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A PRECISION APPARATUS FOR THE RAPID DETERMINATION OF INDICES OF REFRACTION AND DISPERSION BY IMMERSION

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ABSTRACT

A new immersion method for determining indices of refraction and ν values, employing the double-diaphragm method for securing oblique illumination, is described. The average error in the determination of indices of refraction based upon 144 measurements is $2\text{x}10^{-5}$; the maximum error is $5\text{x}10^{-5}$. Complete measurements of the indices of refraction for the sodium *D* line and the hydrogen *F* and *C* lines may be made in approximately 1½ hours, and from these measurements the ν values may be calculated with an average error of 0.1 and a maximum error of 0.8.

CONTENTS

I. INTRODUCTION

In the production of optical glass, careful control over the optical properties, especially the index of refraction (n_D) and the Abbe value (ν), of the product is essential. Most methods for measuring indices of refraction are sufficiently accurate for the control of the n_D value. The ν value, on the other hand, presents a special problem in index measurement. It is defined as $(n_D-1)/(n_F-n_C)$, and in order to determine this value within certain specified limits, the index of refraction must be measured very accurately. Some typical tolerances which have to be met and the errors in the index measurements corresponding to these tolerances are shown in table 1.

Table 1.—*Typical tolerances for n_D and ν values*

Glass	Typical values and tolerances		Error corresponding to tolerance in ν
	n_D	ν	
Borosilicate crown	1.517 ±0.001	64.5 ±0.3	$\pm2\times10^{-5}$
Light barium crown	1.572 ±0.001	57.4 ±0.3	$\pm3\times10^{-5}$
Dense flint	1.649 ±0.0015	33.8 ±0.3	$\pm9\times10^{-5}$

It is obvious from table 1 that the required accuracy of index measurements is determined mainly by the ν value tolerances. If the errors in index measurements are as great as those listed in column three, the ν value may be in error by the amount of the permissible tolerances. In order to be certain that the ν values are well within the required limits, the best spectrometer measurements [1] are necessary for final control of crown glasses, and are desirable for flint glasses. However, there are many useful routine checks and measurements that can be made on refractometers.

In normal times at the optical glass plant of the National Bureau of Standards, the problem of control was solved by making two independent sets of measurements. The immersion method of Faick and Finn [2] was used for immediate production control work. The final and more accurate values of optical constants were later measured on a spectrometer and a precision refractometer.

During wartime, manufacturers are called upon to make new glasses and a greater variety of them. This requires frequent changes in batch compositions and makes necessary more frequent and careful determination of index of refraction and dispersion. Under these conditions the above-mentioned immersion control method was deficient both as to speed and accuracy. The more accurate index measurements as they are generally made, on spectrometers or on carefully calibrated Pulfrich refractometers, require ground and pitch-polished samples. The time-consuming operations of grinding and polishing cause delay in the processing of the glass or in the revision of batch compositions, especially in glass plants that are not equipped with grinding and polishing facilities for the preparation of samples. Most immersion methods permit the use of rough, unprepared samples, but either do not yield sufficiently accurate results or lack the simplicity and speed desired.[3] It was, therefore, felt desirable to develop an immersion method for rough samples which would have the required accuracy and permit speed and ease of manipulation.

II. METHOD OF MATCHING

One of the chief requirements of immersion refractometry is a sensitive method for ascertaining the match between the indices of the sample and the immersion liquid. Of the various methods that could be used for this, one embodying the principles of oblique illumination, well known to petrographers, seemed the most promising both for sensitivity and simplicity of operation. A system of oblique illumination, using double diaphragms, has been described by Saylor [4] and shown to be superior to usual methods, at least when used on a microscope. Another double-diaphragm method, employing a telescope and a long focus lens as parts of the optical system, has been used by Cheshire [5] for measuring the index of refraction of glass by immersion. This system is too cumbersome for rapid control work.

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[1] H. L. Gurewitz and L. W. Tilton, J. Research NBS **32**, 39 (1944) RP1572.

[2] C. A. Faick and A. N. Finn, BS J. Research **6**, 993-1002 (1931) RP320.

This method consists essentially in matching the index of refraction of the glass with that of a suitable liquid in which the sample is immersed and then measuring the index of the liquid. The apparatus consists of a telescope and a grid, illuminated by a sodium light, which are placed on opposite sides of a water-cooled tank containing the immersion liquid and sample. The match between the glass and the liquid is obtained by altering the index of the liquid until no distortion of the grid pattern is observed through the telescope. Accuracies within $\pm 3\times10^{-4}$ are obtainable with this method.

[3] For a discussion of immersion refractometry, see Glazebrook's Dictionary of Applied Physics **4**, 130 (The Macmillan Co., New York, N. Y., 1923).

[4] C. P. Saylor, J. Research NBS **15**, 277-294 (1935) RP829.

[5] R. W. Cheshire, Phil. Mag. [6] **32**, 409-420 (1916).

Basically, the method finally chosen consists in inserting at the focus of a condensing lens (fig. 1, A) a diaphragm which shades out almost one-half of the beam of light. The emerging beam is then passed through the immersion liquid and again brought to a focus by means of a second converging lens. At this focus (fig. 1, B) a second diaphragm is placed so that most of the remaining bright portion of the beam is shaded out. If a telescope is then inserted in the system, the field, as seen through the eyepiece, will appear dimly but uniformly lighted. Now, when a specimen is immersed in a liquid of the same index of refraction as that of the specimen, the light rays passing through the specimen will not be deviated and, since the second

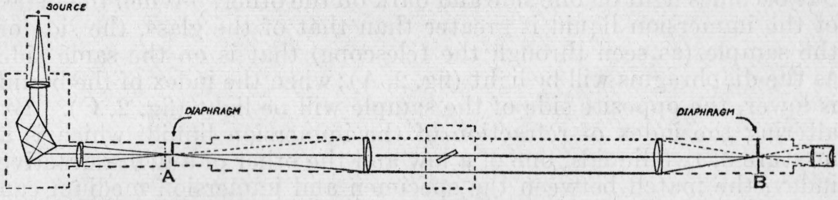

Figure 1.—*Schematic drawing of the optical system for matching the indices of the glass and immersion liquid.*

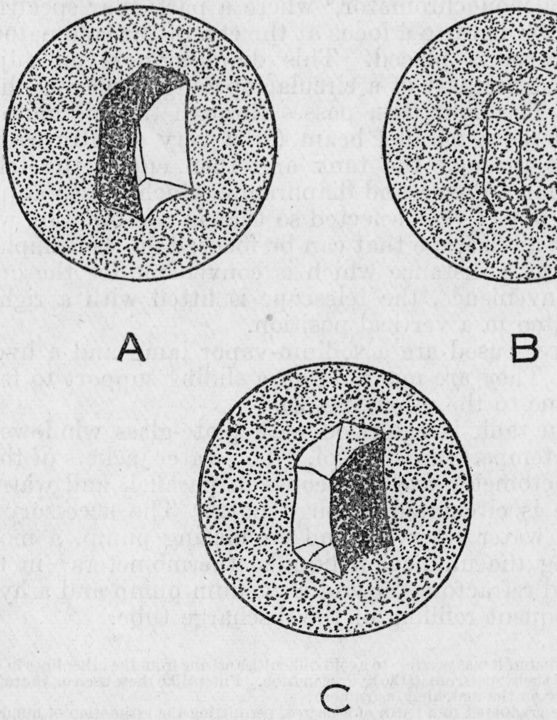

Figure 2.—*Field through the eyepiece of the telescope—diaphragm on right side of the instrument.*

A, When the index of the liquid is greater than the index of the sample; B, When the liquid and sample have the same index; C, When the index of the liquid is lower than that of the sample.

diaphragm takes out the bright portion of the beam, the sample will be practically invisible against the dimly lighted field seen through the eyepiece (fig. 2, B). However, if the match between the sample and liquid is altered, the path of light passing through the sample will be deviated so that the sample no longer blends with the field, but becomes light on one side and dark on the other. When the index of the immersion liquid is greater than that of the glass, the side of the sample (as seen through the telescope) that is on the same side as the diaphragms will be light (fig. 2, A); when the index of the liquid is lower, the opposite side of the sample will be light (fig. 2, C). By altering the index of refraction of the immersion liquid, which is a mixture of two liquids, one of a low and the other of a high refractive index, the match between the specimen and immersion medium can be easily established. After a match has been obtained, the index of refraction of the liquid is measured on a precision refractometer by comparison with a standard of approximately the same index.

III. APPARATUS

The apparatus (figs. 1 and 3) consists essentially of light sources, a monochromator with the exit slit removed, a collimator, an immersion tank, a telescope, a tested precision refractometer, and accessory equipment. Referring to figure 1, the light from the source is first passed through a monochromator,[6] where a particular spectrum line is isolated and is brought to a focus at the end of the collimator where the first diaphragm is placed. This diaphragm is an adjustable knife-edge that covers part of a circular opening, replacing the usual collimator slit. The light then passes through the collimator, from which it emerges as a parallel beam (or a very slightly converging one) that passes through the tank and then enters the telescope. Near the focal plane, the second diaphragm, which is also adjustable, is located. The objective is selected so that in combination with the eyepiece it forms a telescope that can be focused on the sample in the immersion tank at a distance which is convenient for the operator. As an added convenience, the telescope is fitted with a right-angle prism and mounted in a vertical position.

The light sources used are a sodium-vapor lamp and a hydrogen-discharge tube. They are mounted on a sliding support to facilitate changing from one to the other (fig. 3).

The immersion tank is equipped with plate-glass windows and a water jacket for temperature control. The water jackets of this tank and of the refractometer are connected in parallel, and water from the same source is circulated through both. The accessory equipment includes a water reservoir and circulating pump, a motorized stirrer for mixing the immersion liquids, thermometers[7] in the immersion tank and refractometer and a vacuum pump and a hydrogen generator for frequent refilling of the discharge tube.

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[6] By use of a monochromator, it was possible to avoid difficulties arising from the other lines in the sodium source, and from the continuous spectrum in the hydrogen tube. Filters like those used on the refractometer were found unsatisfactory on the matching instrument.

[7] Mercury-in-glass type graduated to a tenth of a degree, permitting the estimation of hundredths of a degree. The thermometers should be carefully checked against each other, since the difference in temperature is the important factor.

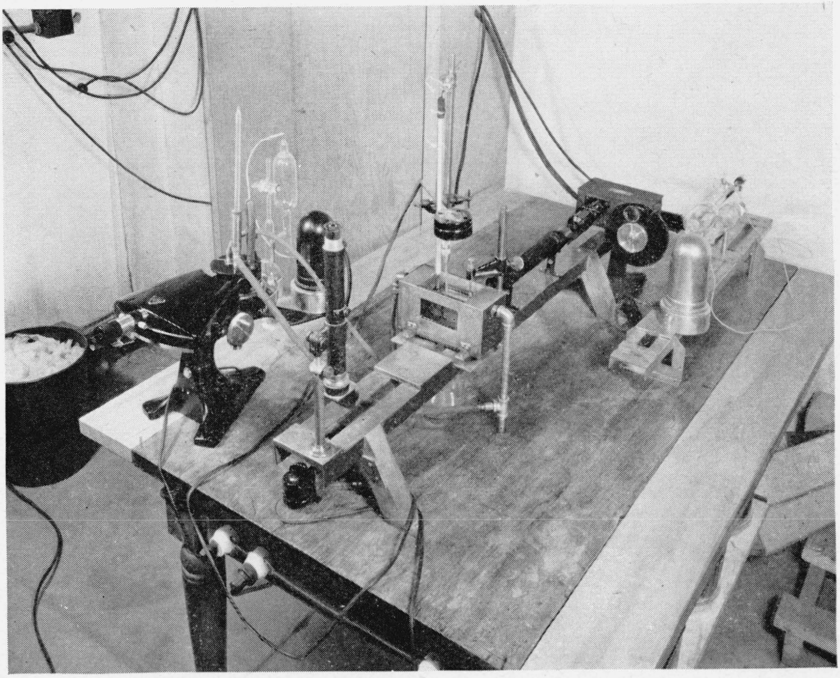

FIGURE 3.—*Complete immersion apparatus.*

IV. PROCEDURE

Six immersion liquids were prepared. The indices of refraction of these liquids were approximately 1.517, 1.572, 1.604, 1.617, 1.620, and 1.649, respectively. The first five liquids were mixtures of liquid petrolatum ($n_D=1.456$) and α-chloronaphthalene ($n_D=1.632$). The sixth liquid was a mixture of α-chloronaphthalene and α-bromonaphthalene ($n_D=1.656$). After isolating a particular spectrum line by means of the monochromator, the sample was immersed in the appropriate liquid, and the two were matched for index by adding small quantities of one of the pure liquids. When a good match had been obtained, the index of the immersion liquid was measured on the refractometer. This process was repeated four times for each sample. In each series of measurements, care was taken to approach the correct value from both a higher index immersion liquid and a lower one. Measurements of the temperature of the immersion liquid and of the refractometer prism block were made with a precision of ±0.02° C. Corrections for very small temperature differences were applied by using an average change of -0.00035 in index per degree rise in temperature.

In order to obtain the highest possible accuracy in index measurements, a comparison method recommended by Tilton[8], was used. By this method, instrumental errors, such as a shift in the zero setting, can be readily compensated. Essentially this method consists in checking the refractometer, before and after each series of immersion measurements, with a standard of known index approximately the same as the rough sample being measured and making the necessary corrections. This check was made for each type of glass and for each wavelength used, by measuring the standard as a Pulfrich slab on the refractometer. In several cases brought to the attention of the authors, considerable difference was found in the readings of refractometers when measuring solid and liquid samples of the same index. Should this be the case, an alternative procedure should be used in checking the refractometer. In the alternative procedure a liquid which in each case is matched with a known sample in index is used as the "standard" instead of the solid sample.

A series of 12 glass samples was tested to obtain the accuracy of the instrument. These samples were cut from prisms that had been previously measured on a spectrometer. The indices of refraction of these test pieces were known to the fifth decimal place. Since these samples were in the shape of rectangular prisms approximately 2 by 2 by 0.2 cm in size and had polished faces, and since the instrument was designed for measuring rough, irregularly shaped specimens, it was necessary to compare the accuracies of measurement obtainable with rough pieces and finished samples. This was done by measuring the indices of refraction of four rough pieces of glass and then grinding these pieces to the same shape as the standards and then remeasuring the indices. The results of this test show that any difference that does exist is within the error of measurement.

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[8] L. W. Tilton, J. Opt. Soc. Am. **32**, 371–381 (1942).

TABLE 2.—*Comparison of measurements made on rough, irregularly shaped samples and on fine-ground, rectangular specimens*

Index of refraction (n_D)			Index of refraction (n_D)		
Rough sample		Ground sample	Rough sample		Ground sample
SAMPLE I			SAMPLE III		
1	1.51580	1.51578	1	1.60179	1.60178
2	1.51582	1.51581	2	1.60182	1.60180
3	1.51578	1.51583	3	1.60180	1.60180
4	1.51585	1.51580	4	1.60184	1.60181
Average	1.51581	1.51581	Average	1.60181	1.60180
SAMPLE II			SAMPLE IV		
1	1.57187	1.57188	1	1.61960	1.61960
2	1.57189	1.57187	2	1.61961	1.61963
3	1.57190	1.57186	3	1.61963	1.61961
4	1.57188	1.57190	4	1.61962	1.61962
Average	1.57189	1.57188	Average	1.61962	1.61962

V. RESULTS

The values obtained in the test of the rough blanks are given in table 2. A total of 48 measurements were made on the 12 regular samples for each of the wavelengths, corresponding to the sodium *D* line, and hydrogen *F* and *C* lines. The results of these measurements are tabulated in table 3. Table 3 also shows the errors in the measured

TABLE 3.—*Comparison of immersion determinations of index of refraction and dispersion with spectrometer values*

Method	n_D	Error ×10⁵	n_F	Error ×10⁵	n_C	Error ×10⁵	μ	Error
SAMPLE I								
Spectrometer	1.51651	-------	1.52211	-------	1.51411	-------	64.6	-------
Immersion 1	1.51653	+2	1.52213	+2	1.51412	+1	64.6	0
Immersion 2	1.51653	+2	1.52209	−2	1.51413	+2	64.9	+0.3
Immersion 3	1.51650	−1	1.52210	−1	1.51410	−1	64.6	0
Immersion 4	1.51651	0	1.52208	−3	1.51412	+1	64.9	+.3
Average	1.51652	1	1.52210	2	1.51412	1	64.8	0.2
SAMPLE II								
Spectrometer	1.51748	-------	1.52316	-------	1.51507	-------	64.0	-------
Immersion 1	1.51746	−2	1.52318	+2	1.51509	+2	64.0	0
Immersion 2	1.51747	−1	1.52316	0	1.51510	+3	64.2	+0.2
Immersion 3	1.51750	+2	1.52317	+1	1.51508	+1	64.0	0
Immersion 4	1.51746	−2	1.52313	−3	1.51508	+1	64.3	+.3
Average	1.51747	2	1.52316	2	1.51509	2	64.1	0.1
SAMPLE III								
Spectrometer	1.57205	-------	1.57906	-------	1.56913	-------	57.6	-------
Immersion 1	1.57202	−3	1.57909	+3	1.56914	+1	57.5	−0.1
Immersion 2	1.57202	−3	1.57904	−2	1.56909	−4	57.5	−.1
Immersion 3	1.57206	+1	1.57905	−1	1.56911	−2	57.6	0
Immersion 4	1.57206	+1	1.57906	0	1.56911	−2	57.5	−.1
Average	1.57204	2	1.57906	2	1.56911	2	57.5	0.1

TABLE 3.—*Comparison of immersion determinations of index of refraction and dispersion with spectrometer values*—Continued

Determination	n_D	Error $\times 10^5$	n_F	Error $\times 10^5$	n_C	Error $\times 10^5$	ν	Error
SAMPLE IV								
Spectrometer	1.57151	------	1.57850	------	1.56859	------	57.7	------
Immersion 1	1.57150	−1	1.57849	−1	1.56858	−1	57.7	0
Immersion 2	1.57151	0	1.57853	+3	1.56860	+1	57.6	−0.1
Immersion 3	1.57154	+3	1.57847	−3	1.56861	+2	58.0	+.3
Immersion 4	1.57151	0	1.57846	−4	1.56858	−1	57.8	+.1
Average	1.57152	1	1.57849	3	1.56859	1	57.8	0.1
SAMPLE V								
Spectrometer	1.60428	------	1.61419	------	1.60027	------	43.4	------
Immersion 1	1.60431	+3	1.61416	−3	1.60025	−2	43.4	0
Immersion 2	1.60430	+2	1.61416	−3	1.60024	−3	43.4	0
Immersion 3	1.60431	+3	1.61420	+1	1.60023	−4	43.3	−0.1
Immersion 4	1.60428	0	1.61419	0	1.60028	+1	43.4	0
Average	1.60430	2	1.61418	2	1.60025	3	43.4	0
SAMPLE VI								
Spectrometer	1.60454	------	1.61446	------	1.60053	------	43.4	------
Immersion 1	1.60454	0	1.61450	+4	1.60056	+3	43.4	0
Immersion 2	1.60457	+3	1.61449	+3	1.60054	+1	43.3	−0.1
Immersion 3	1.60456	+2	1.61450	+4	1.60053	0	43.3	−.1
Immersion 4	1.60454	0	1.61447	+1	1.60057	+4	43.5	+.1
Average	1.60455	1	1.61449	3	1.60055	2	43.4	0.1
SAMPLE VII								
Spectrometer	1.61804	------	1.63018	------	1.61320	------	36.4	------
Immersion 1	1.61804	0	1.63020	+2	1.61322	+2	36.4	0
Immersion 2	1.61803	−1	1.63019	+1	1.61323	+3	36.4	0
Immersion 3	1.61801	−3	1.63019	+1	1.61324	+4	36.5	+0.1
Immersion 4	1.61806	+2	1.63017	−1	1.61325	+5	36.5	+.1
Average	1.61804	2	1.63019	1	1.61324	3	36.5	0.1
SAMPLE VIII								
Spectrometer	1.61731	------	1.62942	------	1.61249	------	36.5	------
Immersion 1	1.61734	+3	1.62941	−1	1.61250	+1	36.5	0
Immersion 2	1.61729	−2	1.62943	+1	1.61252	+3	36.5	0
Immersion 3	1.61728	−3	1.62937	−5	1.61252	+3	36.6	+.1
Immersion 4	1.61729	−2	1.62940	−2	1.61252	+3	36.6	+.1
Average	1.61730	3	1.62940	2	1.61252	3	36.6	.1
SAMPLE IX								
Spectrometer	1.62024	------	1.63248	------	1.61536	------	36.2	------
Immersion 1	1.62023	−1	1.63249	+1	1.61536	0	36.2	0
Immersion 2	1.62024	0	1.63251	+3	1.61536	0	36.2	0
Immersion 3	1.62027	+3	1.63252	+4	1.61540	+4	36.2	0
Immersion 4	1.62027	+3	1.63246	−2	1.61539	+3	36.3	+0.1
Average	1.62025	2	1.63250	3	1.61538	2	36.2	0
SAMPLE X								
Spectrometer	1.62023	------	1.63247	------	1.61536	------	36.2	------
Immersion 1	1.62022	−1	1.63246	−1	1.61539	+3	36.3	+0.1
Immersion 2	1.62024	+1	1.63247	0	1.61535	−1	36.2	0
Immersion 3	1.62025	+2	1.63242	−5	1.61537	+1	36.4	+.2
Immersion 4	1.62025	+2	1.63248	+1	1.61536	0	36.2	0
Average	1.62024	2	1.63246	2	1.61537	2	36.3	+0.1

TABLE 3.—*Comparison of immersion determinations of index of refraction and dispersion with spectrometer values*—Continued

Method	n_D	Error $\times 10^5$	n_F	Error $\times 10^5$	n_C	Error $\times 10^5$	μ	Error
				SAMPLE XI				
Spectrometer	1.64918	--------	1.66296	--------	1.64373	--------	33.8	--------
Immersion 1	1.64918	0	1.66300	+4	1.64375	+2	33.7	−0.1
Immersion 2	1.64917	−1	1.66296	0	1.64370	−3	33.7	−.1
Immersion 3	1.64915	−3	1.66296	0	1.64371	−2	33.7	−.1
Immersion 4	1.64917	−1	1.66293	−3	1.64373	0	33.8	0
Average	1.64917	1	1.66296	2	1.64372	2	33.7	0.1
				SAMPLE XII				
Spectrometer	1.64845	--------	1.66220	--------	1.64301	--------	33.8	--------
Immersion 1	1.64845	0	1.66215	−5	1.64301	0	33.9	+0.1
Immersion 2	1.64847	+2	1.66217	−3	1.64299	−2	33.8	0
Immersion 3	1.64842	−3	1.66218	−2	1.64296	−5	33.7	−.1
Immersion 4	1.64845	0	1.66218	−2	1.64306	+5	33.9	+.1
Average	1.64845	1	1.66217	3	1.64300	3	33.8	0.1

values as compared with the spectrometer values. An examination of these errors shows that the maximum error, or degree of accuracy, for any single measurement is $\pm 5 \times 10^{-5}$, but the average error for the 144 measurements is only $\pm 1.9 \times 10^{-5}$. The reproducibility, or precision, of these measurements, expressed as the average of the deviations from the mean value of each set of four immersion measurements, is $\pm 1.5 \times 10^{-5}$.

A summary of the average accuracy and precision of all the determinations is given in table 4.

TABLE 4.—*Summary of accuracy and precision of determinations listed in table 3*

	Average error	Average deviation from mean values
n_D	1.6×10^{-5}	1.4×10^{-5}
n_F	2.1×10^{-5}	1.6×10^{-5}
n_C	2.1×10^{-5}	1.6×10^{-5}
All values	1.9×10^{-5}	1.5×10^{-5}

The accuracy attained in these measurements is almost as high as the precision attainable on a refractometer. This indicates that the errors introduced in determining the match between the indices of the glass and liquid are for the most part negligible. According to Tilton,[9] it would be very difficult to obtain results of this accuracy without the strict use of a comparison or substitution method. The success attained may be largely attributed to the fact that in measuring a known standard, on the refractometer both before and after each group of four immersion measurements, a comparison method was used that approximates a substitution method. Other factors which should be mentioned are (1) temperature differences between the refractometer and immersion tank were rarely greater than 0.1°, and these temperature effects were considered in obtaining the final

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[9] L. W. Tilton, J. Research NBS **30**, 311–328 (1943) RP1535.

values, and (2) the recommended tests to eliminate shielding of rays, setting on false edges, etc. (see footnote 9) were frequently made.

Although the errors introduced in matching are, in general, negligible, the experience of the operator plays a part in these determinations. This is due to surface effects that interfere with the determination of a match. The ideal condition is one which produces a uniformly illuminated field against which the sample is invisible. This exists very rarely in actual practice. Usually, under conditions of match, the main body of the sample will blend with the field, but the edges of the sample will be outlined by very thin brilliant lines. In these cases, a match is determined by rotating the sample through 180°. There should be no change in the thickness or brilliance of these lines when this is done. Thus any confusion due to surface effects may be avoided. The experience of the operator will also determine the speed with which measurements are made. An experienced person can measure about six samples in an 8-hour day. Thus it is possible to secure complete information on the index of refraction and ν value for a particular melt within 1½ hours after a pot of glass is opened.

VI. SUMMARY

An improved apparatus for determining indices of refraction and dispersion of glass by immersion is described. The results of 144 measurements indicate that the index of refraction may be determined with an average error of $\pm 1.9 \times 10^{-5}$ and a maximum error of $\pm 5 \times 10^{-5}$. Dispersion as represented by ν values may be calculated from these measurements with an average error of 0.1 and a maximum error of 0.8. This apparatus may, therefore, be used both for routine production control work and for the determination of the optical constants of finished optical components, especially of flint glasses. It may also be applied profitably to research in the determination of the effect of composition on the optical properties of glass, especially where the glasses studied may be unstable. The full possibilities of the matching instrument have not been determined. More accurate refractometers and temperature controls are necessary for this purpose.

WASHINGTON, October 1, 1943.

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MATHEMATICAL TABLES

Attention is invited to a series of publications prepared by the *Project for the Computation of Mathematical Tables* conducted by the Federal Works Agency, Work Projects Administration for the City of New York, under the sponsorship of the National Bureau of Standards. The tables which have been made available through the National Bureau of Standards are listed below.

There is included in this list a publication on the hypergeometric and Legendre functions (MT15), prepared by the Bureau.

MT1. Table of the First Ten Powers of the Integers From 1 to 1000:

(1938) VIII+80 pages; heavy paper cover. Out of print.

MT2. Tables of the Exponential Function e^x:

The ranges and intervals of the argument and the number of decimal places in the entries are given below:

Range of x	Interval of x	Decimals given
—2.5000 to 1.0000	0.0001	18
1.0000 to 2.5000	.0001	15
2.500 to 5.000	.001	15
5.00 to 10.00	.01	12

(1939) XV+535 pages; bound in buckram, $2.00.

MT3. Tables of Circular and Hyperbolic Sines and Cosines for Radian Arguments:

Contains 9 decimal place values of sin x, cos x, sinh x and cosh x for x (in radians) ranging from 0 to 2 at intervals of 0.0001.
(1939) XVII+405 pages; bound in buckram, $2.00.

MT4. Tables of Sines and Cosines for Radian Arguments:

Contains 8 decimal place values of sines and cosines for radian arguments ranging from 0 to 25 at intervals of 0.001.
(1940) XXIX+275 pages; bound in buckram, $2.00.

MT5. Tables of Sine, Cosine, and Exponential Integrals, Volume I:

Values of these functions to 9 places of decimals from 0 to 2 at intervals of 0.0001.
(1940) XXVI+444 pages; bound in buckram, $2.00.

MT6. Tables of Sine, Cosine, and Exponential Integrals, Volume II:

Values of these functions to 9, 10, or 11 significant figures from 0 to 10 at intervals of 0.001, with auxiliary tables.
(1940) XXXVII+225 pages; bound in buckram, $2.00.

MT7. Table of Natural Logarithms, Volume I:

Logarithms of the integers from 1 to 50,000 to 16 places of decimals.
(1941) XVIII+501 pages; bound in buckram, $2.00.

MT8. Tables of Probability Functions, Volume I:

Values of these functions to 15 places of decimals from 0 to 1 at intervals of 0.0001 and from 1 to 5.6 at intervals of 0.001.
(1941) XXVIII+302 pages; bound in buckram, $2.00.

MT9. Table of Natural Logarithms, Volume II:

Logarithms of the integers from 50,000 to 100,000 to 16 places of decimals.
(1941) XVIII+501 pages; bound in buckram, $2.00.

MT10. Table of Natural Logarithms, Volume III:

Logarithms of the decimal numbers from 0.0001 to 5.0000, to 16 places of decimals.
(1941) XVIII+501 pages; bound in buckram, $2.00.

MT11. Tables of the Moments of Inertia and Section Moduli of Ordinary Angle Channels, and Bulb Angles with Certain Plate Combinations:

(1941) XIII+197 pages; bound in green cloth. $2.00.

[Continued on p. 4 of cover]

[Continued from p. 3 of cover]

MT12. TABLE OF NATURAL LOGARITHMS, VOLUME IV:

Logarithms of the decimal numbers from 5.0000 to 10.0000, to 16 places of decimals.

(1941) XXII+506 pages; bound in buckram, $2.00.

MT13. TABLE OF SINE AND COSINE INTEGRALS FOR ARGUMENTS FROM 10 TO 100:

(1942) XXXII+185 pages, bound in buckram, $2.00.

MT14. TABLES OF PROBABILITY FUNCTIONS, VOLUME II:

Values of these functions to 15 places of decimals from 0 to 1 at intervals of 0.0001 and from 1 to 7.8 at intervals of 0.001.

(1942) XXI+344 pages; bound in buckram, $2.00.

MT15. The hypergeometric and Legendre functions with applications to integral equations of potential theory. By Chester Snow, National Bureau of Standards. Reproduced from original handwritten manuscript.

(1942) VII+319 pages; bound in heavy paper cover. $2.00.

MT16. TABLE OF ARC TAN x:

Table of inverse tangents for positive values of the angle in radians. Second central differences are included for all entries.

$$x=[0(.001)7(.01)50(.1)300(1)2{,}000(10)10{,}000;12D]$$

(1942) XXV+169 pages; bound in buckram, $2.00.

MT17. Miscellaneous Physical Tables:

Planck's radiation functions (Originally published in the Journal of the Optical Society of America, February 1940); and

Electronic functions.

(1941) VII+58 pages; bound in buckram, $1.50.

MT18. Table of the Zeros of the Legendre Polynomials of Order 1–16 and the Weight Coefficients for Gauss' Mechanical Quadrature Formula.

(Reprinted from Bul. Amer. Mathematical Society, October 1942.)

5 pages with cover. 25 cents.

MT19. On the Function $H(m, a, x)=\exp(-ix)\,F(m+1-ia, 2m+2; ix)$; with table of the confluent hypergeometric function and its first derivative.

(Reprinted from Journal of Mathematics and Physics, December 1942.) 20 pages, with cover. 25 cents.

MT20. Table of Integrals $\int_0^x J_0(t)dt$ and $\int_0^x Y_0(t)dt$:

Values of the two integrals are given for $x=0(.01)10$ to 10 decimal places. (Reprinted from Journal of Mathematics and Physics, May 1943.) 12 pages, with cover, 25 cents.

MT21. Table of $Ji_0(x)=\int_x^\infty \frac{J_0(t)}{t}dt$ and Related Functions:

Table I: $Ji_0(x)$ to 10 decimal places and $F(x)=Ji_0(x)+\log_e 1/2x$ to 12 decimal places for $x=0(.1)3$ with even central differences of $F(x)$.

Table II: $Ji_0(x)$ to 10 decimal places, for $x=3(.1)10(1)22$ with even central differences up to $x=100$.

Table III: "Reduced" derivatives of $F(x)$ for $x=10(1)21$ and $n=0(1)13$, to 12 decimal places.

(Reprinted from Journal of Mathematics and Physics, June 1943.) 7 pages, with cover, 25 cents.

MT22. Table of Coefficients in Numerical Integration Formulae:

The values of $B^{(n)}_n(1)/n!$ and $B^{(n)}_n/n!$ where $B^{(n)}_n(1)$ denotes the nth Bernoulli polynomial of the nth order for $x=1$ and $B^{(n)}_n$ denotes the n^{th} Bernoulli number of the n^{th} order, were computed for $n=1, 2, \ldots 20$. The quantities $B^{(n)}_n(1)/n!$ are required in the Laplace formula of numerical integration employing forward differences, as well as in the Gregory formula. The quantities $B^{(n)}_n/n!$ are used in the Laplace formula employing backward differences.

(Reprinted from Journal of Mathematics and Physics, June 1943.) 2 pages, with cover, 25 cents.

Payment is required in advance. Make remittance payable to the "National Bureau of Standards," and send with order, using the blank form facing page 3 of the cover.

A mailing list is maintained for those who desire to receive announcements regarding new tables as they become available.