity, according to the choice of an arbitrary real factor tacitly made in setting up the definitions of V_1, I_1

(eq 2, and 3). Further, when the load impedance is considered real, the arbitrary factor contained in this quantity may be chosen so that the normal operating load impedance is also represented by the value unity. These two conditions lead to useful, simplified formulas for the calculation of efficiency. Since input impedance $z=1$ corresponds to load impedance $w=1$, the parameters a, b, c, d must satisfy the relation

$$a+b=c+d.$$

Thus the efficiency for load impedance $w=1$, obtained by the appropriate specialization of 18, is given by

$$1/\eta=N=|a+b|^2=|c+d|^2. \tag{20}$$

The efficiency can of course be expressed directly in terms of w_i, z_i ($i=1, 2, 3$), so that efficiency may be calculated without first calculating the four-pole parameters. It is convenient to incorporate the condition that $w=1$ is transformed into $z=1$ by taking w_3, z_3, say, as 1, 1. Then, by using eq 16 and 17 and the associated definition of γ to evaluate $|c+d|^2$, it is readily found that

$$\eta=\left|\frac{(1-z_1)(1-z_2)(w_2-w_1)}{(1-w_1)(1-w_2)(z_2-z_1)}\right|. \tag{21}$$

Although eq 20 and 21 are useful for the purpose intended, it should be remembered that they are based upon special conditions.

A number of measurements have been made, on the basis of the method described, at frequencies in the range 300 to 3,000 Mc/s on bolometer mounts having coaxial-line inputs and at approximately 9,000 Mc/s on mounts having rectangular-waveguide inputs. Values of efficiency calculated from the data range from 0.7 to 0.95 for different mounts, which included auxiliary tuning equipment in all cases and additional components in some cases. In all measurements made so far, it has been found that for suitably located input terminal surfaces, the four-pole could be represented in terms of real parameters. In the 9,000-Mc measurements, bolometers of the types and dimensions mentioned in footnote 5 were used, so that a moderate test of the approximations that have been discussed was provided. It was found that an equation of the form 12 represents these experimental data rather well.

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