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MEASUREMENT OF THE REFRACTIVE INDEX AND DISPERSION OF OPTICAL GLASS FOR CONTROL OF PRODUCT

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ABSTRACT

Commercial critical-angle refractometers are inadequate for acceptance tests on optical glass for precision uses. To facilitate spectrometer determinations, coefficients [1] have been devised which, together with a table of natural sines and slide-rule operations, permit the computation of refractive indices of glass with an accuracy of $\pm 3 \times 10^{-6}$ from minimum-deviation data taken on prisms having angles of $60° \pm 30'$.

The refractive index and dispersion of optical glass that is used for lenses and for other refractive purposes are the most important characteristics of such glass, and considerable attention should be given by the manufacturer to their appropriate measurement. Adequate measurements of dispersion are more difficult than those of refractive index as such. The value $\nu=(n_D-1)/(N_F-N_C)$, used for expressing the dispersion of optical glass, is often specified by purchasers with tolerances of two or three tenths of a unit. The critical factor in its determination is the partial dispersion (N_F-N_C), which ranges from 0.00800 for borosilicate crown with $n_D=1.517$ and $\nu=64$, to 0.02100 for extra-dense flint with $n_D=1.671$ and $\nu=32$. It is evident that the difficulties exist chiefly for crown glass, where errors of ± 0.000012 in measuring both n_C and n_F can result in errors of ± 0.2 in ν value.

Since it is obviously desirable that the errors of acceptance and rejection tests should be appreciably smaller than the tolerances specified for acceptance, and because the measurement of refractive index with errors of appreciably less than ± 0.000012 is exceedingly difficult, there is a logical demand that the close tolerances sometimes specified for ν values, especially for crown glass, should be eased whenever it is possible to do so.

Following progress in the easing of acceptance tolerances, there is a tendency to abandon efforts to get optimum accuracy in index measurements. This tendency is unfortunate and surprising because tolerances for optical glass are usually comparable with, and sometimes smaller than, the limits within which a manufacturer controls his product, and under these conditions both seller and buyer should be interested in keeping the range of uncertainties in acceptance tests appreciably smaller than the tolerances specified for acceptance. Assuming only chance errors, and assuming definite acceptance for

[1] The computational process and the tables here described, with particular reference to glasses with indices in the range 1.509 to 1.532, have been extended for glasses up to 1.677 in index. Copies of the full table of coefficients, *C*, have been duplicated for distribution upon request.

rejection according to the results of relatively imprecise tests, the volume of acceptable product that is rejected will necessarily tend to exceed that of the unacceptable product that is delivered. This means great loss by the manufacturer when the specified tolerances approximate the limits within which he is able to control the major portion of the product. The interest of the purchaser increases as the specified tolerances become narrow in comparison with the degree of control, because the accepted glass will necessarily include much that lies outside the tolerances, and thus the accepted product will be much less homogeneous than is desired or supposed.

The importance of precision and accuracy in refractive index measurements for control of product is recognized by European manufacturers. According to the Jena glass catalog, spectrometer measurements are made by Zeiss for every melt with an accuracy of ± 5 in the fifth decimal place for each refractive index and ± 2 in the fifth decimal for each partial dispersion. This corresponds to ± 0.15 in ν value for crown glass.

At the Chance-Parsons plant, a spectrometer is used to determine the dispersions listed in the catalog, and "routine measurements of refractive indices of meltings are made with a Pulfrich refractometer which has been standardized by reference prisms carefully measured on an accurate spectrometer." The "Pulfrich" refractometers used in England for such purposes are not the Pulfrich refractometers as made by Zeiss but improved models, specially built in England and calibrated at the National Physical Laboratory. Even so, it is admitted that the errors are ± 3 in the fifth decimal place for the partial dispersions, which may correspond to ± 0.23 in ν.

At the National Bureau of Standards, with a Zeiss Pulfrich refractometer, it is found that errors in dispersion measurement are from 50 to 100 percent higher than those cited in the Chance-Parsons catalog for their Pulfrich refractometer measurements. There are no standard or "precision" refractometers that are commercially obtainable and sufficiently accurate to determine the ν values of light crown glass, in a manner satisfactory to either manufacturer or purchaser, when making acceptance tests where the ν values are specified to ± 0.3 or ± 0.5. The highest accuracy attainable with the best of selected and tested precision refractometers approaches but certainly does not fully equal the accuracy that is desirable. The errors in such refractometric determinations result in an uncertainty of approximately ± 0.5 in ν value for crown glass, and thus it could happen that almost all decisions reached by such procedures would be erroneous for crown glass.

Consequently, spectrometer measurements and the use of prisms are imperative for control of product in the manufacture of optical glass, if the constants of glass are to be known with the degree of accuracy that characterizes data commonly supplied with optical glass that is imported for high precision uses. Nevertheless, because of the time required for accurate spectrometer measurements, and also because the ν value of glass varies very little from melt to melt made with a given batch composition and manufacturing procedure, it may be advisable, especially during wartime, and for certain purposes, to rely to some extent on precision refractometer measurements for the values of the refractive indices themselves, and for the detection of gross errors in composition.

Whenever critical-angle refractometers are employed for these purposes they should be used strictly as comparison instruments through continual use of representative specimens, including some from each type of glass, that have previously been standardized by use of a spectrometer.[2] It is suggested that the ν value of a fine annealed sample from every fifth melt of crown glass of each type, all melts being made with a given batch composition and manufacturing procedure, and likewise from every tenth melt of flint glass, should be measured by means of an accurate spectrometer in order to obtain accurate information on ν values and to check the over-all accuracy of the refractometer procedures.

The time required for making prisms and for the spectrometer determinations can be safely reduced by slight easing of the tolerances on flatness of prism surfaces and by using a constant tube length for the collimator,[3] also by approximate temperature control for all crown prisms. Moreover, the time required for the precise computation of refractive index to six or seven significant figures by the well-known formula

$$n = \sin 0.5(A+D)/\sin 0.5A, \qquad (1)$$

where A is the refracting angle of the prism, and D is the angle of minimum deviation, can be materially shortened and the chance of error reduced by use of a single-entry table of index as a function of modified or pseudo values of $2D$. One table, made for prisms of exactly 60°, serves for all wavelengths, (visible spectrum) and for all prism angles in the neighborhood of 60°.

In making the table, the proper entries are readily computed as $n=2\sin(30°+0.5D)$, which gives the index for a prism of exactly 60°. Thus for a tabulation at intervals of 1 minute of arc in $2D$, which proves ample for precise work to six decimal places, one merely doubles the values read from a table of natural sines at intervals of 15 seconds of arc. Table 1 is an example of such a table. It includes the limited refractive-index range $n=1.509419$ to $n=1.531995$, especially useful with prisms of borosilicate glass.

The uniformity with which the sine changes in the neighborhood of 60° and of $2D$ (for glass prisms) permits the precise use of such a table for all prism angles within $60° \pm 30'$ by merely modifying, in each case, the actually observed $2D_A$ by a quantity $\Delta 2D$ that closely compensates for the particular existing $\Delta A = A - 60°$. In order to compute an auxiliary table for getting $\Delta 2D$ for particular values of ΔA, equation 1 is differentiated, and the desired relationship is expressible as

$$\Delta 2D = 2\,[\sin 0.5D \operatorname{cosec} 0.5A \sec 0.5(A+D)]\ \Delta A, \qquad (2)$$

where for crown glass the quantity in brackets is so near 1 that it is convenient to rewrite equation 2 as

$$\Delta 2D = (1+C)\,(2\Delta A) \qquad (3)$$

and tabulate in table 2 only the small coefficients, $C = (\Delta 2D/2\Delta A) - 1$. These can be used with a slide rule in evaluating for any crown-glass

[2] L. W. Tilton, J. Opt. Soc. Am. **32**, 371 (1942); J. Research NBS **30**, 311-328 (1943) RP1535.
[3] L. W. Tilton, BS J. Research **11**, 25-27 (1933) RP575.

prism of $60°+\Delta A$ the equivalent double deviation, $2D_{60}$, for a prism of exactly $60°$ in accord with the equation

$$2D_{60}=2D_{A}-(2\Delta A)-C(2\Delta A). \tag{4}$$

With the values of $2D_{60}$ thus computed, the refractive index of the prism that is being measured is found by linear interpolation in table 1.

In selecting a value of C for use in equation 4 it is necessary to take a C that is computed for the midrange of the particular ΔA and $\Delta 2D$ in question. Accordingly, in making table 2, the C values computed for $A_c=60°+\Delta A/2$ are tabulated under the observed values of $A=60°+\Delta A$. Values of C computed for $2D_c=2D_A-(1+C)\Delta A=2D_{60}+(1+C)\Delta A$ are tabulated on lines designated by values of $2D$.

The computation of index can be quickly made with an accuracy of $\pm 3\times 10^{-6}$, usually without logs, other tables, or a computing machine. The necessary steps are (1) find C from table 2, and (2) find $-\Delta A=(60°-A)$. Then (3) mentally double $-\Delta A$ and (4), usually with a slide rule, form product $-C(2\Delta A)$ in order (5) to add the terms $-2\Delta A$ and $-C(2\Delta A)$ to $2D_A$ and thus obtain $2D_{60}$ with which (6) to enter table 1 for n. Two examples of the use of these tables are appended, and for one of them the corresponding solution by logarithms is given.

TABLE 1.—*Indices of refraction, 1.5094 to 1.5320 for 60° prism*

[For other prisms, $A=60° \pm 30'$, find $2D_{60}=2D_A-2\Delta A-C\,(2\Delta A)$. See table 2. Read initial digits in 0 column on same line with tabulated values unless asterisk refers to initial digits on next line below.]

$2D_{60}$	Minutes of arc									
	0	1	2	3	4	5	6	7	8	9
76° 0′	1.509 419	515	610	705	801	896	992	*087	*182	*278
10′	1.510 373	468	564	659	754	850	945	*040	*135	*231
20′	1.511 326	421	517	612	707	802	897	993	*088	*183
30′	1.512 278	373	468	564	659	754	849	945	*040	*135
40′	1.513 230	325	420	515	610	705	800	895	990	*085
50′	1.514 180	275	370	465	560	655	750	845	940	*035
77° 0′	1.515 130	225	320	415	509	604	699	794	889	984
10′	1.516 079	173	268	363	458	553	648	743	837	932
20′	1.517 027	122	217	311	406	501	595	690	785	880
30′	1.517 974	*069	*164	*258	*353	*448	*542	*637	*732	*827
40′	1.518 921	*016	*110	*205	*299	*394	*488	*583	*678	*772
50′	1.519 867	961	*056	*150	*245	*339	*434	*528	*623	*717
78° 0′	1.520 812	906	*001	*095	*190	*284	*378	*473	*567	*661
10′	1.521 756	850	944	*039	*133	*228	*322	*416	*511	*605
20′	1.522 700	794	888	982	*076	*171	*265	*359	*453	*548
30′	1.523 642	736	830	924	*019	*113	*207	*301	*395	*490
40′	1.524 584	678	772	866	960	*054	*148	*242	*336	*430
50′	1.525 524	618	712	807	901	995	*089	*183	*277	*371
79° 0′	1.526 465	559	653	747	841	934	*028	*122	*216	*310
10′	1.527 404	498	592	686	779	873	967	*061	*155	*249
20′	1.528 343	437	530	624	718	811	905	999	*093	*186
30′	1.529 280	374	468	561	655	749	843	936	*030	*124
40′	1.530 218	311	405	498	592	685	779	873	966	*060
50′	1.531 153	247	340	434	528	621	715	808	902	995

	96	95	94	93
1	9.6	9.5	9.4	9.3
2	19.2	19.0	18.8	18.6
3	28.8	28.5	28.2	27.9
4	38.4	38.0	37.6	37.2
5	48.0	47.5	47.0	46.5
6	57.6	57.0	56.4	55.8
7	67.2	66.5	65.8	65.1
8	76.8	76.0	75.2	74.4
9	86.4	85.5	84.6	83.7

TABLE 2.—*Coefficients for use with table 1 when A differs from 60°*

[Divide values listed below by 100 to get $C=(\Delta 2D/2\Delta A)-1$.]

Observed $2D$	Observed values of A						
	59°30′	59°40′	59°50′	60°00′	60°10′	60°20′	60°30′
76°00′	+0.25	−0.08	−0.41	−0.75	−1.08	−1.41	−1.74
10	.55	+.22	−.11	−.45	−0.79	−1.12	−1.46
20	.85	.51	+.17	−.16	−.50	−0.84	−1.18
30	1.15	.81	.47	+.13	−.21	−.55	−0.89
40	1.45	1.10	.76	.42	+.08	−.26	−.60
50	1.74	1.40	1.06	.72	.37	+.03	−.31
77°00′	2.03	1.69	1.35	1.01	0.66	0.32	−0.02
10	2.33	1.99	1.65	1.31	.96	.62	+.28
20	2.63	2.29	1.95	1.60	1.26	.92	.58
30	2.94	2.60	2.25	1.90	1.55	1.21	.87
40	3.24	2.90	2.55	2.20	1.85	1.51	1.17
50	3.54	3.20	2.85	2.50	2.15	1.81	1.47
78°00′	3.84	3.50	3.15	2.80	2.45	2.11	1.76
10	4.14	3.80	3.45	3.10	2.74	2.40	2.06
20	4.45	4.10	3.75	3.40	3.04	2.70	2.36
30	4.76	4.40	4.05	3.70	3.35	3.00	2.65
40	5.07	4.71	4.35	4.00	3.65	3.30	2.95
50	5.37	5.02	4.66	4.30	3.94	3.59	3.24
79°00′	5.68	5.32	4.96	4.60	4.24	3.89	3.54
10	5.99	5.62	5.26	4.90	4.54	4.19	3.84
20	6.30	5.94	5.57	5.20	4.85	4.49	4.13
30	6.61	6.24	5.87	5.50	5.15	4.79	4.43
40	6.91	6.54	6.17	5.80	5.45	5.09	4.73
50	7.21	6.85	6.48	6.11	5.75	5.39	5.03

COMPUTATION OF REFRACTIVE INDEX, n

Two examples by use of tables 1 and 2:

Remarks	Example 1 ($A<60°$)		Example 2 ($A>60°$)	
Observed A		59°45.38′		60°17.04′
Observed $2D_A$		77°47.59′		77°31.56′
C (table 2)	0.0295		0.0136	
$-\Delta A=(60°-A)$	14.62′		−17.04′	
$-(2\Delta A)$		29.24		−34.08
$-C(2\Delta A)$	(.0295 x 29.2) =	0.86	−(.0136 x 34.1) =	− 0.46
$2D_{60}$		78°17.69′		76°57.02′
n (table 1)	1.522482		1.514847	

COMPUTATION OF REFRACTIVE INDEX, n

One example by use of logarithms:

Observed A	59°45′22.6″	
Observed $2D_A$	77°47′35.4″	
1/2 Observed A	29°52′41.3″	
1/4 Observed $2D_A$	19°26′53.9″	
0.5 $(A+D)$	49°19′35.2″	
Log sin 49°19′30″	9.8799091−10	
5.2 x 0.0000181	0.0000094	
Log sin 0.5 $(A+D)$	9.8799185−10	(1)
Log sin 29°52′40″	9.6973614−10	
1.3 x 0.0000367	0.0000048	
Log sin 0.5A	9.6973662−10	(2)
Log n=(1)−(2)	0.1825523	(3)
Log (n=1.522400)	.1825288	(4)
(3)−(4)	0.0000235	
0.0000235÷0.285	.000082	
	1.522400	
n	1.522482	

WASHINGTON, November 19, 1943.

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[Continued from p. 3 of cover]

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