ANALYSIS OF DIAPHRAGM SYSTEM FOR THE X-RAY STANDARD IONIZATION CHAMBER

By Lauriston S. Taylor

ABSTRACT

Several observers have shown that an arbitrary choice of diaphragm system, for defining the X-ray beam in standard measurements, may introduce an uncertain error in the fundamental determination of the International *r* X-ray unit. A geometrical analysis shows that under certain experimental conditions the X-ray tube focus may be considered as a point source of radiation, while under other conditions it must be considered as an extended source. The difference between the two is considerable, and can not be neglected. Curves are given showing the experimental conditions which must be used with a given X-ray tube and a standard ionization chamber. An approximate measurement is made of the radiation coming from parts of the target face other than the focus, and it is shown how this may introduce error in the measurements. The energy distribution over the area of the beam was calculated, and measurements made photographically gave good agreement with the calculations—indicating clearly the region of the beam that must be used to insure uniformity within the prescribed limits of accuracy. An analysis of the several standard ionization chambers used in other laboratories showed with one exception that their error in defining the beam was less than 0.5 per cent.

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I. INTRODUCTION

In measuring the ionization in an *unrestricted volume of atmospheric air*, according to the internationally adopted definition of the *r* unit,[1] the accuracy of the determination depends principally on, first,

[1] The *r* unit is defined as "the quantity of radiation which, when the secondary electrons are fully utilized and the wall effect of the chamber is avoided, produces in 1 cubic centimeter of atmospheric air at 0° C. and 76 cm mercury pressure such a degree of conductivity that 1 electrostatic unit of charge is measured at saturation current."

the electrical measurements of the ionization and control of the X-ray output; secondly, the determination of the air volume of which the ionization is being measured. The first has been treated by Behnken,[2] Glasser,[3] Duane,[4] and more recently by the author;[5] the second is effectively the object of the present paper.

Behnken[6] in a comparison of the *r* unit between the laboratories of Glasser, Duane, Grebe, Küstner, Holthusen, and the Physikalisch-Technische Reichsanstalt found a 4 per cent discrepancy between the different laboratories. In his publication of the results of this comparison no explanation is offered for the disagreement, it being assumed due to faulty experimental technique rather than to the theory of the experiment. In a study by the author[7] of the various measuring methods, a source of error sufficient to account for Behnken's results was traced to the measurement of the ionization current. However, it is recognized that even an agreement between two laboratories might arise from the errors present compensating each other, so that it is very desirable to study the possible errors in the various other controlling factors.

In practice the ionization is measured in several cubic centimeters and the ionization current divided by the corresponding air volume, the ionization throughout the entire volume being assumed uniform. However, this is not, in general, the case, and so we must determine the effective volume corresponding to uniform ionization. In some recent work Failla[8] has measured air ionization in a standard chamber, using widely varying diaphrams, and found that, according to the particular diaphram system used, there may be a considerable uncertainty in the determination of the air volume.

Behnken, Glasser, and Failla have used circular diaphragms of the order of 1 cm diameter, while Duane uses a rectangular slit system where his chamber diaphragm is 0.5 by 5.0 cm. Most of the observers have placed a screening diaphragm close to the tube to prevent radiation from the target stem entering the standard chamber. Depending upon its relative size and position with respect to the focal spot, this diaphragm may or may not have a direct bearing on the ionization readings.

It therefore seemed advisable to analyze the effects of the diaphragm system carefully so as to obtain some quantitative measure of the error introduced by certain assumptions regarding it. We shall, therefore, treat the problem first considering the focus as a

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[2] H. Behnken and R. Jaeger, Zeit. f. Tech. Phys., **7**, p. 564; 1926.
[3] O. Glasser and U. V. Portmann, Am. J. Roent., **19**, p. 47; 1928.
[4] W. Duane and E. Lorenz, Am. J. Roent., **19**, p. 461; 1928.
[5] L. S. Taylor, B. S. Jour. Res., **2**, p. 771; 1929.
[6] H. Behnken, Strahlentherapie, **29**, p. 192; 1928.
[7] See footnote 5.
[8] G. Failla, Am. J. Roent., **21**, p. 47; 1929.

point source (Sec. II, 1); then as an extended source (Sec. II, 2); and finally show the experimental justification (or failure) for the assumption made (Sec. III, 2).

II. GEOMETRICAL ANALYSIS OF THE DIAPHRAGM SYSTEM

1. BEHNKEN'S ANALYSIS, ASSUMING A POINT SOURCE OF RADIATION

In this it is assumed that the focus is a point source of radiation, and that the X-ray flux density is uniform over the cross-sectional area of the beam in the ionization chamber. When the diaphragms are

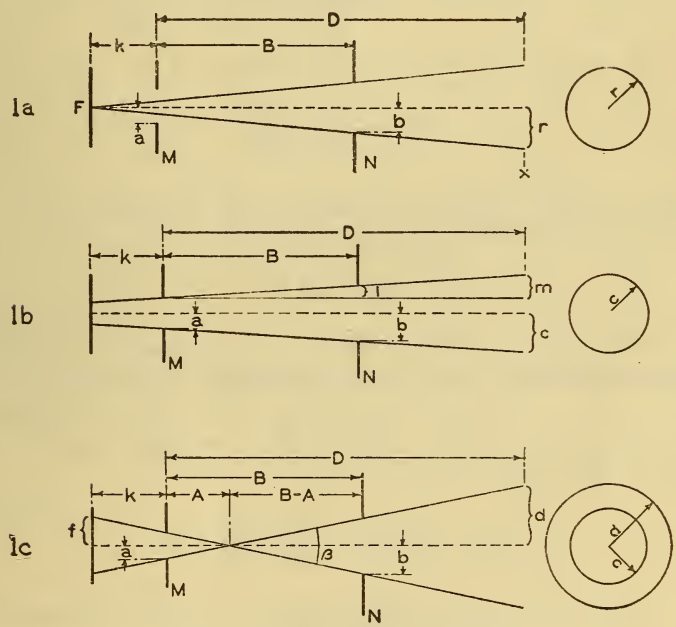

FIGURE 1.—*Diaphragm system*

very small compared with their separation, these two assumptions may be considered valid; but, in general, they are contradictory, since the radiation from a point source gives a flux density in the direction of the beam which is proportional to the cosine of the angle that the given ray makes with this axis.

In presenting Behnken's analysis,[9] we shall use for ease in the later development a nomenclature differing slightly from his. Referring to Figure 1(*a*), *F* is the focus of the X-ray tube, *M* the first diaphragm, radius *a; N* the ionization chamber diaphragm, radius *b;* and *x* a plane in the ionization chamber normal to the axis of the beam.

[9] H. Behnken, Strahlentherapie, 26, p. 79; 1927.

The focus is assumed to be a point, the X-ray intensity of which is represented by a flux density of I_o at unit distance. Then on the axis of the beam at x, the intensity, or flux per unit area, is given by

$$I_x = \frac{I_o}{(D+k)^2} \tag{1}$$

provided, of course, that the radiation be sufficiently penetrating to permit loss by absorption in the length $(D+k)$ to be neglected. Letting r equal the radius of the cross section of the beam at x, we see that

$$\frac{r}{b} = \frac{D+k}{B+k}$$

so that the cross-sectional area of the beam at x is

$$\pi r^2 = \frac{\pi(D+k)^2 b^2}{(B+k)^2} \tag{2}$$

The volume of an element of length dx of the beam is $dV = \pi r^2 dx$. The energy dE absorbed therein per unit time is

$$dE = \kappa I_x dV \tag{3}$$

where κ is a constant. The number of ions produced by this absorbed radiation being proportional to dE, the electrical measurement of the ionization gives a quantity proportional to the X-ray intensity I_x.

From equations (1), (2), and (3) above we find that the energy absorbed in the element at x is

$$dE_1 = \frac{\kappa I_o}{(D+k)^2} \cdot \frac{\pi(D+k)^2}{(B+k)^2} b^2 dx \tag{4a}$$

$$= \kappa \frac{\pi b^2}{(B+k)^2} I_o dx \tag{4b}$$

Equation (4b) expresses, however, the energy absorbed in a volume of cross-sectional area πb^2 and thickness dx, located at a distance $(B+k)$ from the point source (that is, at the diaphragm N). Therefore, for any length L_o of the chamber. the total energy E_1 absorbed in the chamber is given by

$$E_1 = \frac{\kappa \pi b^2}{(B+k)^2} I_o L_o \tag{4c}$$

As a consequence we are justified in considering the effective air volume as having a radius b, a length equal to the effective length of the measuring electrode, and located at a distance $(B+k)$ from the

focus. The question as to what part of the ionized air volume the distance must be measured, in applying the inverse square law, is thus answered. In a previous paper the author has given the necessary electrical conditions to determine accurately this effective length L_0 of the air column from which the ionization is being measured.

2. ANALYSIS ASSUMING AN EXTENDED SOURCE OF RADIATION

(a) GENERAL TREATMENT

Mayneord [10] and others have considered the focus as an extended source, but exaggerated as shown below the importance of the error introduced by assuming a point source. To estimate this error another assumption will be made which, while not quite correct, is indicated by experimental evidence to be a closer approach to the true conditions.

Consider the focus to be an extended source having an area at least as great as the projected area of the diaphragm N upon the target. (Fig. 1(c).) Let further the "brightness" (in the optical sense) of the focal spot be uniform both as to area and direction viewed. Then over the plane x the distribution of flux density varies in a manner determined by the diaphragm system. There will be a *central area* of radius c (fig. 1(b)) having a nearly uniform distribution, and a surrounding *ring area* of outer radius d (fig. 1(c)) having a nonuniform distribution. Their dimensions are readily obtained.

To find the radius c of the central area, consider the system of Figure 1(b)

$$\frac{b-a}{c-a}=\frac{B}{D}$$

from which

$$c=\frac{Db-Da+Ba}{B} \tag{5}$$

For the outer radius d of the penumbra, Figure 1(c) gives,

$$\frac{a}{b}=\frac{A}{B-A} \text{ or } A=\frac{Ba}{a+b}$$

also

$$\frac{B-A}{b}=\frac{D-A}{d}$$

whence

$$d=\frac{D-A}{B-A}b$$

substituting the value of A above

$$d=\frac{Da+Db-Ba}{B} \tag{6}$$

[10] W. V. Mayneord, Brit. J. Rad., N. S., **1**, p. 125; 1928.

It is seen that the values of c and d are independent of the distance k from the focus to the first diaphragm M, and as experimentally demonstrated in Section III, 2 we are thus justified in considering the first diaphragm M, as the source of radiation, whose brightness is also, like that of the target, uniform with both direction and area for the small angles under consideration.

If now, the *aperture* of the system be defined as the solid angle β (fig. 1(c)) between the most divergent rays passing through the two diaphragms, we see that the first diaphragm may be considered as the source of radiation only if the actual area of the focal spot fills the aperture β of the system. This limiting condition is seen from Figure 1(c) where f is the radius of the focal spot. Its magnitude is taken from

$$\frac{f}{a}=\frac{A+k}{A}$$

from which

$$f=\frac{ka+kb+Ba}{B} \tag{7}$$

by substituting the value for A as derived in equation (6).

To determine the flux density at any point on the plane x due to the total emission from the focus, we may use the cosine law of emission and illumination as expressed by

$$dI_x=\frac{I_0 \cos^2\alpha \, da \, dc}{D_{ac}} \tag{8a}$$

where I_0 is the flux density at the target in the direction along the axis of the beam and likewise also the corresponding flux density through the area at M; dI_x is the flux density through an element of area dc in the plane x, and in a direction normal thereto; the angle which the line joining da and dc makes with the normal to these areas and D_{ac} the distance between the elements. The total flux I_x at any point on the plane is obtained by integrating the flux received from the various elements of the surface πa^2 of the diaphragm M. This has been evaluated by Foote [11] for the illumination from a radiating disk.

If β_0 is the angle between the axis and the most divergent ray, then referring to Figure 1 (c), for $\cos^2\beta_0=0.9950$, $\cos\beta_0$ can be assumed equal to unity without introducing an error greater than 0.5 per cent in the flux density at any point. This condition is true for the experimental arrangement of most observers. With a system such as used by Duane, however, it is not negligible.[12]

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[11] P. D. Foote, B. S. Bulletin, 12 (No. 263), p. 583; 1916.

[12] It must be pointed out that the cosine law can not be applied rigorously to X-radiation. However, by considering the energy distribution about an X-ray tube target, we see that over small ranges the error introduced by assuming this law is very small. If we consider the change in flux density over a plane, from a point or extended source, as simply due to the change in distance from the source, we arrive at an equation identical with equation (8a).

In the case where $\cos^2 \beta_o = 0.9950$, we may assume the flux density I_x uniform over the central area (fig. 1 (*b*)) and given by

$$I_x = \frac{I_o}{D^2} \tag{8b}$$

where, since D is the distance from the plane x to the first diaphragm M, the effect at x is as if the source were located at the diaphragm M.

The energy dE_2 absorbed in the central volume element dv of cross sectional radius c and thickness dx is then

$$\begin{aligned} dE_2 &= \kappa I_x \, dv \\ &= \kappa \frac{I_o}{D^2} \, \pi \, c^2 \, dx \\ &= \frac{\kappa \pi \, (Db - Da + Ba)^2}{B^2} \cdot \frac{I_o}{D^2} \, dx \end{aligned} \tag{9a}$$

Like dE_1 in equation (4a) dE_2 is proportional to the ionization current measured. In the special case where the two diaphragms are equal $a = b$ and thus

$$dE'_2 = \frac{\kappa \, \pi b^2}{D^2} I_o \, dx \tag{9b}$$

It will be noticed that dE'_2 in equation (9b) has the same form as dE_1 in equation (4b), the only difference lying in the replacement of $(D+k)$ by D. It remains to determine the ionization in the penumbra, which may be neglected when using very small diaphragms very far apart, but not in general.

The ionization produced in a volume element of the surrounding ring area or penumbra of outer radius d can not be easily calculated, owing to the fact that from any given point on the ring only a fraction of the first diaphragm is visible. Thus, at the inner radius of the ring the X-ray flux density is equal to that of the inner area while at the outer edge it is zero.

(B) EFFECTIVE VOLUME OF AIR IONIZED

Having found the effect in the central area as above, one might proceed by using Foote's derivation to evaluate that in the penumbra, but a simpler and more direct method is suggested by the derivation above for the effect of a point source.

We proceed, therefore, to determine the total energy absorption within the chamber, regardless of its distribution. Since it is permissible to neglect the loss in flux arising from absorption along the path, it is obvious that the ionization in a length dx in any cross section of the beam at any plane x within the ionization chamber is proportional to the total flux through that cross section. This is given, however, by the total flux entering the diaphragm N. The

combined result along any length L of the beam in the ionization chamber is equal then to that of a beam having a cross section equal to that of the entrant diaphragm N and a total flux density equal to that at N. This gives for each element of length

$$dE_3 = \frac{\kappa\pi\ b^2\ I_o}{B^2}\ dx \tag{10}$$ [13]

Summing up for the whole effective length L_o of the measuring electrode, we have

$$E_3 = \frac{\kappa\pi\ b^2\ I_o}{B^2}\ L_o \tag{11}$$

Equation (11) has the same form as that of Behnken, equation (4c), the difference being in the location of the effective source of the X-radiation at the diaphragm instead of the focus.

We have thus established that for either a point source (equation (4b)), or an extended source within fixed limits (equation (10)) the product of the area of the ionization chamber diaphragm by L determines the *effective air volume*; with the condition that in the first case the inverse square law is used with the source at the focus, while for the second it is used with the source considered at the first diaphragm M. The experimental justification for this is given in Section III.

Equation (11) should express actual conditions within experimental limits so long as $\cos^2 \beta_o$ differs from unity by less than the permissible experimental error. Having fixed the maximum permissible variation in flux density we may determine the range of magnitudes over which the equations above are valid. This, of course, depends upon the aperture of the system so that for any given system the region over which an accurate determination of the r unit is possible may be determined.

As indicated in Figure 1(c), the focal spot must fill the aperture if the assumptions made are valid; that is, from equation (7) we must have

$$f > \frac{ka + kb + Ba}{B} \tag{12a}$$

If in a particular case

$$f < \frac{ka + kb + Ba}{B} \tag{12b}$$

for a part of the range of B, then the inverse square law should be applied using the distance from diaphragm N to the focus instead of to M; the diaphragm M serving simply to reduce the radiation coming from points other than the focus. If we have a sufficiently small focal spot (one which never fills the aperture) and a diaphragm

[13] Equation (10) may also be obtained from equation (9a) by the process of decreasing D until it is equal to B, at which point the penumbra disappears with consequent simplification of the problem.

system such that $\cos^2 \beta_0 \geqq 0.995$, we can determine the volume of ionized air as accurately as the experimental precision warrants by assuming a point focus. These conditions have not been realized by all observers.

III. EXPERIMENTAL STUDY OF DIAPHRAGM SYSTEM

1. TYPE OF DIAPHRAGMS USED

First, a study was made to determine if the form of the edge of the circular diaphragms used affected the measured magnitudes. One-quarter inch lead diaphragms having the same diameter (10 mm) were prepared, all having different edges. The first group were tapered ranging from 0° to 15°. The next group were beveled, leaving a half thickness of lead as an edge. In no case was a measurable difference produced in the ionization current. The diaphragms subsequently used were all of this beveled type.

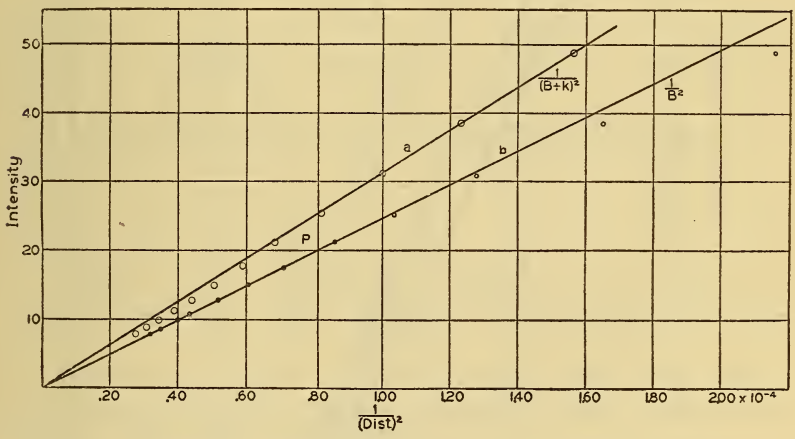

FIGURE 2.—*Intensity of measured ionization as a function of 1/(distance)²*

2. TEST OF INVERSE SQUARE LAW

According to Section II, 2, so long as the aperture of the diaphragm system is filled (equation (12a)) by the extended focal area the distance B between diaphragms M and N should be used and the inverse square law applied thereto. When it is not filled, the distance $(B+k)$ from the focus should be used.

To test this the ionization chamber was mounted on a track of such length that the distance between focus and limiting diaphragm N could be varied from 40 to 200 cm; the radius of the diaphragm M could be varied from 2 to 6 mm and that of N from 2 to 10 mm. A null reading electrostatic system was used for measuring the ionization currents and a constant voltage generator for the X-ray tube.[14]

[14] See footnote 5, p. 808.

To align the system accurately, the ionization chamber track was leveled and approximately adjusted by sighting through the diaphragms at the target. A fine wire was then strung along the axis of the system and the track further adjusted until the wire remained in the center of the front and back chamber diaphragms for all positions of the chamber along the track. This was accomplished even more accurately by inserting in the diaphragms disks having very small holes in the center.

Curve b in Figure 2 gives (for $a=b=0.5$ cm, $k=12$ cm, and $f=$ about 0.62 cm) the observed X-ray intensity or ionization current i,

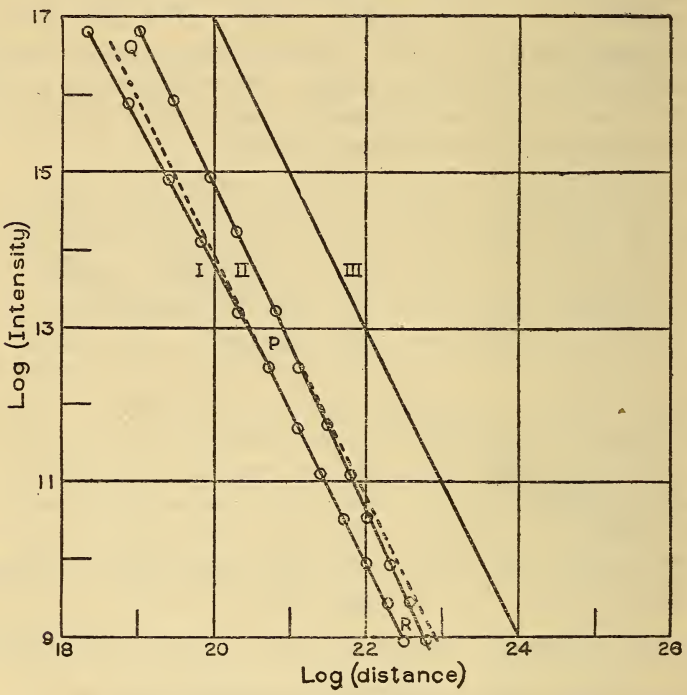

FIGURE 3.—*Validity of inverse square law*

plotted against the inverse square of the distance B between the tube focus and the ionization chamber diaphragm N. As should be expected, up to a certain value of $1/B^2$ corresponding to a distance $B=$ 105 cm, the intensity is found to be strictly proportional to $1/B^2$. According to equation (7), a break in this proportionality should occur when

$$\frac{ka+kb+Ba}{B}=0.62$$

Thus in this case the calculated value of f is 0.62 as compared with the observed value 0.61 obtained when the break occurred at $B=105$ cm.

Likewise if i is plotted against $1/(B+k)^2$ a break occurs at the point corresponding to $B=110$ cm (curve a) except that in this case the portion of the curve corresponding to distances near the tube obeys the inverse square law.

The inverse square law may be expressed as

$$\log i = \log c - 2 \log H \tag{13}$$

where c is a constant and H is the distance, either B or $(B+k)$ as the case may be. By plotting (fig. 3) the data, the log of the intensity (proportional to log i) against that of the distances B or $(B+k)$, as the case may be, the applicability of equation (13) is tested. For Curve I, H is taken as the distance B between diaphragms M and N; while in Curve II, H is taken as the distance $(B+k)$ between N and the focus. Curve III represents the slope (minus 2) which Curves I and II should have if the law expressed in equation (13)

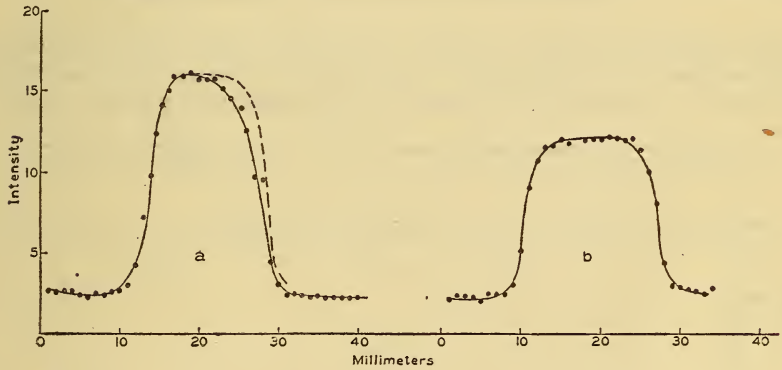

FIGURE 4.—*Distribution of intensity across the standard X-ray beam*

(The relative height of curves *a* and b has no significances.) Curve *a*, for displaced diaphragms; curve *b*, for properly aligned diaphragms.

is obeyed. It is to be seen that, of Curve I, the portion PR is parallel to III; while of Curve II, the portion PQ is parallel to III. Hence the experimental results bear out the conclusions that for distances within a certain range near the tube the inverse square law should be applied with the distance $(B+k)$; while over the range farther from the tube, the distance B is used instead.

It is thus clear that the assumption of neither a point source nor an extended source is valid under all conditions; hence, special care must be taken in any experimental set-up to decide which may be used.

3. UNIFORMITY OF X-RAY FLUX

To check if the flux obeyed the conditions assumed for deriving the equations of Section II, the uniformity of the flux over the cross section of the beam in the center of the ionization chamber was tested

by exposing photographic plates in this plane, the exposure being varied over wide limits of time. With a proper alignment of the diaphragm system, the distribution over the central area should be uniform. That this proved true is shown in Figure 4 (*b*) where the photographic density, obtained by measuring the resulting photographic plate with a Marten's photometer, is plotted against position across the beam. With the X-ray tube and first diaphragm displaced laterally about 0.5 cm. from the central position, a photographic plate exposed inside the ionization chamber, gave the distribution shown in Figure 4 (*a*)—the dotted line giving the expected density for a perfectly symmetrical beam. As expected, the densities were greater on the side toward which the source was shifted. The irregularity with which the observed curve fluctuates about the smooth curve can well be ascribed to the inhomogeneity of the photographic emulsion.

4. CHOICE OF OPERATING CONDITIONS

Two special cases may be given which illustrate how the geometrical analysis of the diaphragm system permits conditions to be chosen that yield an accurate determination of the ionization per unit volume.

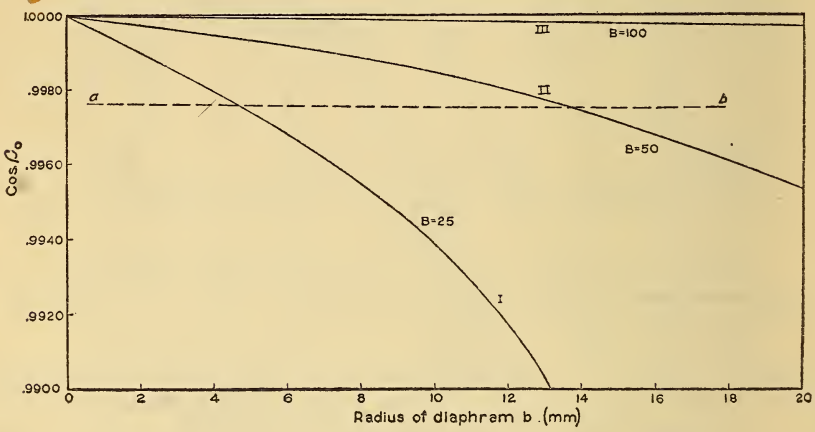

FIGURE 5

Letting β_0 be the maximum angle of divergence between the axis and the rays passing through the diaphragm system, then Curves I, II, and III (fig. 5) give the values of cos β_o plotted against the radial position in the ionization chamber for three cases where $B=25$, 50, and 100 cm, respectively, where $a=0.4$ cm. The values of cos β_o are obtained from the relation

$$\frac{\beta_o}{2}=\tan^{-1}\frac{a+b}{B}$$

from Figure 1 (*c*).

If we stipulate that $\cos^2 \beta_o=0.9950$ introduces the maximum permissible error—which means an error of less than 0.25 per cent in

r—it is seen that any physical conditions whereby $\cos \beta_o = 0.9975$ may be used. Thus the intersection of any curve in Figure 5 with the line $ab = 0.9975$, gives at once the upper limit within which our approximations would be valid. In Curve I for $B = 25$ cm it is seen that b must not exceed about 0.5 cm; in Curve II for $B = 50$ cm, b must not exceed 1.4 cm; while Curve III for $B = 100$ cm indicates that b may become extremely large before exceeding the given limit.

From equation (7) giving the radius of the focal spot necessary to fill the aperture, we may plot curves which indicate the diaphragms and distances which may be used and yet fulfill the condition that $f > (ka + kb + aB)/B$. If for instance, we have a focal spot

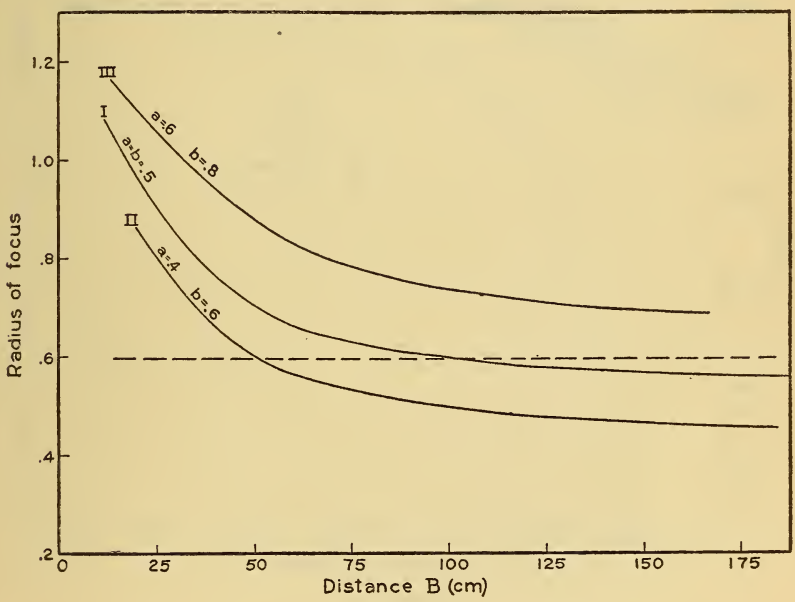

FIGURE 6

of radius 0.6 cm, any set of conditions composing the portion of the curve (fig. 6) below $f = 0.6$ may be used. Thus in Case I, where the diaphragms M and N have a radius of 0.5 cm each, the distance B must never be less than 100 cm. In Case II where $a = 0.4$ and $b = 0.6$ cm, B may be reduced to 50 cm; while for $a = 0.6$ and $b = 0.8$ cm no value of B whatever will suffice.

In the standardization equipments used by Behnken, Duane, and Glasser, each has a diaphragm corresponding to M. However, it is probable that their focal spots did not completely fill the aperture of the system, so that for the purpose of indicating the allowable deviation in each, we may consider the aperture as determined by the radii of the focal spot and diaphragm N and the distance $(B + k)$. We

may then compute cos β_o as a function of $(B+k)$ for different radii of the diaphragm N and so determine the range over which the allowable error in measuring the air volume is not exceeded. A set of these relations are plotted in Figure 7. All conditions falling on the curves above the line at cos $\beta_o = 0.9975$ may then be used without introducing more than the stipulated error.

Consider now three of the well-known X-ray standardization equipments:

I. Behnken.[15] (Physikalische Technischen-Reichsanstalt). $(B+k) =$ 105 cm; the radius of the focal spot f and diaphragm N are about the same and equal to 0.35 cm making cos β_o fall between Curves *I*

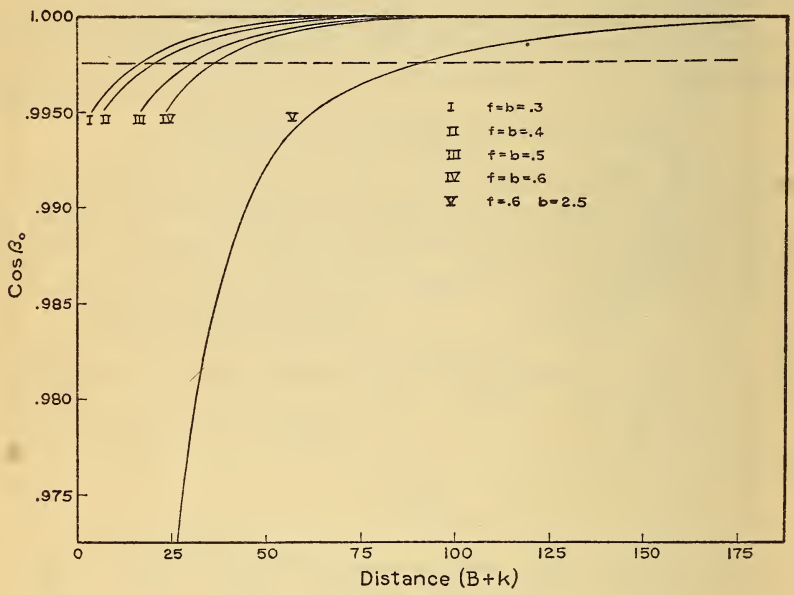

FIGURE 7

and *II* in Figure 7, thus indicating a very small error which may be neglected.

II. Glasser.[16] (Cleveland Clinic). $(B+k) = 45$ cm and $f = b = 0.4$ cm. The point fulfilling these conditions falls on Curve *II* well above cos $\beta_o = 0.9975$, also indicating a very small error.

In the two cases above, the focal spot may be slightly greater than indicated, but, even so, the maximum error will be less than 0.5 per cent.

III. Duane.[17] (Harvard University). This system has for N a rectangular diaphragm 0.5 by 5.0 cm, and $(B+k)$ is approximately

[15] H. Behnken, Strahlentherapie, 79, p. 26; 1927.
[16] O. Glasser and U. V. Portmann, Am. J. Roent., 19, p. 47; 1928.
[17] W. Duane and E. Lorenz, Am. J. Roent., 19, p. 461; 1928.

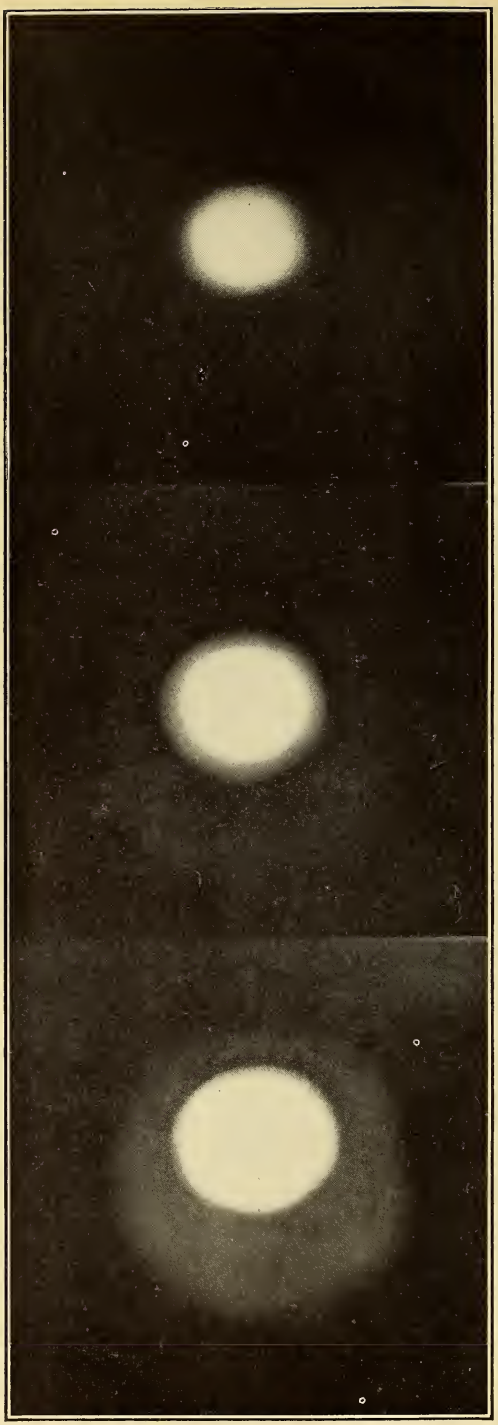

FIGURE 8.—*Pinhole photographs to indicate the magnitude of off-focus radiation*

60 cm. The energy distribution across the beam in the 0.5 cm direction will be uniform within the necessary limits. Along 5.0 cm direction, however, the variation in cos β_o with $(B+k)$ is shown in Curve V, whence it is seen that for $(B+k)=60$, the maximum error in $\cos^2 \beta_o$ is about 1.6 per cent and, therefore, can not be neglected. In order to reduce this error to within the prescribed limits, we must make $(B+k)=90$ cm.

IV. INFLUENCE OF "OFF-FOCUS" RADIATION

Off-focus radiation may be defined as that *radiation from an X-ray tube which originates from points on the target face other than the sharply defined focal spot.* If a diaphragm M be used which is sufficiently small, "off-focus radiation" does not enter. In this radiation lies another possible source of error in standardization measurements, and on account of the difficulty in accurately measuring its magnitude most observers have avoided a quantitative study of the question. Glasser [18] has indicated the importance of screening the ionization chamber diaphragm (N) from radiation coming from the stem and the body of the target other than the face. Likewise, Behnken,[19] Failla,[20] Duane [21] and most others have taken similar precautions, although in every case the method was entirely arbitrary.

A pinhole photograph of the target taken through the diaphragm system will indicate any source of off-focus radiation. Especial care, however, must be used in making such photographs, since an underexposed plate may apparently indicate a good focal spot and little off-focus radiation; an overexposure indicates general radiation from the whole face; whereas an exposure commensurate with the limiting error would give some indication of the actual off-focus radiation in proper proportion to the focal spot radiation. These three cases are shown, respectively, in Figures 8 (a), (b), and (c). While the amount of off-focus radiation will usually be but a small percentage of the focal spot radiation, it may at times be of too great a magnitude to be neglected.

To measure the off-focus radiation, the diaphragm N was fixed and the ionization chamber placed at a fixed distance from the tube such that the aperture of the diaphragm system did not introduce more than 0.5 per cent maximum deviation in the flux density across a plane within the chamber. Ionization current readings were made as the diaphragm M was varied from a diameter of 0.4 to 1.2 cm. The curve in Figure 9 shows the intensity (α to ionization current)

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[18] See footnote 16, p. 820.
[19] See footnote 15, p. 820.
[20] See footnote 8, p. 808.
[21] See footnote 17, p. 820.

plotted against a^2 (α to area of M) for a 200 k. v. Coolidge tube having a focus of radius about 0.4 cm. It is seen that the intensity increases very rapidly up to a point P which corresponds to the condition where the sharply defined focus just fills the aperture, while beyond this point the ionization increases very slowly and somewhat irregularly. Nevertheless, a continual increase is evident and no saturation value of the X-ray flux was indicated over the range studied. It is thus seen that for this par-

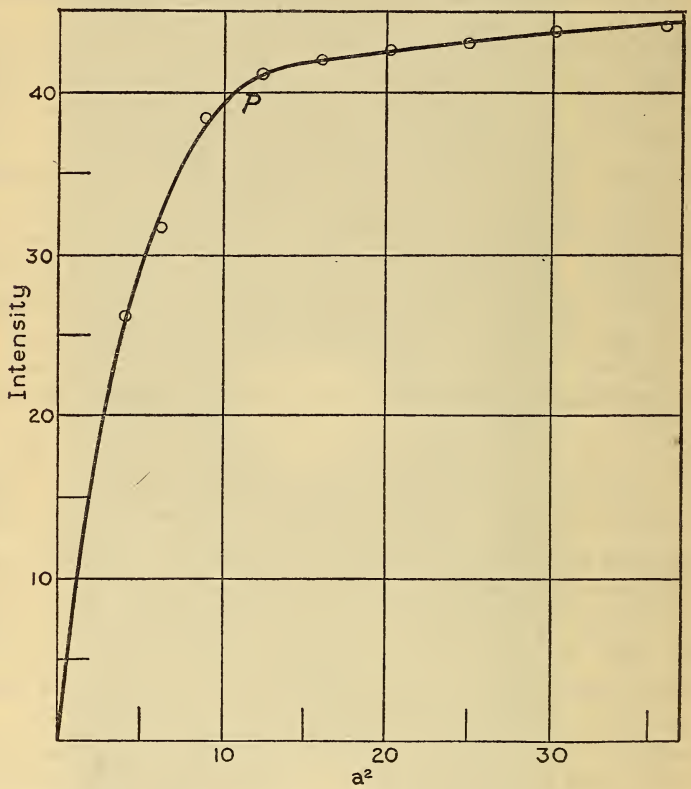

FIGURE 9.—*Measure of the proportion of off-focus radiation to focal spot radiation for a particular tube*

ticular tube and diaphragm system, the magnitude of this off-focus radiation can not be neglected.

It is probable that if two observers were to use, respectively, diaphragms M of 0.9 and 1.5 cm diameter for such a tube that some correction would have to be applied to make their measured r units agree. Similar curves obtained for two other tubes showed differences in their respective off-focus radiation.

Referring to Curve *II* in Figure 5, showing the validity of the inverse-square law, we have seen that the upper portion PQ gave

the nearest approach to the proper slope of -2. However, there is a very slight but definite divergence which may probably be attributed to the off-focus radiation, since the quantity of this radiation entering the ionization chamber varies with the distance B. Over the lower part PR of Curve I the slope is exactly -2 indicating good agreement with the universe square law. (Equation (13).)

Behnken and Glasser have diaphragms M of 15 and 12.5 mm diameter, respectively, placed in each case about 15 cm from the target. Their X-ray tubes have focal spots of different size. It is therefore evident that both may have quite a different ratio of on-focus to off-focus energy, and thus introduce discrepancies between their r units. Again, Failla has no diaphragm close to his X-ray tube, but restricts his scattered radiation by a diaphragm some 30 cm away from the focus. He has given a detailed discussion [22] of the changes in ionization due to varying the size and positions of his diaphragms. These differences in experimental technique require that proper allowance be made for off-focus radiation or else it must be completely shielded from entering the ionization chamber, which may be accomplished by using a diaphragm system wherein the focus always fills the aperture.

V. ERRORS DUE TO ENERGY DISTRIBUTION ABOUT THE TARGET

In three cases noted above (Failla's, Behnken's, and Glasser's) the radiation is taken off the target face at an angle of 45°, and at 90° to the cathode stream. Duane's diaphragm system is similar to that of Behnken and Glasser, except in being rectangular. However, it is dissimilar in that the radiation is taken off the target at a small angle of some 10° to 15°, and, consequently, it is important to determine whether there is any possible error due to the uneven distribution of energy with angular direction about the focus.

The energy distribution about a massive target has been studied by Kaye,[23] Coolidge,[24] and others and is shown in Figure 10 for a 45° angle between cathode stream and normal to the target; the data being taken from the paper by Coolidge and Kearsley and replotted on polar instead of rectangular coordinates. Figure 11 likewise shows the energy distribution about the focus in a plane perpendicular to the cathode stream. It is seen from the curves in Figure 10 that the distribution is nearly uniform in the direction used by Behnken, Glasser, and the Bureau of Standards. This is the direction in which radiation is usually taken in practice. At glancing angles to the

[22] See footnote 8, p. 808.

[23] G. W. C. Kaye, Proc. Roy. Soc. (London), **83**, p. 189; 1910. Kaye found that as the angle between normal and cathode stream changed the distribution about the normal did not vary very much.

[24] W. D. Coolidge and W. K. Kearsley, Am. J. Roent., **9**, p. 77; 1922.

target, however, the change in distribution is very rapid, so that the flux density across a plane x as considered in Section II should decrease appreciably in passing from one side of the beam to the other.

In Duane's case, where long rectangular slits are used, we should expect, therefore, an appreciable change in the flux density across the beam. Of course, if the diaphragms are sufficiently small such a change would be very small and could be neglected. A photographic determination of this effect was easily obtained. A rec-

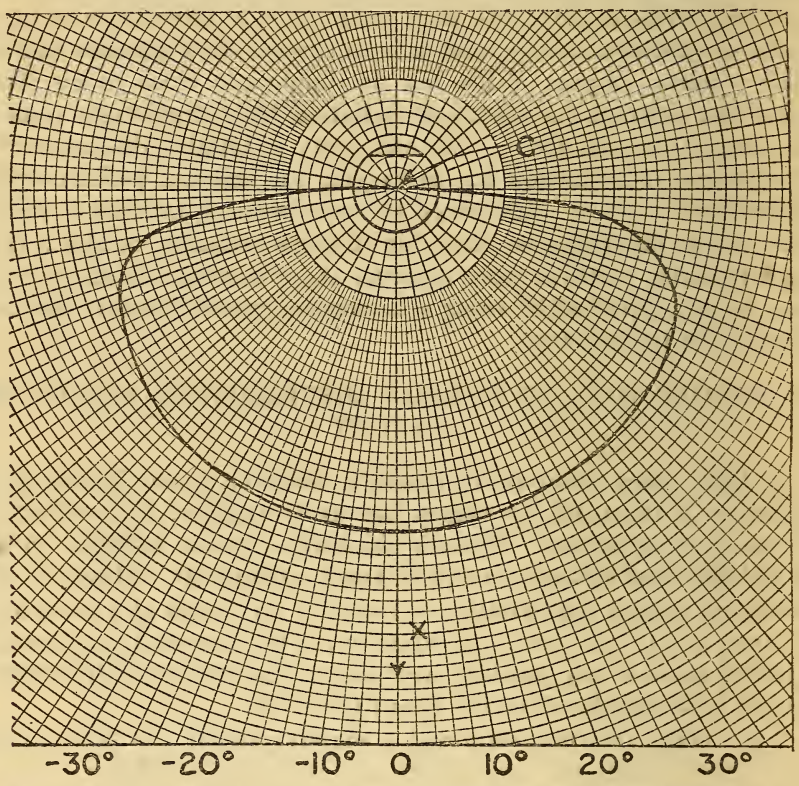

FIGURE 10.—*Distribution of energy about 200 k. v. deep therapy X-ray tube target in a plane normal to the tube axis*

tangular diaphragm system was set up and carefully aligned so as to correspond to that used by Duane, the tube tilted to an angle of about 35° and rotated about +70° to 75° from its normal position, and the cross section of the beam photographed behind the second diaphragm. The resulting plate densities are indicated in Figure 12, curve *a* being taken lengthwise of the rectangular diaphragm, and curve *b* crosswise. As should be expected, the lower part of the field is more intense than the upper and likewise the right side more intense than the left. These inequalities were obtained likewise in

the opposite direction when the tube was rotated through −70° to 75°.

The variation in flux density here considered is entirely independent of that due to the cosine law; it is, however, possible that the two may be present at the same time. In an accurate comparison of the *r* unit between two laboratories care must be taken that such errors are small enough to be neglected. An ideal standardization

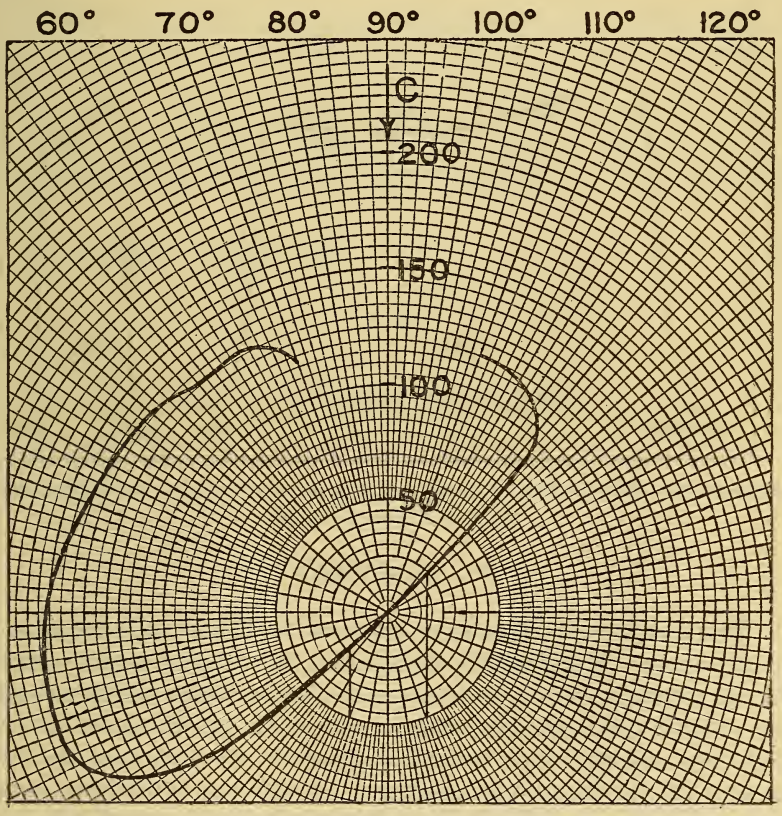

FIGURE 11.—*Distribution of energy about target in a plane through the tube axis and the normal to the target face*

equipment would have the X-ray tube so constructed that the radiation is taken normally from the target.

In the author's first paper,[25] a careful study was made of the several methods used in measuring air ionization with particular reference to the accuracy of each method. As a consequence of the differences found, an improved method was tested and found to be satisfactory. In the present paper, the methods for obtaining the

[25] L. S. Taylor, B. S. Jour. Research, 2, p. 771; 1929.

effective volume of air ionized, have been analyzed; the results being summarized below. The next logical problem is that of analyzing the methods used in the comparison of the *r* unit between the different laboratories having a standard ionization chamber. This problem is being undertaken at present at the Bureau of Standards.

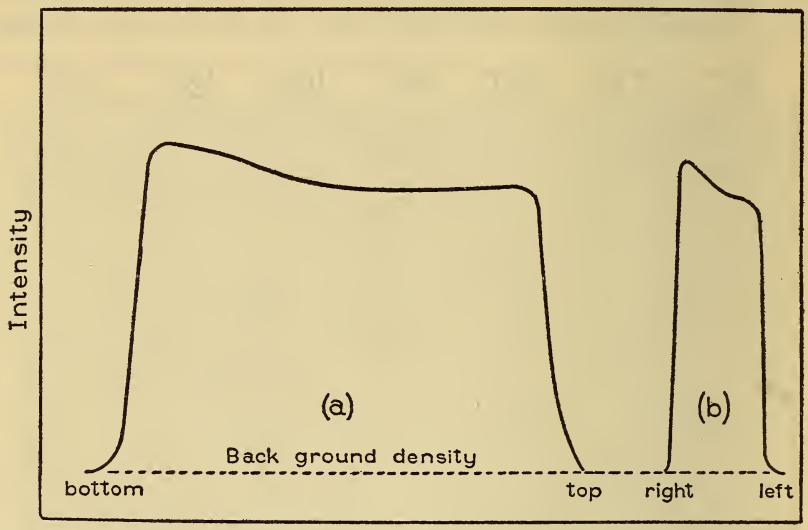

FIGURE 12.—*Distribution of energy across a rectangular beam taken from the target at a glancing angle*

VI. SUMMARY

1. Previous investigators have assumed the X-ray tube focus to be a point source. Experiments described in the present article indicate that the assumption of a uniform and extended focus yields instead a more reliable determination of the *r* unit when the distances between X-ray tube and standard ionization chamber are such that the focal spot area fills the diaphragm aperture. In this case the inverse square law is valid when applied to the distance between the X-ray tube diaphragm and the entrant diaphragm of the ionization chamber.

When the X-ray tube diaphragm is not filled the inverse square law should be used with the distance between the target and the entrant diaphragm of the ionization chamber. Off-focus radiation may impair, however, the accuracy of the results in this case.

2. The use of a limiting diaphragm close to the X-ray tube may introduce an error in the measurement of the ionization in the effective volume, this being due to the off-focus radiation entering the chamber. Such error may introduce discrepancies between the *r* unit as established in various laboratories.

3. By having incorrect alignment of the diaphragm system serious error may be introduced in measuring the effective volume.

4. Due to nonuniform energy distribution about the target, the use of narrow rectangular diaphragms is inadvisable except under carefully determined conditions.

5. Using diaphragms of the same size, but having edges beveled at different angles introduces no measurable difference in the ionization.

6. The discrepancy in measurements of the air volume under the conditions used by other observers is shown to lie within expected error.

Discussion of these problems with the various investigators cited has been very helpful. Recognition is due G. Singer and C. G. Malmberg, of the bureau, for their assistance in the experimental work.

WASHINGTON, April 1, 1929.