U. S. Department of Commerce — National Bureau of Standards

RESEARCH PAPER RP1498

Part of Journal of Research of the National Bureau of Standards, Volume 29, September 1942

CHARACTERISTICS OF WIDE-ANGLE AIRPLANE-CAMERA LENSES

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ABSTRACT

The relative illumination in the focal plane was measured for a number of wide-angle airplane-camera lenses, using a method depending upon the determination of the light-transmitting area of the lens effective at definite orientations of the lens. A new factor dependent on the lens design was found to be operative in reducing the values of the relative illumination in the unvignetted portion of the field for certain types of lenses. Determinations of the resolving power were also made and showed considerable variation in performance with type of lens. The effect of basing the distortion values upon the calibrated focal length instead of the equivalent focal length was determined.

CONTENTS

I. INTRODUCTION

In the production of aerial photographs for use in mapping, the negative size established in the United States Department of Agriculture is 9 by 9 inches, and a lens of 8¼-inch focal length is recommended.[1] This combination produces negatives that are suitable for both planimetric and topographic mapping. However, some Government mapping agencies, such as the United States Geological Survey, are interested in producing more accurate topographic maps than can be obtained with this combination, and they have encouraged the development of short-focal-length lenses capable of producing usable imagery over a greater angular field for the same, or smaller, negative size than obtains for the typical 8¼-inch lens. Negatives produced by these wide-angle lenses are especially useful in topographic mapping, because the stereoscopic relief for contour work increases as the angle separating the image from the center of the negative increases. In the past 2 years numerous short-focal-length lenses, whose half-angular field ranged from 40° to 50°, have been submitted to this Bureau for examination to determine their suitability. An unusual opportunity was accordingly offered for studying the characteristics of a great variety of wide-angle lenses, and the results are presented in this paper.

[1] United States Department of Agriculture Specification A-APC-1102 (as approved March 1940).

II. RELATIVE ILLUMINATION IN THE FOCAL PLANE

The present trend toward the use of wide-angle lenses in airplane photography has renewed interest in one aspect of lens performance that previously did not demand serious attention, namely, illumination in the focal plane. The latitude in film sensitivity is such that, when the half angle of the area imaged by the lens does not exceed 30°, the difference in negative density between 0° and 30° is not sufficiently great to occasion concern. However, if the half angle of the lens increases to 45°, the difference in illumination between the center and extreme portion of the negative becomes serious, since the center of the negative is overexposed, whereas the extremes are underexposed.

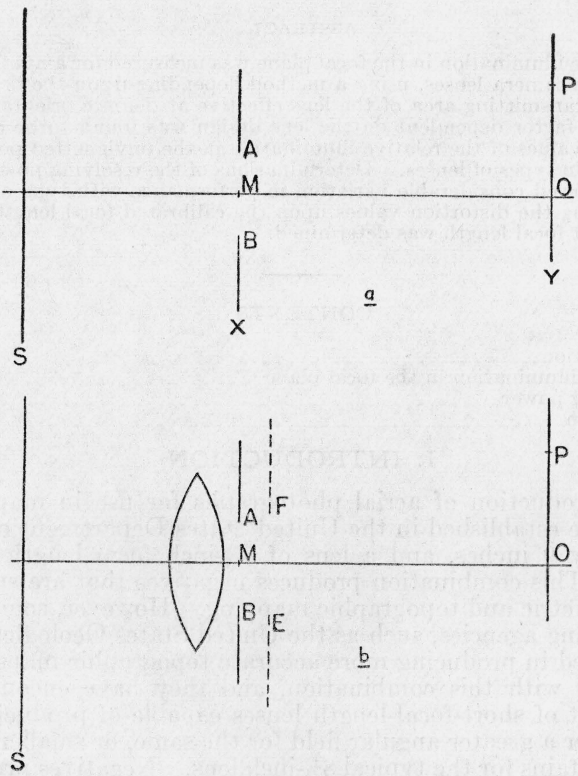

FIGURE 1.—*Schematic drawing to illustrate change of illumination with angle θ.*
(a) *AB*, opening in plane *X*; (b) *AB*, opening in plane *X*; *EF*, entrance pupil of lens, *L*.

Several factors are operative in producing a reduction in the amount of light received by a given unit area in the focal plane. The first of these is illustrated in figure 1, *a*. Let light from a distant uniform extended source, *S*, pass through the circular opening, *AB*, in the opaque plane, *X*. Let *M* be the midpoint of the opening, and *O* the intersection of the normal *MO* drawn to the plane *Y* that is parallel to plane *X*. Let *P* be any point in plane *Y*, such that a line joining *M* and *P* makes angle θ with *OM*; then it can be shown to a satisfactory degree of approximation that the relationship between the illumination E_p at point P and E_0 at point O is

$$E_p = E_0 \cos^4 \theta. \tag{1}$$

It has been customary to extend the coverage of this relationship to the case where the entrance pupil, EF, of the lens serves as the circular opening, figure 1, *b*. This is a reasonable assumption, because the entrance pupil of the lens is simply the image of the diaphragm stop opening as formed by that part of the optical system lying between the diaphragm and the object, and as such should behave as the simple circular opening illustrated in figure 1, *a*. It is shown later in this paper that this is not always true. The second major factor is the encroachment of the lens boundary into the light path. This reduction of the effective diaphragm area by the lens mounting is called vignetting. Until the development of wide-angle lenses, vignetting was the principal factor in reducing the illumination at the edge of the picture. The illumination at 30° is 56 percent of the axial illumination (from eq. 1), and this reduction is not sufficient to impair seriously the quality of the negative if no vignetting is present. However, when the half angle is increased to 45°, the illumination at 45° is 25 percent of that on the axis when no vignetting is present. Hence the vignetting must be kept small indeed if good negatives are to be made with wide-angle lenses. It is difficult to make good use of negatives made when the ratio of illumination in different parts of the focal plane exceeds 8 to 1. It was therefore considered worthwhile to determine by measurement the relative illumination in the focal plane for a number of wide-angle mapping lenses. During the course of this investigation, an additional factor affecting the illumination in the focal plane was discovered and found to be associated with the design of the lens.

An indirect method is used in determining the relative illumination in different parts of the focal plane. Referring to figure 1, *a*, it is evident that the expression for relative illumination may be written:

$$E_p = (A_\theta / A_0) E_0 \cos^3 \theta, \tag{2}$$

where A_0 is the area of the opening AB in the plane X, and A_θ is the projected area of AB as viewed at the angular obliquity θ. For figure 1, *a*, $A_\theta = A_0 \cos \theta$, but for a lens whose effective light-transmitting area is reduced by vignetting or other causes $A_\theta < A_0 \cos \theta$. A method was therefore devised to measure A_θ and A_0.

The lens was mounted on a stand that permitted rotation about a vertical axis. The lens diaphragm was set at the selected stop opening, and the edges of this opening were illuminated from the rear of the lens by a fixed source. Photographs of the stop opening were made with an auxiliary camera in front of the lens. The lens under test and the camera lens were made as nearly coaxial as possible for the zero setting of the lens. The position of the lens was adjusted with respect to the vertical axis of rotation until no transverse displacement of the diaphragm image was noticeable when the lens under test was rotated from the zero position to the limit of its range. Photographs of the diaphragm opening were taken at 5° intervals from the centered position to 45° from the axis. Prints were made from these negatives, and the area of each image was measured with a planimeter. The ratio of the area, A_θ, at any angle, θ, to the area, A_0 of the axial image is taken as equivalent to the ratio of the effective aperture area at the given angle to the axial aperture area. This ratio, A_θ/A_0, is therefore the same as that appearing in the expression for relative illumination in the focal plane, as written in equation 2. The values obtained are shown in table 1 and given graphically in figure 2.

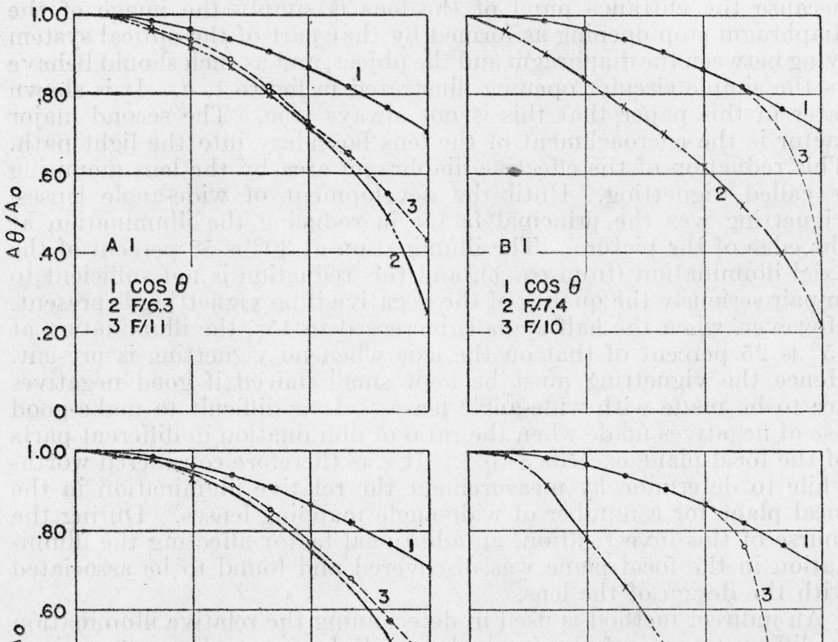

FIGURE 2.—*Ratio of effective aperture area, A_θ, to axial aperture area, A_0.*

TABLE 1.—*Ratio of A_θ to A_0 for four lenses*[1]

Manufacturer and lens type number	Relative aperture	Angular separation from axis									
		0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
A1	6.3	1.000	0.995	0.962	0.923	0.867	0.799	0.725	0.637	0.487	0.247
	11	1.000	------	.969	.944	.886	.820	.731	.647	.550	.431
B1	7.4	1.000	.959	.906	.844	.775	.698	.617	.520	.402	.218
	10	1.000	.982	.974	.959	.928	.903	.880	.819	.704	.390
C1	5.5	1.000	.988	.878	.934	.900	.835	.741	.660	.520	.343
	11	1.000	------	------	------	.916	.852	.772	.675	.574	.464
C2	4	1.000	.993	.913	.783	.659	.507	.352	.192	.037	------
	11	1.000	------	.974	.957	.927	.893	.876	.748	.244	------

[1] A_0=area of lens diaphragm opening; A_θ=area of effective diaphragm opening at angle θ from lens axis.

Vignetting by the lens boundary becomes effective at a small angle for lenses *B1* and *C2* when used at the maximum *f*/value, and consequently the curve of A_θ/A_0 departs from cos θ at the start, as may be seen in the number 2 curves in figure 2. However, when *B1* is stopped down to *f*/10, the cosine law is followed out to 35°, at which point vignetting begins and curve 3 falls away from curve 1. For *C2* at *f*/11, the cosine law is followed to 30°, and vignetting becomes effective at that point, as evidenced by curve 3, branching down from curve 1.

Since the relation $A_\theta/A_0=\cos\theta$ was found to be followed in the unvignetted region for lenses *B1* and *C2*, it is somewhat disconcerting to find that for lenses *A1* and *C1* the values of A_θ/A_0 are consistently below the corresponding values of cosine θ. This departure is present at both aperture ratios, although it is somewhat less pronounced at the smaller aperture ratio. For lens *A1*, vignetting begins at 35° for aperture *f*/6.3 and at 40° for aperture *f*/11. For *C1*, vignetting begins at 40° for aperture *f*/5.5, and at 45° for aperture *f*/11. Hence this departure of A_θ/A_0 from the corresponding value of cosine θ is present in the unvignetted region, and the amount of this departure is greater than can reasonably be assigned to experimental error. This is shown for a lens of the *A1* type in table 2, where the percentage departure of values of A_θ/A_0 from corresponding values of cosine θ increases progressively with increasing angle. Since the errors in measurement do not on the average exceed ± 1.5 percent, this departure of A_θ/A_0 from cosine θ is markedly greater than can be attributed to error. For example, the departure at 30° is -16.3 percent at aperture *f*/6.3. This is definitely in the unvignetted region and must arise from some cause that has not heretofore been generally recognized. This condition did not prevail for all lenses studied. It therefore seemed probable that such variation in behavior could only arise from differences in the lens design. To verify this hypothesis, a series of computations was made for a lens of the *A1* type. These computations consisted in tracing rays that were parallel and of various linear separations in the object space through that portion of the lens lying between the diaphragm opening and the object space. This was done for axial rays and for rays in the plane of the optic axis that made an angle of θ with the axis. The linear separation of each ray from the axis at the point where it passed through the plane of the diaphragm was determined, and pairs of rays were selected that passed through the diaphragm at the same distance from the axis. The ratio of the separation d_θ for each pair at angle θ to the separation d_0 for each corresponding axial pair in the object space was obtained. It was found that this ratio, d_θ/d_0, was less than the cosine θ. For $\theta=30°$, the percentage departure at *f*/6 was -10.3; at *f*/11, -9.5; and at *f*/20, -9.2. These values indicate that the effect is a maximum at large aperture but decreases slightly with diminishing aperture. Similar behavior is seen in table 2 for $A_{30°}/A_0$, where it is apparent that the percentage departure from the cosine value is greater at *f*/6.3 than at *f*/11.

The departure from the cosine value is greater for $A_{30°}/A_0$ than for $d_{30°}/d_0$. This discrepancy is undoubtedly due to the differences in the objects projected. In the first instance we are dealing with a circular area, whereas in the second only a single diameter is considered. It is probable that the projection of additional diameters at several inclinations to the initial diameter would permit an evaluation of the areal effect. However, the present computations for a single diameter clearly indicate that the ratio A_θ/A_0 is affected by the lens design.

TABLE 2.—*Deviation of A_θ/Af from cosine θ for lens A1*

Relative aperture	Quantity	Angular separation from the axis								
		0°	10°	15°	20°	25°	30°	35°	40°	45°
6.3	Cosine θ	1.000	+0.985	+0.966	+0.940	+0.906	+0.866	+0.819	+0.766	+0.707
	A_θ/A_0	1.000	+.962	+.923	+.867	+.799	+.725	+.637	+.487	+.247
	Cos θ—A_θ/A_0	0.000	+.023	+.043	+.073	+.107	+.141	+.182	+.279	+.460
	Percentage of deviation	.0	−2.3	−4.4	−7.8	−11.8	−16.3	−22.2	−36.4	−65.1
------	A_θ/A_0	1.000	+.969	+.944	+.886	+.820	+.731	+.647	+.550	+.431
	Cos θ—A_θ/A_0	0.000	+.016	+.022	+.054	+.086	+.135	+.172	+.216	+.276
	Percentage of deviation	.0	−1.6	−2.3	−5.7	−9.5	−15.6	−21.0	−28.2	−39.0

Finally, the relative illumination in the focal plane is obtained by multiplying the values of A_θ/A_0 shown in table 1 by $\cos^3\theta$. These values are given in table 3, and a graphic illustration for each lens is given in figure 3. Curve 1 in each box shows the value of $\cos^4\theta$ and is

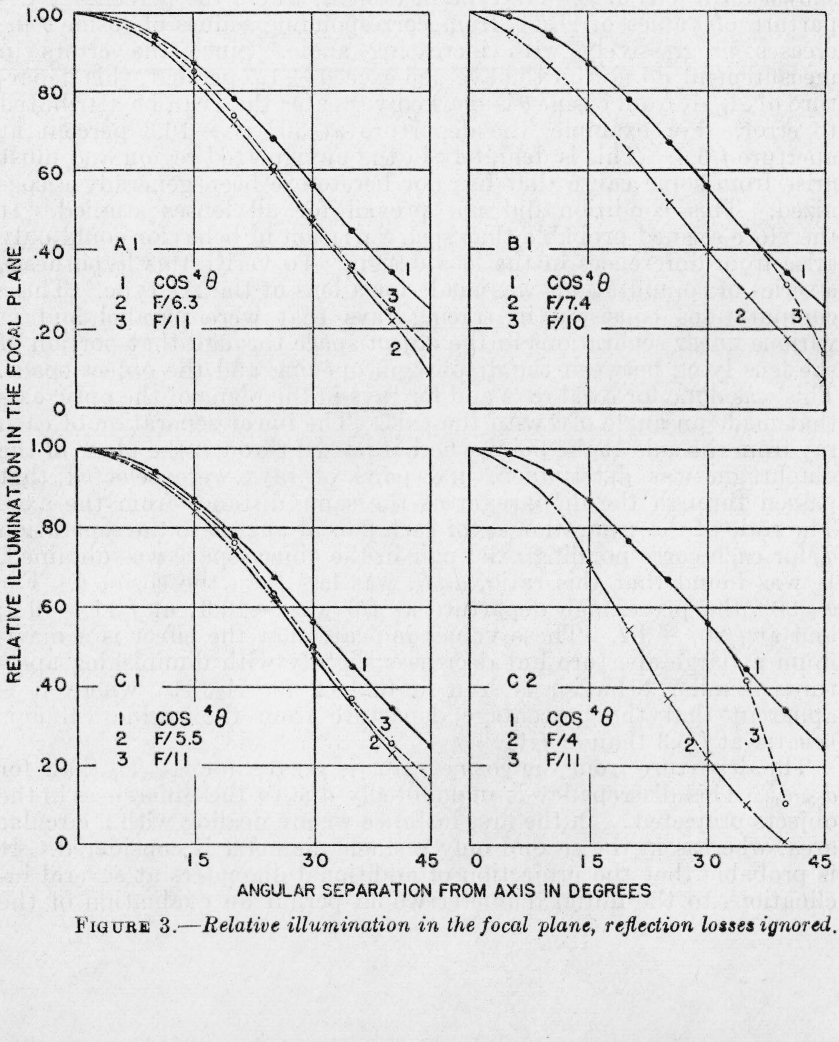

FIGURE 3.—*Relative illumination in the focal plane, reflection losses ignored.*

inserted for ready reference. The relation $E_p = E_0 \cos^4\theta$ is followed within the limits of experimental errors by lenses *B1* and *C2* until vignetting begins, whereas $E_p < E_0 \cos^4\theta$ for lenses *A1* and *C1* throughout the entire range. It is accordingly clear that the loss in illumination inherent in the lens design is greater for *A1* and *C1* than the loss resulting from vignetting.

TABLE 3.—*Relative illumination in the focal plane*

Manufacturer and lens type number	Relative aperture	Angular separation from the axis									
		0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
A1	6.3	1.000	0.984	0.919	0.832	0.719	0.595	0.471	0.350	0.219	0.087
	11	1.000	--------	.926	.851	.735	.610	.475	.356	.247	.152
B1	7.4	1.000	.948	.865	.761	.643	.520	.401	.286	.181	.077
	10	1.000	.971	.930	.864	.770	.672	.572	.450	.316	.138
C1	5.5	1.000	.977	.934	.842	.747	.622	.481	.363	.234	.121
	11	1.000	--------	--------	--------	.760	.634	.501	.371	.258	.164
C2	4	1.000	.982	.872	.706	.547	.377	.229	.106	.017	--------
	11	1.000	--------	.930	.862	.769	.665	.569	.411	.110	--------

The values of the relative illumination obtained by this method of measurement do not take into account possible loss in light resulting from absorption in the glass or from reflections at the surfaces. However, the values present a clear picture of the best possible performance of a given lens.

III. RESOLVING POWER

In the study of resolving power and distortion, the precision lens-testing camera described by Gardner[2] and Case was used. The method of selecting the image plane of best average definition was the same as that described in a previous paper.[3] The principal modification in the equipment was the addition of three collimators that extended the image region studied from 30° to 45° from the axis.

The results of the measurements on resolving power are shown in table 4. In all cases, except for lenses 15, 17, 19, 22, 23, and 24, the *f*/values, listed in column 4, are the maximum. For lenses of a given type and focal length, the highest value of the axial resolving power is nearly constant. The variations on the axis that appear in table 4 resulted from the necessity of selecting an image plane that yielded usable imagery throughout the entire field. Under these conditions, the resolving power on the axis sometimes falls below the maximum of which it is capable.

[2] J. Research NBS **18**, 449 (1937) RP984.
[3] J. Research NBS **22**, 729 (1939) RP1216.

TABLE 4.—*Resolving power in the plane of best average definition for 26 wide-angle airplane-camera lenses*

Manu-facturer and lens type number	Lens number	Equivalent focal length	Relative aperture or f/value	Tangential resolving power in lines per millimeter (angular distance from axis)										Radial resolving power in lines per millimeter (angular distance from axis)									
				0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
A1	1	132.49	6.3	63	63	63	63	32	22	16	16	8	8	63	63	63	63	63	44	44	44	44	22
	2	132.55	6.3	24	32	32	32	32	24	16	16	16	12	24	32	48	32	32	32	32	32	32	24
	3	132.33	6.3	56	56	56	40	40	28	20	7	7	10	56	56	56	56	56	56	40	28	20	14
	4	132.21	6.3	63	63	44	44	44	32	32	32	22	11	63	63	44	44	44	32	32	32	22	32
	5	132.63	6.3	32	32	22	22	32	22	22	32	32	16	22	22	16	16	16	16	22	44	44	32
	6	131.05	6.3	44	44	44	44	44	44	32	16	8	11	63	44	44	63	44	44	32	22	16	16
	7	132.11	6.3	64	64	44	32	16	16	16	11	11	11	64	64	44	44	44	32	32	32	32	22
	8	131.77	6.3	44	44	44	44	32	32	32	32	22	16	44	44	44	44	44	44	32	32	32	22
	9	101.29	6.3	58	58	58	58	58	58	41	29	21	14	58	41	41	41	58	58	41	29	21	14
	10	152.27	6.3	56	56	40	28	20	20	20	20	20	14	56	56	40	40	40	28	28	28	20	14
A2	11	166.23	11	40	40	40	40	28	20	10	7	14	------	40	40	56	40	20	20	40	28	------	------
	12	166.11	11	50	50	71	50	35	25	18	13	13	------	50	50	71	35	25	25	50	18	------	------
B1	13	129.23	10	23	32	45	45	45	45	32	32	32	11	23	32	65	23	11	8	8	11	32	11
B2	14	145.42	10	40	80	40	20	10	14	20	20	14	7	40	58	58	20	14	14	40	29	------	------
B3	15	109.01	11	56	56	40	28	20	20	20	20	14	3.5	56	56	40	28	14	7	5	20	30	3.5
	16	151.44	8	28	28	55	19	19	14	14	7	------	------	28	39	55	14	7	7	7	7	14	------
	17	149.23	16	20	39	56	56	28	20	28	28	20	------	20	39	56	28	10	7	7	20	14	------
	18	163.39	8	36	36	51	18	13	13	13	18	------	------	36	36	56	18	9	7	7	18	7	------
	19	165.10	11	56	56	28	20	14	10	10	10	3.5	------	56	56	28	14	5	5	5	5	20	------
C1	20	115.99	5.5	51	72	51	25	25	25	36	25	13	13	51	72	51	51	36	25	36	36	36	36
C2	21	153.90	4	54	76	54	27	19	19	14	14	10	------	54	76	27	10	10	7	19	14	------	------
D1	22	158.28	22	18	36	54	54	54	24	14	10	20	2	36	54	54	54	18	14	6	8	6	8
	23	158.07	22	18	36	36	54	42	28	28	28	18	6	14	18	36	54	42	14	14	14	48	6
	24	158.28	22	36	36	54	54	36	36	28	28	24	4	18	36	54	54	36	12	8	12	42	6
E1	25	98.92	6.3	85	85	59	42	42	30	30	30	21	15	85	85	85	59	59	59	59	42	42	21
F1	26	87.22	22	48	48	48	48	34	34	24	24	17	17	48	48	48	48	48	34	34	34	24	24

The curves shown in figures 4 and 5 illustrate the tangential and radial resolving powers for the lenses described in table 4. Identifications are given in the upper right-hand corner of each box. Where more than one lens of a given type has been studied under identical conditions, the curves show the averaged values for the group instead of for a single lens. The solid-line curves show the tangential and the dotted-line curves, the radial resolving power. The separation of the two curves illustrates the astigmatic difference in the given image plane at each angle.

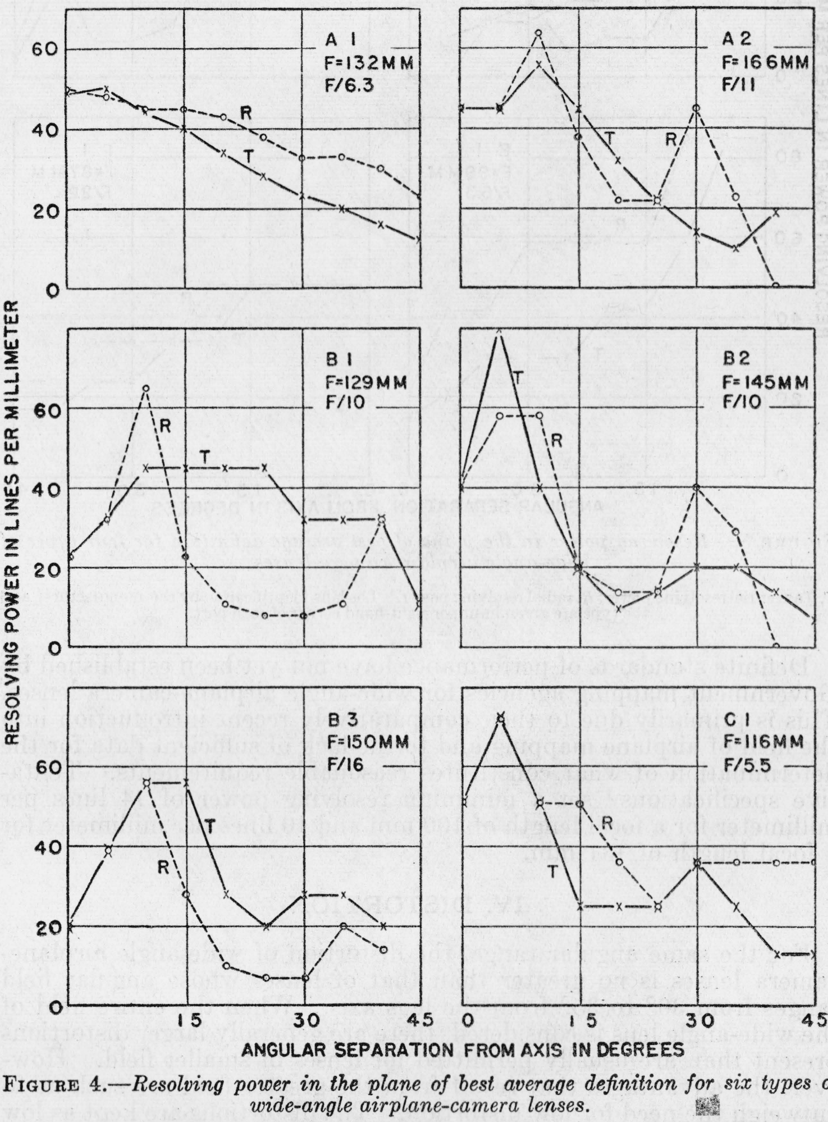

FIGURE 4.—***Resolving power in the plane of best average definition for six types of wide-angle airplane-camera lenses.***

T, Tangential resolving power; *R*, radial resolving power. Lens identification by manufacturer and type in upper right-hand corner of each part.

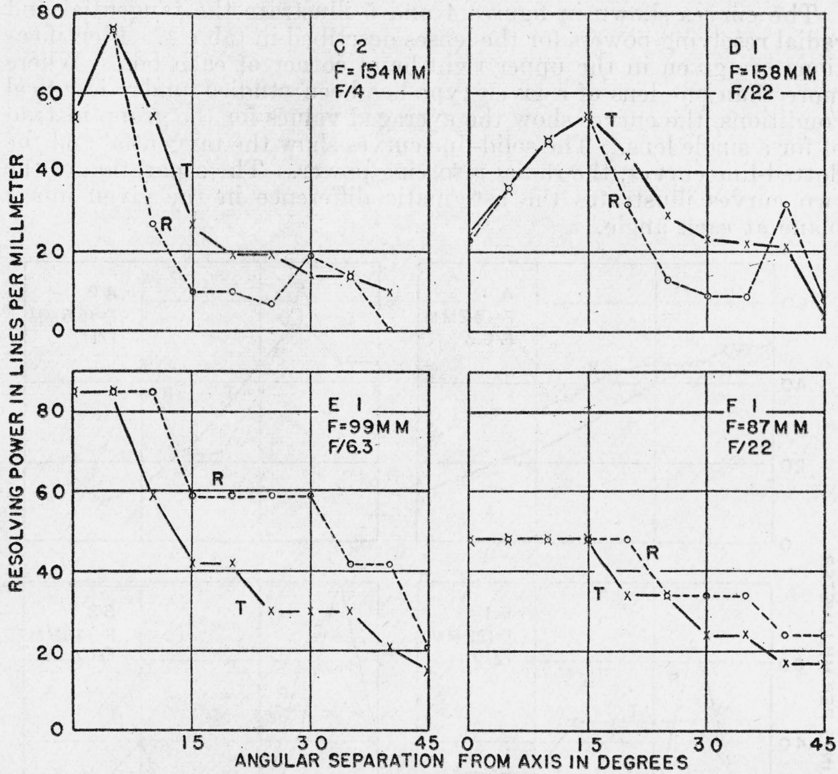

FIGURE 5.—*Resolving power in the plane of best average definition for four types of wide-angle airplane-camera lenses.*

T, Tangential resolving power; *R*, radial resolving power. The lens identification by the manufacturer and the type are given in upper right-hand corner of each part.

Definite standards of performance have not yet been established by Government mapping agencies for wide-angle airplane-camera lenses. This is primarily due to their comparatively recent introduction into the field of airplane mapping and to the lack of sufficient data for the determination of what constitutes reasonable requirements. Tentative specifications [4] set a minimum resolving power of 14 lines per millimeter for a focal length of 100 mm and 10 lines per millimeter for a focal length of 131 mm.

IV. DISTORTION

For the same angular range, the distortion of wide-angle airplane-camera lenses is no greater than that of lenses whose angular field ranges from 30° to 35° from the lens axis. When the entire field of the wide-angle lens is considered, there are generally larger distortions present than are usually permitted for lenses of smaller field. However, the advantages that result from the greater field are such as to outweigh the need for low distortion. The distortions are kept as low

[4] These specifications are contained in a communication to this Bureau from the Department of the Interior, Geological Survey, in July 1940.

as possible and as more of the lenses are made, it is reasonable to suppose that means of reducing the distortion will be found. In the tentative specifications mentioned earlier (see footnote 4), the maximum allowable distortion referred to the equivalent focal length is ± 0.20 mm for 100-mm lenses and ± 0.30 mm for 130-mm lenses.

Table 5 lists the distortion for the lenses whose resolving powers are listed in table 4. For a given lens whose distortion is shown in table 5, the resolving power is given under the same number in table 4; column 3 gives the equivalent focal length, on the basis of which the values of the distortion shown in columns 4 to 12 are computed. Column 13 gives the calibrated focal length, on the basis of which the values of the distortion shown in columns 14 to 22 are computed.

TABLE 5.—*Distortion of 26 wide angle airplane camera lenses*

Manufacturer and lens type number	Lens number	Equivalent focal length	Distortion referred to the equivalent focal length									Calibrated focal length	Distortion referred to the calibrated focal length								
			5°	10°	15°	20°	25°	30°	35°	40°	45°		5°	10°	15°	20°	25°	30°	35°	40°	45°
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
		mm	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*	*mm*
	1	132.49	0.00	0.00	+0.01	+0.01	+0.02	+0.03	+0.06	+0.04	-0.03	132.51	0.00	0.00	+0.01	0.00	+0.01	+0.02	+0.05	+0.02	-0.05
	2	132.55	.00	.00	.00	+.01	+.02	+.04	+.07	+.06	-.01	132.59	.00	-.01	-.01	-.01	.00	+.02	+.04	+.03	-.05
	3	132.33	.00	.00	+.01	+.02	+.04	+.06	+.07	+.04	-.08	132.33	.00	.00	+.01	+.02	+.04	+.06	+.07	+.04	-.08
	4	131.93	.00	.00	.00	+.01	+.02	+.04	+.05	+.16	-.09	131.97	.00	-.01	-.01	-.01	.00	+.02	+.02	+.13	-.13
	5	132.63	.00	.00	.00	+.02	+.04	+.07	+.09	+.08	-.04	132.66	.00	.00	-.01	+.01	+.03	+.05	+.07	+.06	-.07
A1	6	131.05	.00	.00	+.01	+.03	+.05	+.09	+.10	+.04	-.12	131.04	.00	.00	.00	+.03	+.05	+.10	+.11	+.05	-.11
	7	132.11	.00	.00	.00	+.01	+.02	+.04	+.04	-.01	-.18	132.03	+.01	+.01	+.02	+.04	+.06	+.09	+.10	+.06	-.10
	8	131.77	.00	.00	+.01	+.03	+.06	+.09	+.13	+.11	-.03	131.83	.00	-.01	-.01	+.01	+.03	+.06	+.09	+.06	-.09
	9	101.29	.00	.00	+.01	+.03	+.05	+.09	+.11	+.11	.00	101.35	.00	-.01	-.01	+.01	+.02	+.06	+.07	+.06	-.06
	10	152.27	.00	.00	+.01	+.01	+.02	+.02	.00	-.10	-.40	152.03	+.02	+.04	+.07	+.10	+.13	+.16	+.17	+.10	-.16
A2	11	166.33	.00	.00	+.01	.00	-.01	-.01	+.07	+.26	------	166.51	-.02	-.03	-.04	-.07	-.09	-.11	-.06	+.11	------
	12	166.11	.00	.00	.00	-.02	-.05	-.04	+.05	+.21	------	166.23	-.01	-.02	-.03	-.06	-.11	-.11	-.03	+.11	------
B1	13	129.23	.00	-.01	-.01	-.02	-.02	-.03	-.02	.00	+.06	129.25	.00	-.01	-.01	-.03	-.03	-.04	-.03	-.02	+.04
B2	14	145.42	.00	.00	+.02	+.02	.00	-.04	-.07	+.09	+.41	145.62	-.02	-.04	-.03	-.05	-.09	-.16	-.21	-.08	+.21
	15	109.01	.00	+.01	+.02	+.03	+.04	+.05	+.08	+.17	+.38	109.28	-.02	-.04	-.05	-.07	-.09	-.11	-.11	-.06	+.11
	16	151.44	.00	.00	+.02	+.05	+.08	+.10	+.15	+.25	------	151.68	-.02	-.04	-.04	-.04	-.03	-.04	-.02	+.05	------
B3	17	149.23	.00	.00	+.01	+.02	+.03	+.04	+.10	+.42	------	149.56	-.03	-.06	-.08	-.10	-.12	-.15	-.13	+.14	------
	18	163.39	.00	.00	+.02	+.05	+.07	+.10	+.20	+.35	------	163.71	-.03	-.06	-.06	-.07	-.08	-.08	-.02	+.08	------
	19	165.10	.00	.00	+.02	+.05	+.08	+.11	+.16	+.26	------	165.35	-.02	-.04	-.04	-.04	-.04	-.03	-.02	+.05	------
C1	20	115.99	.00	.00	+.02	+ 06	+.10	+.17	+.24	+.31	+.32	(1)	.00	.00	+.01	+.03	+.05	+.07	+.07	+.04	-.10
C2	21	153.90	.00	.00	.00	+.01	-.01	.00	+.02	+.19	------	154.03	-.01	-.02	-.03	-.04	-.07	-.08	-.07	+.08	------
	22	158.28	.00	.00	-.01	-.01	-.01	+.01	+.07	+.20	+.50	158.60	-.03	-.06	-.09	-.13	-.16	-.17	-.15	+.18	------
D1	23	158.07	.00	.00	.00	.00	.00	+.02	+.08	+.23	+.60	158.47	-.04	-.07	-.10	-.14	-.18	-.21	-.20	-.11	+.20
	24	158.28	.00	.00	-.01	-.01	-.01	+.01	+.06	+.21	+.56	158.64	-.03	-.06	-.11	-.14	-.18	-.20	-.19	-.09	+.20
E1	25	98.92	.00	.00	+.02	+.04	+.08	+.14	+.21	+.26	+.21	99.17	-.02	-.04	-.04	-.05	-.04	+.00	+.03	+.05	-.04
F1	26	87.22	.00	.00	.00	-.01	-.01	.00	.00	+.02	+.06	87.25	.00	.00	-.01	-.02	-.02	-.02	-.02	.00	+.03

[1] Corrected by a ⅛-in. glass plate.

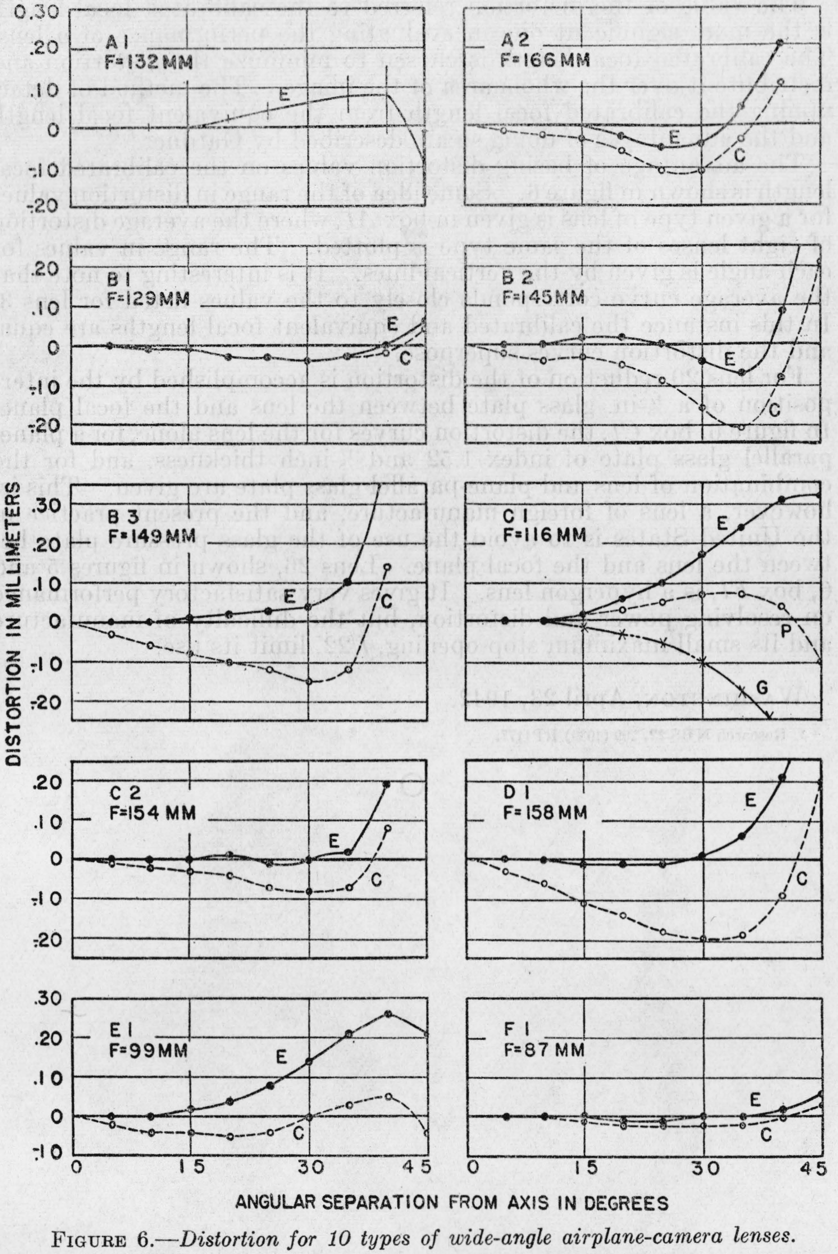

FIGURE 6.—*Distortion for 10 types of wide-angle airplane-camera lenses.*

E, Distortion referred to the equivalent focal length; *C*, distortion referred to the calibrated focal length. For lens *C1*; *G*, distortion for a plane-parallel plate of glass of index 1.52 and ⅛-in. thickness; *P*, distortion for combination of lens and plane-parallel glass plate. The lens identification by the manufacturer and the type are given in the upper center of each part.

The value of the distortion referred to the calibrated focal length is the more significant one in evaluating the performance of a lens. The calibrated focal length is chosen to minimize the distortion and distribute it over the whole area of the image. The method of determining the calibrated focal length from the equivalent focal length and the advantages of doing so are described by Gardner.[5]

The advantage of basing distortion values on the calibrated focal length is shown in figure 6. Some idea of the range in distortion values for a given type of lens is given in box *A1*, where the average distortion of eight lenses of the same type is plotted. The range in values for each angle is given by the vertical lines. It is interesting to note that the average curve corresponds closely to the values listed for lens 3. In this instance the calibrated and equivalent focal lengths are equal and the distortion curves superpose.

For lens 20 reduction of the distortion is accomplished by the interposition of a ⅛-in. glass plate between the lens and the focal plane. In figure 6, box *C1*, the distortion curves for the lens alone, for a plane-parallel glass plate of index 1.52 and ⅛-inch thickness, and for the combination of lens and plane-parallel glass plate are given. This is, however, a lens of foreign manufacture, and the present practice in the United States is to avoid the use of the glass pressure plate between the lens and the focal plane. Lens 26, shown in figures 5 and 6, box *F1*, is a hypergon lens. It gives very satisfactory performance on resolving power and distortion, but the difficulty of manufacture and its small maximum stop opening, *f*/22, limit its use.

WASHINGTON, April 23, 1942.

[5] J. Research NBS **22**, 209 (1939) RP1177.

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[Continued on p. 4 of cover]

[Continued from p. 3 of cover]

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