U. S. Department of Commerce
National Bureau of Standards

RP1807
Research Paper ~~RP1815~~
Volume 39, July 1947

Part of the Journal of Research of the National Bureau of Standards

Solution of Problem of Producing Uniform Abrasion and Its Application to the Testing of Textiles [1]

By Herbert F. Schiefer

A general mathematical solution to the problem of producing uniform abrasion over a plane area of a specimen from every azimuthal direction was worked out. This solution requires that the abradant and the specimen rotate in the same direction and with the same angular velocity. The specimen may revolve about the center of the abradant with any angular velocity in the same or opposite direction as its direction of rotation. The specimen must not extend beyond the boundary of the abradant. Simple revolution of abradant about specimen and of specimen about abradant without rotation of either are special solutions. The special solution in which the specimen does not revolve about the center of the abradant is the simplest one from mechanical considerations. A machine based upon this special solution is described, and preliminary results obtained with it are discussed.

I. Introduction

Resistance to abrasion is one of the factors that affect the serviceability of many organic and inorganic products, such as textiles, rubber, plastics, leather, linoleum, ceramics, mastic tile, concrete, stone, wood, and metals. An analysis of the problem of measuring the resistance to abrasion shows clearly that it can be resolved into three major phases, namely, (1) the design and development of an abrasion machine that will abrade a plane area of a specimen uniformly over the entire area and from every direction in its plane, (2) the design and development of an abradant that remains constant not only during a test but also between different tests, and (3) the development of a quantitative method for the evaluation of the amount of abrasion. Research has been actively pursued on all three phases in the Textiles Section of the National Bureau of Standards, as part of the broad and long-term fundamental program of research on abrasion sponsored and supported by the Office of the Quartermaster General, War Department.[2] A complete mathematical solution to the first phase has been worked out. This solution is presented in this paper, and a machine based upon a special case of the general solution is described. The solutions to the second and third phases depend necessarily to some degree upon the type and nature of the material being tested. These two phases have also received a great deal of attention, and considerable progress has already been made toward finding their solutions for testing textiles.

A great many abrasion-testing machines have been developed [1, 2].[3] A review of the literature indicates that in only one case is the relative motion between the specimen and the abradant the same for the whole area of the specimen that is abraded, and in no case is it the same from every direction of the abraded surface. It is not possible, therefore, to attribute unequal wear to a structural difference in the specimen, or "form factor."

In an attempt to approximate the conditions that produce more uniform abrasion, machines with complicated mechanisms have been developed. Haven [3] designed a special cam to produce uniform linear velocity over a major

[1] Presented at the February 1947 meeting of The Fiber Society. Published also in Textile Research J. (July 1947).

[2] Permission to publish this paper was granted by the Office of the termaster General.

[3] Figures in brackets indicate the literature references at the end of this paper.

portion of the stroke in a translatory reciprocation. The acceleration and deceleration is confined to 1½ in. at each end of the reciprocation. Repenning [2] developed a machine in which the rotary reciprocation of the specimen was of a different period than the translatory reciprocation of the abradant. The relative motion between the specimen and abradant is a variable figure eight or Lissajou figure. Martindale [4] developed a machine in which the specimen is given two perpendicular translatory reciprocations of different periods. The motions relative to a stationary abradant correspond to variable Lissajou figures. In the Shawmut wear-testing machine [5] special cams move the specimen in two perpendicular directions according to a specific pattern while a rotating abradant is periodically lifted and dropped. Gavan, Eby, and Schrader [6] described a new sandpaper abrasion tester in which the specimen is abraded uniformly unidirectionally. In their machine, eight specimens are mounted on an endless conveyor in such a manner that in each revolution the specimens move over the sandpaper abradant. The sandpaper moves continuously in the opposite direction so that new abrasive paper is constantly supplied as the abradant, a feature used earlier by Sigler and Holt [7] and also by Larose [8].

II. Problem

The problem that is presented for solution by the first phase is to find the type of motion of the abradant and of the specimen and the conditions for these motions so that a plane area of the specimen is abraded uniformly over its entire area and from every direction in its plane. The problem will be solved if the following two conditions can be satisfied: (a) The instantaneous relative velocity between the abradant and the specimen is constant in magnitude and direction for every point of the abraded area, schematically shown in *A* of figure 1, and (b) the instantaneous relative velocity between the abradant and the specimen at any point of this area is constant in magnitude but changes in direction relative to the specimen from instant to instant in a continuous and uniform manner, schematically shown in *B* of figure 1. These conditions simply imply that every point is abraded equally from every azimuthal direction and that this process is repeated over and over again until the test is terminated.

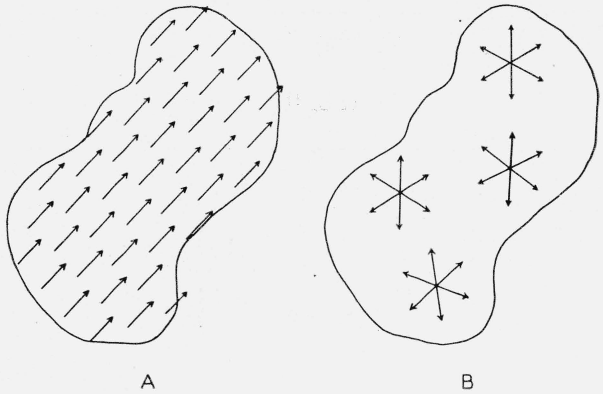

FIGURE 1.—*Schematic diagram showing uniform unidirectional abrasion, A, and uniform abrasion from every azimuthal direction, B.*

It is assumed that the abradant is uniform and remains constant.

III. Solution

1. Discussion

It is apparent that motions of simple translation of the abradant and of the specimen, such as are used by Gaven, Eby, and Schrader [6] meet the first but not the second requirement of the problem. Uniform unidirectional abrasion obtained with their machine is no doubt sufficient for testing a number of materials for such special uses where all the wear is in one direction. The second requirement could be approximated by rotating the specimen through a small angle after each contact with the abradant. Motions of reciprocation, either of translation, rotation, or combinations of translation and rotation, do not meet the conditions of the problem and must be excluded as the relative velocity between the specimen and abradant varies greatly with time. If the problem has a general solution, then the motions of the abradant and specimen must consist of rotations and revolutions. From the conditions of the problem, it follows that the area to be abraded must at all times be in contact with the abradant. From these remarks it suffices, for a general solution, to consider a large plane abradant, which rotates about its perpendicular axis, and a smaller plane specimen, which simultaneously revolves in the plane of the abradant about the axis of the abradant and rotates in the same plane about its own perpendicular axis in such a way that it is always completely contained

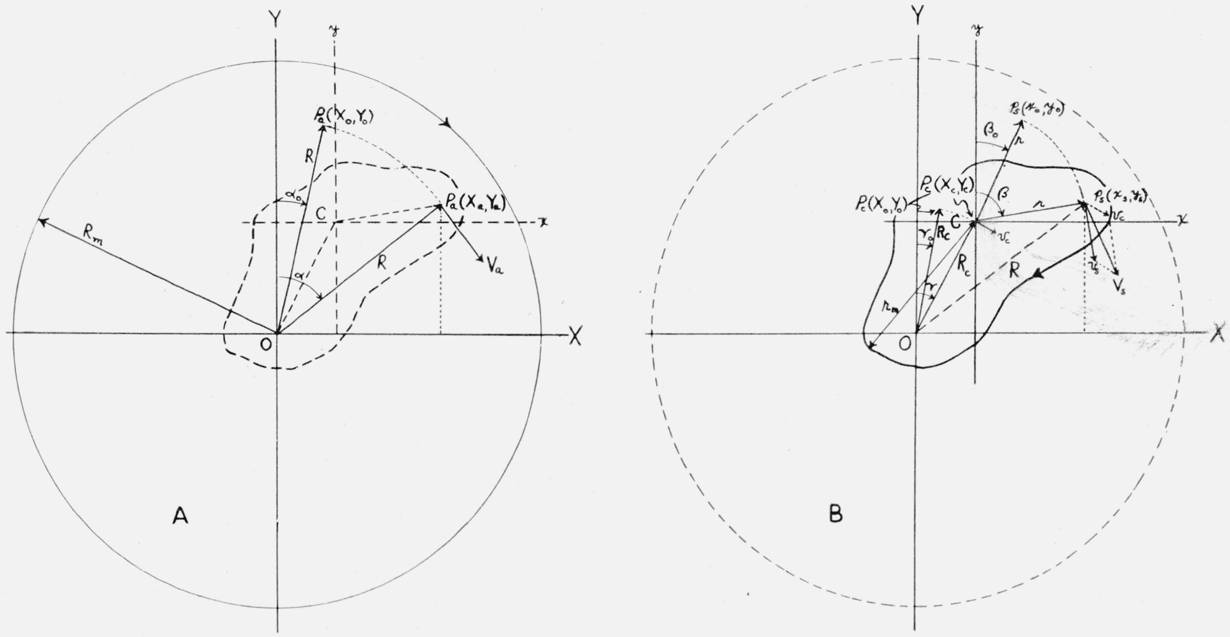

FIGURE 2.—*Schematic diagram illustrating motion of abradant, A, and of Specimen, B.*

within the boundary of the abradant. The abradant is represented schematically in *A* of figure 2, and the specimen is represented schematically in *B* of figure 2.

2. Rotation of Abradant

Let $P_a(X_0,Y_0)$ in *A* of figure 2 be any point of the abradant defined with respect to the *XY* coordinate axes by $X_0=R\sin\alpha_0$ and $Y_0=R\cos\alpha_0$, when $t=0$. The point P_a rotates about the center O with a constant angular velocity, W. The position of this point at time t is $P_a(X_a,Y_a)$ and is given by the simultaneous equation

$$\left.\begin{aligned} X_a&=R\sin\alpha\\ Y_a&=R\cos\alpha\\ \alpha&=\alpha_0+Wt\end{aligned}\right\}. \tag{1}$$

The X and Y components of the velocity of the point $P_a(X_a,Y_a)$ at any time t, and the magnitude and slope of the resultant velocity, are given by the equation

$$\left.\begin{aligned} V_{X_a}&=\frac{dX_a}{dt}=WY_a\\ V_{Y_a}&=\frac{dY_a}{dt}=-WX_a\\ |V_a|&=W\sqrt{X_a^2+Y_a^2}=W|R|\\ m_{V_a}&=-\frac{X_a}{Y_a}\end{aligned}\right\}. \tag{2}$$

The slope m_{V_a} is the negative reciprocal of the slope of the radius vector R, which is $m_R=Y_a/X_a$. The velocity vector V_a is thus at right angles to R.

3. Rotation of Specimen

Let $p_s(x_0,y_0)$ by any point of the specimen defined with respect to the *xy* coordinate axes by $x_0=r\sin\beta_0$ and $y_0=r\cos\beta_0$, when $t=0$. The point p_s rotates about the center C with a constant angular velocity, w. The position of this point at time t is $p_s(x_s,y_s)$ and is given by the simultaneous equation

$$\left.\begin{aligned} x_s&=r\sin\beta\\ y_s&=r\cos\beta\\ \beta&=\beta_0+wt\end{aligned}\right\}. \tag{3}$$

The x and y components of the velocity of the point $p_s(x_s,y_s)$ at any time t and the magnitude and slope of the resultant velocity are given by the equation

$$\left.\begin{aligned} v_{x_s}&=wy_s\\ v_{y_s}&=-wx_s\\ |v_s|&=w\sqrt{x_s^2+y_s^2}=w|r|\\ m_{v_s}&=-\frac{x_s}{y_s}\end{aligned}\right\}. \tag{4}$$

The slope m_{v_s} is the negative reciprocal of the slope of the radius vector, r, which is $m_r=y_s/x_s$.

The velocity vector v_s is thus at right angles to the radius vector r.

4. Revolution of Specimen

Let $P_c(X_0,Y_0)$ be the position of the center of rotation, C, of the specimen defined with respect to the XY coordinate axes by $X_0=R_c \sin \gamma_0$ and $Y_0=R_c \cos \gamma_0$, when $t=0$, which revolves about the center O with a constant angular velocity Ω. The position of this point at time t is $P_c(X_c,Y_c)$ and is given by the simultaneous equation

$$\left.\begin{aligned} X_c&=R_c \sin \gamma \\ Y_c&=R_c \cos \gamma \\ \gamma&=\gamma_0+\Omega t \end{aligned}\right\}. \tag{5}$$

The X and Y components of the velocity of the point $P_c(X_c,Y_c)$ at any time t and the magnitude and slope of the velocity are given by the equation

$$\left.\begin{aligned} v_{X_c}&=\Omega Y_c \\ v_{Y_c}&=-\Omega X_c \\ |v_c|&=\Omega\sqrt{X_c^2+Y_c^2}=\Omega|R_c| \\ m_{v_c}&=-\frac{X_c}{Y_c} \end{aligned}\right\}. \tag{6}$$

The slope m_{v_c} is the negative reciprocal of the slope of the radius vector R_c, which is $m_{R_c}=Y_c/X_c$. The velocity vector v_c is thus at right angles to the radius vector R_c.

5. Rotation and Revolution of Specimen

The position of the point $p_s(x_s,y_s)$ referred to the XY coordinate axes is $P_s(X_s,Y_s)$, where $X_s=X_c+x_s$ and $Y_s=Y_c+y_s$. From eq 4 and 6 it follows that the X and Y components of the velocity of the point $P_s(X_s,Y_s)$ at any time t and the magnitude and slope of the resultant velocity are given by the equation

$$\left.\begin{aligned} V_{X_s}&=\Omega Y_c+wy_s \\ V_{Y_s}&=-(\Omega X_c+wx_s) \\ |V_s|&=\sqrt{\Omega^2(X_c^2+Y_c^2)+w^2(x_s^2+y_s^2)+2w\Omega(x_sX_c+y_sY_c)} \\ m_{V_s}&=-\frac{\Omega X_c+wx_s}{\Omega Y_c+wy_s} \end{aligned}\right\}. \tag{7}$$

6. Relative Velocity

If the abradant A of figure 2 and the specimen B of figure 2 are superimposed and the values of $|R|$, $|R_c|$, $|r|$, α_0, β_0, and γ_0 are so chosen that at any time t the points $P_a(X_a,Y_a)$ and $P_s(X_s,Y_s)$ coincide at the point $P(X,Y)$ in A of figure 3, that is, $X=X_a=X_s$ and $Y=Y_a=Y_s$, then the

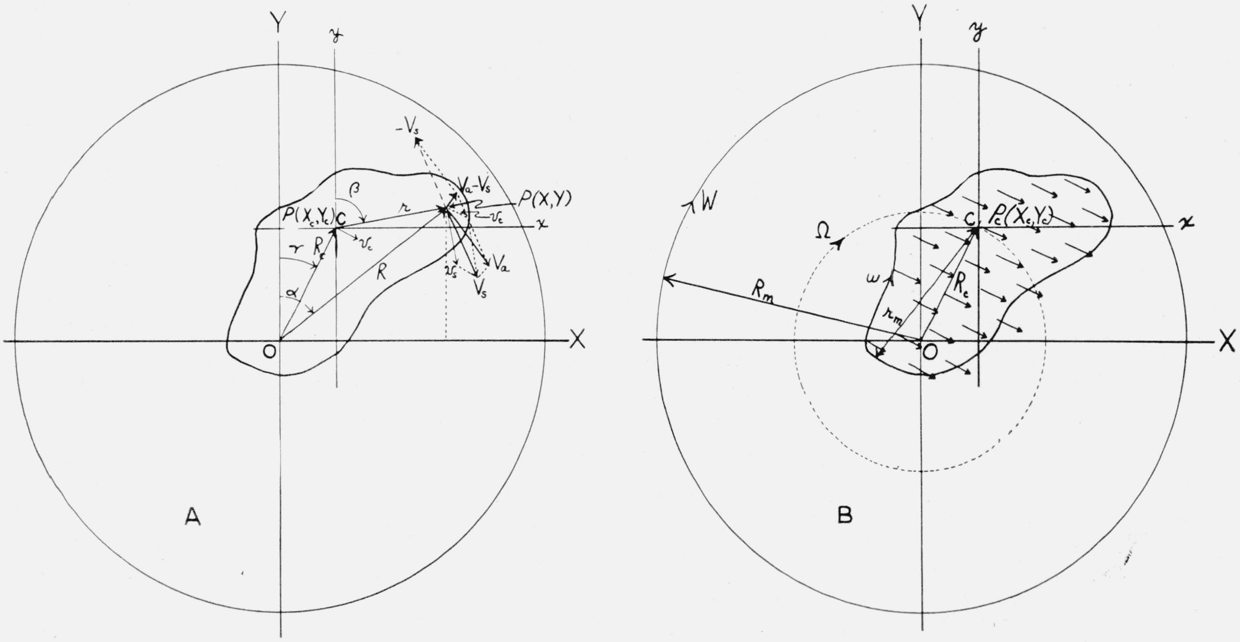

FIGURE 3.—*Schematic diagram showing relative velocity between abradant and specimen, A, and uniform instantaneous relative velocity, B.*

relative velocity between the abradant and the specimen at the point $P(X,Y)$, which is any point common to both the abradant and the specimen at any time t, is given by the vector difference between the velocities for the abradant and for the specimen at the point $P(X,Y)$. The X and Y components of the relative velocity at the point $P(X,Y)$ at any time t and the magnitude and slope of the resultant relative velocity are given by the equation

$$\left.\begin{aligned} V_{X_a}-V_{X_s}&=(W-w)Y+(w-\Omega)Y_c \\ V_{Y_a}-V_{Y_s}&=-(W-w)X-(w-\Omega)X_c \\ |V_a-V_s|&=\sqrt{(W-w)^2(X^2+Y^2)+(w-\Omega)^2(X_c^2+Y_c^2)+2(W-w)(w-\Omega)(X_cX+Y_cY)} \\ m_{(V_a-V_s)}&=-\frac{(W-w)X+(w-\Omega)X_c}{(W-w)Y+(w-\Omega)Y_c}. \end{aligned}\right\} \quad (8)$$

As W, w, and Ω are constants that may have any real values, we may write $W=nw$ and $\Omega=Nw$, where n and N are any real numbers. Substituting these values for W and Ω in eq 8 results in eq 9.

$$\left.\begin{aligned} V_{X_a}-V_{X_s}&=[(n-1)Y+(1-N)Y_c]w \\ V_{Y_a}-V_{Y_s}&=-[(n-1)X+(1-N)X_c]w \\ |V_a-V_s|&=w\sqrt{(n-1)^2(X^2+Y^2)+(1-N)^2(X_c^2+Y_c^2)+2(n-1)(1-N)(X_cX+Y_cY)} \\ m_{(Va-Vs)}&=-\frac{(n-1)X+(1-N)X_c}{(n-1)Y+(1-N)Y_c}. \end{aligned}\right\} \quad (9)$$

7. General Solution

The conditions of the problem, namely, uniform abrasion over the entire abraded area of the specimen at any time t, require that both $V_{X_a}-V_{X_s}$ and $V_{Y_a}-V_{Y_s}$ are constant for every point $P(X,Y)$ of the abraded area. It follows, therefore, that the identities given by the equation

$$\left.\begin{aligned} (n-1)Y+(1-N)Y_c&\equiv k_1/w=\text{constant} \\ (n-1)X+(1-N)X_c&\equiv k_2/w=\text{constant}, \end{aligned}\right\} \quad (10)$$

must be satisfied, where k_1 and k_2 are constants. From these identities it follows that $(n-1)$ must equal zero, that is, $n=1$ and therefore $W=w$. In other words, for the general solution of the problem for uniform abrasion over the entire area of the specimen, both the abradant and the specimen must rotate in the same direction and with the same angular velocity. No restrictions, whatsoever, are imposed upon the values of N, X_c, Y_c, and w. They can be varied at will, provided the relationship $|r_m|\leqq|R_m|-|R_c|$ is satisfied. This relationship merely implies that the specimen is always in contact with the abradant. The specimen may have any shape, and it may revolve about the center of rotation of the abradant with any angular velocity in the same or in the opposite direction as the direction of rotation of the abradant. The specimen may be placed in any position relative to the abradant, provided the distance between the two centers of rotation $|R_c|$ satisfies the expression $|R_c|\leqq|R_m|-|r_m|$.

In this general solution of the problem, that is, when $n=1$, $W=w$, and $\Omega=Nw$, eq 9 reduces to the equation

$$\left.\begin{aligned} V_{X_a}-V_{X_s}&=(1-N)wY_c \\ V_{Y_a}-V_{Y_s}&=-(1-N)wX_c \\ |V_a-V_s|&=(1-N)w\sqrt{X_c^2+Y_c^2}=(1-N)w|R_c| \\ m_{(V_a-V_s)}&=-\frac{X_c}{Y_c} \end{aligned}\right\} \quad (11)$$

The slope of the relative velocity vector $m_{(V_a-V_s)}$ is the negative reciprocal of the slope of the radius vector R_c, which is $m_{R_c}=Y_c/X_c$. The relative velocity vector, V_a-V_s, is therefore always at right angles to the radius vector R_c, which joins the centers of rotation of the abradant and of the specimen. This condition is shown schematically in B of figure 3 at the time t.

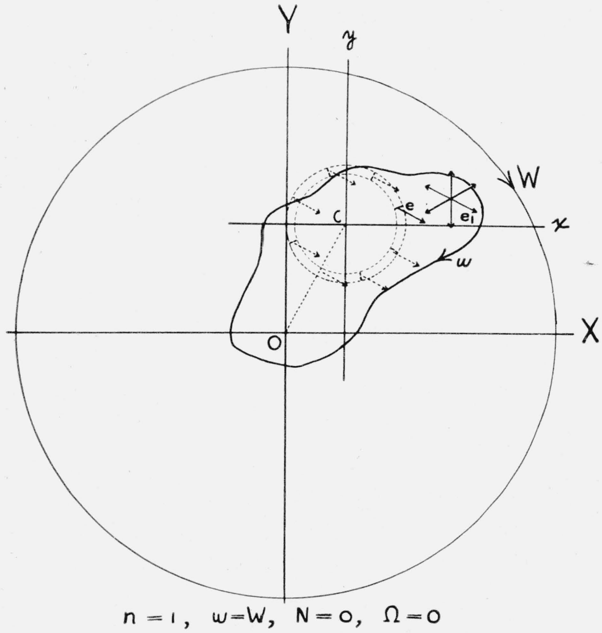

FIGURE 4.—*Schematic diagram showing attainment of uniform abrasion from every azimuthal direction for a special solution of problem.*

The slope angle of R_c changes 360 degrees in each revolution of point C about O and the slope angle of r also changes 360 degrees in each rotation of the specimen about C. Therefore, the direction of the relative velocity vector at every point of the specimen changes through 360 degrees with respect to the specimen for every change of 360 degrees in the angular difference $|\beta-\gamma|$. But $|\beta-\gamma|=|\beta_0-\gamma_0|+|w-\Omega|t=|\beta_0-\gamma_0|+|1-N|wt$. From this expression it can be seen that the relative velocity vector at every point of the specimen changes once through 360 degrees in $1/\|1-N\|$ rotations of the specimen. This condition is shown schematically in figure 4, where any small element of the specimen, e, assumes all of the different indicated positions in one rotation for the special case when $N=0$ that is, for the case where the specimen does not revolve about the center O. It is clear that the element e is abraded equally from all directions in one rotation, as indicated on the element e_1. Obviously, if $N>2$, or if the direction of revolution is opposite to the direction of rotation, i. e., $N<0$, then the relative velocity vector at every point of the specimen will change through 360 degrees more frequently than for the special case when $N=0$ and for $0<N\leqq 2$.

A complete general solution of the problem of uniform abrasion from every azimuthal direction has thus been obtained. It requires that the abradant and the specimen rotate about their respective centers with the same angular velocity, w. The specimen may be revolved about the center of the abradant with any angular velocity Nw, where N is any real number, including 0.

It can be seen by inspection of the expression of the relative velocity given by $|V_a-V_s|=(1-N)w|R_c|$ and from the slope of the relative velocity vector given by $m_{(V_a-V_s)}=-(X_c/Y_c)$, that the relative motion between the abradant and the specimen for the general solution of the problem is equivalent to a mere revolution of the specimen about the center of the abradant, or of the abradant about the center of the specimen, with an angular velocity equal to $(1-N)w$. This equivalence can be shown in another way by referring the motions to a system of coordinate axes that rotate about the center of the abradant with an angular velocity $w=W$. The abradant is at rest with respect to this system of rotating axes, and the specimen does not rotate but merely revolves with the angular velocity $(1-N)w$. Similarly, if the motions are referred to a system of coordinate axes that rotate about the center of the specimen with an angular velocity, $w=W$, then the specimen is at rest with respect to this system of rotating axes and the abradant does not rotate but merely revolves with the angular velocity $(1-N)w$. Thus uniform abrasion from every direction can be obtained by a simple motion of revolution of either the abradant or of the specimen.

8. Special Solution

The simplest motion, however, from mechanical consideration is the special solution when $N=0$. This solution corresponds to one where the specimen does not revolve about the center of the abradant, that is, the center of the specimen is at rest relative to the center of the abradant. The motions of abradant and of specimen consist merely of rotation in the same direction and with the

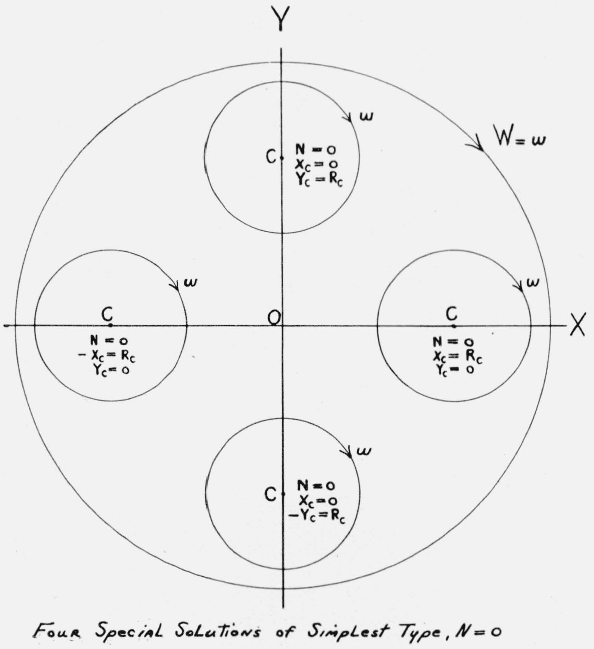

FIGURE 5.—*Schematic diagram showing four special solutions of simplest type, $N=0$.*

same angular velocity.[4] This arrangement is shown schematically in figure 5 for testing four specimens simultaneously. In this arrangement each specimen is abraded against the same area of the abradant in each rotation of the specimen and abradant.

9. Degenerate Solutions

The special solution when $N=1$ corresponds to one where the direction and angular velocity of revolution of the specimen about the center of the abradant are the same as the direction and angular velocity of rotation of both the abradant and the specimen. For this case the relative velocity between the specimen and the abradant is zero for every point of the specimen. In other words the specimen is at rest relative to the abradant and no abrasion is produced. This special case is a degenerate solution of the problem for uniform abrasion, namely no abrasion.

The special solution when $X_c=0$ and $Y_c=0$ corresponds to one where the abradant and the specimen are coaxial and the motion of revolution of the specimen vanishes as $|R_c|=0$. Because the abradant and the specimen rotate in the same direction and with the same angular velocity, the relative velocity is again zero for every point of the specimen, that is, the specimen is at rest relative to the abradant and no abrasion is produced. This special case is another degenerate solution of the problem for uniform abrasion, namely, no abrasion.

The special solution when $w=0$ corresponds to one in which both the abradant and the specimen are at rest and no abrasion is produced. It is still another degenerate solution of the problem for uniform abrasion, namely, no abrasion.

IV. Abrasion Machine, Special Solution $N=0$

1. Description

An abrasion machine based upon the special solution when $N=0$, that is, when the specimen does not revolve about the center of the abradant is shown in figure 6. It consists of a motor speed-reducer with a pulley attached to each end of a double shaft. The pulley at the left rotates the abradant, and the pulley at the right rotates the specimen. The direction and speed of rotation is the same for the abradant and the specimen. Several additional pulleys were provided so that

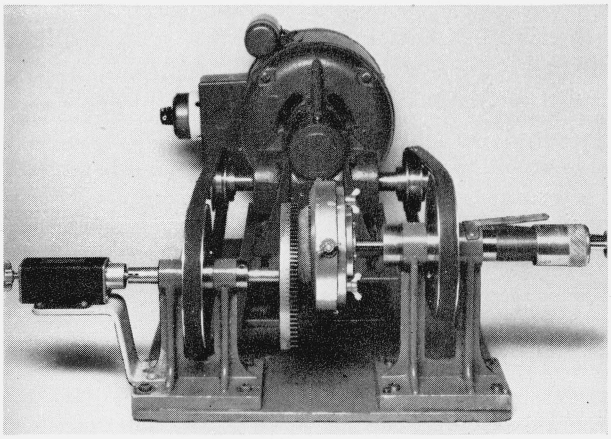

FIGURE 6.—*Abrasion machine based upon special solution of simplest type, $N=0$.*

[4] In the preparation of this paper it was discovered that Colonel Charles Dévé [9] had found experimentally, in polishing plane surfaces of glass, that uniform wear is obtained if the glass and abradant rotate in the same direction and with the same angular velocity, provided that no part of the glass surface extended beyond the boundary of the abradant. He showed, that if a small glass disk is placed off center on a rotating abradant and is held in a fixed position at its center by a pin that permits free rotation of the glass disk about this pin, then the glass disk commenced to rotate slowly in the same direction as the abradant and eventually attained the same angular velocity as the abradant. Intuitive reasoning led him to conclude that the system of free rotation on a rotating abradant is a system of uniform wear provided the glass disk does not extend beyond the boundary of the abradant.

Kessler [10] developed an abrasion machine for stone in which the abradant and the specimen were positively rotated in the same direction but at different angular velocities. If he had made the angular velocities equal, then he would have had a machine conforming to the special solution $N=0$ of the general solution presented in this paper.

tests could also be made with the speed of the abradant greater or less than that of the specimen.

The specimen can be mounted in a number of ways. The different steps of one method are shown in the foreground of figure 7, a, b, and c. In figure 7, a, a circular specimen and two parts of a clamp are shown in the left foreground. One part of the clamp is shown in place on a wooden holder with a slightly curved wooden hub projecting upward through the central opening. The circular specimen is placed over this hub and the second part of the clamp is securely fastened to the first part, as shown in the right foreground of figure 7, a. When the clamp is removed from the wooden holder, an outer annular ring, about 1 inch in width, of the specimen is securely held by the clamp and the central portion is lax and assumes a more or less spherical shape as shown in the foreground of figure 7, b. The clamp is then attached to the large disk shown in the right foreground. In doing this the small disk is forced against the lax specimen by the small spring that is compressed to a given amount. The lax specimen is thus placed under a given tension that is distributed over its entire periphery, as shown in the foreground of figure 7, c. When the shaft to which the specimen and clamp are attached is in place as shown in figure 6, then the spring shown in figure 7, c, can be compressed a known amount by the micrometer screw at the right end of the spring. The force of compression presses the specimen against the abradant. Both the tension in mounting the specimen and the force of compression can be controlled and varied in magnitude. If the specimen stretches during a test, the stretch is automatically taken up by the tensioning device. Similarly, as the thickness of the specimen decreases during the test, the change of thickness is automatically taken up by the pressure unit. The slight decreases in tension and pressure from these automatic adjustments appear to be negligible. The change in the latter can be compensated for by turning the micrometer head an amount equal to the decrease in thickness of the specimen.

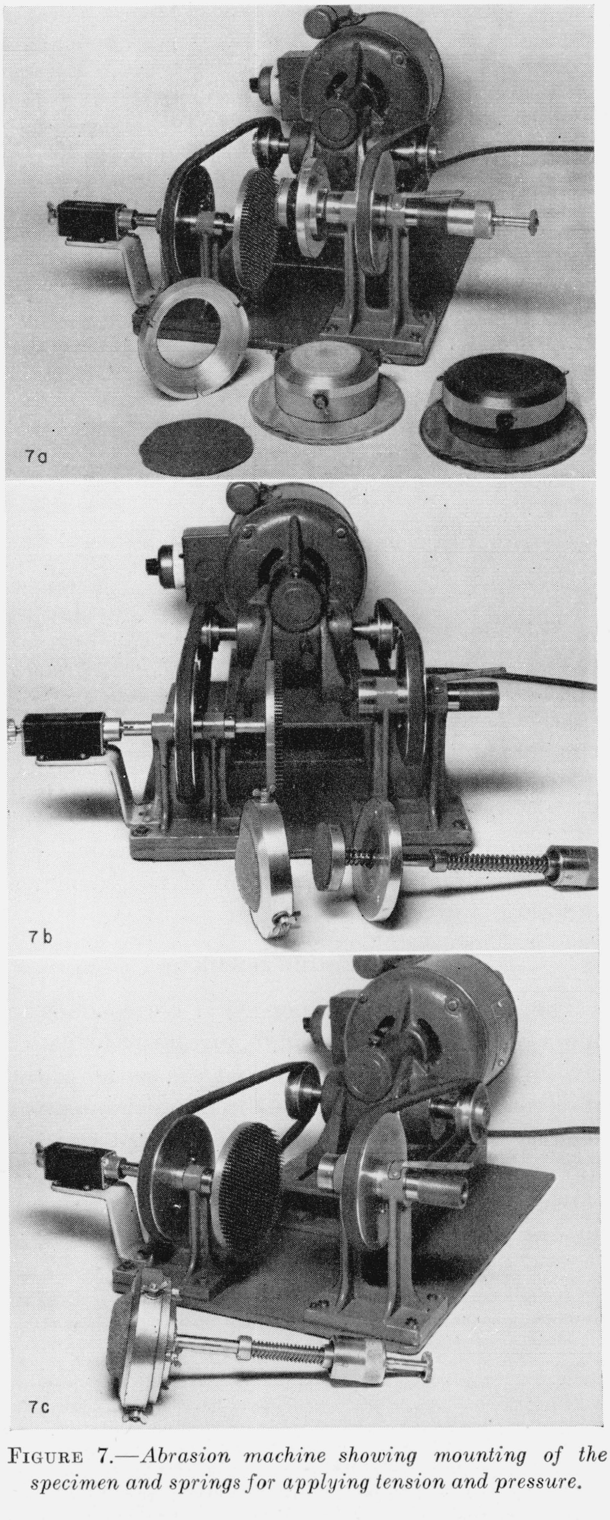

Figure 7.—*Abrasion machine showing mounting of the specimen and springs for applying tension and pressure.*

2. Abradant

Any of the usual abradants used in abrasion testing for textiles can be used with the machine. Most of these, however, change considerably during a test. In the development of an abradant that remained constant, many different abradants were considered. The abradant shown in figure 7, c, was finally decided upon as representing the one most likely to remain constant. It consists of a disk 4½ inches in diameter, in one face of which are inserted a series of parallel strips of clock springs, spaced ⅛ inch apart and each strip being 0.025 inch thick and protruding ¼ inch. The edges of these springs were ground and lapped to a plane surface, which constitutes the abradant for textile materials. From the tests made to date, this abradant appears to remain constant. Obviously it is not entirely uniform over its area.

3. Results of Preliminary Tests

Exploratory tests have been made with the abrasion machine on a great variety of different textile materials. By appropriate choice of total pressure between the specimen and the abradant, it was found that the materials tested could be abraded to destruction in a reasonable time. Tests were also made by rotating the specimen at a much higher or lower angular velocity than that of the abradant. For such tests the abrasion was not uniform over the abraded area, confirming the prediction of the mathematical solution. Uniform abrasion was obtained when the two angular velocities were equal. The rate of abrasion increased with the angular velocity.

A series of different felts were tested under a variety of different testing conditions. The specimens shown in figure 8 are of one felt. Specimen No. 1 is an untested specimen. Specimens 2, 3, 4, 5, and 6 were tested for 5,000, 10,000, 15,000, 20,000, and 25,000 rotations, respectively. They show that the abrasion over a circular area 2 inches in diameter is uniform. The decrease in thickness with the number of rotations appears to be a promising method for a quantitative index of abrasion for felts.

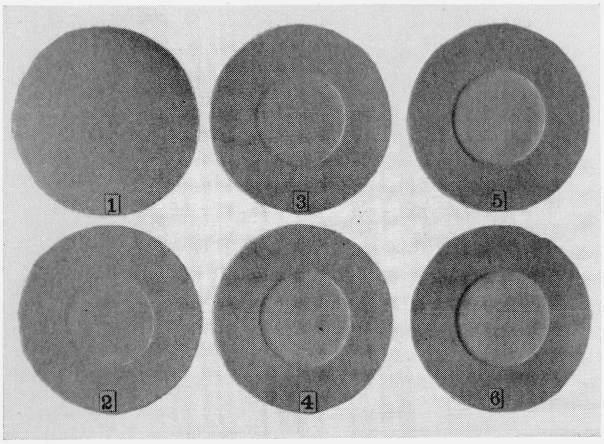

FIGURE 8.—*Felt specimens showing abrasion produced by the machine in increments of 5,000 rotations of the specimen.*

The fact that the abradant was not uniform over its entire area did not affect the uniformity of the abrasion. This result was not expected. A close check on the speeds indicated that, owing to a slight difference in the belt drive, the abradant made one more rotation than the specimen in every 240 rotations, or 1 minute. The effect of nonuniformity of the abradant was automatically eliminated by this slight difference in speed. As a matter of fact, as abradants are as a rule nonuniform, it is expedient to deliberately make the speed of rotation of the abradant slightly different from that of the specimen.

V. Conclusions

The advantages of abrasion machines based upon the mathematical solutions presented in this paper are many. Such machines are applicable not only when testing the resistance to abrasion of textiles, but also appear to be applicable to many organic and inorganic materials, as well as in grinding, lapping, and polishing of plane surfaces. The structural details of the machines and of the abradants would have to be designed and selected specifically for each type of material. The optimum testing technic and method of evaluation of the test results would also have to be worked out for each material.

The author is especially grateful to Philip Miller, who assisted in the preliminary experiments that led to the solution of this very interesting problem; to John Krasny, who has carried out tests on many different textiles and for this assistance in the development of special clamps; and to J. M. Blandford, who has prepared the illustrations for this paper.

VI. References

[1] H. S. Hall and E. R. Kaswell, Wear-resistance of apparel textiles: Literature survey, Textile Research J. **15,** 178 (1945).

[2] H. F. Schiefer, L. Fourt, and R. T. Kropf, Textile testing in Germany. To be published by Textile Research Institute.

[3] G. B. Haven, New Abrasion machine, Textile World **76,** 2654 (1929).

[4] J. G. Martindale, New design cloth wearing tester, J. Textile Inst. **33,** T151 (1942).

[5] H. F. Schiefer, Wear testing carpets, J. Research NBS **29,** 33 (1942) RP1505.

[6] F. M. Gavan, S. W. Eby, and C. C. Schrader, New sandpaper abrasion tester, ASTM Bul. No. 143, 23 (1946).

[7] P. A. Sigler and W. L. Holt, Simple abrasion test machine for rubber, India Rubber World **82,** 63 (1930).

[8] P. Larose, New form abrasion tester, Can. J. Research **18,** 161 (1940).

[9] Colonel Ch. Dévé, Guide de L'Ouvrier en verres d'optique de precision, Revue d'Optique Théorique et Instrumentale (Paris, 1930). (Translation by Thomas L. Tippell, Optical Workshop Principles, Adam Hilger Ltd., London, 1943).

[10] D. W. Kessler, Wear Resistance of natural stone, BS J. Research **11,** 635 (1933) RP612.

WASHINGTON, May 2, 1947