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PRECISION OF TELESCOPE POINTING FOR OUTDOOR TARGETS

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ABSTRACT

The probable error of a single pointing (PE_s) is measured for a single telescope with a variety of targets. This investigation shows that, although some change in PE_s with distance does occur, the distribution of PE_s as a function of distance can usually be neglected and a value of 0.62 second assigned as a practical average. The values of PE_s for an indoor target usually show a small variation from one experienced observer to another, and from right to left eye of the same observer. There is also a measurable systematic difference in pointing between the right eye and the left eye of the same observer. In outdoor pointing, a long-period error or drift is usually superposed upon the short-period errors.

CONTENTS

I. INTRODUCTION

When pointings are made with a telescope, the probable error of a single setting is of interest. In this paper the error, associated with the optical characteristics of the problem, is discussed. The errors present may be either accidental or systematic. The study of the accidental errors is emphasized herein. These accidental errors arise partly from the conditions prevailing along the optical path and partly from limitations on the part of the observer. The former are greater in magnitude. In outdoor pointing, the image of the target formed by the objective and seen through the ocular is constantly oscillating with respect to the cross hairs mounted in the focal plane of the objective. Setting the cross hairs into apparent coincidence with the image of the target formed by the objective and seen through the ocular is, for the most part, a matter of deciding upon the mean position of the image of the target and setting the cross hairs thereon.

Even when the image of the target appears stationary, an error of pointing still persists and is measurable.

In the present work, the pointing errors for a given telescope have been measured for a number of outdoor targets and an average value determined for the probable error of a single pointing. The figure so determined is of interest in that it is the limit of precision that can be attained in outdoor pointing under average daylight conditions and over a terrain that consists partly of park area and partly of city residential and business sections. When, therefore, measurements are being made, under similar conditions, that involve pointing accuracy, it is profitless to attempt, by further refinement of the equipment, to better the precision if this limit has already been reached. For outdoor pointing, the average value of the probable error of a single setting found here is 0.62 second.

It is the purpose of this paper to present a concise account of the experimental results. A more detailed account, including a bibliography and summary of previous work, is being published elsewhere.[1]

II. APPARATUS AND METHOD OF MEASUREMENT

The apparatus consists of a telescope from a surveying level with an effective aperture of 39 millimeters and a magnification of 37 diameters, and a means of varying the pointing without disturbing the telescope. This is accomplished by a weak prism, placed in front of the telescope objective, capable of rotation about its axis. As the deviation of a parallel beam of light caused by a prism is a function of the angle of incidence, the image of the target can be made to move with respect to the telescope cross hairs by rotating the prism.

Observations are made in groups of 10 and the probable error of a single pointing is determined from these data. Five 10-groups are usually taken in succession to constitute a single run. The total error of calibration is kept under 2 percent, and the least reading in seconds of the instrument is less than one-fifth the magnitude of the least probable error of a single pointing. The probable error of a single pointing, PE_s, measured in seconds, is computed from the approximate formula

$$PE_s = \frac{0.8453k}{\sqrt{n(n-1)}} \sum r \qquad (1)$$

where n is the number of observations, $\sum r$ is the sum of the residuals, and k is the calibration constant of the prism.

III. RESULTS OF MEASUREMENT

1. INDOOR TARGET

Although the main purpose of the present work is the determination of PE_s for outdoor targets, the PE_s for the given telescope was first measured with an indoor target. Thus when, as is later found, the PE_s for an outdoor target is greater than the value obtained with an indoor target, one can be sure that the limit on PE_s is being placed by the conditions of outdoor pointing and not by limitations of the

[1] F. E. Washer and H. B. Williams, J. Opt. Soc. Am. **36** (June, 1946).

telescope itself. Accordingly, the experiment was first performed indoors with a vertical wire mounted in the focal plane of a collimator as target.

It is found that, for the conditions under which these measurements are made, the accuracy of pointing is not affected by aperture or color of light and that the value, $PE_s=0.25$ second, is the optimum value that can be expected for indoor pointing with this instrument.

2. OUTDOOR TARGETS

The outdoor test objects consist of a number of distant targets that are readily discernible from the laboratory. Several series of observations were made on each target. In most cases, measurements were made independently by two observers identified as W and B. The results of measurement are assembled in table 1.

TABLE 1.—*Probable error of a single pointing for each of two observers with distant targets*

Target	Range	Observer W		Observer B		Average	
		PE_s	*N*	PE_s	*N*	PE_s	*N*
	m	*sec*		*sec*		*sec*	
Collimator	2	0.25	600	----------	----------	0.25	600
Radio	100	----------	----------	0.45	50	.45	50
Tilden	323	.46	100	.42	150	.44	250
Sedgwick	445	.36	200	.50	200	.43	400
Broadmoor	703	----------	----------	.57	300	.57	300
Ordway	905	.61	300	.44	100	.57	400
Klingle	1,153	.60	200	.57	100	.59	300
Swank	1,399	.58	300	.58	200	.58	500
Majestic	2,722	.51	150	.52	100	.51	250
Baptist	3,000	----------	----------	.73	200	.73	200
Unity	3,080	.75	600	.67	200	.73	800
Howard	4,505	.90	50	.72	50	.81	100
Clock	4,779	.66	500	.71	150	.67	650
Trinity	5,553	.65	200	----------	----------	.65	200
Capitol	7,554	.92	100	.69	100	.80	200
Bradbury	13,624	.86	50	.97	50	.92	100
Average		0.64	2,750	0.60	1,950	0.62	4,700

It is evident from table 1 that the values of PE_s for a given target do not differ greatly from one observer to the other when n is large. Furthermore, assuming that any effect of distance can be neglected, the grand average of 2,750 observations by W differs by only 0.04 second from the grand average of 1,950 observations by B. The differences between the PE_s values for the two observers on a given target when n is small are no greater than those that sometimes occur between successive groups for a single observer. The average PE_s for 4,700 observations by two observers is 0.62 second.

3. EFFECT OF DISTANCE

In the preceding section it has been assumed that PE_s is independent of the distance separating observer and target and a value assigned to it on that basis. This procedure is justified practically because of the seemingly random scattering of the PE_s values with respect to distance. Also, so many variables enter into pointing accuracy that the effect of distance is screened by the greater influence of other factors that are not subject to control, such as state of the weather and conditions of visibility.

Notwithstanding these handicaps, least squares solutions were made on the basis of several assumptions. The solution that appeared most satisfactory is

$$PE_s = 0.064d^{\frac{1}{4}} + 0.19, \tag{2}$$

where d is the distance separating target and observer. In figure 1, the values of PE_s predicted by equation 2 are plotted as a function of distance. The observed values of PE_s are shown as circles. It is

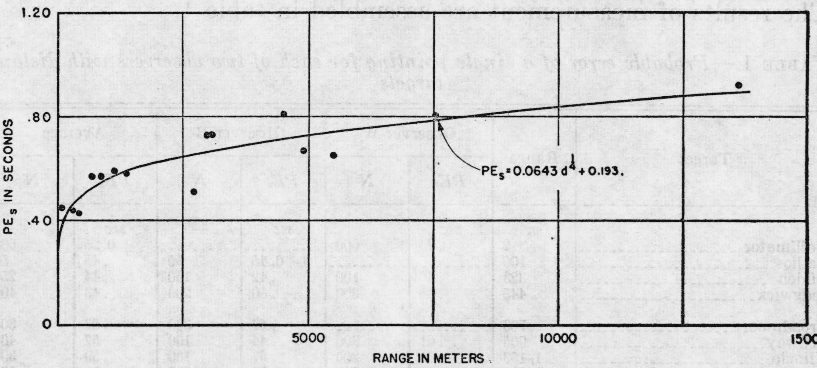

FIGURE 1.—*Probable error in a single pointing* (PE_s) *versus distance.*

The points indicate observed values. The solid curve is the relation computed from the formula $PE_s = 0.064d^{1/4} + 0.19$.

evident that on the whole there is fairly good agreement. In contemplating the points of disagreement, it may be mentioned that there may not have been enough variation in the viewing conditions to bring about a truly average value for each target.

4. CONSISTENCY

It was early noted that, when a set of 50 observations, comprising five 10-groups, is made, the mean values of deviations in the line of sight for the various 10-groups in a 50-set are not consistent. In other words, the means of the 10-groups appeared to drift or oscillate as if a long slow period of change was superposed on the short-period changes that occur in a 10-group. For example, such a change would be introduced by variation in a horizontal air-temperature gradient perpendicular to the line of sight. To test for the existence of such a long-period error, each of the five 10-group means in a 50-set is treated as a single observation, and the probable error, C, of such a single mean is computed from the formula

$$C = \frac{0.8453}{\sqrt{n(n-1)}} \sum r. \tag{3}$$

If the distribution of the observations contained in the 50-set is normal then the value of C should be approximately equal to the average probable error of the mean, $\overline{PE}_m$, for the five 10-groups, which is obtained from the formula

$$\overline{PE}_m=\frac{PE_s}{\sqrt{10}}. \tag{4}$$

The values of $\overline{PE}_m$ and $\overline{C}$ for some of the targets are given in table 2, where, however, these data are averaged for a number of 50-sets. Values are also listed of the difference $\overline{C}-\overline{PE}_m$ to facilitate comparison. If no long-period error is present, the values of $\overline{C}-\overline{PE}_m$ ought to be relatively small, and the average over all the targets ought to be approximately zero. However, the table shows that $\overline{C}$ is greater than $\overline{PE}_m$ in all cases for observer B and in 10 out of 12 cases for observer W. Therefore, it may be stated that a long-period error does exist. For observer W, the average value of $\overline{C}-\overline{PE}_m$ is 0.08 second, and for observer B, the value is 0.36 second.

TABLE 2.—*Consistency of observations for each of two observers with distant targets*

Target	Range	Observer W				Observer B			
		$\overline{PE}_s$	$\overline{PE}_m$	$\overline{C}$	$\overline{C}-\overline{PE}_m$	$\overline{PE}_s$	$\overline{PE}_m$	$\overline{C}$	$\overline{C}-\overline{PE}_m$
	m	*sec*	*sec*	*sec*	*sec*	*sec*	*sec*	*sec*	*sec*
Radio	100	------	------	------	------	0.45	0.14	0.86	0.72
Tilden	323	0.49	0.15	0.14	−0.01	.42	.13	.62	.49
Sedgwick	445	.36	.11	.16	.05	.50	.16	.63	.47
Broadmoor	703	------	------	------	------	.57	.18	.70	.52
Ordway	905	.62	.20	.29	.09	.44	.14	.50	.36
Klingle	1,153	.62	.20	.30	.10	.57	.18	.42	.24
Swank	1,399	.55	.17	.31	.14	.58	.18	.42	.24
Majestic	2,722	.51	.16	.16	.00	.52	.16	.21	.05
Baptist	3,000	------	------	------	------	.73	.23	.59	.36
Unity	3,080	.75	.24	.34	.10	.67	.21	.63	.42
Howard	4,504	.90	.28	.30	.02	.72	.23	.45	.22
Clock	4,779	.67	.21	.42	.21	.71	.22	.53	.31
Trinity	5,533	.65	.21	.40	.19	------	------	------	------
Capitol	7,554	.92	.29	.31	.02	.69	.22	.54	.32
Bradbury	13,624	.86	.27	.37	.10	------	------	------	------
Average	------	0.66	0.21	0.29	0.08	0.58	0.18	0.55	0.36

The quantity $\overline{C}-\overline{PE}_m$ is very much greater for observer B than for observer W, and as the values of PE_s are closely comparable for both, this indicates the presence of yet another error in addition to drift. In seeking an explanation for this large difference between observers W and B, it was recalled that observer B used the left and right eyes for alternate 10-groups in a 50-set, whereas observer W used the right eye only. At the time this practice was initiated it was believed that no difference should arise therefrom. However, on analyzing the averages of the means for right and left eyes, a systematic difference in the direction of pointing for right and left eyes appears to be definitely present for observer B. This difference, which is designated

OD−OS, although showing some difference from one 50-set to another, is generally positive. It may be mentioned that systematic differences between the settings made by the right and left eye are evident in tables of data shown in a paper by Labitzke,[2] which deals with bisection errors. On the basis of Labitzke's work, it appears that the average of settings made by either eye is likely to show a systematic error but that the magnitude is not the same for each eye, thus an *OD−OS* value exists for each individual, although it is improbable that the value will be the same for all individuals.

5. OBSERVER DIFFERENCES WITH AN INDOOR TARGET

Having noted the pointing differences for the right and the left eye with outdoor targets in the case of observer B, it seemed worth while to investigate this matter further by using the indoor target to eliminate the uncertainty arising from the long air column present with the outdoor target. A series of observations was made by each of six observers [3] at various apertures of the viewing telescope, ranging from the full aperture of 39 millimeters to an aperture of 6.8 millimeters. Whereas the outdoor observations by observer B usually had three 10-groups of left-eye pointing to two 10-groups of right-eye pointing, the indoor work involved an equal number of 10-groups for each eye. Accordingly, each observer made at least 60 observations in groups of 10, alternating the use of the right and the left eye.

(a) PROBABLE ERROR OF A SINGLE POINTING

The values of PE_s for the six observers are shown in figure 2. PE_{OD} and PE_{OS} are the probable errors of single pointings with right and left eyes, respectively, and PE_s is the average for the two eyes. It may be noted in figure 2 that there is usually a difference between the two eyes in the magnitude of the probable error of a single pointing. Whether this difference is sufficiently great to be significant in most cases is problematical, as the average departure of PE_{OD} or PE_{OS} from PE_s for all observations shown is 8.5 percent. However, it seems reasonable that each eye should be considered as a separate individual, and, on this assumption, perhaps some significance may be attached to this difference. In any event, it is interesting to note that the lowest PE_s is not always obtained with the eye most generally used for observing by each individual. For example, although observers C and W, who normally observe with their left and right eye, respectively, do have lower probable errors for those eyes, observer B, who generally observes with the left eye, and observers T, E, and F, who normally use the right eye, all have a little greater error at most apertures when observing with the eye normally used.

It should be pointed out that observers E and F, whose probable errors range much higher than those of other observers and whose differences in probable error for the right and the left eye are more pronounced, were at this time very inexperienced. Observers W, T, B, and C, the most experienced, have generally the lowest and least erratic probable errors. These patterns, then, of individual

[2] P. Labitzke, Untersuchung uber psychologisch—und physiologische Bisektionsfehler, Z. Instrumentenk. **44**, 61 and 165 (1924).

[3] The four additional observers cooperating in this work were L. W. Tilton, T; F. A. Case, C; E. H. Samuels, E; and F .C. Samuels, F.

differences in observation are good evidence that experience is conducive to precision in pointing.

In the upper graph of figure 3, the probable errors of a single observation of pointing made with right and left eyes are averaged for

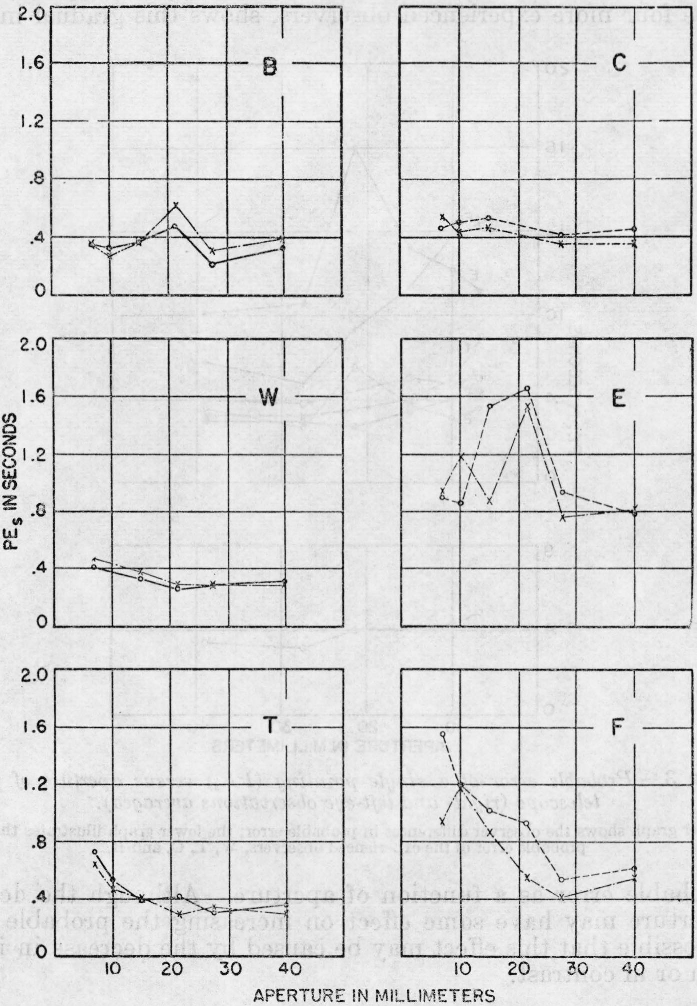

FIGURE 2.—*Probable error of a single pointing* (PE_s) *versus aperture of the viewing telescope.*

Values of PE_s for right-eye pointing are marked ○ and those for left eye pointing are marked X.

each individual. Here may be seen more strikingly the difference in the magnitude of the probable error between the more experienced and the inexperienced observers. For each observer the probable error tends to decrease slightly when the aperture is decreased from 39 to 27.5 millimeters. Then the probable error begins to increase gradually as the aperture is decreased progressively from 27.5 to 6.8

millimeters. This decrease in probable error followed by an increase when the aperture is stopped down progressively was also found by Tilton [4] in his investigation of pointing accuracy in connection with prism size in minimum deviation refractometry. The lower graph of figure 3, in which the probable errors at each aperture are averaged for the four more experienced observers, shows this gradual increase

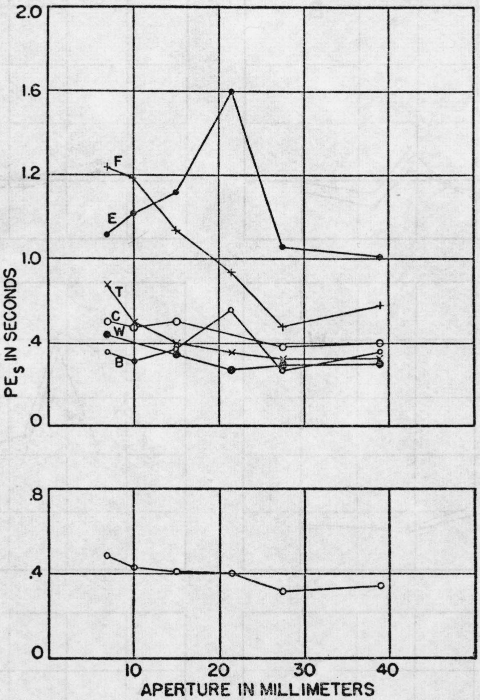

FIGURE 3.—*Probable error of a single pointing (PE_s) versus aperture of viewing telescope (right- and left-eye observations averaged).*

The upper graph shows the observer differences in probable error; the lower graph illustrates the average probable error of the experienced observers, W, T, C, and B.

in probable error as a function of aperture. Although the decrease in aperture may have some effect on increasing the probable error, it is possible that this effect may be caused by the decrease in illumination or in contrast.

(b) POINTING DIFFERENCES

Successive 10-groups of observations were made with right and left eyes alternately, and it was found that the mean of the 10-group settings for the right eye differed from the mean of the 10-group settings for the left eye; that is, the observers tended to point more to one side of the target with one eye than with the other. These eye differences

[4] L. W. Tilton, Prism size and orientation in minimum deviation refractometry, BS J. Research **6**, 59 (1931) RP262.

are tabulated under the column *OD−OS* in table 3 for each observer. M_D and M_S are the probable errors of the mean for right and left eye, respectively. *R* is the probable error of the difference in the means for right and left eyes. It may be seen in table 3 that in 28 out of 34 instances, the values of *OD−OS* are greater than *R;* moreover, the value of *OD−OS* ranges from 1 to 17 times as great as *R*, and in 12

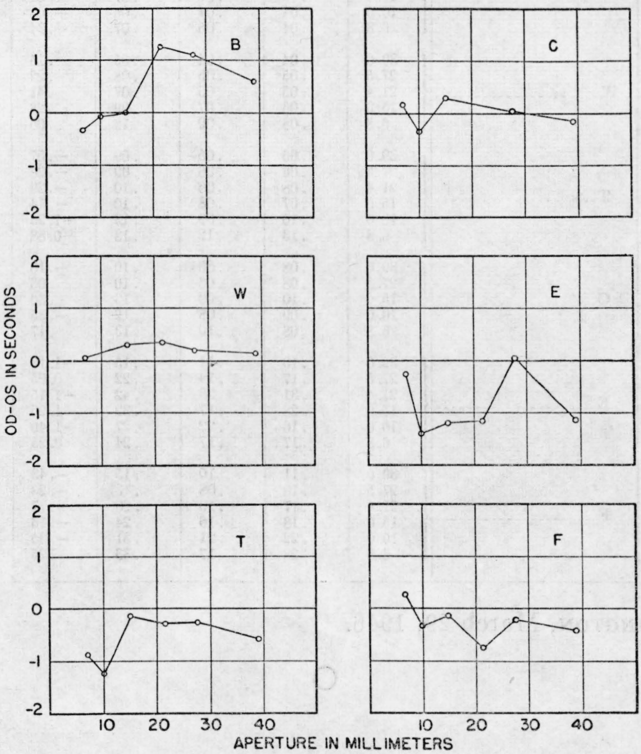

FIGURE 4.—*Right- and left-eye differences (OD−OS) in pointing as a function of aperture for individual observers.*

instances is 4 or more times greater than *R*, which indicates definitely that there is a real difference between the settings made with the right eye and the left eye. There is no attempt here to explain the cause of this difference but merely to give evidence thereof. Figure 4 illustrates values of *OD−OS* for each observer. It is interesting to note that observer T has all negative values for the differences and observers E and F have predominately negative, whereas observer W has all positive values, and observers B and C have a majority of positive values.

TABLE 3.—*Individual eye differences in pointing and their probable errors*

Observer	Aperture	M_D	M_S	R	$OD-OS$
	mm	*sec*	*sec*	*sec*	*sec*
B	39.0	±0.03	±0.04	±0.05	0.61
	27.5	.04	.06	.07	1.09
	21.4	.09	.12	.14	1.26
	15.0	.05	.05	.07	−0.01
	10.0	.04	.04	.06	−.08
	6.8	.04	.05	.07	−.34
W	39.0	.04	.04	.05	.15
	27.5	.05	.05	.08	.22
	21.4	.05	.05	.07	.41
	15.0	.06	.07	.09	.28
	6.8	.08	.09	.12	.06
T	39.0	.06	.05	.09	−.57
	27.5	.06	.06	.09	−.26
	21.4	.08	.06	.10	−.29
	15.0	.07	.08	.10	−.14
	10.0	.10	.09	.13	−1.23
	6.8	.13	.12	.18	−0.88
C	39.0	.08	.06	.10	−.16
	27.5	.08	.06	.10	.03
	15.0	.10	.09	.13	.30
	10.0	.09	.08	.12	−.35
	6.8	.08	.10	.13	.17
E	39.0	.10	.11	.15	−1.13
	27.5	.17	.14	.22	0.05
	21.4	.31	.28	.42	−1.15
	15.0	.28	.17	.32	−1.20
	10.0	.16	.22	.27	−1.40
	6.8	.17	.17	.24	−0.25
F	39.0	.11	.10	.15	−.43
	27.5	.10	.08	.13	−.23
	21.4	.17	.10	.20	−.77
	15.0	.18	.16	.24	−.16
	10.0	.22	.21	.31	−.35
	6.8	.28	.17	.33	.25

WASHINGTON, March 29, 1946.