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FRESNEL REFLECTION OF DIFFUSELY INCIDENT LIGHT

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ABSTRACT

The reflection factor of a plane boundary between two media has been computed by the Fresnel formula for unpolarized, perfectly diffused incident light as a function of the relative index of refraction of the media. Because of total internal reflection, the factors depend importantly upon whether the diffuse flux is incident externally or internally. For example, diffuse light in air incident on the plane surface of glass of index 1.5 is 9.2 percent reflected; but if the perfectly diffuse light is incident internally, the reflection factor is 60 percent.

In the optical specification of light-scattering materials[1] by means of the absorption and scattering coefficients of the Kubelka-Munk theory,[2] it is frequently necessary to consider the effect of both external and internal reflection of light diffusely incident on a plane boundary between two media of different indices of refraction.[3] The reflectance at such a boundary has therefore been computed by application of the well-known Fresnel formula. The results are shown in table 1 as a function of the ratio, m, of the index of refraction of the denser medium to that of the rarer. The reflectance for perpendicular incidence, r, computed from the formula

$$r=(m-1)^2/(m+1)^2 \tag{1}$$

is also shown in the table.

The reflectance for completely diffused unpolarized light incident externally was computed from the approximate formula given by McNicholas:[4]

$$r=\sum_0^{\pi/2} r_\phi \sin 2\phi\Delta\phi \Big/ \sum_0^{\pi/2} \sin 2\phi\Delta\phi, \tag{2}$$

where r_ϕ is the reflectance by the Fresnel formula for unpolarized light incident on the surface from the rarer medium at an angle, ϕ, from the perpendicular to the surface. The angular interval, $\Delta\phi$, used in these summations was 0.04 radian.

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[1] Deane B. Judd (with the collaboration of W. N. Harrison, B. J. Sweo, E. F. Hickson, A. J. Eickhoff, Merle B. Shaw, and George C. Paffenberger), *Optical specification of light-scattering materials,* J. Research NBS **19**, 287 (1937) RP1026.

[2] P. Kubelka and F. Munk, *Ein Beitrag zur Optik der Farbanstriche,* Z. tech. Physik **12**, 593 (1931).

[3] J. W. Ryde and B. S. Cooper, *Scattering of light by turbid media, I,* Proc. Roy. Soc. (London) [A] **131,** 451 (1931).

B. W. King, Jr., *Effect of particle size and index of refraction on reflectance,* J. Ceramic Soc. **23,** 221 (1940).

J. S. Preston, *The properties of diffusing glasses with special reference to surface effects,* Proc. Intern. Illumination Cong. **1,** 373; 1931 (London, 1932).

[4] H. J. McNicholas, *Absolute methods in reflectometry,* BS J. Research **1,** 29 (1928) RP3. See equation 18, p. 50.

TABLE 1.—*Reflectance of unpolarized light at a plane boundary between two media as a function of their relative index of refraction, m*

m	Reflectance for perpendicular incidence	Reflectance for completely diffuse incidence		*m*	Reflectance for perpendicular incidence	Reflectance for completely diffuse incidence	
		External reflection	Internal reflection			External reflection	Internal reflection
1.00	0.00000	0.0000	0.000	1.31	0.01801	0.0628	0.454
1.01	.00002	.0028	.022	1.32	.01902	.0644	.463
1.02	.00010	.0055	.044	1.33	.02006	.0660	.472
1.03	.00022	.0082	.064	1.34	.02111	.0676	.480
1.04	.00038	.0108	.084	1.35	.02218	.0692	.489
1.05	.00059	.0134	.103	1.36	.02327	.0707	.497
1.06	.00085	.0158	.122	1.37	.02437	.0723	.505
1.07	.00114	.0183	.140	1.38	.02549	.0738	.513
1.08	.00148	.0206	.158	1.39	.02663	.0754	.520
1.09	.00185	.0230	.175	1.40	.02778	.0769	.528
1.10	.00227	.0252	.192	1.41	.02894	.0784	.536
1.11	.00272	.0274	.208	1.42	.03012	.0800	.543
1.12	.00320	.0294	.224	1.43	.03131	.0815	.550
1.13	.00372	.0314	.240	1.44	.03252	.0830	.557
1.14	.00428	.0334	.254	1.45	.03373	.0845	.564
1.15	.00487	.0353	.269	1.46	.03497	.0860	.571
1.16	.00549	.0371	.283	1.47	.03621	.0875	.577
1.17	.00614	.0389	.296	1.48	.03746	.0890	.584
1.18	.00682	.0407	.309	1.49	.03873	.0904	.590
1.19	.00753	.0425	.322	1.50	.04000	.0919	.596
1.20	.00826	.0443	.335	1.51	.04129	.0934	.602
1.21	.00903	.0461	.347	1.52	.04258	.0948	.608
1.22	.00982	.0478	.359	1.53	.04389	.0963	.614
1.23	.01064	.0496	.371	1.54	.04520	.0977	.619
1.24	.01148	.0513	.382	1.55	.04652	.0992	.624
1.25	.01235	.0530	.393	1.56	.04785	.1006	.630
1.26	.01323	.0546	.404	1.57	.04919	.1020	.635
1.27	.01415	.0563	.414	1.58	.05054	.1035	.640
1.28	.01508	.0579	.424	1.59	.05189	.1049	.645
1.29	.01604	.0596	.434	1.60	.05325	.1063	.650
1.30	.01701	.0612	.444				

The reflectance for completely diffused unpolarized light incident internally was computed from the similar formula:

$$r = \sum_{0}^{\pi/2} r_\theta \sin 2\theta\Delta\theta \Big/ \sum_{0}^{\pi/2} \sin 2\theta\Delta\theta, \tag{3}$$

where r_θ is the reflectance by the Fresnel formula for unpolarized light incident on the surface from the denser medium at an angle, θ, from the perpendicular to the surface. The angular interval, $\Delta\theta$, used in these summations was 0.04 radian except for the interval of 0.04 radian containing the critical angle ($\theta_c = \sin^{-1} 1/m$) for which it was taken as 0.0005 radian.

Values of r_ϕ were read directly from tables published by Moon.[5] Values of r_θ for θ less than the critical angle were found by reading the value of r_ϕ from Moon's table for ϕ equal to the angle whose sin is $m \sin \theta$. For θ greater than the critical angle, r_θ is, of course, equal to 1.

The summations were carried out for $m = 1.1$, 1.2, 1.3, 1.4, 1.5, and 1.6. Intermediate values were found by third-difference osculatory interpolation[6] with the terminal intervals filled in by extrapolation of the third-differences. The uncertainty is estimated to be less than 2 in the last figure given. The values of reflectance for external

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[5] Parry Moon, *A table of Fresnel reflection*, J. Math. Phys. **19**, 1 (1940).

[6] J. W. Glover, *Derivation of the United States Mortality Table by osculatory interpolation*, Quart. Pub. Am. Statistical Assoc. **12**, 90 (1910).

D. B. Judd, *Extension of the standard visibility function to intervals of 1 millimicron by third-difference osculatory interpolation*, BS J. Research **6**, 465 (1931) RP289.

diffuse incidence agree well with those computed by Ryde and Cooper (see footnote 3) from a formula derived by Walsh.[7]

Figure 1 shows as a function of *m*, reflectance for perpendicular incidence, reflectance for external diffuse incidence, and reflectance for internal diffuse incidence. It may be seen that there is considerable difference between these functions not only in amount but also in the shapes of the curves which represent them.

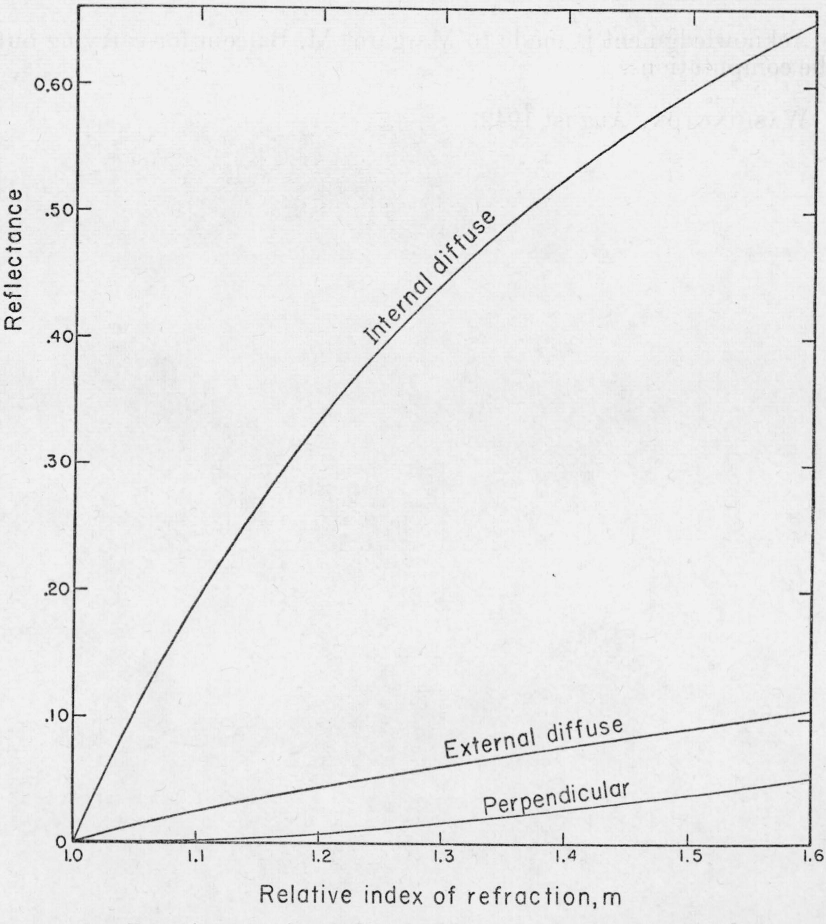

FIGURE 1.—*Fraction of incident light reflected from a plane interface for three angular conditions of illumination: (a) perpendicular incidence, (b) perfectly diffuse incidence from the less dense medium (external), and (c) perfectly diffuse incidence from the denser medium (internal).*

Note that internal reflection of diffusely incident light takes place to a considerable degree even at boundaries between media of only slightly differing index of refraction (*m* approaching 1.0).

Preston,[3] Ryde,[3] and Duntley[8] have correctly warned that internal reflectance is sharply dependent on the degree of approach to complete diffusion of the incident light. For the same reasons it is

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[7] J. W. T. Walsh, *The reflection factor of a polished glass surface for diffused light*, Dept. Sci. Ind. Res. (Brit.), Illumination Research Tech. Pap. 2, 10 (1926).

[8] S. Q. Duntley, *The optical properties of diffusing materials*, J. Opt. Soc. Am. 32, 61 (1942).

also sharply dependent upon the degree of approach of the surface to a perfect plane. A slightly wavy or scratched surface bounding a diffusing medium, or a surface exhibiting an "orange-peel" texture should be expected to have values of internal reflectance considerably lower than those for a perfect plane.

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