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A Study of the Damaging Effect of Fatigue Stressing on SAE X4130 Steel

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The damaging effect of fatigue stressing above the endurance limit was investigated with notched specimens of SAE X4130 steel. The damage was measured by the decrease in endurance at another stress. A deflection method for detecting the formation of a fatigue crack permitted the damage measurement to be limited chiefly to the precrack stage. The results showed that the apparent rate of damage depends on the stress history. If the prestress is higher than the test stress, the damage occurs rapidly at first, then more slowly. The reverse is true if the damaging stress is lower than that used to measure the damage.

Tests also were made with smooth specimens in an effort to determine the cumulative damage caused by fatigue at more than one stress. Two methods were developed for extrapolating to determine the point at which fatigue cracking starts, so the damaging cycle ratios could be based on the life of the specimens prior to cracking. Complete *S-N* curves were determined for specimens after each of eight different damaging treatments. With these curves a method of expressing damage was developed that permitted the direct addition of damage occurring at different stresses. The reliability of this method was checked by testing specimens after fatigue loading at two or more different stresses and comparing the results with predictions based on the addition of the indicated damage. The agreement was within the experimental error.

Contents

I. Introduction

Most ferrous metals exhibit a fatigue limit. If a sample of the metal is subjected to a pulsating stress greater than the fatigue limit, a crack will form that eventually will lead to fracture. Prior to formation of the first crack a change is taking place in the metal that is detectable only by means of a fatigue test. For example, if a specimen is subjected to an overstress for one-half the number of cycles required to cause fracture, the mechanical properties of the material, other than fatigue, appear to be unchanged. However, the fatigue properties, both the endurance at stresses above the fatigue limit and the fatigue limit itself, will be lowered.

The purpose of the work described in this report was to study some of the phenomena associated with this damaging process, particularly with a view to making the information more readily applicable to steel structures against failure under alternating loads.

Previous investigations of fatigue damage have dealt largely with smooth specimens, although the majority of fatigue failures in service propagate from points of stress concentration. Therefore, it was thought worthwhile to make a thorough study with notched specimens for comparison with the results with smooth specimens. This study comprised the first part of the present investigation, and the results are discussed in section IV of this report. The work was done under the auspices of and with financial assistance from the National Advisory Committee for Aeronautics, and has been described in a publication of this committee [1].[1]

A large proportion of the highly stressed machine elements in service probably are stressed above their fatigue limit, occasionally during their life. It is important, therefore, to know how much damage is done by this overstressing. As the rate of damage in a simple test is known [2] to be a complicated function of the stress history, it has not been possible to predict systematically the performance of material in service where the stress variation follows a random schedule. The second part of this investigation dealt with the study of the damage caused by combinations of different ranges of stress. It was hoped that the laboratory tests could be made to conform more closely to the random fluctuation of stress usually encountered in service by making the loading schedule more and more complicated. The results of this part of the investigation are described in section V of this report.

[1] Figures in brackets indicate the literature references at the end of this paper.

II. Materials and Specimens

All the fatigue tests included in this report were made on R. R. Moore machines. The specimens were of normalized SAE X 4130 steel. A different heat was used for each part of the investigation. The mechanical properties of the steels are listed in table 1.

TABLE 1.—*Mechanical properties of the steels*

	M 390 used for notched specimens	M 357 used for smooth specimens
Yield strength, 0.2-percent offset, lb/in.^{2}	62,500	63,000
Ultimate tensile strength, lb/in.^{2}	104,200	98,000
Elongation in 1 in., percent	27	29
Reduction of area, percent	57	60
Rockwell hardness, B scale	90	89

The notched specimens used in the tests described in section IV had a cylindrical test section 0.35 in. in diameter, into which was cut a semicircular circumferential notch 0.05 in. deep and 0.05 in. in radius. The stresses listed for these specimens were calculated for the smallest section, 0.25 in. in diameter, without regard to stress concentration. The notches were finished with a copper wire, slightly smaller in diameter than the notch, charged with a slurry of No. 302 emery in water. The specimens were rotated slowly in a lathe while the wire, held at right angles to the axis of the specimen, was rotated rapidly. Thus the fine polishing scratches at the bottom of the groove were substantially parallel to the axis of the specimen.

The smooth specimens used in the tests described in section V had a minimum section 0.25 in. in diameter, with a contour radius of 9⅞ in. They were polished with Aloxite polishing paper belts mounted on a rubber drum 5½ in. in diameter. The specimens were mechanically rotated about 15 rpm while being held lightly against the rotating drum, the axis of the specimen being at right angles to the axis of the drum. The drum speed was 1,750 rpm, and the specimens usually could be polished satisfactorily in less than 1 minute. The only precaution necessary was to avoid heavy pressure of the specimen against the abrasive. This polishing method has proved economical in preparing standard R. R. Moore specimens. Although the resulting surface is not highly polished, the results obtained with these specimens are as consistent as those with a higher polish. All the specimens were examined with a low-power microscope to make sure that none with circumferential scratches were tested.

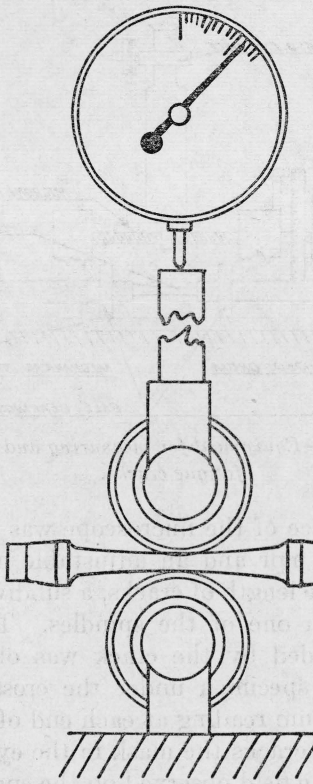

FIGURE 1.—*Specimen comparator.*

The minimum diameter of the specimens was measured, with a probable error of about 0.0001 in. For the notched specimens this was done with the aid of calibrated gage wires slightly less than 0.1 in. in diameter placed in the groove, the distance between the outsides of the wires being measured with micrometer calipers. The smooth specimens were measured with a vertical comparator using a 1/10,000-in. dial gage. A sketch of this instrument is shown in figure 1. In place of the usual anvils, contact with the specimen was made with two ball bearings about 1⅝-in. outside diameter. The specimen was placed between the bearings with the axis of the specimen at right angles to those of the bearings. In this way the specimen could be moved back and forth parallel to its length to locate the minimum section without danger of scratching the surface.

For some of the work described in section V it was necessary to have a reference mark at the minimum section of the specimens. To locate this point, a light source was set up so that an edge of the projected beam coincided with the line between the centers of the comparator bearings. Thus when a minimum reading was obtained on the dial, the edge of the beam passed across the minimum section of the specimen. A reference mark was made at this position with a very small drop of rubber cement placed on the specimen with a fine wire.

III. Testing Machines and Special Apparatus

The R. R. Moore machines used in this investigation were equipped with ball-bearing spindles, and all tests were run at 3,600 rpm. A variable autotransformer was used to reduce the speed of the 10,000-rpm motors with which some of the machines were equipped.

1. Deflection Measuring Micrometer

This has been described in detail in a previous publication [3]. Briefly, the upper one of a pair of contacts was fastened to the specimen end of a bearing housing on the R. R. Moore machine, the lower contact being carried on a micrometer screw mounted on the bed of the machine. The contacts operated a signalling device through a tube circuit, so the position of the upper contact could be determined by raising the lower contact and noting the reading of the micrometer drum when the circuit closed. The contacts were set slightly apart so that any increase in the deflection of the specimen during test would close the circuit, operating the signal. The best results with this apparatus were obtained by mounting the fatigue-testing machine on springs to minimize vibrations from extraneous sources. No specimen was used that caused excessive vibration when the machine was running. With these precautions, it was possible to detect a deflection of 0.001 to 0.002 mm.

2. Stroboscopic Viewing Device

One of a pair of contacts was mounted in a Bakelite disk and fitted to the shaft of the R. R. Moore machine, the other (stationary) contact being mounted so as to be rotatable about the center line of the machine. These contacts were used to flash a stroboscope, and by adjusting the stationary contact circumferentially the flash could be made to occur at any position in the revolution of the specimen. The flashing light was directed, by means of a condenser and mirror, onto the bottom surface of the specimen, and a low-power microscope fitted with a prism in front of of the objective was used to view the illuminated area. Because this area was stressed in tension, the fatigue cracks were opened and made readily visible.

3. Apparatus for Counting and Measuring Fatigue Cracks

A traveling microscope was used for this work, figure 2. The specimen was removed from the fatigue machine and mounted in spindles on the stage so that it was parallel to the traversing screw of the microscope. Weights hung on the ends of the spindles produced a bending moment which stressed the upper surface in tension. The area being examined was illuminated by an intense beam incident at a grazing angle in the plane of the axes of the specimen and microscope. As the scratches seen under the microscope were also in this plane, little light was reflected into the microscope. A fatigue crack, being at right angles to the incident beam, showed up brightly against the generally dark background.

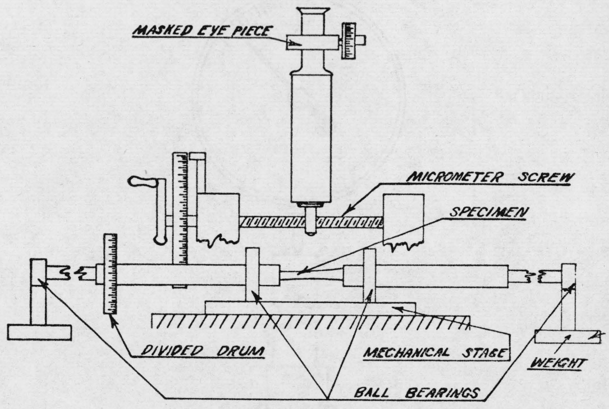

FIGURE 2.—*Equipment for measuring and counting fatigue cracks.*

The eyepiece of the microscope was fitted with both a cross hair and an adjustable mask. For measuring the length of cracks, a subdivided drum was fitted on one of the spindles. The central angle subtended by the crack was obtained by rotating the specimen under the cross hair and noting the drum reading at each end of the crack. For counting cracks the mask in the eyepiece was set so that the field observed on the specimen was just one millimeter wide (the edges of the mask were at right angles to the length of the specimen). The surface could then be scanned in 1 mm wide bands, starting from the reference mark at the minimum section, by traversing the microscope 1 mm after each revolution of the specimen. A ratchet was set to give a click to indicate the completion of each revolution.

IV. Test Results with Notched Specimens

Previous work, both at this Bureau and elsewhere, revealed two significant factors for satisfactorily evaluating fatigue damage. First, the dispersion of the experimental results necessitates an average of about eight specimens to give a sufficiently precise result for any one condition. Second, the evaluation of damage by running tests to complete fracture is limited to cycle ratios [2] sufficiently small so that the chance of crack formation during prestressing is negligible; otherwise, the damage in some cases would be dependent only on the size of an initial fatigue crack rather than on the effect of prestressing.

The methods followed in the study of fatigue damage with notched specimens were chosen on the basis of these considerations. In order to guard against crack formation it was necessary to have a means of detecting fatigue cracks at an

[2] "Cycle ratio" is defined as the ratio of the number of cycles run at a given overstress to the number necessary to cause failure at that stress.

early stage without stopping the test. Such a method was developed based on the fact that the deflection of a rotating beam specimen under constant load increases when a crack forms. The apparatus and technic used to detect and measure this deflection are described in section III of this report. The deflection is a function of the size of crack, and the criterion of failure chosen was a deflection of 0.005 mm, corresponding to a crack area about 12-percent of the original cross section [3]. The specimens were actually run until the deflection had increased to 0.01 mm in order to make certain that the deflection was due to a crack, but the data in this section are based on the number of cycles run at the time the specimen deflected 0.005 mm. This number is referred to as N_c.

Tests were conducted in the conventional manner (to fracture) on a group of notched specimens to determine the *S–N* curve and the fatigue limit. Four stresses in the range between the fatigue limit ($\pm$39,000 lb/in.^{2} for notched specimens) and the yield strength of the steel (62,500 lb/in.^{2}) were used in the investigation. The four stresses chosen were $\pm$42,000, $\pm$48,000, $\pm$54,000, and $\pm$60,000 lb/in.^{2}

Tests were made to determine N_c at each of these four stresses. At least 10 specimens were tested at each stress, and the median of each group was used as the value of endurance at that stress. The median was considered to be more representative of a group of endurance values than the average, as it is less affected by an occasional excessively divergent value. Also the median can be determined in cases where some elements of the group cannot be expressed numerically but are known to lie in a certain range. For example, in determining the endurance at a given stress, if only one specimen of the group did not fail, the average endurance of the group would be infinite, whereas the median value would be finite.

As the scatter of the results was not uniform in the four groups, it was necessary to test more specimens in some cases than in others in order to obtain approximately the same precision for each value of N_c. The number of tests in each group ranged from 11 to 17.

The values of N_c (the number of cycles to form a crack of definite size) for each selected stress are listed in table 2.

In the next stage of the investigation each specimen was first stressed for a predetermined number of cycles at one of the four chosen stresses, then the stress was changed as given in table 3 and the number of cycles to failure determined. The difference between this value and the median (see footnote 3) value of endurance for the original material at the same stress (table 2) gave a measure of the damage caused by the prestressing. This difference was expressed as a percentage of the undamaged endurance.

For each of the combinations in table 3, tests were made with the prestressing carried to 10, 25, 50, 75, and 90 percent of the median value of N_c. The number of tests made for each prestress condition was between 6 and 10, depending on the scatter of the individual values. The median values for each group are listed in table 4.[3] The variation of damage with percentage of prestress is shown graphically in figures 3 and 4 for a test stress of $\pm$48,000 lb/in.^{2}, and in figures 5 and 6 for a test stress of $\pm$54,000 lb/in.^{2} In the graphs the broken lines join the upper and lower quartile points of each group of values and thus give an indication of the scatter of the data. The curves from figures 3 to 6 are combined in figure 7 for comparison.

In the test groups in which prestressing was carried to 75 and 90 percent of the endurance, some specimens failed before the prestressing was completed, that is, the damage was greater than 100-percent. The inclusion of these results caused no difficulty in determining the median, although it did make the comparison of precision between the groups more uncertain.

While the results of the damage tests as shown in figure 7 are not directly comparable with those obtained with smooth specimens [1, 2], there are certain similarities. The tendency with both types of specimen was for the curve representing a prestress above the test stress to lie above the 45° line. The opposite tendency was noted when the magnitude of the prestress was below that of the test stress. (The 45° line may be considered as the damage curve for a prestress equal to the test stress.)

[3] The middle number of a series arranged in order of magnitude is the median. The middle one of the numbers that lie below the median is the lower quartile, the middle one of those above is the upper quartile. Half the difference between the upper and lower quartiles is the semi-interquartile (SIQ) range. This is a measure of the dispersion of the individual values which expresses the same limits relative to the median as the probable error expresses relative to the average.

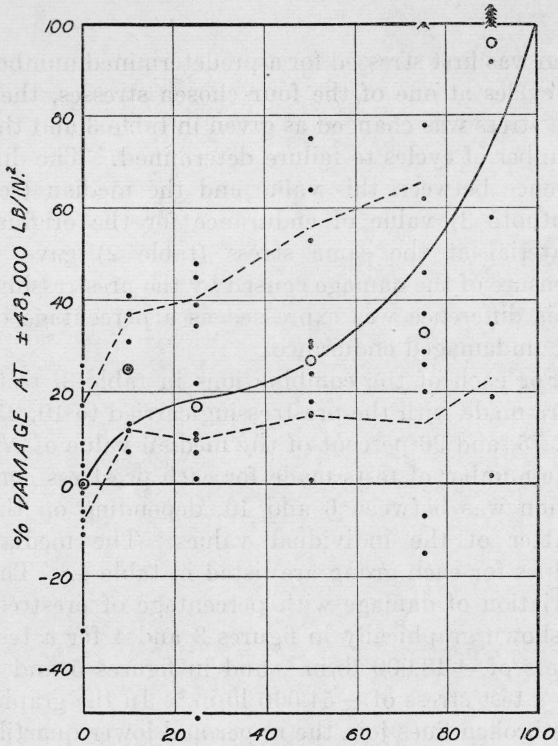

FIGURE 3.—*Damage at ±48,000 lb/in.² due to stressing at ±42,000 lb/in.²*

Notched specimens. Cycle ratios and damage based on N_e.

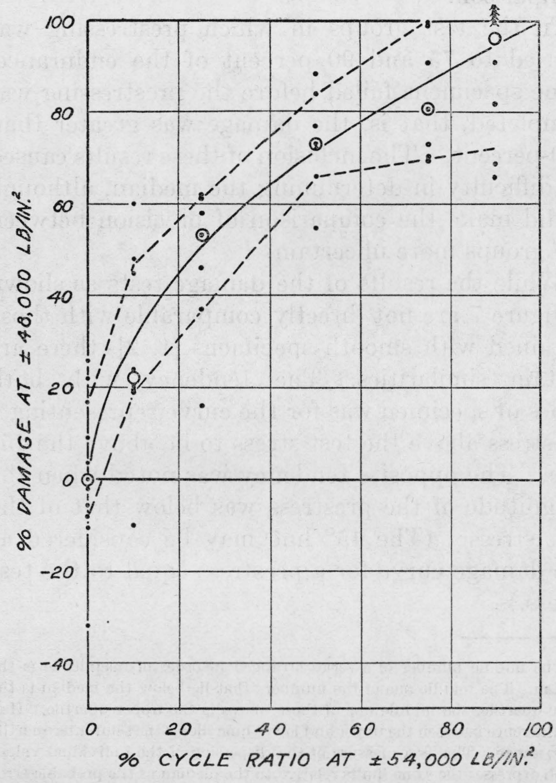

FIGURE 4.—*Damage at ±48,000 lb/in.² due to stressing at ±54,000 lb/in.²*

Notched specimens. Cycle ratios and damage based on N_e.

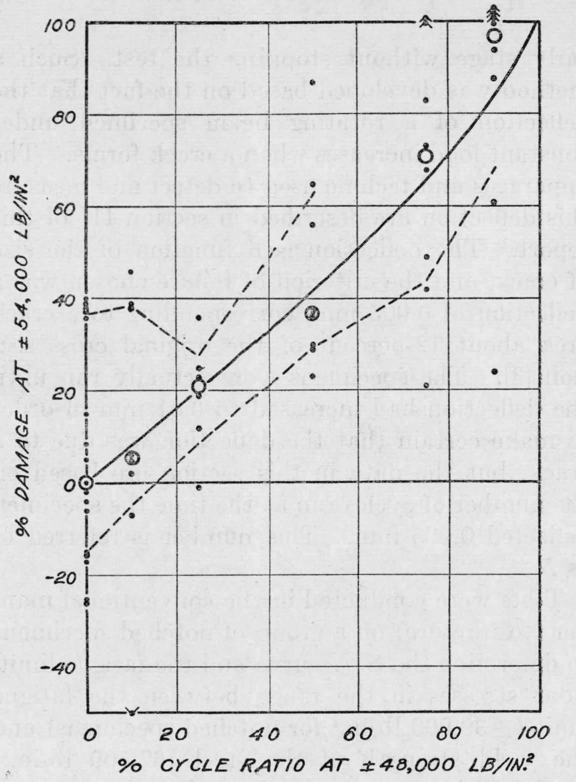

FIGURE 5.—*Damage at ±54,000 lb/in.² due to stressing at ±48,000 lb/in.²*

Notched specimens. Cycle ratios and damage based on N_e.

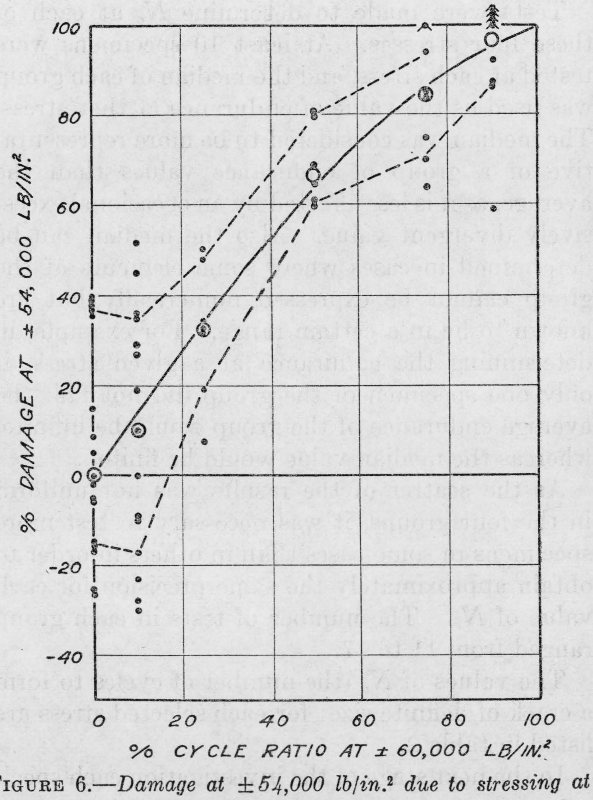

FIGURE 6.—*Damage at ±54,000 lb/in.² due to stressing at ±60,000 lb/in.²*

Notched specimens. Cycle ratios and damage based on N_e.

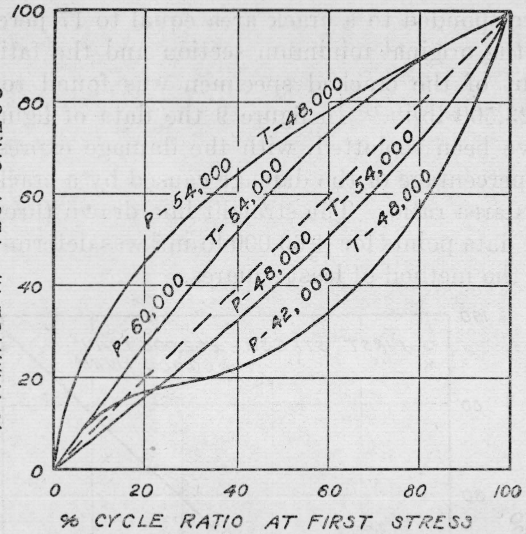

FIGURE 7.—*Damage as measured by loss of endurance* (N_c).

Curves from figures 3 to 6, notched specimens. P=Prestress in pounds per square inch. T=Test stress in pounds per square inch.

TABLE 2.—*Endurance of notched specimens*

Stress, lb/in.²	Number of cycles to crack
±42,000	963,000
±48,000	264,000
±54,000	93,000
±60,000	44,000

TABLE 3.—*Damage test conditions*

Prestress, lb/in.²	Test stress lb/in.²
±42,000	±48,000
±54,000	
±48,000	±54,000
±60,000	

In order to provide a basis of comparison with the results of fatigue damage reported by others (usually based on the fatigue limit), some tests of this type were conducted with two values of prestress, ±42,000 and ±60,000 lb/in.² The effect of various amounts of prestressing on the fatigue limit subsequently determined by fracture tests is shown in table 5 and figure 8. The number of cycles of prestress used for the 75-percent and 90-percent cycle ratios was the same as the number used in the previous phase of the investigation, but a correction was made in determining the cycle ratios represented by these amounts of prestressing. The method of making this correction will be shown for the case of the highest cycle ratio at ±42,000 lb/in.²

TABLE 4.—*Effect of prestress on endurance of notched specimens of SAE X4130 steel stressed as rotating beams*

Number of specimens	Test stress	Prestress	Cycle ratio, percent of N_c value	Endurance, 10³ cycles	Damage	SIQ [1]
	lb/in.²	*lb/in.²*			%	%
17	±42,000	--------	0	963	0	25
11	±48,000	--------	0	264	0	13
7	±48,000	±42,000	10	198	25	12.5
10	±48,000	±42,000	25	220	16.7	18
10	±48,000	±42,000	50	192	27.3	20
10	±48,000	±42,000	75	177	32.9	26
10	±48,000	±42,000	90	11	95.8	----
8	±48,000	±54,000	10	205	22.3	14
7	±48,000	±54,000	25	125	52.6	12
7	±48,000	±54,000	50	72	72.7	9
7	±48,000	±54,000	75	50	81.1	15
8	±48,000	±54,000	90	12	95.5	----
17	±54,000	--------	0	93	0	26
8	±54,000	±48,000	10	87	6.4	23
6	±54,000	±48,000	25	73	21.5	10
8	±54,000	±48,000	50	59	36.6	16
8	±54,000	±48,000	75	27	71	----
8	±54,000	±48,000	90	3	96.8	----
15	±54,000	±60,000	10	84	9.7	21
7	±54,000	±60,000	25	63	32.2	14
6	±54,000	±60,000	50	31	66.7	10
8	±54,000	±60,000	75	15	83.9	12
8	±54,000	±60,000	90	3	96.8	----
15	±60,000	--------	0	44	0	23

[1] SIQ=Semi-interquartile range.

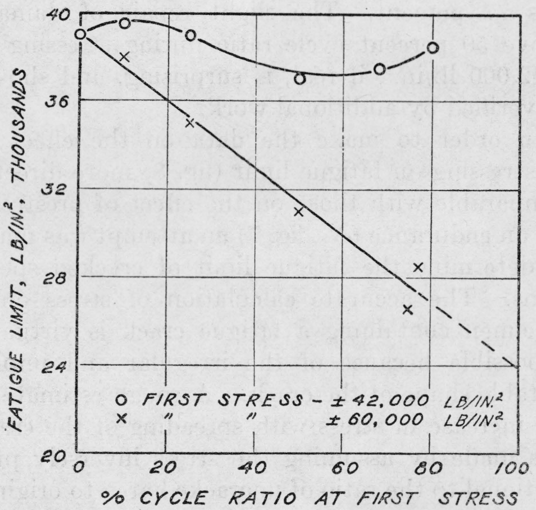

FIGURE 8.—*Effect of prestress on fatigue limit.*

Notched specimens.

TABLE 5.—*Effect of prestress on fatigue limit of notched specimens of SAE X4130 steel stressed as rotating beams*

Prestress ±42,000 lb/in.²			Prestress ±60,000 lb/in.²		
Cycle ratio	Fatigue limit	Damage, percentage of maximum	Cycle ratio	Fatigue limit	Damage, percentage of maximum
%	*lb/in.²*		%	*lb/in.²*	
10	±39, 500	−3	10	±38, 000	6
25	±39, 000	0	25	±35, 000	26
50	±37, 000	13	50	±31, 000	52
68	±37, 500	10	75	±26, 500	81
80	±38, 500	3	77	±28. 500	68

The median N_c for this stress was 963×10^3. When it was desired to prestress a group of specimens to 90 percent of this value, or 867×10^3 cycles, some of the specimens failed before prestressing was completed. Only those which did not fail in prestressing could be used for the determination of fatigue limit, so this determination was not truly representative. Therefore, in the original series of values of N_c, the median of all values greater than 867×10^3 was taken as being representative of the specimens which were used to determine the fatigue limit after prestressing. The cycle ratio as plotted in figure 8 is based on this second median ($1{,}057\times10^3$ cycles), and is consequently less than 90 percent. A similar correction was made for the 75 percent prestress at ±42,000 lb/in.², and for the 90 percent value at ±60,000 lb/in.² The corrected values of cycle ratio at ±42,000 lb/in.² were 68 percent and 80 percent; at ±60,000 lb/in.² the highest value was 77 percent. The slight repair of damage above 50 percent cycle ratio during stressing at ±42,000 lb/in.², if real, is surprising, and should be verified by additional work.

In order to make the data on the effect of prestressing on fatigue limit (fig. 8) more directly comparable with those on the effect of prestressing on endurance (N_c, fig. 7) an attempt was made to determine the fatigue limit of cracked specimens. The accurate calculation of stress on a specimen containing a fatigue crack is virtually impossible because of the irregular and unpredictable shape of the crack. A rough estimate of the increase in stress with spreading of the crack was made by assuming the stress inversely proportional to the ratio of uncracked area to original area.

The specimens used for this determination had deflected 0.010 mm in the fatigue test. This corresponded to a crack area equal to 17 percent of the original minimum section and the fatigue limit of the cracked specimen was found to be ±23,500 lb/in.² In figure 9 the data of figure 8 have been replotted, with the damage expressed as percentage of the damage caused by a crack of this area ratio. The straight line drawn through the data points for ±60,000 lb/in.² was determined by the method of least squares.

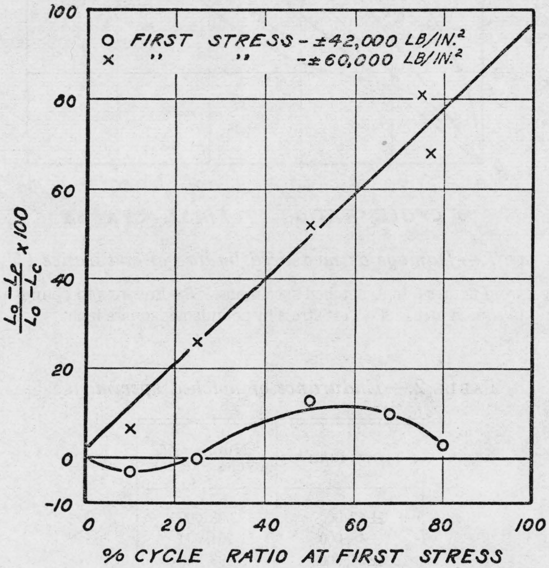

FIGURE 9.—*Damage as measured by loss of fatigue limit, notched specimens.*

L_o=fatigue limit of original specimen. L_p=Fatigue limit of prestressed specimen. L_c=fatigue limit of cracked specimen.

Figure 10 shows the *S-N* curves for specimens with three types of stress concentration: smooth (zero concentration), notched, and cracked (maximum concentration). The points marked *X* or + in figure 10 are the results from cracked specimens having crack areas other than 17 percent. It will be noted that the value for a specimen having an area ratio of 50 percent lies very far above the 17 percent ratio curves, while the values for specimens having a 12 percent ratio are in the same range or lower. This suggests that the stress concentration at high stresses is less for large than for small cracks. More experimental work would be required before any general statement could be made with certainty.

In table 6 are shown the values of stress concentration factors given by the ratios of the fatigue limits of the specimens under three conditions of stress concentration. Almen [4] has shown that the slope of the falling part of the *S-N* curve increases with increasing stress concentration. If the slope of the falling part of the *S-N* curve is a measure of effective stress concentration, then the ratios of the slopes of the three lines of figure 10 should give the same values for these factors as obtained from the fatigue limits. There was substantial agreement of these factors as determined by the two methods (table 6). The value of theoretical stress concentration factor given in table 6 for the notched specimen was calculated from a table given in reference [5].

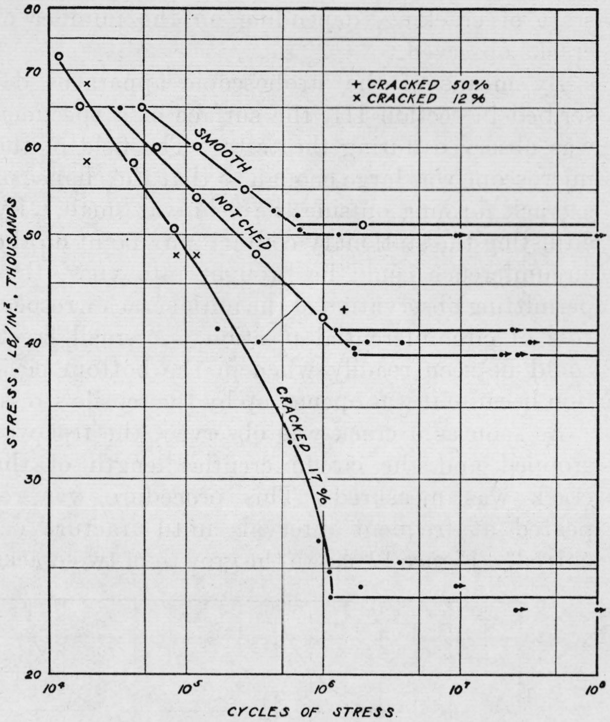

Figure 10.—*S-N curves for SAE X4130 steel under three conditions of stress concentration.*

Large open points represent the median values from a group of specimens tested at the same stress. Small closed points are the results of individual tests.

Table 6.—*Stress concentration factors in specimens of SAE X4130 steel stressed as rotating beams*

Type of specimen	Fatigue limit	SCF[1] from fatigue limit	Slope of *S-N* curve	SCF from slope	Theoretical SCF
	lb/in.²				
Smooth	±50,000	1.0	−0.767	1.0	1.0
Notched	±39,000	1.28	−.983	1.28	1.59
Cracked (17%)	±23,500	2.13	−1.46	1.91	

[1] SCF=Stress-concentration factor.

V. Test Results With Smooth Specimens

1. Methods of Estimating the Start of Cracking

The preceding experiments showed that fatigue damage results are more reproducible when based on the number of cycles to form a small crack than when based on the number of cycles to fracture. It was considered that even greater reproducibility might be obtained if it were possible to determine the number of cycles at which a crack first starts. Then the damaging cycle ratios could be based on the precrack stage only. This was done in the experiments described below, all tests being made on smooth (9⅞-in. contour radius) specimens.

In the discussion of methods for estimating the start of cracking, the relationship shown in figure 11 should be kept in mind. This shows a semilogarithmic plot of test stress against the number of cracks formed prior to fracture. The points plotted are based only on cracks lying in a band 4 mm wide at the center of the specimen. The number of cracks increased rapidly as the stress was raised above the fatigue limit. Because of

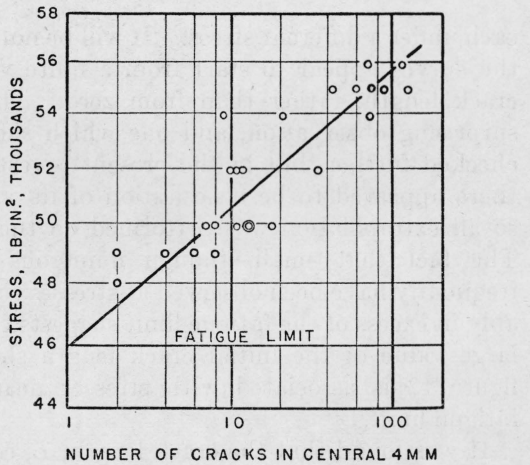

Figure 11.—*Influence of applied stress on the number of cracks formed prior to fracture.*

the large variation in the number of cracks formed, two different methods were used for estimating the start of cracking, depending on the number of cracks observed.

By means of the stroboscopic apparatus described in section III, the surface of a specimen was observed during the tests. The field of the microscope was large enough so that the chance of a crack forming outside the field was small. By adjusting the stationary contact, any point on the circumference could be brought into view, thus permitting observation of the initial crack irrespective of circumferential position. A small crack could be seen readily when in the bottom position because it was opened up by the tensile stress.

As soon as a crack was observed, the test was stopped and the circumferential length of the crack was measured. This procedure was repeated at frequent intervals until fracture occurred. Figure 12 shows the growth of two cracks

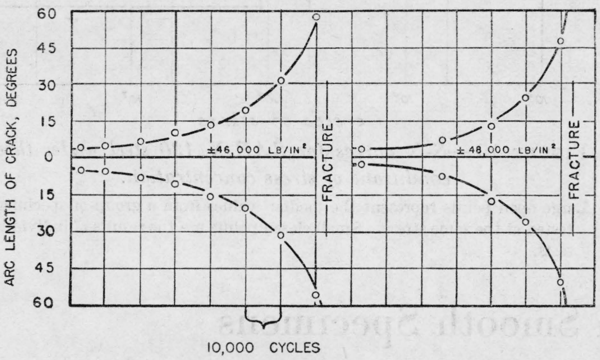

Figure 12.—*Curves showing change of crack length at each of two different stresses.*

The midpoint of the crack when first measured is taken as zero.

each under a different stress. It will be noted that the curves appear to start from a finite value of crack length, rather than from zero. This is a surprising observation, and one which should be checked further, but in the present investigation there appeared to be no question of its validity, so all extrapolation was performed on this basis. The fact that small, though numerous cracks frequently have been observed at stresses considerably in excess of the fatigue limit suggests that the large value of the initial crack length shown in figure 12 is associated with stresses nearer the fatigue limit.

It was found that the crack length, L, could be expressed by the equation

$$\log (L-C)=\alpha N, \tag{1}$$

where C and α are constants, the latter dependent on stress; N is the number of cycles, and A the logarithmic base. This is shown graphically in figure 13 where $(L-C)$ is plotted logarithmically against N for one of the curves shown in figure 12. When curves similar to figure 13 were plotted for various nominal stress values, it was found that the slope was a simple function of the stress, as shown in figure 14.

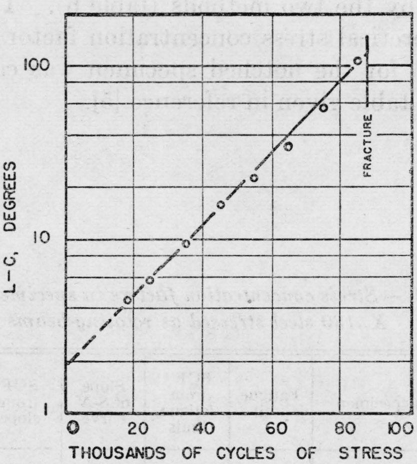

Figure 13.—*Semilogarithmic plot of crack growth curve.*

The ordinate is the arc length (L) of the crack minus a constant (C).

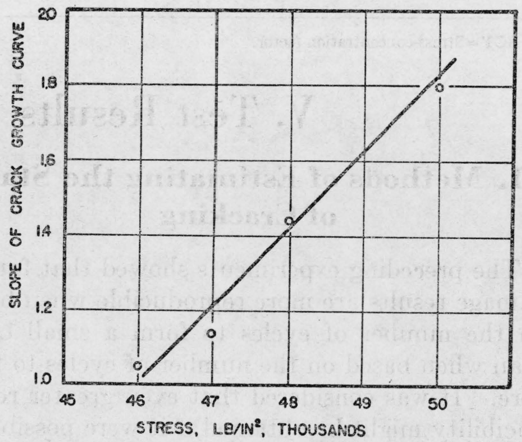

Figure 14.—*Influence of stress on the rate of crack growth.*

Curves of the type shown in figure 13 were used to determine by extrapolation the point at which cracking starts, assuming that the cracks grow exponentially from an original arc length of about 6°, as indicated in figure 12. The deflection measuring apparatus described in section III of this report was used to indicate the presence of a crack, but this did not operate until the crack reached 50° or 60° in the circumferential length, the growth curves (fig. 13) being used to extrapolate back to the start of the crack.

The above method was used only in cases where the total number of cracks was not more than five, since more numerous cracks might change the surface stress sufficiently to invalidate the extrapolation. For specimens which had numerous cracks the extrapolation was performed as follows: By means of the masked eyepiece described in section III, the cracks were counted in circumferential bands 1 mm wide. Practically all the cracks occurred within 6 mm of the center of the specimen and for the purpose of totaling the number of cracks this area was divided into three zones. The first extended 2 mm each side of the center, the second from 2 to 4 mm on each side and the third 4 to 6 mm. The average stress in each zone was calculated from the geometry of the specimen and found to be 99.8, 98.8, and 97.0 percent of the stress at the center for the first, second, and third zones, respectively. The square root of the number of cracks in a band plotted against the average stress in that band, gave a nearly linear relationship which could be extrapolated back to the stress at which no cracks would be formed. Curves of this type are shown in figure 15.

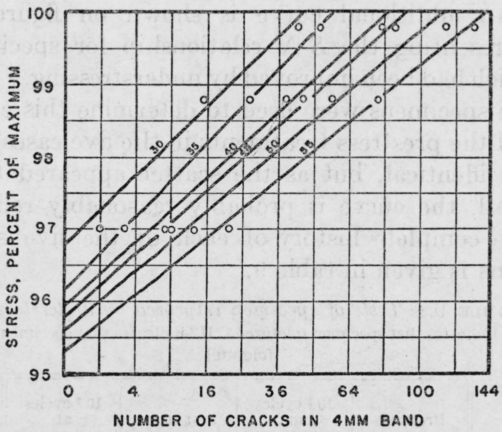

FIGURE 15.—*Results of crack counts on one smooth specimen at various stages between the beginning of cracking and fracture.*

Numbers on curves represent thousands of cycles of stress which the specimen had run when the cracks were counted. Total fracture occured at 59,000 cycles.

Points obtained for the start of cracking by these two methods of extrapolation (figs. 13 and 15) are shown on figure 16, both the stress and number of cycles being plotted logarithmically. The straight line through the points was determined by the method of least squares. A similar graph based on the points representing fracture is also included in this figure (the points for this curve are not shown).

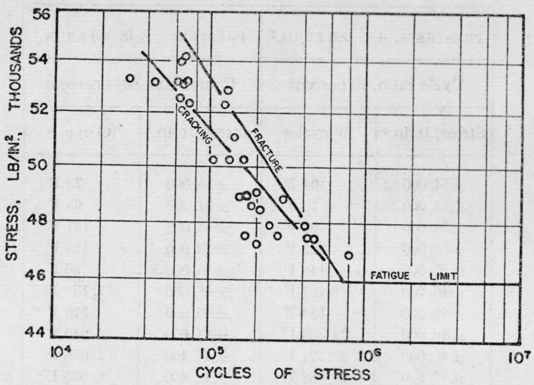

FIGURE 16.—*S-N curves for start of cracking and for fracture in normalized SAE X4130 steel.*

The experimental points are for cracking only.

2. Damage Tests, Including Strength-Loss Line Hypothesis

In the next phase of the work, two stresses were chosen between the fatigue limit and the yield strength of the material, and specimens were stressed for various cycle ratios at these stresses. The stresses chosen were ±48,000 and ±54,000 lb/in.², and the cycle ratios were based on the number of cycles to the start of cracking.[4] Eight prestress conditions were used, as shown in table 7.

TABLE 7.—*Prestress conditions*

Stress	Cycle ratio, percentage of N_c
lb/in.²	
±48,000	10
	33⅓
	66⅔
	90
±54,000	10
	33⅓
	66⅔
	90

The prestressed specimens were used to determine a complete *S-N* curve for each condition, a typical curve of this type being shown in figure 17. This method has been used recently by Kommers [2] to measure fatigue damage. These curves were determined in the usual manner, that is by tests to fracture. The experimental data are

[4] The term N_c will be used to designate the number of cycles to the start of cracking. This value is determined from the cracking curve of figure 16.

TABLE 8.—*Results of damage tests*

[Cycle ratios are based on N_e. "F" indicates that specimen fractured, "U" indicates that the specimen did not break.]

PRESTRESS, ±48,000 LB/IN.²		PRESTRESS, ±48,000 LB/IN.²	
Cycle ratio, 10 percent		Cycle ratio, 66⅔ percent	
Stress, lb/in.²	10³ cycles	Stress, lb/in.²	10³ cycles
±54, 000	102 F	±54, 000	73 F
±54, 000	70 F	±54, 000	63 F
±54, 000	96 F	±54, 000	131 F
±52, 000	225 F	±52, 000	129 F
±50, 000	146 F	±50, 000	96 F
±46, 200	1, 187 F	±46, 000	1, 131 F
±46, 200	514 F	±46, 000	516 F
±46, 000	22, 339 U	±44, 600	240 F
±46, 000	27, 721 U	±44, 400	1, 038 F
±45, 800	44, 988 U	±44, 400	20, 635 U
		±44, 200	36, 233 U
Cycle ratio, 33⅓ percent		Cycle ratio, 90 percent	
±54, 000	61 F	±54, 000	99 F
±54, 000	81 F	±54, 000	24 F
±54, 000	142 F	±54, 000	58 F
±52, 000	86 F	±52, 000	112 F
±50, 000	141 F	±50, 000	111 F
±47, 000	313 F	±46, 000	622 F
±46, 000	743 F	±44, 000	132 F
±45, 300	1, 608 F	±42, 600	410 F
±45, 100	22, 711 U	±42, 400	416 F
±45, 000	831 F	±42, 200	34, 516 U
±44, 900	40, 787 U	±42, 000	17, 709 U
±44, 800	21, 942 U		
±44, 500	20, 027 U		

PRESTRESS, ±54,000 LB/IN.²		PRESTRESS, ±48,000 LB/IN.²	
Cycle ratio, 10 percent		Cycle ratio, 66⅔ percent	
Stress, lb/in.²	10³ cycles	Stress, lb/in.²	10³ cycles
±52, 000	110 F	±52, 000	69 F
±50, 000	318 F	±50, 000	186 F
±48, 000	720 F	±48, 000	132 F
±45, 800	2, 003 F	±48, 000	86 F
±45, 800	834 F	±46, 000	454 F
±45, 600	33, 016 U	±44, 000	19, 227 U
±45, 600	428 F	±44, 000	19, 133 U
±45, 400	21, 033 U	±43, 000	18, 986 U
		±42, 000	459 F
		±40, 400	16, 101 U
Cycle ratio, 33⅓ percent		Cycle ratio, 90 percent	
±52, 000	79 F	±59, 000	12 F
±50, 000	109 F	±52, 000	54 F
±48, 000	267 F	±50, 000	91 F
±46, 000	1, 276 F	±48, 000	128 F
±45, 600	327 F	±45, 000	641 F
±44, 000	853 F	±44, 000	162 F
±43, 800	23, 882 U	±43, 000	25, 304 U
±43, 800	19, 229 U	±42, 000	144 F
±43, 600	21, 451 U	±42, 000	16, 009 U
		±41, 000	575 F
		±40, 400	176 F
		±40, 200	249 F
		±40, 000	21, 338 U
		±40, 000	11, 643 U

shown in table 8. The curves for the eight prestress conditions are shown in figures 18 and 19. The cycle ratio for the largest number of cycles should be less than 90 percent because of the specimens which cracked before reaching the predetermined number of cycles. However, unlike the tests described in section IV, these results could not be accurately corrected for the premature failures, so no attempt was made to do so.

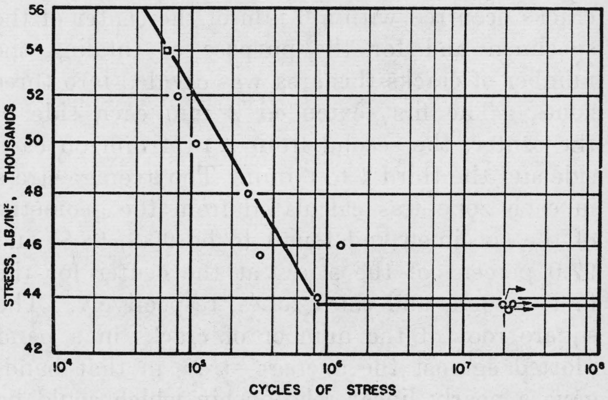

FIGURE 17.—*S-N curve for specimens that had been subjected to 33⅓ percent cycle ratio at ±54,000 lb/in.²*

The square point is at the stress used for prestressing and the number of cycles obtained by subtracting the prestress cycles from expected life at this stress.

An additional curve is shown on figure 18, representing the *S-N* relationship for specimens which had been improved by understressing. Only five specimens were used to determine this curve, and the prestress treatments in the five cases were not identical, but as the scatter appeared to be small, the curve is probably reasonably reliable. The complete history of each of the five specimens is given in table 9.

TABLE 9.—*Tests of specimen improved by understressing*

["F" indicates that specimen fractured, "U" indicates that specimen did not fracture]

Prestress	10³ cycles at prestress	Test stress	10³ cycles at test stress
lb/in.²		*lb/in.²*	
±45, 000	21, 700	±56, 000	80 F
±46, 000	24, 239	±56, 000	79 F
±45, 900	18, 567	±54, 000	144 F
±45, 900	26, 700	±52, 000	438 F
±45, 800	31, 578	±50, 000	3, 933 U

In figure 19, a dashed line represents the *S-N* relationship for specimens which had been damaged 100 percent, that is, specimens which were just ready to crack. This curve was obtained as follows: Points were obtained at ±48,000 and ±54,000 lb/in.² by subtracting the number of

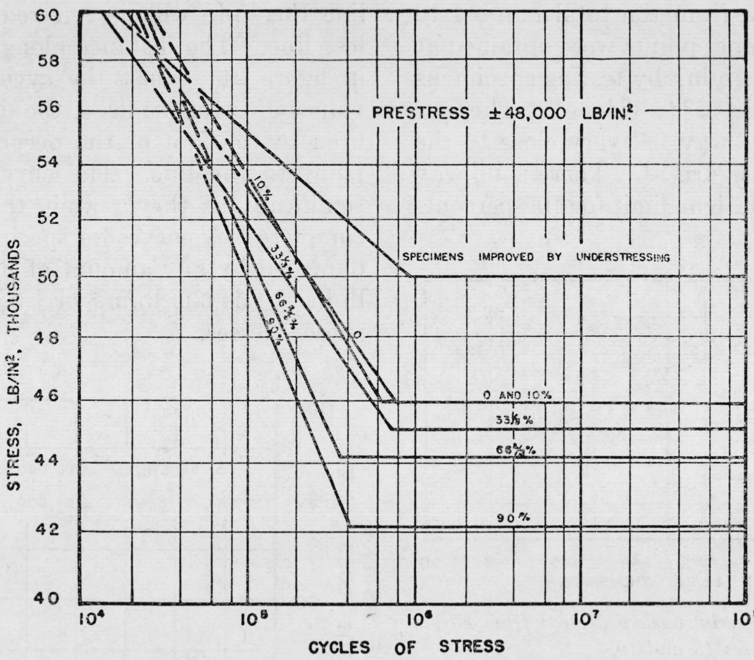

FIGURE 18.—*S-N curves for specimens overstressed at ±48,000 lb/in.² and for specimens improved by understressing.*

The figures on the curves represent the cycle ratio (based on N_o) to which the specimens were run in prestress.

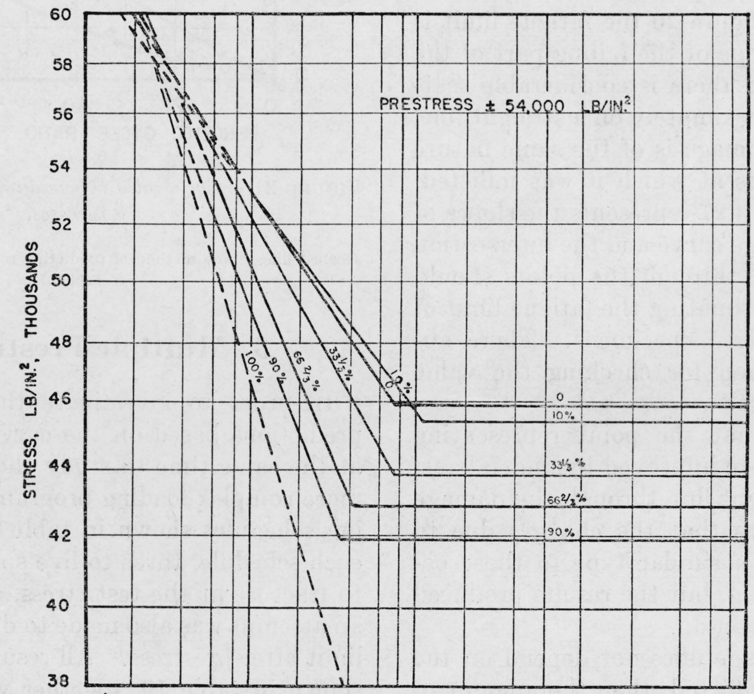

FIGURE 19.—*S-N curves for specimens overstressed at ±54,000 lb/in.²*

The figures on the curves represent the cycle ratios (based on N_o) to which the specimens were run in prestress.

cycles to form a crack from the total number to fracture. Two additional points were obtained at $\pm 42{,}000$ and $\pm 38{,}000$ lb/in.^{2} by testing specimens having very small cracks ($2°$ to $3°$ long). The four points obtained in this way fell very close to the straight line shown in figure 19. There is no way to obtain directly the fatigue limit for 100-percent-damaged specimens.

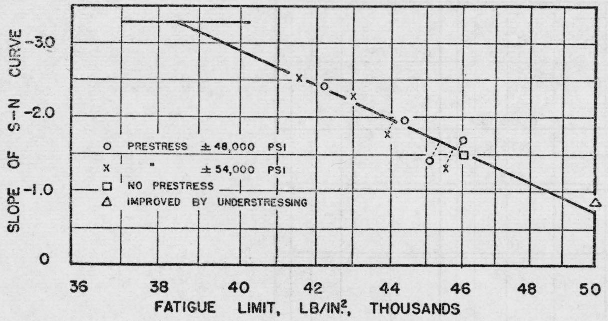

FIGURE 20.—*Strength loss relationship derived from curves of figures 18 and 19.*

It will be noted in figures 18 and 19 that the curves appear to radiate from approximately the same point. This suggests that the damage could be measured by a simple geometrical property of the *S-N* curves. In figure 20 the fatigue limit is plotted against the slope of the falling part of the *S-N* curve. Although there is considerable scatter, the points lie approximately on a straight line, indicating that the damage is of the same nature regardless of the stress at which it was inflicted. The horizontal line at 3.27 represents the slopes of the 100-percent-damage curve and the intersection of this line with that through the points should serve as a means of estimating the fatigue limit of the 100-percent-damaged specimens. There appears to be no method for checking the value obtained.

It will be noted that the point representing specimens that had been improved by understressing lies near the straight line through the damage points. This indicates that the changes due to understressing are of a similar type to those occurring on overstressing, but the results produced are in the opposite direction.

If the type of damage does not depend on the stress at which it is inflicted, then the amount of damage should be represented by distance along the curve of figure 20, being expressed as percent of the distance from the value for the original material to that for 100-percent damage. This line therefore will be referred to as the strength loss line. The distance along this line is plotted in figure 21 against the cycle ratio for the two values of prestress used, the distance being measured to the foot of the perpendicular from each point to the line. The curves of figure 21 are significant as they permit the prediction of the complete *S-N* curves for specimens that have been damaged by any amount of prestress at $\pm 48{,}000$ lb/in.^{2}, $\pm 54{,}000$ lb/in.^{2} or by any combintion ofa these stresses.

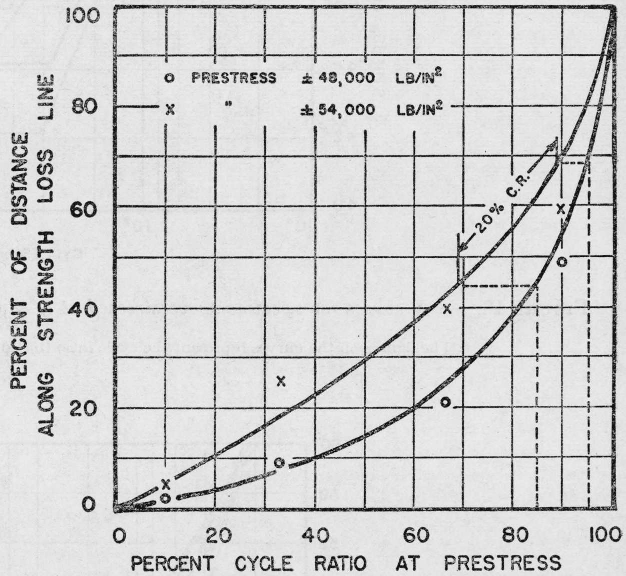

FIGURE 21.—*Cycle ratio versus damage relationship for two stresses.*

Dashed lines show method of predicting results for one complex loadin schedule.

3. Multiple Prestress Tests

In order to investigate the reliability of the predictions based on the curves of figure 21, and at the same time to study the damage caused by more complex loading programs, the six prestressing schedules shown in table 10 were used. For each schedule, three to five specimens were tested to fracture at the test stress, and for schedule IV an attempt was also made to determine the fatigue limit after prestress. All results of these tests are shown in table 10, together with the predictions based on figure 21. The method of making these predictions is illustrated by the dotted lines on the figure, showing the steps involved in schedule V. A cycle ratio of 85 percent at $\pm 48{,}000$

lb/in.^{2} results in 44 percent damage. A 44 percent damage corresponds to 69 percent cycle ratio at $\pm 54{,}000$ lb/in.^{2}, and $69+20=89$ percent cycle ratio at 54,000 lb/in.^{2}, which corresponds to 69 percent damage. A 69 percent damage would be caused by 96 percent cycle ratio at $\pm 48{,}000$ lb/in.^{2}, so a 4 percent cycle ratio should remain after the prestressing is complete. The number of cycles to the start of cracking at this stress is 333,000, and $0.04 \times 333{,}000 = 13{,}000$, which represents the number of cycles to form a crack at the test stress. In addition, it requires 78,000 cycle at this stress for the crack to grow to fracture, so the total predicted life under these conditions is 91,000 cycles.

TABLE 10.—*Multiple prestress tests*

Prestress schedule [a]	Test results, 10^3 cycles to fracture					Median 10^3 cycles	Prediction 10^3 cycles
I	126	304	163	147	207	163	177
II	65	81	57	37	21	57	56
III	7	28	5	-------	-------	7	34
IV [b]	174	80	101	-------	-------	101	148
V	<0	98	<0	102	86	86	91
VI	106	266	75	-------	-------	106	137

[a] I.—33⅓ percent cycle ratio at $\pm 48{,}000$ lb/in.^{2} + 33⅓ percent cycle ratio at $\pm 54{,}000$ lb/in.^{2}, test $\pm 48{,}000$ lb/in.^{2}

II.—33⅓ percent cycle ratio at $\pm 54{,}000$ lb/in.^{2} + 33⅓ percent cycle ratio at $\pm 48{,}000$ lb/in.^{2}, test $\pm 54{,}000$ lb/in^2.

III.—75 percent cycle ratio at $\pm 54{,}000$ lb/in.^{2} + 15 percent cycle ratio at $\pm 48{,}000$ lb/in.^{2}, test $\pm 54{,}000$ lb/in^2.

IV.—50 percent cycle ratio at $\pm 48{,}000$ lb/in.^{2} + 33⅓ percent cycle ratio at $\pm 54{,}000$ lb/in.^{2}, test $\pm 48{,}000$ lb/in^2.

V.—85 percent cycle ratio at $\pm 48{,}000$ lb/in.^{2} + 20 percent cycle ratio at $\pm 54{,}000$ lb/in.^{2}, test $\pm 48{,}000$ lb/in^2.

VI.—15 percent cycle ratio at $\pm 48{,}000$ lb/in.^{2}, + 10 percent cycle ratio at $\pm 54{,}000$ lb/in.^{2}, then repeat twice more. Test $\pm 48{,}000$ lb/in.^{2}.

[b] Determination of fatigue limit for schedule IV.

"F" indicates that specimen fractured. "U" indicates that specimen did not break.

In addition to those listed, three specimens cracked before the prestressing was completed.

Stress, lb./in.^{2}	10^3 cycles	Fatigue-limit prediction, lb./in.^{2}
$\pm 42{,}000$	4,898 U	$\pm 43{,}200$
$\pm 42{,}200$	22,715 U	----------
$\pm 42{,}800$	5,459 U	----------
$\pm 43{,}000$	399 F	----------

The results listed in table 10 show reasonable agreement with the predictions, considering the large probable error of the tests involved. A small change in the rate of damage from one specimen to the next, particularly in the region where the curves of figure 21 are steep, may cause a large variation in the damage due to a certain prestress. Other methods tried for predicting these results, including the simple cumulative addition of cycle ratios suggested by Miner [6], failed to give reasonably close values for more than one or two of the schedules.

4. Repair of Damage

Two exploratory tests were made to verify the work of others on the repair of fatigue damage. In each case the prestress was 90 percent cycle ratio (based on N_e) at $\pm 48{,}000$ lb/in^2. The first specimen was then stressed at $\pm 42{,}200$ lb/in.^{2} (just under the new fatigue limit) for 34×10^6 cycles. When tested at $\pm 48{,}000$ lb/in.^{2} this specimen did not fail in 40×10^6 cycles. The understressing had not only repaired the prior damage but raised the fatigue limit above that of the material in its original condition. As a matter of interest, this same specimen was subsequently "coaxed" by stressing 20 to 30 million cycles at each of seven successively higher stresses. At a stress of $\pm 62{,}000$ lb/in.^{2} the specimen endured 21×10^6 cycles, then failed after 2,500,000 cycles at $\pm 64{,}000$ lb/in^2. Thus a specimen damaged to the extent where the fatigue limit was less than $\pm 42{,}500$ lb/in.^{2} was improved by suitable stress treatment, bringing its fatigue limit above $\pm 62{,}000$ lb/in^2.

The second damaged specimen was normalized in a vacuum, reproducing as closely as possible the temperature cycle of the original heat treatment. When tested at $\pm 48{,}000$ lb/in.^{2} the specimen fractured in 126,000 cycles. The normal expectancy for specimens after this prestress would be 143,000 cycles, indicating that the heat treatment resulted in no repair of the fatigue damage.

VI. Discussion

The life of a specimen under fatigue stressing consists of two distinct phases. During the first, part of the material is progressively damaged until a small crack is formed, and during the second this crack grows until fracture occurs. In this report the term "fatigue damage" is used to designate only the change occurring during the first stage, material being considered 100 percent damaged at the time the first crack forms. If one wishes to study the first-stage damage, some method of determining the start of cracking is required. Lacking this, the effects of the second stage will confuse the results. Within the stress range used in this study, the proportion of the total life represented by the second stage varies from as much as 50 percent to as little as 10 percent, so it can be seen that a large error would be involved if the two stages were not separated.

With the *SAE* X4130 steel used in this investigation, the ratio of the second stage to the total life of the specimen was larger at stresses considerably above the fatigue limit than at stresses close to the fatigue limit. This is contrary to what certain other investigators have reported, but such a discrepancy may be expected, since the ratio depends on many factors such as the slope of the *S-N* curve for the original material, the rate of crack propagation in the metal, and the size of the specimen.

The damage resulting from a particular number of cycles of overstress depends on the past history of the specimen. Thus, the performance cannot be predicted accurately for metals in service which are subjected to various ranges of stress, unless the complete loading schedule is known. Since this generally is not possible, the best that can be done for increasing the safety of a given steel structure is to apply a large number of cycles of stress below the fatigue limit after each period of overstress. This usually is taken care of by the nature of the service, but in some applications involving exceptionally high stresses it might be worthwhile to apply low stresses purposely to counteract the effect of unusually long periods of high stress.

The results of the limited number of tests made with specimens improved by understressing suggest that this improvement can be measured by distance along a strength loss line in a manner similar to that used for measuring damage (fig. 20). If this can be done, then the results of any loading schedule can be predicted, regardless of whether the stresses are above or below the fatigue limit.

VII. Conclusions

The results given in this paper lead to several important conclusions regarding fatigue damage occurring *prior* to the start of cracking.

1. If the damage occurring at one stress was determined by measurement at another, the apparent rate of damage depended on the value of both stresses. When the first was larger than the second, the damage occurred rapidly at first, then more slowly: the reverse was true when the first stress was the smaller.

2. The *S-N* curves for specimens after various damaging treatments showed that with increasing damage, the absolute value of the slope of the falling part of the curves increased and the fatigue limit decreased.

3. These differences in the *S-N* curves provided a measure of damage by which the damage occurring at different stresses could be directly added. The performance of specimens tested under complex loading schedules could be ascertained by the use of cycle-ratio versus damage curves for the stresses involved.

4. The improvement due to understressing apparently can be measured in the same way as damage due to overstress.

5. Fatigue damage prior to the start of cracking was repaired by understressing but not by heat treatment.

Certain other conclusions regarding fatigue phenomena in general may be drawn from the results obtained.

6. The slopes of the falling part of the *S-N* curves for the same material in smooth, notched, and cracked specimens showed approximately the same (stress concentration) ratios as the fatigue limits.

7. The number of cracks formed in a specimen before fracture increased rapidly as the test stress was raised above the fatigue limit.

8. The length, L, of fatigue cracks may be represented by the equation

$$\log (L-C)=\alpha N$$

where C and α are constants, and N is number of cycles.

9. In specimens stressed considerably above the fatigue limit, the number of cracks, M, in a narrow circumferential band of the specimen can be expressed by the equation

$$\sqrt{M}=KS,$$

where K is a constant, and S is the average stress in the band.

10. The development of a small crack in a specimen resulted in a large decrease in the value of the fatigue limit as compared with specimens which had been stressed just short of cracking.

VIII. References

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