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REVIEW OF RECENT ABSOLUTE DETERMINATIONS OF THE OHM AND THE AMPERE

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ABSTRACT

In the decade preceding 1944 there were published the results of eight determinations of the absolute ohm and seven of the absolute ampere. These determinations were made in the national standardizing laboratories of England, France, Germany, Japan, and the United States. However, only preliminary results of some of the researches have been published, and one value of the absolute ampere was obtained in an experiment that was originally developed for another purpose. Hence to obtain the most probable values of the absolute ohm and ampere, it is necessary to ignore some of the published results and to consider some of those remaining as more reliable than others. A critical analysis has been prepared of each of the determinations, and this analysis has been used as a basis for weighting the results.

The most probable value of the absolute ohm is given by the relation

1 mean international ohm = 1.000 490 absolute ohms.

The mean deviation from the mean of the results used in obtaining this value is only 14 parts per million. Further confirmation of the probability of this result is shown by the agreement with the results of two other recent compilers of the absolute-ohm determinations.

The most probable value of the absolute ampere is given by the relation

1 mean international ampere = 0.999 853 absolute ampere.

The mean deviation from the mean of the three results used in obtaining this value is 3 parts per million. Other recent compilers. however, have given larger values, one, 27 parts per million larger, and the other, 119 parts per million larger.

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I. INTRODUCTION

The values of the absolute electrical units are all based on absolute determinations of the ohm and the ampere. The absolute determinations of these two units that have been made in the national laboratories during the past decade are more accurate that those made at any previous time. The national laboratories at which the measurements were made are the National Physical Laboratory, England, (NPL); Laboratoire Central d'Électricité, France, (LCE); Physikalisch-Technische Reichsanstalt, Germany, (PTR); Electrotechnical Laboratory, Japan, (ETL); and the National Bureau of Standards, United States, (NBS). But the recent determinations are not all of equal accuracy, so that the most probable value cannot be obtained merely by taking a mean of all the published values. In fact, only by making a critical analysis of each determination can one judge the relative importance of the different values that have been obtained.

Some definite principles have been used in determining the weight to be applied to the different determinations of the ohm and the ampere. In general, a result which has been classed as preliminary in the article which describes the experimental work is given zero weight. There is one exception, which will be explained in connection with the analysis of the work. One other result has been given zero weight because much more precise results have now been obtained by the same experimenters. Other results have been weighted according to the judgment of the author of this article, giving a weight of 5 to those which seem to be most accurate, and smaller weight to results that are less reliable.

The author realizes the difficulty of making a critical review of a field in which the work of himself and associates must be compared with that of others. This difficulty is not as great as in some fields, because there has been cordial cooperation among all who are working on the values of the absolute electrical units. Each laboratory has welcomed visits from other investigators working in this field. The author has visited all the laboratories where the investigations have been made, except those in Japan.

The results of all recent evaluations of the absolute electrical units have been expressed in terms of the so-called "international ohm" and the "international ampere." Both of these units have slightly different values in different laboratories. The differences between the units of the national laboratories for the same quantity have always been too small to be considered except in measurements of the highest precision. Since the measurements to be reviewed in this paper are of an accuracy which requires that these differences shall be taken into account, a brief statement concerning the establishment and maintenance of the international units is necessary.

The international ohm as established in 1893 was "represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grammes in mass, of a constant cross-sectional area and of a length of 106.3 cms." The mass of mercury was chosen to give a cross-sectional area of 1 square mm and a length of 106.3 cm. These dimensions gave to the international ohm a value which agreed with the absolute ohm as nearly as could be determined from the absolute measurements that had been made up to that time. The international ohm was maintained by means of the mercury ohm for about 20 years. Gradually wire-resistance standards were developed that were so permanent that the international ohm could be maintained more precisely by them than by reestablishing the unit every few years by means of measurements on the mercury ohm. Hence during the last two decades not one of the national laboratories has reestablished its unit for the international ohm by making measurements upon the mercury ohm. Instead each laboratory maintains its international ohm by means of wire resistance standards, the values of which were originally established by reference to the mercury ohm. There are slight differences between the values of the international ohm of the different national laboratories. These differences have been determined on several occasions at the International Bureau of Weights and Measures. Hence the results of all recent measurements of the absolute ohm can be expressed in terms of the mean international ohm, the value of which is the mean of the values of the international ohm as maintained at the important national laboratories. The mean international ohm is herein assumed to have remained constant during the last decade. There is no longer need of making measurements on the mercury ohm, since the absolute ohm can now be used to establish and maintain the unit of resistance with greater accuracy than can be obtained by using the mercury ohm.

The international ampere was defined as the "unvarying current which, when passed through a solution of nitrate of silver in water * * * deposits silver at the rate of 0.001118 grammes per second." The international ampere so defined, in connection with the international ohm has been used to determine the electromotive force of a standard cell. Routine laboratory measurements of current in international amperes have always been made by reference to a standard resistor and a standard cell. Since 1911 the National Bureau of Standards has maintained the international volt by means of a group of standard cells. Similar methods have been used by other national laboratories, at least during the last decade. The results of determinations of the absolute ampere made in the last decade have been expressed in terms of the international ohm and international volt as maintained in the national laboratories where the determination was made. These national units can be reduced to a common basis by the comparisons of standard resistors and standard cells which have been carried out at the International Bureau of Weights and Measures. The results for 1933, 1935, 1937, and 1939 have been published in the Procès-verbaux of the International Bureau. It is assumed that the mean international ampere (the mean of six national laboratories) has remained constant during last decade.

II. RÉSUMÉ OF ABSOLUTE-OHM DETERMINATIONS

Eight independent researches on the value of the absolute ohm have been reported during the last decade. In order to evaluate properly the different results, the following analysis of the eight determinations has been made, taking them in chronological order.

(a) 1936: CURTIS, MOON, AND SPARKS AT NBS [1]

In the method employed in this investigation the value of a self inductance in international henrys was compared with its value in absolute henrys. The ratio of these values is the same as the ratio of the value of a resistance in international ohms to its value in absolute ohms. The inductance of a self inductor in international henrys was obtained by electrical measurements which involved the mean solar second and the international ohm. In the electrical measurements, the self inductance was compared by a bridge method with a capacitance and a resistance, using low-frequency alternating current; and then the capacitance was measured in terms of resistance and time, using a pulsating current. The error in the electrical measurements was probably only a few parts in a million. A large number of electrical measurements were made on three helical inductors having different dimensions and different values of inductance.

The inductance of each helix in absolute henrys was computed from its measured dimensions and the permeability of the surrounding medium. The mechanical measurements of the diameters, which were very uniform, were made with high accuracy. The average pitch of each helix was accurately measured, but the measurements showed that the pitch was not exactly uniform for any of the helices. The basic formula used in computing the inductance in absolute units assumed that the winding was of uniform pitch, although the formula was extended by introducing terms to correct for the displacement of one or more complete turns from a true helical form. The actual helix did not conform exactly to the conditions required for a precise computation of the inductance, even by the extended formula. For example, there may have been displacements within each turn, which would affect the result more than the average displacement of a turn. These displacements would all tend to increase the inductance over that of a uniform helix. Hence, the computed value of the inductance for each of the three inductors used may have been too small. If this were the case, the result given by the authors was too small. Since their result is the smallest of any of those obtained in the last decade, it seems probable that the effect of irregularities in pitch produced an error in the result. Hence the result of this investigation is given a low weight, despite the fact that all electrical and mechanical measurements were carried out with great care.

[1] H. L. Curtis, C. Moon, and C. M. Sparks, *An absolute determination of the ohm*, J. Research NBS **16**, 1 (1936) RP857.

(b) 1936–37: VIGOUREUX AT NPL [2]

Vigoureux used the same apparatus used by Smith [3] in 1914. The fundamental unit in this apparatus was a Lorenz apparatus or homopolar generator for which the induced electromotive force could be computed from measured quantities, namely, the dimensions of the generator, the speed of rotation of its armature, and the current in its field coils. The armature consisted of two disks, each 54 cm in diameter. They were 167 cm apart and connected by a rigid shaft. The field coils for each disk were a pair of short solenoids coaxial with the disk and a short distance from it. The electrical measurements consisted in balancing the induced electromotive force against the fall of potential produced by the field current in a standard of resistance. The value of this resistance was thus determined from the dimensions of the generator and the speed of rotation of its armature.

Vigoureux remeasured all the mechanical dimensions and made 74 sets of electrical measurements. The same methods of making the mechanical measurements were employed as had been used by Smith. The measurement of any individual outside diameter of the helices, which was about 35 cm, was accurate to 0.1 micron (abbreviation μ). However, according to Smith, there were variations of 40μ in the different diameters of one helix, and in the most uniform helix the variation was about 15μ, or 40 parts per million (abbreviation ppm). Individual measurements of the length of the helices, which length was about 16 cm, were accurate to 1μ, or 6 ppm. The pitch, which had a nominal value of 1667μ, had a periodic variation with a total amplitude of about 30μ for the most uniform coil, and 50μ in the most extreme case. The remeasurement showed that, on the average, the diameters had decreased 13 ppm and the lengths increased by 5 ppm.

The electrical measurements were reported in eight groups. The mean of each group varies from the mean of all the groups by a minimum of 5 ppm and a maximum of 13 ppm. Only a single set of coils was available, so that there were no independent checks. Summarizing, the coils were not precisely made, the electrical measurements showed considerable variations even when averaged in large groups and only one set of helical coils was employed. Hence, a low weight has been assigned to the result.

(c) 1937: YONEDA AT ETL [4]

Yoneda compared a fixed standard of mutual inductance, the value of which could be computed from the mechanical dimensions of the inductor, with a variable mutual inductance, using a low-frequency alternating current. This variable was compared with a resistance and capacitance in an alternating-current bridge. Finally, the capacitance was measured in terms of resistance and time by a pulsating current. From this series of measurements, the inductance of the standard was obtained in international henrys. The work has not been described in sufficient detail to permit an evaluation of its merits. The result is preliminary and hence is given zero weight.

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[2] P. Vigoureux, *Determination of the ohm by the method of Lorenz*, Nat. Phys. Lab. Collected Researches **24**, 209 (1936–37).

[3] F. E. Smith, *Absolute measurements of a resistance by a method based on that of Lorenz*, Trans. Roy. Soc. (London) [A] **214**, 27 (1914); Nat. Phys. Lab. Collected Researches **9**, 209 (1914).

[4] Rinkichi Yoneda, *Absolute determination of electrical resistance*, Procès-verbaux, Comité International des Poids et Mesures [2] **18**, 178 (1937).

(d) 1937: HARTSHORN AND ASTBURY AT NPL [5]

These authors used the method of Albert Campbell, by which the value of a mutual inductance was determined in international henrys and in absolute henrys. The standard inductor used in this investigation was a Campbell mutual inductor Its value in absolute henrys was computed from the dimensions of the inductor and the permeability of the surrounding medium. The value in international henrys was obtained by a series of electrical measurements that involved a resistance whose value was known in international ohms and a sinusoidal current whose frequency was precisely known.

For making the electrical measurements, a standard resistor and the primary of an auxiliary mutual inductor were connected in series to the source of the sinusoidal current. The secondary of this inductor was connected to the primary of a second mutual inductor. The electromotive force induced in the secondary of this second mutual inductor was balanced against the fall in potential over the standard resistor. Comparisons of the mutual inductances of the auxiliaries with that of the standard mutual inductor and an evaluation of the frequency of the sinusoidal current were required to determine the mutual indutance of the standard in international henrys. The electrical measurements were made with extreme care. The authors gave careful consideration to every suggested source of error and succeeded in devising an experimental technique that apparently made the errors of measurement negligibly small. They made 23 sets of measurements, each of which required 8 separate observations in order to eliminate correction terms. The maximum deviation of the value obtained in any set of observations from the mean was 9 parts per million. The authors estimated that the electrical measurements did not introduce an error of more than 5 parts in a million in the final result, and this conclusion is well supported by the data.

The computed value of the mutual inductance of the standard mutual inductor was not as accurately known as the measured value. The primary winding of the standard consisted of two helices wound on the same marble cylinder and separated by a distance about twice the length of each helix. The secondary was a multilayer coil having twice the diameter of the helices. It was coaxial with the helices and midway between them. The details of the mechanical measurements have not been published, but they have been published for two very similar inductors made by the NPL for the Japanese Government. The first was destroyed in the Tokio earthquake and was later replaced. It will be assumed as a basis for discussion that the variations in dimensions of the inductor retained at the NPL are about the same as the corresponding variations in dimensions of one of the inductors sent to Japan. The ellipticity of each helix was such that in every plane perpendicular to the axis the major diameter was 70 ppm greater than the minor diameter. Also the form on which the two helices were wound was slightly conical, so that the average diameter of one helix was 22 ppm larger than the average diameter of the other. The pitch of one helix was 70 parts in a million less than its nominal value of 1 mm, whereas at the other end it was 600 parts in a million greater than the nominal value, and there were variations in pitch of

[5] L. Hartshorn and N. F. Astbury, *The absolute measurement of resistance by the method of Albert Campbell* Trans. Roy. Soc. (London) [A] 236, 423 (1937).

30 μ in each helix. A correction of 6 ppm was made in the computed inductance on account of the variations in diameter, and of 16 ppm because of the variation in pitch, but neither of these corrections could be determined with much accuracy. Errors in the dimensions of the secondary and in its location relative to the primary probably did not introduce an error greater than 1 ppm in the computed inductance.

The imperfection in the construction of the inductor might lead to the conclusion that there was considerable uncertainty in the value of the computed inductance. However, the measured difference between the inductance of the NPL standard and that of one of those prepared for the Japanese Government agreed with the computed difference within 5 ppm, and the agreement between the NPL standard and the second Japanese standard was 8 ppm. These agreements in the measured and computed differences do not make certain that the computed inductances are correct, since the same type of systematic errors could have occurred in the measurements upon all three inductors or in the computations of their inductances. The error in the computed inductance may have been more than the 10 ppm estimated at the National Physical Laboratory. Nevertheless, the agreement between the three inductors indicates that the corrections for irregularities are probably correct. It is an excellent piece of work and deserves a high weight.

(e) 1938: JOUAUST, PICARD, AND HÉROU AT LCE [6]

The electrical network employed by Jouaust, Picard, and Hérou was the same as that used by Hartshorn and Astbury but involved different balancing conditions. A new description of the network is required to clarify the difference. A standard resistor and the primary of a mutual inductor were connected in series to the source of sinusoidal current. The resistor was paralleled with another circuit consisting of the primary of a second mutual inductor in series with a moderately high resistance. The electromotive forces induced in the two secondaries were balanced against each other by proper adjustment of the constants of the circuits. The product of the resistances of two standard resistors was equal to the product of the square of the angular velocity of the alternating current, the mutual inductance of the inductor which was connected directly to the a–c source, and the self inductance of the primary of the second mutual inductor. Hence a self and a mutual inductance had to be compared with the self inductance of a standard in the form of a helix approximately 9.5 cm in diameter wound with wire 0.3 mm in diameter, the pitch of the winding being 0.6 mm. The self inductance of this standard was about 18 millihenrys. The diameter of the helix was computed from the measured length of wire at a definite tension required to wind a given number of turns. This computation involved not only the length of wire, but also the elastic constants of the fused quartz form. The variations in diameter were 150 ppm. The pitch, measured on a special comparator, varied by as much as 50 μ from its nominal value of 600 μ. The authors do not estimate the error in the computed inductance.

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[6] R. Jouaust, M. Picard, and R. Hérou, ***Determination of the unit of resistance in the cgs electromagnetic system***, Bul. soc. franç. elec. [3] 8, 1 (1938).

There were 20 determinations in the final series of electrical measurements by which the value of inductance in international henrys was evaluated. Each determination required five different electrical comparisons. The mean variation from the mean was ±20 ppm. The final value is given to 1 part in 100,000. The authors state, "We estimate that the last significant figure can differ from the true value by several units."

The method of measuring the diameter of the primary standard of inductance may have introduced appreciable errors. Also the electrical measurements were difficult and are not given in enough detail to permit an independent analysis. Hence the result has been given a low weight.

(f) 1938: CURTIS, MOON, AND SPARKS AT NBS [7]

The same method was employed in this investigation as was used by the authors in 1936. The principal improvement consisted in constructing an inductor in the form of a helix in which the variations in pitch were much less than in those previously used. The form for the inductor was a glass cylinder, 120 cm long, 35 cm in diameter, having a wall thickness of about 7 cm. In the outside surface a screw thread with a pitch of 1 mm was ground and lapped. In the screw thread was wound a wire having a diameter of 0.72 mm. The variations in the pitch of the winding were so small (the maximum observed was 0.003 μ) that those indicated may have resulted from inaccuracies of measurement rather than from imperfections in construction. The outside diameter of the helix had a maximum variation of 4 μ, or 10 ppm. The average diameter as obtained by one set of measurements differed from that by another independent set by 0.5 μ, or 1.5 ppm, but this was probably caused by uncertainties in the length of the two end standards used. The authors state that the error in the computed inductance is probably less than 4 ppm.

The value of the inductance in international henrys was obtained from 90 sets of observations. The average deviation of the result of each set from the mean of all the sets was 6 ppm. The electrical measurements were not as consistent as those of Hartshorn and Astbury, but the standard inductor of the NBS had much more uniform dimensions. Hence the result of this determination is given the same weight as that of Hartshorn and Astbury.

(g) 1939: ZICKNER AT PTR [8]

The method employed by Zickner was the same as that used by Curtis, Moon, and Sparks. Details of his method have not been published. The result is preliminary and hence given zero weight.

(h) 1939: WENNER, THOMAS, COOTER, AND KOTTER AT NBS [9]

Only a preliminary report has thus far been published. The complete report has been delayed by the war. The author of this

[7] H. L. Curtis, C. Moon, and C. M. Sparks, *A determination of the absolute ohm, using an improved self inductor*, J. Research NBS 21, 375 (1938).

[8] G. Zickner, *On the status of the experiments for the determination of the international ohm in absolute units*, Procès-verbaux, Comité Consultatif d'Electricité, p. E41, 1939 (advance print for Procès-verbaux, Comité International des Poids et Mesures).

[9] F. Wenner, J. L. Thomas, I. L. Cooter, and F. R. Kotter, *Preliminary report on the absolute measurement of a resistance based on the reversal of a direct current in a mutual inductance*, Procés-verbaux, Comité Consultatif d'Electricité, p. E48, 1939 (advance print for Procés-verbaux, Comité International des Poids et Mesures).

digest can give a summary of the method because of his personal contact with the investigators while the work was in progress and because a laboratory report is available to him.

In the method employed in this investigation a direct current was passed through a standard resistor and through the primary winding of a standard mutual inductor. The current through the primary of the mutual inductor was reversed at regular intervals without changing the current through the resistor. A commutator reversed the connections to the secondary of the mutual inductor in such a way that the pulses of induced electromotive force were always in the same direction. This rectified electromotive force was balanced against the constant potential drop through the resistor. A direct-current galvanometer was used to indicate the balance. When a balance was obtained, the equation for computing the resistance was

$$R=4nM,$$

where

R = the resistance of the standard resistor in absolute ohms,
M = the computed mutual inductance of the standard in absolute henrys, and
n = the number of reversals of current in the secondary per second.

The resistance in international ohms of the standard resistor was obtained by direct comparison with the fundamental standards of the National Bureau of Standards.

The primary of the standard mutual inductor was a long helix from which two sections were removed. The porcelain cylindrical form for this helix was 83 cm long, 41 cm in diameter, and had a wall thickness of 6 cm. A screw thread having a pitch of 2 mm was ground and lapped in the outer surface. It was wound with wire 1.7 mm in diameter. The two portions that were removed were each 5.6 cm long and started 2.5 cm from the median plane.

The secondary was wound on a form made by cementing together three glass disks, the two outer ones having larger diameters than the inner one. The cemented structure formed a ring having a channel 8 mm wide and 25 mm deep. The mean diameter of the secondary was 52.5 cm. The median plane of the secondary coincided with the median plane of the primary.

The important dimensions for computing the inductance were the diameter and the pitch of the helix. The maximum variations in diameter were less than 20 ppm. The variations in pitch were negligibly small. The mean pitch was measured to about 10 ppm. The error in the computed inductance was probably less than 10 ppm.

The electrical measurements repeated to 1 or 2 ppm. However, there was the possibility of systematic errors, as very little variation of conditions was introduced.

This investigation suffers only by the fact that it has not been subjected to the critical analysis that is inherent in preparing a report for publication. If published reports only are considered, the result of this investigation is preliminary and should be given zero weight. However, the author of this review, knowing the reasons for the delay in publication, feels justified in giving it an intermediate weight.

III. VALUES OF THE ABSOLUTE OHM IN TERMS OF THE MEAN INTERNATIONAL OHM

The results of all the determinations discussed in this review are given in table 1. Each result has been expressed in terms of that value of the mean international ohm which had been established nearest in time to the determination. The results of Yoneda and Zickner have been given zero weight, as both the authors state that they are preliminary and no additional evidence is available. In Vigoureux's work, there were uncertainties in the electrical measurements and variations in the mechanical dimensions of the apparatus so that a weight of unity was assigned. The same weight was given to the results of Jouaust, Picard, and Hérou, who used novel methods of measuring the mechanical dimensions and of making the electrical measurements, and who did not use sufficient checks to convince the reader that systematic errors were eliminated. The 1936 result of Curtis, Moon, and Sparks has been given a weight of two because of the thorough checking that was employed. The only weakness of this work was the inherent imperfections in the helices. The work of Wenner, Thomas, Cooter, and Kotter, although carried out with great care, may still contain some systematic errors that will be brought to light when it is prepared for publication. It has therefore been given a weight of three. The results of Hartshorn and Astbury and of Curtis, Moon, and Sparks in 1938 are each given a weight of five. The inductor of Hartshorn and Astbury was less perfect than the one of Curtis, Moon, and Sparks, but the electrical measurements of the former showed smaller variations than those of the latter. These two researches appear to have eliminated sources of error more completely than any of the others and hence are given the highest weight.

TABLE 1.—*Results of recent determinations of the absolute ohm*

Year	Author	Value of 1 mean international ohm in absolute ohms	Assigned weight
1936	Curtis, Moon, and Sparks	1.000 454	2
1937	Vigoureux	1.000 499	1
1937	Yoneda	1.000 465	--------
1937	Hartshorn and Astbury	1.000 505	5
1938	Jouaust, Picard, and Hérou	1.000 520	2
1938	Curtis, Moon, and Sparks	1.000 482	5
1939	Zickner	1.000 50	--------
1939	Wenner, Thomas, Cooter, and Kotter	1.000 488	3
	Weighted mean	**1.000 490**	--------

The results given in table 1 indicate that the most probable value of the absolute ohm is

1 mean international ohm = 1.000 490 absolute ohms.

Although the weighting has been made according to the best judgment of the author, actually the method of weighting affects the result but little. The means obtained by various methods of weighting the results are given in table 2.

TABLE 2.—*Weighted means obtained by various methods of weighting the eight results on the absolute ohm*

Method of weighting	Resulting weighted mean	Weighted mean deviation from the mean
		ppm [1]
Weighted as in table 1	1.000490	14
Equal weight to all eight	1.000489	17
Equal weight to 6 not preliminary	1.000491	17
Equal weight to 3 with highest weights	1.000492	9

[1] The mean value from which the deviations are taken is the corresponding value in the preceding column. The weighting is that indicated in the first column for that row.

IV. VALUES OF THE ABSOLUTE OHM IN CERTIFIED INTERNATIONAL OHMS

In certifying standard resistors, each of the national laboratories has used a value of an international ohm that is different from the mean international ohm. In fact, some laboratories do not use the same unit of resistance in their regular certifications as in their reports to the International Bureau, although both units may be called international ohms. Hence a laboratory which maintains its resistance unit by means of standards certified by a national laboratory must know the relationship between the mean international ohm and the unit used in certifying resistors before conversion to absolute ohms can be effected. The necessity will be illustrated by explaining the different units of resistance that are now used at the National Bureau of Standards, giving enough historical background for the reader to appreciate the way in which these units arose.

The international ohm as now certified by the National Bureau of Standards has been maintained, without intentional change, for 35 years by means of standard resistors. A value of resistance based on the mercury ohm was assigned to each of a group of standard resistors in 1908. The unit based on the values of these standard resistors has been used by the Bureau in regular certification of standards since that time and is herein called the "international ohm (certified NBS)."

In 1910 an International Technical Committee met at the National Bureau of Standards to determine the value of the international volt from measurements with silver voltameters. Before proceeding with the determination of the volt, the Committee had to reach an agreement concerning the most probable value of the international ohm. They agreed to use the average of the two latest determinations of the mercury ohm up to that time. This unit was only 7 ppm smaller than the unit which was then in use at the National Bureau of Standards. At that time it did not seem worth while to change the values assigned to the Bureau's standards by this small amount, so that the ohm as used before 1910 has continued to be the basis of certified values up to the present time. However, when international cooperation was again undertaken after some 20 years, this difference of 7 ppm was of some moment. In the international comparisons made during the last 10 years a unit has been used in which a correction for the difference of 7 ppm has been made. The published results of the Bureau's absolute electrical measurements have been made in terms of this unit, which has been referred to as the "NBS international ohm." The value of a resistance in international ohms (certified NBS) may be converted to its value in NBS international ohms by multiplying by 1.000 007.

The comparisons of the units of resistance made at the International Bureau of Weights and Measures in 1939 gave the NBS international ohm as 3.2 parts per million less than the mean international ohm, or

1 mean international ohm = 1.000 003 NBS international ohms.

Hence

1 international ohm (certified NBS) = 1.000 004 mean int. ohms.

It follows from the results given in table 1 that

1 international ohm (certified NBS) = 1.000 494 absolute ohms.

V. RÉSUMÉ OF ABSOLUTE-AMPERE DETERMINATIONS

Seven different researches on the value of the absolute ampere have been reported during the last decade. The following analysis covers all of them. In all the determinations, the value in absolute amperes of a current was obtained by comparing the electromagnetic force on a conductor carrying a current with the gravitational force on a known mass, and the value in international amperes of the same current was obtained by observing the fall of potential which it produced in a standard resistor.

(a) 1934: CURTIS AND CURTIS AT NBS [10]

The Rayleigh current balance used by Rosa, Dorsey, and Miller [11] in 1912 was reassembled and used in this investigation. In this balance the electromagnetic force is exerted between a multilayer moving coil and two multilayer fixed coils of larger diameter, all of which carry the same current. The moving coil, with its axis vertical, is suspended from the pan of a balance; the fixed coils are mounted one above, the other below the moving coil. The electromagnetic force is balanced by the gravitational force on a mass placed on the pan of the balance to which the moving coil is attached.

Several sets of coils were employed, all of which were made by winding round wire in a channel cut in a brass form. The moving coils had diameters of either 20 or 25 cm, and the fixed coils had diameters of 40 or 50 cm. Each coil had an approximately square cross section, a side of which did not exceed one-twentieth of the mean diameter of the coil.

By experimentally adjusting the axial distance between the moving coil and each fixed coil until, with a given current, the force between the coils was a maximum, the method of computing the current was so simplified that, for a first approximation, the only mechanical quantity required was the ratio of the mean radii of the coils. This was obtained by comparing the currents required to produce equal and opposite magnetic fields at their centers when the coils were coaxial and concentric. Hence no precise mechanical measurement of the

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[10] H. L. Curtis and R. W. Curtis, *An absolute determination of the ampere*, BS J. Research **12**, 665 (1934) RP685.

[11] E. B. Rosa, N. E. Dorsey, and J. M. Miller, *A determination of the international ampere in absolute measure*, Bul. BS 8, 269 (1912) S171.

diameter was required. The next approximation was developed by assuming that the current was uniformly distributed over the cross section. Then the computation of the absolute value of the current required in addition to the ratio of the mean radii, the ratios of both the width and depth of each coil to its diameter. These additional ratios were determined by Rosa, Dorsey, and Miller when the coils were constructed by making measurements of each dimension.

In the reassembly, slight changes were made to improve the convenience of operation, but the same wire-wound coils of many layers were employed. The ratios of the radii of the coils were redetermined, but no checks on the additional ratios were possible.

The results indicate that the value obtained with a set of coils, each of which had small ratios of the width and the depth of the cross section to the diameter of the coil, was more dependable than the value obtained when this ratio was larger. However, it appears that the correction for the cross sections of the coils could not in any case be accurately made, as the wires were not uniformly distributed throughout the cross section. In winding the coils, the wire could not be uniformly distributed, this being especially the case where the wire passed from one layer to the next. The coils used in this investigation are so inferior to those used in more recent investigation that the result has been given zero weight.

(b) 1935: DUPOUY AND JOUAUST AT LCE [12]

The authors used a balance of the type proposed by Cotton. In this type of balance, the electromagnetic force between a short conductor carrying a current and a strong magnetic field in which it is placed, is compared with a gravitational force. The computation of the current requires a knowledge of the magnetic induction in the region occupied by the conductor. As there is no independent method of measuring this induction with precision, the absolute value of the current is not accurately determined. The authors state that their result may be in error by a few parts in 10,000. The result has been given zero weight.

(c) 1936: VIGOUREUX AT NPL [13]

This author used the balance designed by Ayrton, Mather, and Smith, but constructed new coils for it.[14] The coils were in the form of helices of wire, wound in screw threads cut in marble cylinders on a lathe. Two moving coils and four fixed coils were employed. One moving coil was hung on each of the pans of a balance. These moving coils were cylinders 20 cm in diameter having a length of winding of 15.2 cm. The two fixed coils, which were used in conjunction with a single moving coil, were wound near the two ends of a hollow cylinder of marble about 32 cm in diameter. Each helix was 11 cm long, and there was a 3-cm gap between them. Each pair of fixed coils was coaxial with its moving coil, and the center of the gap between the fixed coils coincided with the center of the moving coil.

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[12] G. Dupouy and R. Jouaust, *On the absolute measurement of a magnetic field and the determination of the ampere in absolute value*, J. phys. et radium [7] **6**, 123 (1935).

[13] P. Vigoureux, *An absolute determination of the ampere*, Trans. Roy. Soc. (London) [A] **236**, 133 (1936); Nat. Phys. Lab. Collected Researches **24**, 173 (1938).

[14] W. E. Ayrton, T. Mather, and F. E. Smith, *A new current weigher and a determination of the electromotive force of the normal Weston cadmium cell*, Trans. Roy. Soc. (London) [A] **207**, 463 (1908); Nat. Phys. Lab. Collected Researches **4**, 1 (1908).

In deriving the formula for computing the current, a first approximation was obtained by assuming that each coil was a current sheet. Corrections were then applied for the discrete windings of the coils and for the interaction between the two sets of coils.

The helices showed rather large variations in their dimensions. The maximum variation in diameter of each fixed coil was 50 ppm and of each moving coil 100 ppm. All the coils had periodic variations in pitch, the total amplitude being about 10μ, in a pitch of 1,000 μ. The author states, "This periodic error constitutes a serious defect in the coils of this current balance."

The result is given as the electromotive force of the neutral cadmium cell at 20° C. The voltage of the typical cell as realized at the NPL was found to be 1.01816 abs amp $\times$ NPL int. ohm. Some later measurements gave 1.018151. The author states that, neglecting a possible error in the value of gravity at Potsdam, the "accuracy will be assessed" at 20 ppm. In view of the variations in the mechanical dimensions noted above, it seems that a systematic error of more than 20 ppm might be present.

Several steps are required to convert the value given by the author from the units which he used to those herein employed. The result obtained by making this conversion is given in table 3. Because the dimensions of the coils showed large variations, the result is given an intermediate weight.

(d) 1937: YONEDA AND ISHIBASHI AT ETL [15]

The authors used a Rayleigh current balance with wire-wound coils. The apparatus was very similar to that employed by Curtis and Curtis in 1934. No details of the apparatus or of the methods of measurement are available. Only a preliminary result is given. Zero weight is assigned to the result.

(e) 1939: CURTIS, CURTIS, AND CRITCHFIELD AT NBS [16]

This investigation is a continuation of the work of Curtis and Curtis. Two new fixed coils and two new moving coils were used in the Rayleigh current balance, but otherwise the same apparatus was used and the same technique followed as that described in the first review of this series. Both of the new fixed coils and one moving coil were flat spirals of aluminum ribbon. The second moving coil was a short helix. The insulation of the aluminum ribbon consisted of a thin layer of aluminum oxide, which was formed by electrolytic action on the surface of the ribbon. The coils were very compact and uniform. They conformed much more closely to the requirements assumed in the mathematical derivation of the formula for computing the absolute value of the current than was the case with the multiple-layer coils of wire.

The short helix used as a second moving coil consisted of wire wound in a screw thread that was lapped in a glass form. The length of the winding was 2.6 cm. The formula for computing the current when using this moving coil was a modification of the formula for coils with an approximately square cross section. The diameter was not meas-

[15] R. Yoneda and Y. Ishibashi, *Absolute determination of current*, Procès-verbaux, Comité International des Poids et Mesures [2] **18**, 185 (1937).

[16] H. L. Curtis, R. W. Curtis, and C. L. Critchfield, *An absolute determination of the ampere, using improved oils,*J Research NBS **22**, 485 (1939) RP1200.

ured for this research, as the ratio of its diameter to that of each of the fixed coils was measured by the magnetic method.

The most probable value of the absolute ampere as reported by the authors was an average of four values, two obtained when using the coils described in the preceding paragraphs and two from the paper of Curtis and Curtis. It now seems desirable to discard all results obtained with multiple-layer coils of wire such as were used by Curtis and Curtis. Hence the result for this research given in the table which follows is the average of the two values obtained when using spiral fixed coils with two different moving coils, one being a spiral and the other a short helix. It is slightly different from the results given in the paper as the most probable value of the absolute ampere. A high weight is assigned to the result.

(f) 1939: VON STEINWEHR AT PTR [17]

A Rayleigh current balance was used in this investigation. Methods of computation and operation similar to those used by Curtis and Curtis were employed. Most of his coils were wound with ribbon. A pair of fixed coils and a moving coil were made by winding a spiral of copper ribbon in which the turns were insulated by a layer of silk. Two moving coils were made from aluminum ribbon, insulating the turns by a layer of aluminum oxide as described by Curtis, Curtis, and Critchfield. One pair of fixed coils was wound with enamel-covered wire. The author states: "The results . . . must not yet be considered final." Zero weight is assigned to the result.

(g) 1942: CURTIS, DRISCOLL, AND CRITCHFIELD AT NBS [18]

In this investigation the apparatus was the same as that described in the fifth review, except that the fixed coils of aluminum ribbon were replaced by helices which were the two halves of a single helix electrically divided into two parts by attaching a lead at the midpoint of the winding. The method of obtaining the constants of the balance was quite different from that employed when the fixed coils of aluminum ribbon were used, because with the helical fixed coils the magnetic method of obtaining the ratio of radii of fixed and moving coils was not applicable. When the helical moving coil was used, the force per unit current was computed from the dimensions of the fixed and moving helices. When the spiral moving coil of aluminum ribbon was used, its effective radius was obtained by an indirect method.

When the helical moving coil was employed, the balance was similar to the one used by Vigoureux at the NPL, the principal difference being that the moving coil was much shorter.

The variations in the dimensions of the helices were small. The maximum variation in outside diameter of the fixed helix having a diameter of 46 cm was 0.4 μ, or 1 ppm. The length of the two halves, each 14 cm long, did not differ by more than 0.8 μ, indicating very uniform pitch. No periodic error in the pitch could be detected. The

[17] H. von Steinwehr, *Report concerning the progress of the experiments on a determination of the international ampere in absolute units*, Procès-verbaux Comité Consultatif d'Electricité, p. E51, 1939 (advance print for Procès-verbaux, Comité International des Poids et Mesures).

[18] R. W. Curtis, R. L. Driscoll, and C. L. Critchfield, *An absolute determination of the ampere, using helical and spiral coils*, J. Research NBS **28**, 133 (1942) RP1449.

outer surface of the moving coil with the helical winding had the shape of an elliptical cylinder with a mean diameter of 24.5 cm and a length of 2.6 cm. The difference between the major and minor diameters was 13 μ. However, the average diameter as obtained from measurements at six equally spaced generators differed from that obtained from the six interspaced generators by only 0.1 μ, or less than 1 part in a million. The pitch was so uniform that no variation exceeding 0.1 μ could be detected.

The mean diameter of the spiral moving coil was obtained by an indirect comparison with the helical moving coil. The ratio of the diameter of each moving coil to the diameter of each of six fixed coils was obtained by a magnetic method. The mean deviation in the six ratios of the diameters of the two moving coils was 5 ppm.

The authors include in their final value the results of the investigation that was completed in 1939, as well as those obtained with the coils described in the preceding paragraphs. However, the value for the 1942 investigation as given in table 3 is the average of only the two values obtained with the subdivided helix for the fixed coils and with either a spiral or a short helix as the moving coil. A high weight is given to this result.

VI. VALUES OF THE ABSOLUTE AMPERE

The results of all determinations of the absolute ampere herein discussed are given in table 3. Weights have been arbitrarily assigned to the result of each determination.

TABLE 3.—*Results of recent determinations of the absolute ampere*

Year	Author	Value of 1 mean international ampere in absolute amperes	Assigned weight
1934	Curtis and Curtis	0. 999 934	
1935	Dupouy and Jouaust	. 999 8$_9$	
1936	Vigoureux	. 999 863	3
1937	Yoneda and Ishibashi	. 999 938	
1939	Curtis, Curtis, and Critchfield	. 999 860	5
1939	Von Steinwehr	1. 000 04	
1942	Curtis, Driscoll, and Critchfield	0. 999 856	5
	Weighted mean	[a] 0. 999 859	

[a] This value is corrected on page 252 for the new value of the acceleration of gravity.

The weighting markedly affects the results. This is shown by the values given in table 4. This is in contrast with the results for the ohm, as shown in table 2.

TABLE 4.—*Weighted means obtained by various methods of weighting the seven results on the absolute ampere*

Method of weighting	Resulting weighted mean	Weighted mean deviation from the mean
		ppm [1]
Weighted as in table 3	0.999 859	3
Equal weight to all seven	.999 912	51
Equal weight to 4 not preliminary	.999 878	28
Equal weight to 3 with highest weights	.999 860	2

[1] The mean value from which the deviations are taken is the corresponding value in the preceding column. The weighting is that indicated in the first column of the corresponding line.

The results of Curtis and Curtis and of Yoneda and Ishibashi, both of which were obtained with multilayer coils of round wire, have been given zero weight. The result of Dupouy and Jouaust was incidental to another investigation, and the authors do not claim an accuracy of much more than 1 part in 10,000, so it also has been given a zero weight. The result of Von Steinwehr is given by the author as preliminary, and the wide variation of his results when using different coils supplies an added reason for giving zero weight. The weight for the result of Vigoureux is not the highest because of the imperfections of the coils in his balance. Only two determinations can be given the highest weight, and both of these are from the laboratories of the National Bureau of Standards.

The value of the acceleration of gravity, which enters into all seven results, was in every case based on that obtained at Potsdam, Germany, by an absolute measurement carried out during the first decade of this century. Transfers by pendulums of invariable length have been made from Potsdam to the laboratories making the absolute measurement of current. In the last few years two absolute determinations of the value of gravity have been made, one at the NPL and the other at the NBS. Moreover, the methods used in reducing the data of the Potsdam determination have been reexamined.[19] The revised Potsdam value was compared with the two more recent determinations. The conclusion was reached that the most probable value of the acceleration of gravity at the gravity station of the National Bureau of Standards is

$$980.083 \text{ cm/sec}^2.$$

At the pan of the current balance, the value becomes

$$980.082 \text{ cm/sec}^2.$$

The value of gravity used in the three recent determinations of the absolute ampere at the NBS was

$$980.095 \text{ cm/sec}^2.$$

This difference of 13 ppm in the numerical value of the acceleration

[19] H. L. Dryden, *A reexamination of the Potsdam absolute determination of gravity*, J. Research NBS **29**, 303 (1942) RP1502.

of gravity makes a decrease in value of the absolute ampere of 6.5 ppm. Presumably, the value of the absolute ampere obtained at other laboratories requires an identical correction. Making this correction to the weighted mean value obtained from table 3,

1 mean international ampere=0.999 853 absolute ampere

From relationships given elsewhere between the mean international ohm and volt and the international ohm and volt as certified by the National Bureau of Standards, it follows that

1 international ampere (certified NBS)=0.999 838 absolute ampere.

VII. MAINTENANCE OF THE VOLT AND ITS ABSOLUTE VALUE

The volt is maintained at the national standardizing laboratories by groups of standard cells.[20] The group at the National Bureau of Standards consists of about 20 cells, several of which were made in 1906. In 1911 the average of the group was assigned a value in international volts that was determined from the international ohm as obtained from a column of mercury, and the international ampere as obtained from deposits of silver in a silver voltameter. The unit thus established has been maintained until the present time and has been used both in international comparisons and in certifications. Comparisons made at the International Bureau in 1939 showed that this unit of electromotive force is 11 ppm less than the mean international unit. From the weighted mean values of the absolute ohm and ampere in terms of the mean international ohm and ampere, it follows that

1 mean international volt=1.000 343 absolute volts.

Hence

1 international volt (certified NBS)=1.000 332 absolute volts.

VIII. THE UNITS OF INDUCTANCE AND CAPACITANCE

The units of inductance and capacitance are maintained at each of two of the national laboratories by reference to a standard inductor whose inductance can be computed from its mechanical dimensions. In two other national laboratories, the units are not maintained independent of the other electrical units but standards of inductance and capacitance are frequently measured in terms of resistance and time. The latter method is employed at the National Bureau of Standards.

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[20] For methods used by the different national laboratories to maintain the units of resistance and of electromotive force, see H. L. Curtis, The Establishment and Maintenance of the Electrical Units, Bul. Nat. Research Council No. 93 (1933). So far as the author knows, the only change in methods which has been adopted since the above was written is the use of improved standard resistors at the National Bureau of Standards.

Measurements of time can be made with extreme accuracy. Hence the relationships between the various units of inductance are identical with those of the units of resistance, but between the units of capacitance the relationships are the reciprocals of those of the units of resistance.

Hence

1 international henry (certified NBS) = 1.000 007 NBS international henrys.
= 1.000 004 mean international henrys.
= 1.000 494 absolute henrys.

Also

1 international farad (certified NBS) = 0.999 993 NBS international farad.
= 0.999 996 mean international farad.
= 0.999 506 absolute farad.

IX. SUMMARY

The factors for converting the value of a standard as certified in international units by the National Bureau of Standards to its value in absolute units according to the analysis of this paper are given in table 5.

TABLE 5.—*Factors for converting the value of a standard in international electrical units (certified NBS) to its value in absolute units*

Quantity	Unit	Conversion factor [1]
Resistance	Ohm	1.000 494
Electromotive force	Volt	1.000 332
Current	Ampere	0.999 838
Quantity	Coulomb	.999 838
Inductance	Henry	1.000 494
Capacitance	Farad	0.999 506
Energy	Joule	1.000 170
Power	Watt	1.000 170

[1] The figures in the last decimal place have little significance.

X. COMPARISONS OF RESULTS WITH THOSE OF OTHER RECENT COMPILATIONS

Two other compilations of the absolute values of the electrical units have recently been made. The values of the ohm and ampere as given by the three compilers are collected in table 6. Dunnington [21] gave the relation of the NBS international ohm and ampere to the absolute ohm and ampere but did not indicate the analysis by which he arrived at

[21] F. G. Dunnington, *The atomic constants: A revaluation and an analysis of the discrepancy*, Rev. Modern Phys. **11**, 65 (1939).

the published results. For this compilation his values have been reduced to mean international ohms and amperes and his value of the ampere changed to correspond to the recent determinations of gravity. Stille [22] made a complete and impartial analysis of the recent determinations of the ohm and the ampere. He has modified the results as published by the different authors, so that all are expressed as the value in absolute units of 1 mean international unit. His final value for the ampere is carried only to 1 part in a hundred thousand, but he has given several possible interpretations in which the value is carried to 1 part in a million. The value which he appears to favor, corrected to correspond to the value of gravity used in this compilation, has been chosen for inclusion in table 6.

TABLE 6.—*Conversion factors given by various compilers*

Compiler	Value in absolute units of 1 mean international unit	
	Ohm	Ampere
Dunnington	1.000 488	0.999 972
Stille	1.000 493	.999 880
Curtis	1.000 490	.999 853

The value of the ohm seems to be well established, since the maximum difference between different compilers is 3 ppm. The value of the ampere is not so well established as there is a maximum difference of 119 parts in a million. This large difference can be explained by the following considerations. Dunnington did not have available one, and probably two, of the recent determinations of the National Bureau of Standards. Probably he would now give a value lower than the one he used in 1939. Stille did not have available the paper by Curtis, Driscoll, and Critchfield, which would have lowered his average. However, the difference is largely due to the adoption by the writer in this compilation of a system of weighting results. Stille has done the equivalent of this by giving for one author several values corresponding to the different sets of coils he used, whereas for others he gave only the average of two or more sets of coils. The weighing in this paper is of necessity arbitrary, but it represents the best judgment of the author of this review.

WASHINGTON, May 15, 1944.

[22] U. Stille, *The conversion factors from the international to the absolute electrical units*, Z. Physik. **121**, 34 (1943).