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FRICTIONAL PROPERTIES OF RUBBER

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ABSTRACT

Laboratory measurements of coefficients of friction of soft rubber compounds were made by towing specimens on horizontal tracks and by allowing them to slide down inclined tracks. The specimens were prepared by attaching the rubber to a metal backing and molding it against glass surfaces having different degrees of roughness. The coefficients increase markedly with speed, ranging from about 1 at 10^{-4} cm/sec to more than 4 at 5 cm/sec. The occurrence of vibrations prevented observations at higher speeds. Static friction is greater than dynamic friction for speeds appreciably less than 10^{-3} cm/sec and less than dynamic friction for greater speeds. The coefficients decrease slightly with increasing pressures and are independent of the size of the specimen. Except at very low speeds the smoother surfaces yield the higher coefficients. Materials such as talc or bloom on the sliding surfaces cause large decreases in the coefficients.

Attention is called to the dependence of the coefficients of friction on the speed, which is shown in several previous investigations on rubber and other materials.

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I. INTRODUCTION

The coefficients of friction of soft vulcanized rubber compounds have been studied by numerous investigators. The data reported for many of the investigations previous to March 1934 have been collected and tabulated by Dawson and Porritt [1].¹ Most of the studies of the frictional properties of rubber deal with particular kinds of rubber goods, such as tires, power-transmission belts, soles and flooring, and water-lubricated bearings. In such studies the experimental conditions are those under which the products are commonly used. For example, the motion between the friction surfaces for tires or belts is a combination of rolling and sliding. Foreign materials, many of which act as lubricants, are nearly always present between the surfaces. Often, sliding is accompanied by vibration of the rubber, as in the case of the skidding of tires, where the vibration is distinctly audible.

In the present investigation, which is a part of a more general investigation on the abrasive wear of rubber, the specimens and tracks were prepared in the laboratory. This procedure makes it possible to employ a relatively wide range of experimental conditions and to exercise control over the composition and the surface of the specimens. No attempt was made to determine the skidding resistance of tires; instead the study was limited to the frictional characteristics of clean, soft rubber compounds, particularly those of the type used in tire treads, when sliding on clean smooth tracks. The effects of the presence of certain types of lubricants were briefly studied in order that the results might be compared with those for relatively clean surfaces. Conditions of slide under which the specimens vibrate or chatter were not studied except to note that the average force required to pull the specimen decreased considerably when chattering began. Because of chattering of the specimens, the maximum speeds for which friction was studied were limited to 10 cm/sec. Also, because of difficulties experienced in preventing the specimens from buckling or otherwise deforming greatly, the maximum normal pressures between the specimens and the tracks were limited to 40 lb/in.² for most of the work.

II. EXPERIMENTAL PROCEDURE

Since this investigation was begun several years ago and then was discontinued for nearly 2 years before it was again resumed, the experi-

¹ Figures in brackets indicate the literature references at the end of this paper.

Experimental procedure and data are presented in two parts. The first part was more in the nature of an exploratory investigation consisting in brief studies for a relatively large variety of experimental conditions. The second part was a more detailed study of a rather limited field. Different samples, different apparatus, and different experimental techniques were employed in these two parts of the investigation but the results are similar in many respects and lead to the same general conclusions.

1. EARLIER EXPERIMENTS

(a) APPARATUS AND PROCEDURE

The rubber specimens for most of this work were in the form of three circular disks, which were cemented to one surface of a rectangular metal plate in such a way that they shared equally the weight of the plate and the additional load. The disks were $\frac{1}{16}$ in. thick and the diameters for different sets ranged from $\frac{5}{16}$ to $\frac{1}{2}$ in. For some of the work the rubber disks were replaced with steel and the track was cut from the rubber to be studied. In some of the tests the metal plate and disks were towed along the friction track by means of a cord which was wound on a drum. A motor and reduction gear served to rotate the drum, and a spring scale in the tow line served to measure the frictional force. In other tests a circular friction track about 2 ft. in diameter was rotated at the desired speed by means of the motor, and the specimens were held at rest by the tow line and spring scale.

(b) RUBBER COMPOUNDS INVESTIGATED

The compositions of the rubber compounds used in the specimens are given in table 1. One is a typical pure-gum compound; the other is the standard abrasion compound described in the Federal Specification for Rubber Goods No. ZZ-R-601a. These compounds were selected for study because of their great difference in resistance to abrasion and in order to determine whether the coefficients were characteristic of the filler or of the rubber. Since specimens made from the soft pure-gum compound distorted greatly under conditions of slide, the larger part of the study was made on the standard abrasion compound and on specimens cut from a rubber conveyor belt the exact composition of which was not known.

TABLE 1.—Composition of rubber compounds

[The standard abrasion and pure-gum compounds were press-vulcanized 60 and 30 minutes, respectively, at 142° C.]

Ingredients	Rubber compounds	
	Standard abrasion	Pure-gum
	<i>Parts by weight</i>	<i>Parts by weight</i>
Rubber (smoked sheets).....	100	100
Sulfur.....	3.5	3
Zinc oxide.....	20	5
Stearic acid.....	2	1
Diorthotolylguanidine.....	1.25
Mercaptobenzothiazole (Captax).....	1
Phenylbetanaphthylamine.....	1
Channel-black (Micronex).....	30
Total.....	157.75	110

(c) RUBBER SLIDING ON TRACKS OF DIFFERENT DEGREES OF ROUGHNESS

Coefficients of friction for the standard abrasion compound sliding on steel, and on different grades of abrasive materials are plotted against the logarithm of the sliding speed in figure 1. The logarithmic scale is used for the speed in this and subsequent figures for convenience in showing the coefficients over a large range of speeds. The smoothest track was steel polished with No. 0 polishing paper. Various grades of abrasive papers, fixed on a rigid base, served as rough tracks. Figure 1 shows that for every track the coefficient of friction increased with the speed. At the higher speeds the smoother tracks yielded the greater coefficients; but for speeds approaching zero, the rough tracks yielded the greater coefficients. Each point represents the average of several observations. Except for the steel

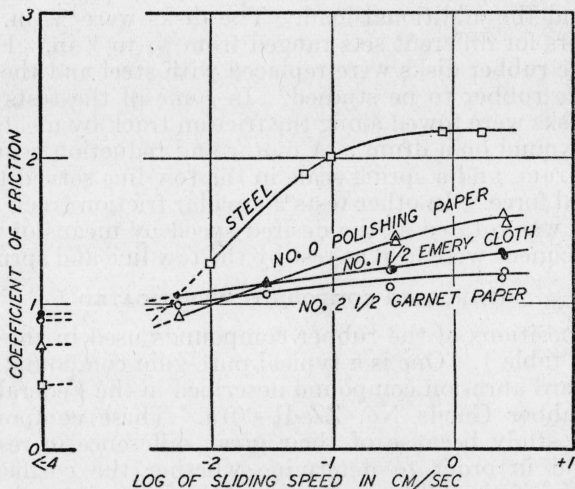


FIGURE 1.—Coefficient of friction of rubber sliding at different speeds on surfaces of varying degrees of roughness.

The composition of the rubber is that given for the standard abrasion compound in table 1.

track the data for the different points in each curve, particularly where two points occur for one speed, were obtained at various times over a period of more than a year. The values shown for the lowest speed were obtained by observing the spring scale after stopping the motor. The specimens continued to move on the track under the action of the spring for some time after the motor was stopped. Readings were taken when the specimen had apparently stopped. The normal pressures between the specimens and the track were 25 to 30 lb/in.²

Figure 2 shows coefficients of friction obtained with the pure-gum compound on the steel track. The values for the standard abrasion compound shown in figure 1 are also reproduced here for purposes of comparison. For these two compounds the increases of the coefficients with speed are similar. At speeds approaching 10 cm/sec the values of the coefficients became equal; but since the specimens vibrated at these speeds, the values were not plotted.

(d) STEEL SLIDING ON A RUBBER TRACK

For the study of steel sliding on rubber, the rubber disks were replaced by steel disks of about the same size. The track, which was circular in form, was cut from a rubber-covered conveyor belt. Por-

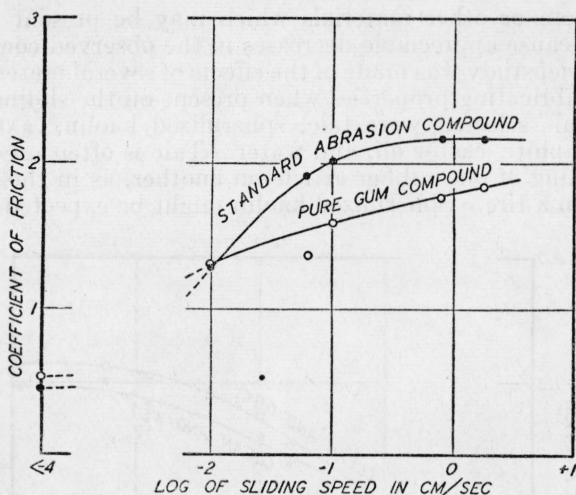


FIGURE 2.—Comparison of the coefficients of friction of the two rubber compounds described in table 1 when sliding at different speeds on a steel track.

The data for the standard abrasion compound are those shown for the steel track in figure 1.

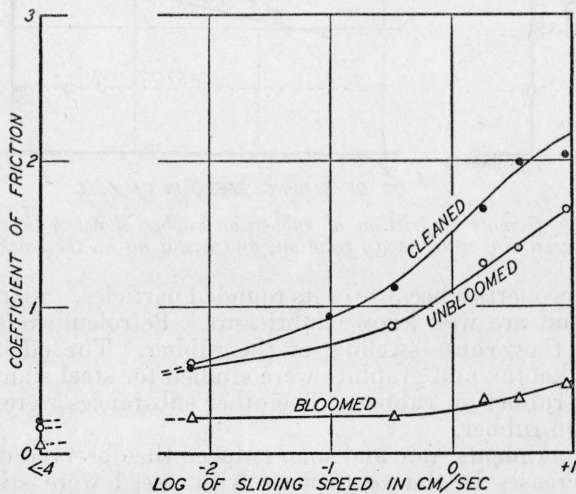


FIGURE 3.—Coefficients of friction of steel on rubber, showing the effect of the presence of bloom on the rubber surface.

tions of this belt showed a decided tendency to bloom. The curves in figure 3 show, respectively, the coefficient of friction plotted against the logarithm of the speed for the steel disks sliding on a bloomed portion of the rubber, on a portion which showed no apparent bloom, and on a portion which had been cleaned with acetone the day before

the observations were made. As in the previous experiments, the coefficients increased with speed in every case, although the presence of bloom sharply reduced the observed values.

(c) EFFECTS OF CERTAIN LUBRICANTS

Since bloom or other materials which may be present on rubber compounds cause appreciable decreases in the observed coefficients of friction, a brief study was made of the effects of several materials which may have lubricating properties when present on the sliding surfaces. The materials selected were talc, spherulized kaolin,² a thick soap solution, graphite, castor oil, and water. Talc is often used to facilitate the sliding of one rubber article on another, as in the case of an inner tube in a tire. Spherulized kaolin might be expected to possess

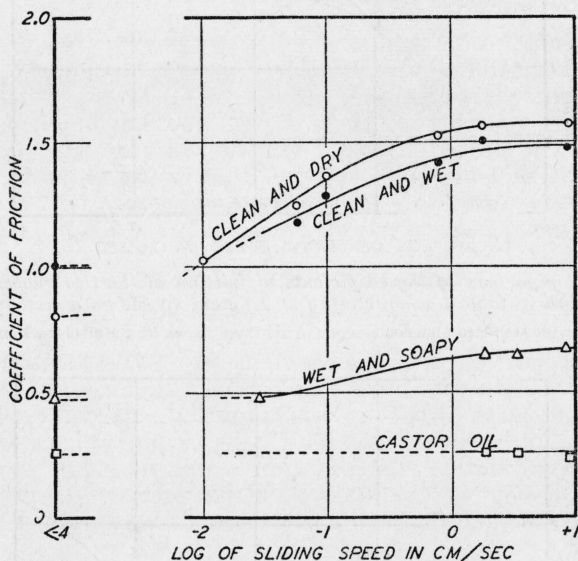


FIGURE 4.—Coefficients of friction of rubber on rubber showing the effects of the presence of water, soap solution, and castor oil on the track.

lubricating properties because of its rounded particles. Soap, graphite, and castor oil are well known lubricants. Petroleum oils were not tried, since they cause swelling of the rubber. The effects of talc, spherulized kaolin, and graphite were studied for steel sliding on rubber and for rubber on rubber. The other substances were tried only for rubber on rubber.

In our experiments, talc and soap reduced the observed coefficients, but the increases in the coefficients with speed were still evident. Spherulized kaolin reduced the coefficients to a greater extent than talc did, but slight increases of the coefficients with speed still persisted. The presence of graphite or castor oil on the sliding surfaces further reduced the coefficients, and the increases of the coefficients with speed disappeared completely.

² The spherulized kaolin was prepared by blowing kaolin powder through an oxy-hydrogen flame and collecting the rounded particles thus produced. The particles were of the order of 1 micron in diameter.

For rubber sliding on rubber, wetting the surfaces with water caused a small decrease in the coefficients for the higher speeds and a small increase as the speed approached zero. No general conclusions can be drawn from a single experiment of this kind. Observations reported later in this paper show that when rubber slides on glass, the presence of water causes a sharp decrease in the observed coefficients. Reports on the frictional characteristics of rubber sliding on glass and on roadway materials [2], of tires on roads [3], and of water-lubricated bearings [4] show that the effect of water between the sliding surfaces depends markedly on the thickness of the water film and on other factors—such as the speed, the normal pressure, the nature and smoothness of the surfaces, and the presence of foreign materials in the water film.

(f) ABRASION OF SPECIMENS ON A SMOOTH TRACK

In an attempt to determine whether any rubber was abraded from the specimens in sliding on a relatively smooth metal track, the specimens and plate to which they were cemented were weighed to 0.1 mg before and after towing them several hours at about 3 cm/sec on the track. The procedure was repeated for increasing normal pressures between the specimens and track up to the point at which the specimens were torn apart by the frictional forces. No definite loss by abrasion could be detected for pressures up to 260 lb/in.²

2. LATER EXPERIMENTS

(a) COMPOUNDS INVESTIGATED

A large proportion of the measurements of coefficients of friction were carried out on specimens made from a channel-black compound containing 49 parts of carbon black to 100 parts of crude rubber. The composition is shown in table 2. This compound is one of those used in an investigation of stress-strain properties at different rates of stretch [5]. Except for the omission of softeners or plasticizers which would facilitate manufacturing processes, it is a typical tire-tread compound.

TABLE 2.—Composition of rubber compounds

[The specimens were press-vulcanized for 15 minutes at about 142° C.]

Ingredients	Rubber compounds	
	Channel-black	Clay
	<i>Parts by weight</i>	<i>Parts by weight</i>
Rubber (smoked sheets).....	100	100
Sulfur.....	3	3
Zinc oxide.....	5	5
Stearic acid.....	2	2
Benzothiazyl disulfide (Altax).....	1	1
Tetramethylthiuram disulfide (Tuads).....	0.1	0.1
Phenyl-beta-naphthylamine.....	1.5	1.5
Channel-black (Micronex).....	49	-----
Clay (Dixide).....	-----	140
Total.....	161.6	252.6

Several specimens were also made of a compound in which the channel-black was omitted. Because of distortions of the soft rubber under frictional forces, it was impossible to obtain satisfactory results with these specimens. Specimens were then made with a body of the channel-black compound and a thin layer of the soft compound on the sliding surface. These specimens chattered at the higher speeds, and at low speeds the frictional forces repeatedly increased to a maximum and dropped to a lower value.

Specimens were also made from a compound in which 70 parts of clay filler was substituted for the channel-black. This amount of clay is equal in volume to the amount of channel-black which it replaced. These specimens jumped and chattered for speeds of 0.01 cm/sec or greater.

A number of satisfactory measurements of the coefficient of friction were obtained, however, on specimens made from a compound containing 140 parts of clay instead of the channel-black. This compound, referred to in table 2 and elsewhere as the clay compound, possesses a hardness measured by resistance to indentation which is about equal to that of the channel-black compound.

(b) APPARATUS AND PROCEDURE

(1) *General description.*—A schematic diagram of the two arrangements used in determining the coefficients of friction is shown in figure 5. The horizontal track was suspended by four wires from points about 6 feet above it, and the specimen was pulled along the track at a constant speed by means of a motor and reduction gears. The frictional force was determined by measuring the force required to hold the track at rest while the specimen was being moved. The coefficient of friction in this arrangement is the ratio of the measured force to the total weight on the specimen.

In the inclined-track arrangement the specimen was allowed to slide down the track under the action of the combined weights of the specimen carriage and the additional load which was hung from it. In this case the frictional force remained constant, and the speed of slide was measured. The coefficient of friction is equal to the tangent of the angle made by the plane of the track and the horizontal.

(2) *Specimens.*—Attempts were made to use specimens which were cut from a sheet of rubber and fitted snugly into a recess in the under side of the carriage. As these specimens slid along the track, they became shorter and thicker, leaving an open space at the leading end of the recess, and finally they buckled. This excessive distortion was eliminated by vulcanizing the rubber stock to rectangular pieces of steel $\frac{1}{8}$ in. thick which fit into the recess. The rubber in these specimens was usually about $\frac{1}{16}$ in. thick, but increasing the thickness to $\frac{1}{8}$ in. produced no noticeable difference in the coefficients of friction. Most of the specimens had dimensions of either $\frac{5}{16}$ by $1\frac{1}{8}$ in. or $\frac{3}{4}$ by $1\frac{1}{2}$ in. The direction of slide was parallel to the larger dimension, and the leading end was beveled to prevent the edge from rolling under.

The sliding surfaces of the specimens were of three types. The first, a smooth surface, was obtained by molding the rubber against plate glass. When these specimens were pressed against a glass surface, only part of their areas made actual contact with the glass.

Interference fringes were observed around the boundaries of the contact areas, and considerable force was required to lift the specimens off the glass. The coefficients of friction observed differed greatly between individual specimens and between different observations on the same specimen. Examination of the specimens after they were allowed to slide on the track showed that only parts of their surfaces

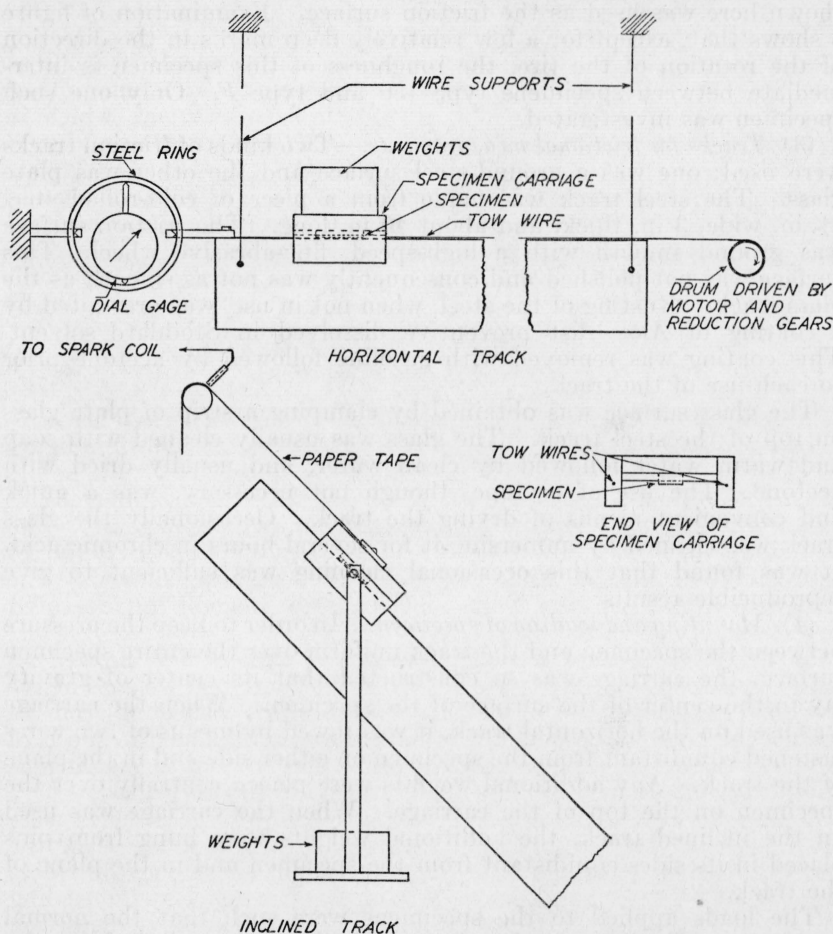


FIGURE 5.—Schematic diagram of the two arrangements used in determining the coefficients of friction.

made contact and that the proportion of each specimen which made contact varied considerably between individual specimens.

In order to obtain more uniform conditions of contact and to get the pressure between the specimen and the track distributed more uniformly over the entire surface, specimens were made by molding the rubber against plate glass which had been roughened with an abrasive. Two degrees of roughness were obtained, one by roughening the glass with carborundum flour and the other by the use of 150-mesh carborundum. Much of the work reported here was done using these two types of specimens, which are referred to as specimens

type *F*, and type 150, respectively. Photomicrographs of two specimens of type *F* and one of type 150 are shown in figure 6. The specimen represented by the photomicrograph labeled *D* was obtained by cutting a piece of rubber from the tread of a used tire and cementing it to a rigid fiber back. The rubber was about $\frac{3}{32}$ -in. thick, and care was taken to get the thickness nearly uniform. The tread surface shown here was used as the friction surface. Examination of figure 6 shows that, except for a few relatively deep marks in the direction of the rotation of the tire, the roughness of this specimen is intermediate between specimens type 150 and type *F*. Only one such specimen was investigated.

(3) *Tracks for frictional measurements.*—Two kinds of friction tracks were used; one was a ground-steel surface and the other was plate glass. The steel track was made from a piece of cold-rolled steel $1\frac{1}{2}$ in. wide, 3 in. thick, and about 40 in. long. The friction surface was ground smooth with a high-speed, fine-abrasive wheel. This surface was not polished and consequently was not as smooth as the glass track. Rusting of the steel, when not in use, was prevented by a coating of Alox rust preventive dissolved in Stoddard solvent. This coating was removed with gasoline followed by acetone prior to each use of the track.

The glass surface was obtained by clamping a strip of plate glass on top of the steel track. The glass was usually cleaned with soap and warm water followed by clean water, and usually dried with acetone. The use of acetone, though not necessary, was a quick and convenient means of drying the track. Occasionally the glass track was cleaned by immersing it for several hours in chromic acid. It was found that this occasional cleaning was sufficient to give reproducible results.

(4) *Mounting and loading of specimen.*—In order to keep the pressure between the specimen and the track uniform over the entire specimen surface, the carriage was so constructed that its center of gravity lay in the center of the surface of the specimen. When the carriage was used on the horizontal track, it was towed by means of two wires fastened equidistant from the specimen on either side and in the plane of the track. Any additional weights were placed centrally over the specimen on the top of the carriage. When the carriage was used on the inclined track, the additional weights were hung from pins placed in its sides equidistant from the specimen and in the plane of the track.

The loads applied to the specimens were such that the normal pressure on the sliding surfaces ranged from 2.5 to 15 lb/in.² for the large specimens and from 7.5 to 40 lb/in.² for the small ones. A large proportion of the work was done using 10 lb/in.² on the large specimens and 20 lb/in.² on the small ones.

(5) *Measurement of the frictional forces.*—The force to be measured in the horizontal-track arrangement was that required to keep the track from moving when the specimen was pulled. Since it was desired to measure this force with a minimum of displacement of the track, it was determined by measuring the distortion it produced when applied to a steel ring. The ring had an outside diameter of about 3.6 in., and an inside diameter of about 3.3 in., and was about 0.6 in. wide. It was fastened between the track and a fixed support, as shown in figure 5, so that the force to be measured acted along the

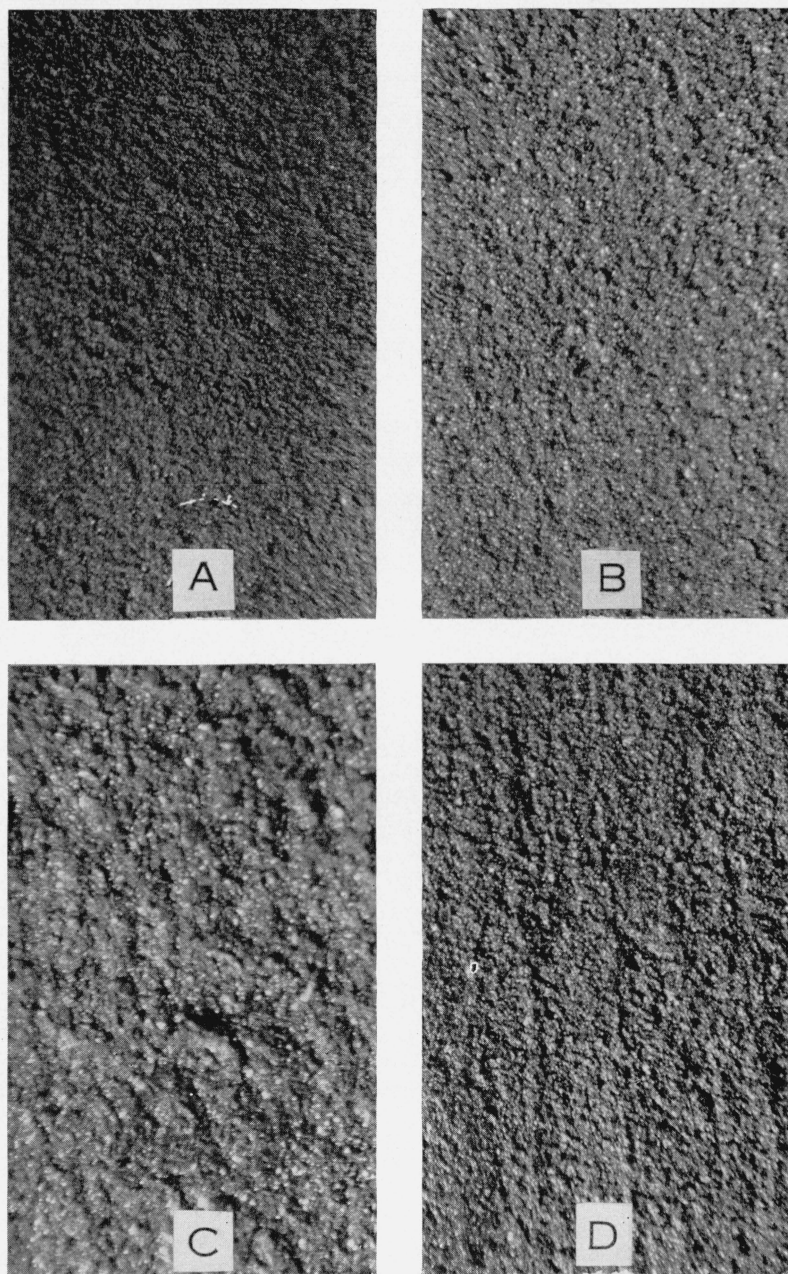


FIGURE 6.—Photomicrographs of the sliding surfaces of the rubber specimens. A and B are type F, C is type 150, and D is the specimen from a used-tire tread. Magnification 48X.

horizontal diameter. The distortion in the direction of the vertical diameter was measured by means of a sensitive dial gage graduated to ten-thousandths of an inch. The distortion shown by the gage rarely exceeded 0.008 in. (80 divisions), and calibration over this range showed that it was proportional to the force applied, the calibration factor being 305 g per division.

(6) *Sliding speeds.*—The speeds on the horizontal track ranged from 0.0001 to 7 cm/sec. Since observations became increasingly tedious as the speed was decreased, and since the coefficients at very low speeds were markedly affected by vibrations in the building, no attempt was made to extend the investigation on the horizontal track to speeds below this range. The maximum speeds, from 1 to 7 cm/sec, were limited by jumping and chattering of the specimens on the track.

In the case of the inclined track, the speed was the quantity to be measured, while the forces acting on the specimen remained constant. As a convenient means of determining the speed, a paper tape was fastened to the specimen carriage and towed through a spark gap. Sparks produced at desired intervals punctured the tape, making a permanent record of the position of the specimen as a function of time. The force required to pull the tape was negligible.

Usually the angle of incline was adjusted before the specimen was placed on the track; but when the equilibrium speed was over 0.01 cm/sec, it was often necessary to allow the specimen to begin sliding with the track at a relatively low angle of incline, and to increase the angle as the speed decreased. Determinations of the speed were not made until the desired angle of incline was reached.

III. RESULTS

1. CHANGES IN FRICTIONAL FORCES AT THE START OF SLIDE

It was noted throughout the later experiments (section II-2) that when a specimen was towed at constant speed on a horizontal track, the force of friction did not become constant until the specimen had moved several centimeters. Likewise, a specimen sliding down an inclined track moved several centimeters before it attained a constant equilibrium speed. Figure 7 shows the results of a study of these changes for typical specimens of type 150 and type *F* when sliding on a glass track. For the horizontal track the points on the graphs refer to readings of the force of friction which were taken at predetermined intervals of time. Care was taken at the start of slide to allow all the slack and stretch in the apparatus to be taken up before observations were begun. For the low speeds, readings were not taken until the specimen was allowed to move about half of a millimeter. In the case of the inclined track the position of the specimen was recorded at various intervals of time and the average speed for each interval was plotted.

These changes in the frictional characteristics of the specimens at the start of slide were first thought to be due to nonuniformities in the friction track, but this explanation was ruled out by the fact that the phenomenon occurred on any portion of the track and on all the glass tracks which were used in the tests. Also, when the specimen was stopped during its slide and was started again, it repeated the changes which were observed at the beginning of the test and approached the same equilibrium conditions of slide. Furthermore, similar results were observed for specimens sliding on the steel track.

It appears that these changes of frictional properties at the start of slide are transitions from static conditions of contact between the friction surfaces to dynamic conditions and are regarded as such in the more detailed discussions which follow, but it is not clear why the effect should extend over such a large distance of slide.

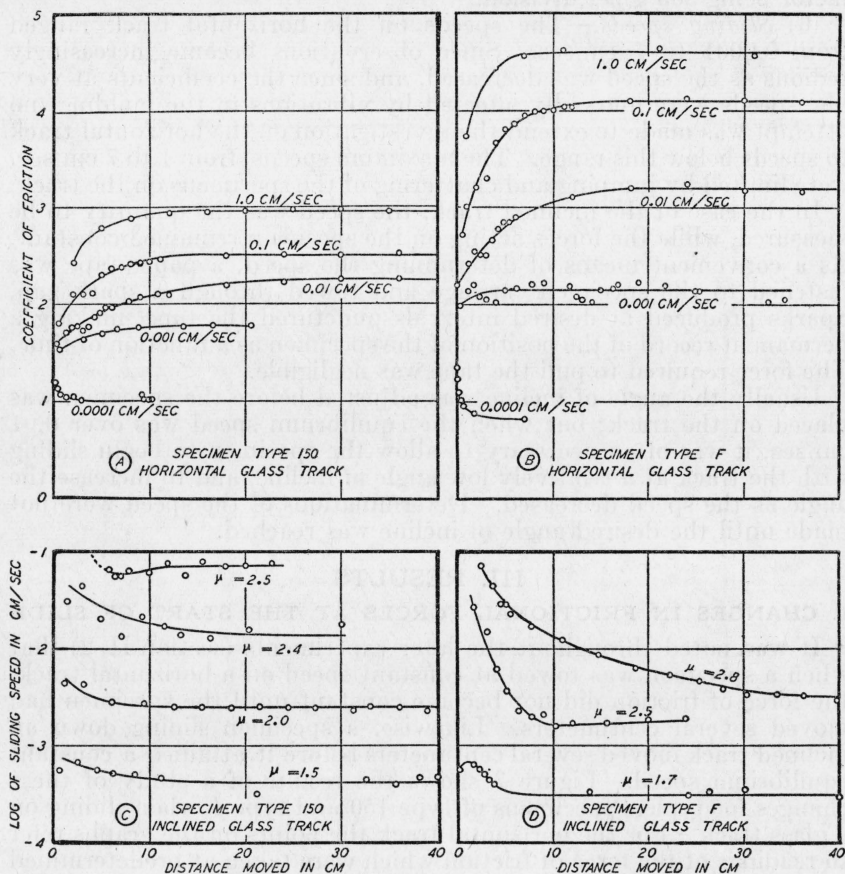


FIGURE 7.—Changes in frictional properties of rubber at the start of slide.

Results are shown for two types of surfaces on horizontal and inclined glass tracks. μ is the tangent of the angle of incline and is equal to the coefficient of friction.

2. COEFFICIENT OF STATIC FRICTION

If the curves in figure 7 (A and B) are extended back, one would expect them to intersect the zero ordinate at a point which corresponds to the coefficient of static friction. This point appears to lie somewhere between the values 1.2 and 1.5 for the specimen type 150 and between 1.5 and 2.0 for type F. Accordingly, if the specimens are placed on the inclined glass track, the maximum angle of repose should be between $\tan^{-1} 1.2$ and $\tan^{-1} 1.5$ for the specimen type 150 and between $\tan^{-1} 1.5$ and $\tan^{-1} 2.0$ for type F. It was found that the specimen type 150 always started sliding when the angle of incline was $\tan^{-1} 1.3$ or greater, seldom started when the angle was $\tan^{-1} 1.2$,

and never started for angles appreciably less than $\tan^{-1} 1.2$. The maximum angle of repose for the specimen type *F* was similarly about $\tan^{-1} 1.6$.

3. COEFFICIENT OF DYNAMIC FRICTION

If one regards the changes of the frictional forces during the first several centimeters of slide on the horizontal track as transitions from static to dynamic conditions, the relatively constant coefficient finally reached is the coefficient of dynamic friction for that particular speed. Likewise the nearly constant speed which is attained after the first several centimeters of motion down the inclined track represents the equilibrium speed for that particular angle of incline.

The data from the equilibrium conditions shown in figure 7 have been plotted in figure 8 to show the relation between the coefficient of dynamic friction and the speed, and to show the correlation between tests made on the horizontal track and those made on the inclined

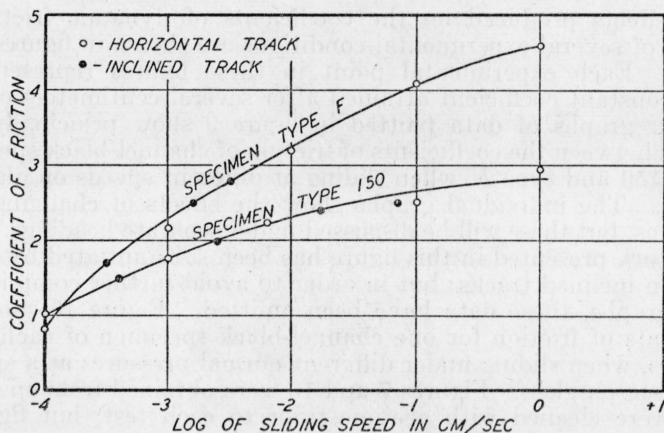


FIGURE 8.—Coefficients of friction of channel-black rubber sliding at different speeds on a glass track.

The data were taken from the equilibrium conditions of slide in figure 7 and show the correlation between tests made on the horizontal and inclined tracks.

track. This graph indicates that the coefficients obtained by use of the horizontal and inclined tracks are in agreement, that the coefficients of friction for both specimens increase as the speeds increase, and that except for the lowest speed the coefficients for the surface with the finer texture, type *F*, are greater than those for type 150. These observations have been substantiated by results from several similar specimens (see fig. 9).

4. RELATION BETWEEN STATIC AND DYNAMIC COEFFICIENTS

It is evident from figure 7 that the coefficients of dynamic friction for speeds of 10^{-3} cm/sec or more are greater than the coefficient of static friction, but it appears that at a speed of 10^{-4} cm/sec the dynamic coefficients are the smaller. This phenomenon leads one to believe that if a specimen were once sliding under nearly equilibrium conditions, it should continue to slide down a track which is inclined at an angle less than the angle of repose. Accordingly, a specimen of type

150, for which several determinations showed that the angle of repose was not less than $\tan^{-1} 1.2$, was placed on the inclined glass track and allowed to slide about 2 hours with the angle of incline at $\tan^{-1} 1.3$. The angle was then decreased by small amounts at intervals of from 1 to several hours until it reached a value of $\tan^{-1} 1.1$, where it was left for nearly 3 days. During this time the specimen slid about 12 cm, with an average and nearly constant speed of slide of 4.8×10^{-5} cm/sec. The angle was again decreased by small steps until a value of $\tan^{-1} 1.0$ was reached. With the track at this angle the specimen slid continuously for about 9 days, moving 16.5 cm at an average speed of 2.1×10^{-5} cm/sec. Thus experiments with both horizontal and inclined tracks show that at very low speeds the coefficient of dynamic friction is less than the static coefficient.

5. EFFECTS OF VARIOUS FACTORS ON THE COEFFICIENTS OF DYNAMIC FRICTION

The effects produced on the coefficients of dynamic friction by changes of several experimental conditions are shown in figures 9, 10, and 11. Each experimental point in these figures represents the nearly constant coefficient attained after several centimeters of slide. The four graphs of data plotted in figure 9 show principally comparisons between the coefficients of friction of channel-black specimens of type 150 and type *F*, when sliding at different speeds on glass and on steel. The individual graphs show the effects of changing other conditions, but these will be discussed under separate headings. Much of the work presented in this figure has been substantiated by experiments on inclined tracks; but in order to avoid further complications of the graphs, these data have been omitted. Figure 10 shows the coefficients of friction for one channel-black specimen of each of the two sizes, when sliding under different normal pressures at a speed of 0.1 cm/sec on glass. Figures 7 and 10 were obtained from specimens which were cleaned with acetone prior to each test, but figure 11 shows the effect of omitting this precaution.

(a) SLIDING SPEED

The data for coefficients of friction of rubber presented in figures 8, 9, and 11 have been plotted as a function of the speed. In every case the coefficient increases with the speed. This increase becomes relatively more pronounced for smoother and for cleaner sliding surfaces. In most cases the rate of increase of the coefficient in respect to the speed becomes less for speeds over 0.1 cm/sec.

Similar increases in the coefficients are also shown throughout the earlier experiments (section II-1), where the conditions were quite different from those in the later experiments. The only cases in which the coefficients are independent of the speed are those for which the sliding surfaces were lubricated with graphite or castor oil.

Large decreases in the coefficients of all the specimens were observed when the speed reached a value at which the specimen began to vibrate or chatter. Chattering usually occurred between 5 and 10 cm/sec for the channel-black specimens of types *F* and 150. Specimens of softer compounds and those with smoother surfaces began to chatter at lower speeds.

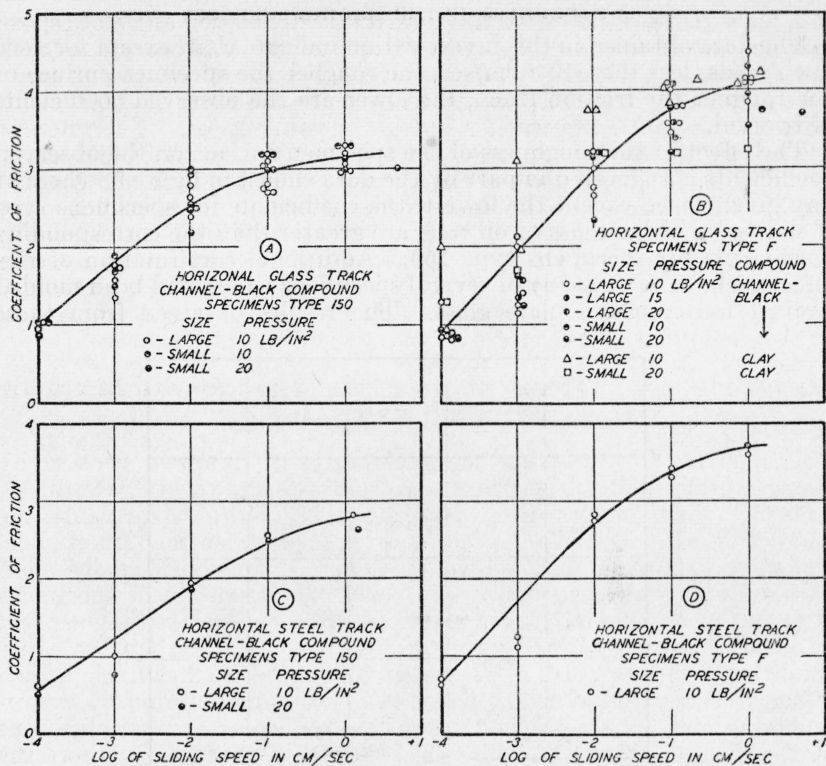


FIGURE 9.—Comparison between the coefficients of friction of channel-black specimens type 150 and type F when sliding on glass and on steel tracks.

The individual graphs show variations in the coefficients for nominally identical specimens, for specimens of different sizes, and for various pressures. The dimensions of the large and small specimens are, respectively, $\frac{3}{4}$ by $1\frac{1}{2}$ in. and $\frac{5}{16}$ by $1\frac{1}{2}$ in. Graph B also compares coefficients obtained for the channel-black and clay compounds.

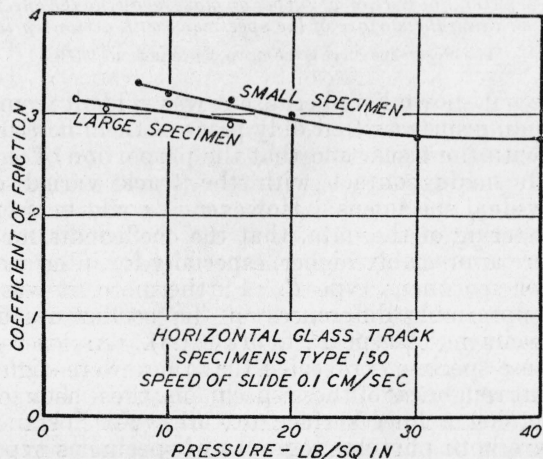


FIGURE 10.—Coefficients of friction of rubber when sliding under different normal pressures on a glass track.

(b) ROUGHNESS OF THE SLIDING SURFACES

The data obtained in this investigation indicate that, except for very low speeds, less than 10^{-3} cm/sec, the rougher the specimen surface or the rougher the friction track, the lower are the observed coefficients of friction.

The effect of the roughness of the specimen surface on the observed coefficients is suggested in part by the data shown in figure 9, where at any given speed except the lowest, the coefficients for specimens type *F* sliding on either glass or on steel are greater than the corresponding coefficients for specimens type 150. Additional confirmation of this effect is shown by a study of several specimens which had been molded against unroughened plate glass. The results obtained from these

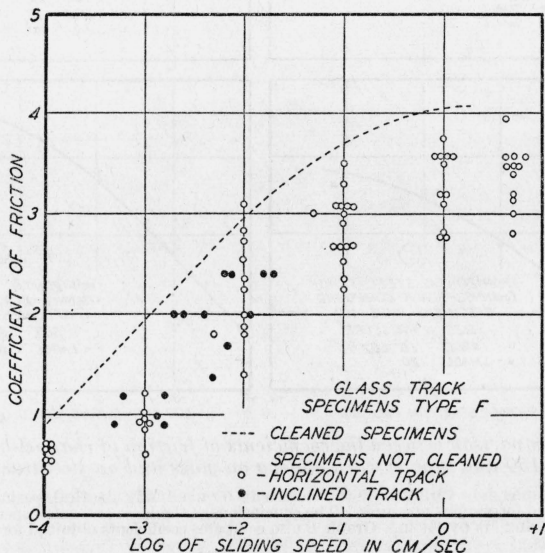


FIGURE 11.—Coefficients of friction of rubber on glass, showing the effect of omitting the precaution of cleaning the surface of the specimens with acetone prior to the test.

The broken-line curve is the curve shown in figure 9 (B).

specimens are not shown here because it was evident from the appearance of the sliding surfaces that only parts of them had actually come into contact with the track, and that the proportion of each specimen surface which made contact with the track varied considerably between individual specimens. However, it could be seen in spite of the great scattering of the data, that the coefficients for the smooth specimens were appreciably higher, especially for intermediate speeds, than those for specimens type *F*. Furthermore, it was found that there were appreciable differences in the coefficients obtained for individual specimens of type *F*, figure 9 (B). A closer examination of some of these specimens revealed that there were slight differences in the apparent roughness of these specimens, those showing the higher coefficients having a finer surface texture. For instance, in figure 6, *A* and *B* are both photomicrographs of specimens type *F*, but the surface of *A* appears to be of a finer texture than that for *B*. Correspondingly, the coefficients obtained from *A* are appreciably higher.

The effect of the roughness of the track is illustrated by the data presented in figure 1. In this work the rubber specimens were drawn over various abrasive tracks, and, except for the lowest speed, the smaller the abrasive particles the higher were the coefficients obtained. Further evidence, though not conclusive in itself, is shown by figure 9, where the coefficients for rubber on steel were found to be slightly less than the coefficients for corresponding specimens at the same speeds on glass. This difference might be due to differences in the nature of the materials, but it seems more likely to be due to the relative smoothness of the surfaces of the tracks, the glass track being somewhat smoother than the steel.

(c) SURFACE AREA AND NORMAL PRESSURE

There seems to be no relationship between the coefficients and the specimen sizes. The data in figure 9 show that differences between nominally identical specimens are of the same order as the differences between specimens of the two sizes employed. Figure 10 shows that the coefficients decrease only slightly with increasing pressure. In this figure the coefficients are plotted as a function of the normal pressure for one channel-black specimen of each of the two sizes when sliding on glass at a speed of 0.1 cm/sec. The range of pressures investigated, 2.5 to 40 lb/in.², was determined by limitations of the apparatus.

(d) VIBRATION OF THE APPARATUS

The values shown in figure 9 (*C* and *D*) for the coefficients of friction at the speed of 10^{-3} cm/sec seem to be somewhat lower than would be expected from the values at the other speeds. A comparison of these coefficients with the results obtained from the inclined track also showed some discrepancy. It was found that the motor was causing vibration in the apparatus at this speed. When this vibration was eliminated, the observed coefficients became somewhat higher and in better agreement with those obtained from the inclined track. The values shown in figures 7 and 8, and part of those shown in figure 9 (*A* and *B*) were obtained after the vibration had been eliminated.

Further indications of the effect of vibration on the observed coefficients of friction is shown by the data for the lowest speeds plotted in figure 7 (*A* and *C*). In the case of the horizontal track, figure 7 (*A*), the seventh point represents an observation taken early in the morning, before general activity in the building began. The value of this coefficient is higher than those observed later in the day and during the previous day. Likewise, the seventh point for the inclined track, figure 7 (*C*) represents the average speed during the night. This point is well below the other points, all of which represent speeds during the 2 days. Observations such as these have been made repeatedly.

(e) COMPOSITION OF THE RUBBER COMPOUND

The frictional properties of a rubber compound seem to be dependent more on the rubber matrix than on the compounding ingredients or fillers. Since cleaned specimens of very soft rubber compounds show a strong tendency to chatter or vibrate on the tracks even at relatively low speeds, only a limited amount of work was done with such compounds. Observations were made of the clay compound

described in table 2, and data for three specimens of this compound were plotted in figure 9 (B) for the purpose of comparing them with the data for channel-black specimens which had similar sliding surfaces. The surfaces of the specimens of both compounds were produced by molding them against plate glass which had been roughened with carborundum flour. The size of the clay specimens was the same as for the larger channel-black specimens, and the normal pressure was 10 lb/in.² The values of the coefficients for the clay specimens are rather widely scattered, but for any one specimen the general changes in the coefficients in respect to speed are quite similar to those obtained for the channel-black specimens.

A specimen taken from the tread of a used tire also showed frictional properties which were similar to those obtained from the prepared channel-black specimens. The surface of this specimen, shown in figure 6 (D), had some rather high and low regions, but the general surface texture seemed to be not far different from some of the prepared specimens. Because the cement bond between the rubber and the rigid back for this specimen was not so strong as the bond between the rubber and the steel on the other specimens, the pressure was limited to about 6 lb/in.². The values of the coefficients of dynamic friction obtained on the horizontal glass track for this specimen, after cleaning it with acetone, were 1.3, 1.7, 2.1, 2.4, 3.0, and 3.4, respectively, for speeds of 0.0001, 0.001, 0.01, 0.1, 1.0, and 5 cm/sec. These values are in approximate agreement with those given in figure 9 (A) for specimens of type 150. The coefficients observed before cleaning ranged from 0.7 at 0.01 cm/sec to 1.1 at 1.0 cm/sec. The frictional forces for the clean specimen showed the usual increase in value during the first several centimeters of slide at speeds over 0.001 cm/sec. and a decrease in value during the first several millimeters of slide at 0.0001 cm/sec. Chattering of the specimen occurred during part of the sliding at speeds of 1 and 5 cm/sec.

Another comparison between coefficients of friction of two widely differing compounds is shown in figure 2. Because of the differences in the experimental procedures, the actual values of the coefficients in this figure cannot be compared to those obtained in the later experiments (section II-2), but it can be seen that the coefficients for these two compounds do not differ greatly over the range of speeds investigated.

(f) CLEANING THE SPECIMENS

The specimens which were used for obtaining the data shown in figures 7 to 10 were cleaned with acetone prior to each test. This procedure made possible more uniform results than could be obtained otherwise, and the actual values observed for the coefficients were not greatly different from those obtained when the specimens were tested immediately after removing them from the molds. Cleaning the specimens also permitted using them repeatedly with no observable change in frictional properties, whereas uncleaned specimens showed progressively lower coefficients as blooming progressed. Consequently, by cleaning the specimens it was possible to determine the equilibrium coefficients of dynamic friction of the same specimen for various conditions of slide (different speeds, normal pressure, horizontal and inclined tracks, etc.) instead of using a new specimen for each test and introducing unavoidable differences in surface roughness.

As an illustration of the effects of omitting the precaution of cleaning the specimens, a number of coefficients of dynamic friction for uncleaned specimens are plotted as a function of the logarithm of the speed in figure 11. The specimens for this work were left in the mold about 2 days after vulcanizing and were placed on the friction apparatus as soon after removing them from the mold as possible. This procedure kept the specimens relatively clean and free from bloom, though probably not entirely so. As a means of comparing these values with those obtained from specimens which were cleaned with acetone, the curve from figure 9 (*B*) is shown as a broken line. The data for both graphs were obtained from specimens with the same type of surface, namely type *F*. Figure 11 shows that the values of the coefficients for the uncleaned specimens scatter rather widely, but approach the values of the cleaned specimens as an upper limit. Similar observations have been made for specimens type 150.

(g) MOISTURE

Although the relative humidities in the laboratory ranged from 10 to 80 percent during the investigation of the coefficients of friction of rubber on steel and rubber on glass, there was no observable correlation between the coefficients observed and the relative humidity. In the case of rubber sliding on glass, the presence of visible water on the track or the specimen resulted in a sharp decrease in the coefficients observed. When the track was submerged in water, the values of the coefficients decreased to about 4 or 5 percent of the values obtained for dry surfaces. Coefficients obtained after wetting the track and the specimen with water and allowing it to evaporate until all the visible water disappeared were not observably different from coefficients obtained after drying the surfaces with acetone. No attempts were made to remove adsorbed water from the glass surface.

The effects of water on the coefficient of friction of rubber on steel were not studied, because of the possibility of changes in the nature of the steel surface due to corrosion.

6. SUMMARY OF RESULTS

The experimental results obtained from this investigation may be briefly summarized as follows:

1. Dynamic friction increases with speed up to a certain critical speed at which the specimens chatter. The dynamic friction is less than static friction only at speeds less than about 10^{-3} cm/sec.
2. Clean rubber specimens slide several centimeters before the friction becomes constant.
3. Except for very low speeds, the rougher the surfaces the lower are the observed coefficients.
4. The coefficient of friction of a rubber compound seems to be dependent more on the rubber matrix than on the compounding ingredients and fillers.

IV. DISCUSSION

The four generalizations listed in the foregoing summary of results are discussed in order here under separate headings.

1. SLIDING SPEED

The increase of friction with speed for speeds ranging from 10^{-4} to 10 cm/sec was evident throughout the investigation. At the lowest speeds the values of the coefficients were seldom much over unity and were usually somewhat less. For speeds greater than the order of about 10^{-3} cm/sec. the dynamic friction became greater than the static friction. For the highest speeds at which the specimens slid smoothly, the coefficients reached values ranging from somewhat over 2 to more than 4, depending on the cleanliness and smoothness of the sliding surfaces. The limiting speed of smooth sliding seemed to be determined by the properties of the specimen, being nearly 10 cm/sec for the channel-black compound and less for softer compounds. The effect of the geometry of the specimen on this limiting speed was not studied, although there was no observable difference between the two sizes of specimens employed in the investigation.

(a) CHANGE OF FRICTION WITH SPEED

Observations of coefficients of static friction which were less than dynamic coefficients and observations of dynamic friction which increased with speed have been reported at various times for rubber and for other materials. However, the reports do not include observations of such high values for the coefficients as have been found in the present investigation on rubber. That the static friction for rubber was less than dynamic friction was reported by W. S. James in 1924 [6], and by J. B. Derieux in 1934 [7], for sections of tires sliding on various surfaces. Derieux also reported that the friction increases with speed. R. Ariano [8] found that when a rubber belt was allowed to slip over a pulley, the coefficient of friction increased with the speed of slip. In an investigation of the frictional properties of small rubber blocks sliding on wet and on dry roadway materials, Papenhuyzen [2] also showed coefficients of friction which increased with the speed.

A large proportion of the investigations on the frictional properties of rubber has dealt with the skidding of tires on roads. In many of the earlier investigations the tire was dragged on the road and the forces for starting the motion and for maintaining it at some constant speed were used in determining, respectively, the coefficients of static and dynamic friction. Sliding speeds of less than 3 mph (about 135 cm/sec) were seldom studied. It can be seen that if the coefficient increases sharply with speed and reaches a maximum value at a speed much lower than the one investigated, this maximum coefficient would be reached before the observer was aware that the specimen was moving and consequently would appear to the observer to be the coefficient of static friction. Thus one may readily account for the many observations of static coefficients which were greater than the dynamic coefficients. In more recent investigations [3, 9, 10] the coefficients were determined for various amounts of slip as the tire rolls along the road. In general, when braking forces or accelerating forces are applied, the tires begin to slip, so that the speed of rotation is not equal to the speed of a free-rolling wheel.

When a braking force is applied to the wheel, the amount of this slippage increases with the force until the wheel locks and slides. Moyer [3] showed that for a speed of 20 mph on wet concrete, the coefficients of friction increased sharply with increasing percentage of

slip, reaching a maximum value when the slip reached about 18 percent of the vehicle velocity. The coefficient of friction reached 90 percent of its maximum value at about 5-percent slip. A slip of 5 percent at 20 mph corresponds to a sliding speed of 45 cm/sec. The coefficients decreased as the slip increased above 18 percent. Schmid [9] showed similar results for various types of roads by means of autographic recording of the braking force and the angular speed of the wheel. At 4 kilometers per hour (about 2.5 mph) the coefficient of friction rose sharply with increasing slip up to 10 percent of the speed of the vehicle and then rose less rapidly as the slip increased to about 20 percent. Beyond 20-percent slippage the wheel often locked and the coefficient decreased. Because of the tendency for the wheel to lock, difficulty was encountered in determining coefficients for slippages above the point at which the coefficient reached a maximum value. Similar increases of friction with increasing slip have been obtained for various types of roads at the National Physical Laboratory [10].

(b) VIBRATIONS

It is well known that when rubber slides on a solid surface, as in the case of a tire on the road, a distinctly audible vibration often occurs. In most of the investigations on frictional properties of tires, or of other rubber products, such vibrations seem not to have been given much attention. The coefficient of friction is usually regarded as the ratio of the average towing force to the load irrespective of the condition of slide. Papenhuyzen [2], however, does study such vibrations for rubber sliding on glass and for rubber sliding on roadway materials. Employing an apparatus in which one end of a cylindrical rubber specimen was held coaxially against the face of a rotating disk of glass or of roadway material, he found that vibrations occurred at a critical speed which was greater for the roadway material than for glass. Below this critical speed, sliding was smooth, that is, no vibration occurred. Examination of the vibratory motion at speeds immediately above the critical speed by means of motion pictures showed that the cycle of vibration consisted of two parts, one in which the specimen was distorted and the other in which the specimen "broke free"³ and quickly receded to a position of much less distortion. The time required for the specimen to reach its maximum distortion depended on the speed of the disk and the amount the specimen slipped during the time of the distortion. This slippage was small, but observable. The time of relaxation depended largely on the physical characteristics of the specimen.

Vibrations of this type are known as mechanical relaxation oscillations [11]. Analogous oscillations occur in certain electric circuits. Mechanical relaxation oscillations are observed in oscillating systems of small inertia, where the restoring force increases with the displacement. An extensive study of such oscillations has been made of the bowed violin string [12]. Oscillations of this type have been explained [11, 13, 14] as being due to a decrease in friction with increasing speed. The decrease in friction with increasing speed was not actually observed in the present investigation, but figures 1, 2, 4, and 9 indicate that the friction approached a maximum at the speed at which chat-

³ In our investigation with tracks which were inclined at relatively large angles, specimens often "broke free" from a condition of uniform speed and slid down the rest of the track with greatly accelerated motion.

tering occurred. Thus, it is reasonable to suppose that the chatter was accompanied by a decrease in friction beyond this maximum and that the chattering was a manifestation of relaxation oscillations.

Mechanical relaxation oscillations have been observed for materials other than rubber, metals in particular, but only with measuring apparatus which possesses certain dynamical properties. Kaidanovsky and Haykin [11] studied such oscillations for a lubricated cylindrical journal bearing and stated that in order to obtain such oscillations it was necessary that the dynamic friction decrease with speed. Haykin, Lissovsky, and Solomonsky [15] describe these experiments in English and account for the "stick-slip" phenomenon of Bowden and Leben [16], in which no lubricant was employed, as a manifestation of such oscillations. Extensive studies of mechanical relaxation oscillations for both lubricated and unlubricated metals have been recently reported by Morgan, Muskat, and Reed [17].

In a theoretical discussion of relaxation oscillations, H. Blok [13] points out that these oscillations are not necessarily audible, but that they may be either subsonic or ultrasonic, depending on the characteristics of the sliding system. He also shows that for certain materials and under certain conditions the amplitude of the vibrations may be very small, and he calls such oscillations "micro-vibrations."

2. CHANGES IN FRICTION AT THE START OF SLIDE

It was shown in figure 7 that when a rubber specimen was towed on the friction track, the coefficient of friction changed during the first several centimeters of slide. These changes were apparently from a value which may be regarded as the static coefficient to a constant equilibrium value, which may be regarded as the coefficient of dynamic friction for that particular speed. For speeds greater than 10^{-3} cm/sec the coefficient increased from the initial value, and at speeds appreciably less than 10^{-3} cm/sec the coefficient decreased. This change in friction may be connected with the peculiar properties of thin films of water which may be present on the sliding surfaces. Derjaguin [18] has shown that thin films of water possess definite and measurable shear moduli and yield points. A further confirmation of the rigid properties of thin films of water has recently been reported by Eversole and Lahr [19]. An explanation of this property of the film supposes the existence of long chains of oriented water molecules extending from the solid surfaces into the interior of the liquid. Papenhuyzen [2] supposes that the presence of such films on the sliding surfaces would account for values of friction which are higher than would otherwise be observed. In addition he supposes that the orientation of the molecules and the formation of the chains would be enhanced by sliding of the surfaces, and that a certain time would be required for the formation of these chains. This idea is similar to that presented by Hardy and Doubleday [20] in explanation of the time required to reach equilibrium conditions of slide when certain oils are employed as boundary lubricants.

If one were to apply this idea to the data in figure 7, one would arrive at a chain formation time of about 10 seconds for the speed of 1.0 cm/sec, 100 seconds of 0.1 cm/sec, and so on. At the speed of 10^{-4} cm/sec, however, the chains do not seem to be formed, since the friction decreases to its equilibrium value.

3. ROUGHNESS OF THE SLIDING SURFACES

That the higher coefficients are obtained with the smoother surfaces is in contradiction to frictional characteristics for many commonly observed sliding conditions. This conclusion, however, applies only to the range of speeds from about 10^{-3} cm/sec to speeds at which the specimens vibrate. Below 10^{-3} cm/sec it appears that the rougher surfaces show the higher coefficients. It will be noted that the speed of about 10^{-3} cm/sec is also the critical speed above which the coefficients at the start of slide increase, and below which they decrease.

Since measurements of the area of contact could not be made without considerable difficulty, no attempt was made to correlate the various coefficients with the actual contact area of the specimens. The finer surfaces present more points of contact than do the coarser surfaces, but this fact does not indicate the relative contact areas.

4. COMPOSITION OF THE RUBBER COMPOUND

Since rubber is the external phase in a rubber-filler system, one might expect that the filler should not greatly affect the frictional properties of the compound. Changes in other physical properties, modulus of rigidity for example, would necessarily affect the critical speed at which vibrations in the specimen occur. The inclusion of waxes, oils, or other materials in the compound which come to the surface would be likely to affect the coefficients greatly, and were avoided as far as possible in this work. Cleaning the specimens prevented lubrication due to blooming of the ingredients which were employed.

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