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Absorption by Sound-Absorbent Spheres

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The absorption of a plane wave of sound by a sphere is computed. The calculations are based on the assumption that the complex ratio of sound pressure at a point on the sphere's surface to the normal component of particle velocity is a constant independent of the direction of incidence ("normal impedance assumption"). Absorption measurements were made on hair-felt covered spheres placed in a reverberation room, and were compared with the computed absorption by means of the reverberation room statistics appropriate for spheres. The theory and measurements both show that absorbent spheres can have absorption coefficients greater than unity. The discrepancies between theoretical and experimental coefficients seem to indicate that the normal impedance assumption is not valid for the hair felt used in the experiments.

I. Introduction

In an earlier paper [1],[1] the authors gave the results of a combined theoretical and experimental investigation of the absorption of sound by long circular cylinders having a known normal acoustic impedance. An expression was derived for the absorption coefficient of a cylinder on which is incident a plane wave of sound, having a single frequency, traveling in a direction perpendicular to the axis of the cylinder. The derivation was based on the assumption that the complex ratio of sound pressure at a point on the cylinder's surface to the normal component of particle velocity was a constant independent of the direction of incidence. The ratio was therefore equal to the acoustic impedance of the surface for perpendicularly incidence sound. This will be referred to hereafter as the "normal impedance assumption."

A comparison of the measured and theoretical absorption coefficients for random incidence seemed to indicate, however, that the normal impedance assumption was not satisfied for the hair felt. In other words, it seemed that sound was propagated also through the absorbent covering (hair felt) of the cylinders in directions other than normal to the surface. Similar conclusions were reached more recently by Scott [2] in his study of propagation of sound between walls and through ducts lined with mineral wool.

It was pointed out in the previous paper [1] that a comparison of theoretical and experimental absorption coefficients for sound absorbent spheres would give more definite information on the validity of the normal impedance assumption for hair felt. Consequently, an investigation was carried out on spheres covered with the same type of hair felt as was used for covering the cylinders.

The theoretical calculations on spheres may be divided into two parts: (1) The absorption by a sphere, calculated on the basis of the normal impedance assumption, when a plane wave of sound is incident on it, (2) a statistical calculation of the absorption by a sphere for sound waves having random incidence, the results of which have already been given in reference [1]. The measurements of the acoustic impedance of the hair felt are given in the same reference and are required here for the first part of the calculations. The calculated absorptions for random incidence are compared with measurements made in a reverberation room.

[1] Figures in brackets indicate the literature references at the end of this paper.

II. Scattering by an Absorbent Sphere

A sphere of sound-absorbent material is placed in a plane sound wave of a single frequency whose direction of propagation is parallel to the x-axis (see fig. 1). The scattering and absorption by the sphere are to be computed. The normal impedance assumption is supposed to be valid at all points on the surface of the sphere.

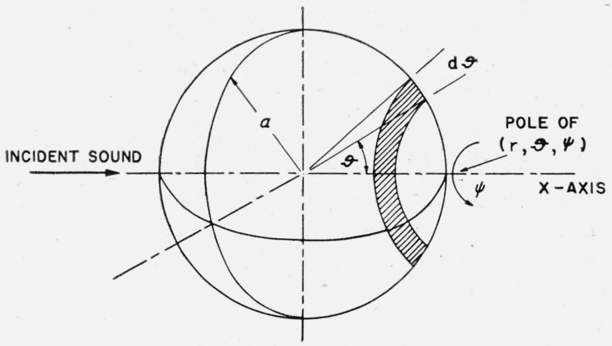

FIGURE 1. *Representation of absorbent sphere.*

The velocity potential of the incident plane wave is

$$\left.\begin{aligned}\Phi_i&=\phi_0 e^{i(\omega t-kx)}=\phi_0 e^{i(\omega t-kr\cos\vartheta)}\\&=\phi_0\left\{\sum_{\mu=0}^{\infty}(2\mu+1)\,(-i)^{\mu}P_{\mu}\,(\cos\vartheta)j_{\mu}(kr)\right\}e^{i\omega t},\end{aligned}\right\}\quad(1)$$

and for simplicity it is assumed that $\phi_0=1$. In this equation $\omega=2\pi\times$frequency of the incident wave and k is the generalized wave number. This expansion of the velocity potential in an infinite series of Legendre polynomials P_μ ($\cos\vartheta$) and spherical Bessel functions $j_\mu(kr)$ in the spherical polar coordinates r, ϑ, ψ, is introduced in order to facilitate satisfaction of the boundary conditions at the surface of the absorbent sphere. It is clear from considerations of symmetry that the velocity potential will be independent of the azimuthal angle, ψ.

The wave scattered by the sphere can be represented by

$$\Phi_s=\sum_{\mu=0}^{\infty}b_\mu(j_\mu-in_\mu)P_\mu(\cos\vartheta)e^{i\omega t},\qquad(2)$$

in which it is understood that j_μ and n_μ are functions of kr. The function n_μ is a spherical Neumann function. Each term in this expression represents an outgoing wave, as can be seen from the asymptotic expansion of $j_\mu-in_\mu$ for large r. The b's are constants to be determined by the boundary conditions at the surface of the sphere.

The total velocity potential is $\Phi=\Phi_i+\Phi_s$. The sound pressure at any point is $p=-\rho(\partial\Phi/\partial t)$ (ρ=density of air), and the radial velocity component, which is normal to the surface of the sphere, is $v_r=(\partial\Phi/\partial r)$. Since the normal impedance assumption is supposed to be valid, the boundary condition at the surface of the sphere is

$$p=-Zv_r,\ (r=a)\qquad(3)$$

where $Z=|Z|e^{i\delta}$ is the acoustic impedance. The negative sign is used because the positive direction of v_r is *out* of the absorbent surface, whereas Z is customarily measured with the positive direction for particle velocity *into* the absorbent surface. Calculation of p and v_r from Φ and substitution into eq 3 will yield equations for determination of the b_μ's.

III. Absorption Coefficient

The total time-averaged acoustic power, P, absorbed by the sphere can be computed by a technique similar to that used in the earlier paper [1] for cylinders. Since the calculation is not basically different, the details will not be given here.

A definition of absorption coefficient is needed. Let the time-averaged acoustic power transmitted through unit area by the incident plane wave be P_0. The power absorbed per unit peripheral area of the sphere is $P/4\pi a^2$. The absorption coefficient α_s for a plane wave is defined here by

$$\alpha_s=\frac{P}{4\pi a^2P_0}.\qquad(4)$$

It is shown in the earlier paper that the random incidence effective absorption coefficient for a sphere measured in a reverberation room is greater than α_s by a factor of 4 if the Sabine reverberation formula is used to compute absorption coefficient from measured decay times. The reverberation-room absorption coefficient α_t is defined by

$$\alpha_t=4\alpha_s=\frac{P}{\pi a^2P_0}.\qquad(5)$$

Both α_s and α_t are expressed in sabins per unit peripheral area.

The final expression for the reverberation-room absorption coefficient in terms of the acoustic impedance is

$$\alpha_t = \frac{4}{(ka)^4}\left|\frac{Z}{\rho c}\right| \cos\delta \sum_{\mu=0}^{\infty} \frac{2\mu+1}{\left|-\frac{Z}{\rho c}(j'_\mu - in'_\mu) + i(j_\mu - in_\mu)\right|^2}. \tag{6}$$

The quantity c in this expression is the velocity of sound in air, and j'_μ, n'_μ are the derivatives of j_μ, n_μ.

IV. Experimental Results and Discussion

1. Description of Spheres

The spheres consisted of solid plaster-of-paris cores covered with a layer of cattle-hair felt. Two pieces of the felt, cut to an interlocking pattern, were sewed together over the spherical core like a cover is sewed on a baseball. The contact edges of the pieces were not cut vertically but on a slant to insure minimum distortion of the hair felt. The average diameter of the cores was 5.02 in. and the average diameter of the finished spheres was 6.33 in.

The felt, nominally ⅞ in. thick and weighing 0.60 lb/ft^2, was of the same type and was manufactured by the same concern as that used in the experiments on cylinders.

2. Impedance of Felt

The results of acoustic impedance measurements on the hair felt with rigid backing, and the method for measuring impedance, were given in the earlier paper (see fig. 5 of [1]).

3. Absorption Measurements

The reverberation room measurements were made on 68 of the felt-covered spheres. In figure 2 are shown some of the spheres suspended from wires in the reverberation room. Except for the volume swept out by a set of rotating vanes in the center of the room, the spheres were hung at least 54 in. apart in a rhombohedral lattice. Ten of the spheres were at a distance of about 24 in. from a wall and the remainder at not less than 36 in. from a wall. Descriptions of the reverberation room and the technique for measurement of decay times were given in reference [1].

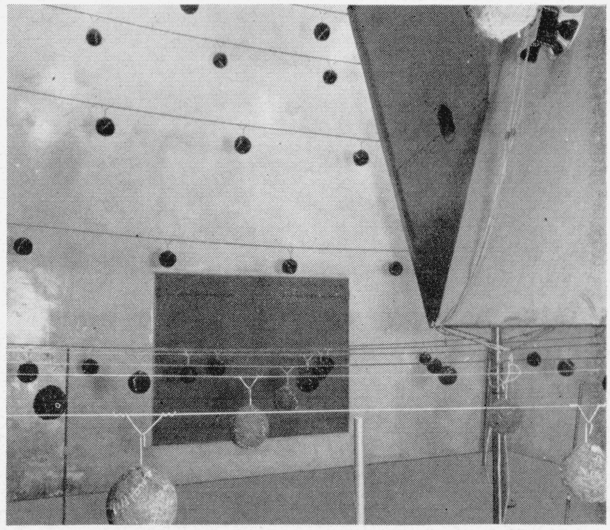

FIGURE 2. *Felt-covered spheres suspended in the reverberation room.*

Results of the reverberation room measurements of the absorption coefficient α_t (defined by eq 5) are plotted in figure 3. The theoretical values of α_t are plotted in the same figure, and were computed from the measured impedances (given in fig. 5 of [1]) by means of eq 6. The measured absorption coefficients for one layer of hair felt laid flat on the floor of the reverberation room are also plotted in figure 3. Comparison of the measured values for the large flat layer with the corresponding measured coefficients for the absorbent spheres shows that a given area of felt has considerably greater absorption, at higher frequencies, when wrapped on a sphere than when applied to a large flat surface. A similar fact was noted in the earlier measurements on absorbent cylinders.

Figure 3 shows that the spheres have absorption coefficients greater than unity at frequencies above 1,200 c/s. This latter frequency corresponds to $ka \cong 2$. However, the maximum theoretical coefficient for the spheres is somewhat greater than the maximum measured value. This was also true in the case of the absorbent cylinders.

There is general agreement between the theoretical and measured absorption curves, but the differences at some frequencies are too great to be ascribed to experimental uncertainty. The theoretical coefficients are too low at low frequencies, too high at frequencies near $ka=2$, but are in fairly good agreement with the measured coefficients at high frequencies. The differences

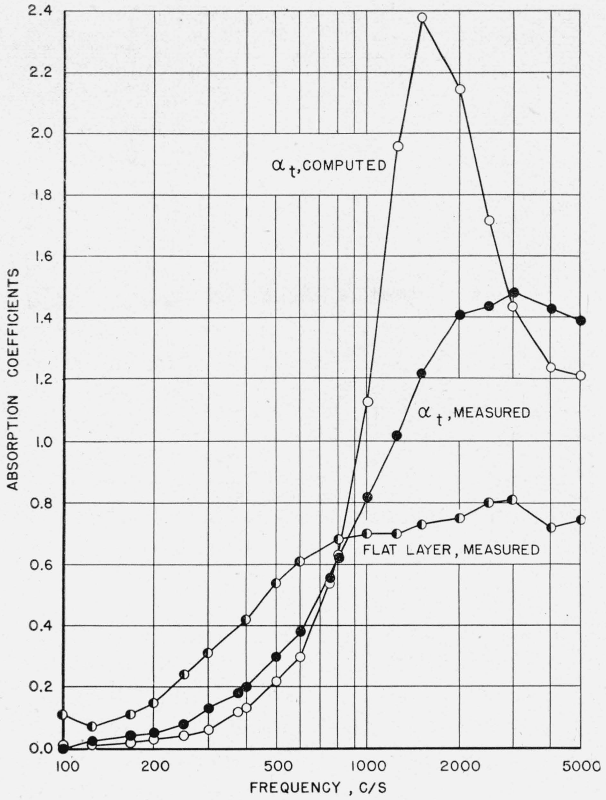

FIGURE 3. *Absorption coefficients for spheres covered with hair felt.*

○, Theoretical values of the absorption coefficient α_t computed by means of equation 6; ●, values of α_t measured in a reverberation room; ◐, absorption coefficients for a flat single layer of hair felt measured in a reverberation room.

between the theoretical and measured values of α_t can only be explained, it seems, on the grounds that the normal impedance assumption is not valid for the hair felt used in the experiments. This implies that sound waves are propagated through the felt in directions other than normal to the surface.

V. Technique of Calculation

Spherical Bessel and Neumann functions of high order and their derivatives were required for calculation of α_t (eq 6). These functions are algebraic combinations of sinusoidal functions and polynomials. At the time these calculations were made (more than 2 years ago), it seemed that the Tables of Circular and Hyperbolic Sines and Cosines [3] prepared by the Mathematical Tables Project of the National Bureau of Standards would readily yield results of the requisite accuracy, and therefore these tables were used. Recently published tables [4, 5] of amplitudes and phase angles for scattering by absorbent spheres and cylinders would probably yield the desired results more expeditiously.

Enough terms in the infinite series of eq 6 were used so that the estimated sum of the remainder terms amounted to less than 0.1 percent of the whole sum. It is estimated that the final computed values of the absorption coefficients α_t have errors (arising from the sequence of calculations) of less than 0.01.

VI. Summary and Proposals

A wave theory has been applied to the absorption of sound by an absorbent sphere. An exact expression, including the effects of diffraction, has been obtained for the absorption coefficient of a sphere on which is incident a plane wave of sound. The derivation of the expression for the absorption coefficient was based on the normal impedance assumption. This assumption is that the complex ratio of sound pressure at a point on the surface of the sphere to the normal component of particle velocity, is a constant independent of the direction of incidence. The ratio is therefore equal to the acoustic impedance of the surface for perpendicularly incident sound. The theoretical coefficients were computed for spheres covered with hair felt by using directly measured acoustic impedances.

The theoretical coefficients were compared with values obtained experimentally for spheres placed in a reverberation room. In order to effect the comparison, it was necessary to use the statistics (developed in an earlier paper) for the absorption by a sphere located in a random wave field. The theory predicts, and measurements confirm, that absorbent spheres can have absorption coefficients appreciably greater than unity. The discrepancies between theoretical and experimental coefficients seem to force the conclusion that wave propagation takes place through the absorbent material in directions other than normal to the surface, which is equivalent to saying that the normal impedance assumption is not valid for hair felt. This confirms the conclusion reached tentatively in an earlier investigation on cylinders covered with the same type of hair felt [1].

Further experiments should be made on the random incidence absorption of sound by isotropically porous spheres. The material of the spheres should absorb sound by viscous damping of the motion of air in the pores. From measurements of the velocity and attenuation (propagation constants) of waves within the porous material, and from measurements of the acoustic impedance, it should be possible to deduce the absorption when the sphere is placed in a plane wave field, and hence the absorption in a reverberation room.

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VII. References

[1] R. K. Cook and P. Chrzanowski, J. Research NBS **36,** 393 (1946) RP1709; J. Acous. Soc. Am. **17,** 315 (1946).

[2] R. A. Scott, Proc. Phys. Soc. **58,** 165, 253, 358 (1946).

[3] Tables of circular and hyperbolic sines and cosines (sponsored by the National Bureau of Standards), published by the Works Project Administration of the Federal Works Agency, New York (1939).

[4] A. N. Lowan, P. M. Morse, H. Feshbach, and M. Lax, Scattering and radiation from circular cylinders and spheres, published by the National Bureau of Standards and the MIT Underwater Sound Laboratory (Feb. 1945).

[5] M. Lax and H. Feshbach, J. Acous. Soc. Am. **20,** 108 (1948).

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