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INTERCOMPARISON OF PLATINUM RESISTANCE THERMOMETERS BETWEEN −190° AND 445° C

By Harold J. Hoge and Ferdinand G. Brickwedde

ABSTRACT

Eight platinum resistance thermometers satisfying the requirements of the International Temperature Scale were calibrated on the International scale and intercompared from −190° to 445° C. The δ's of the Callendar-Van Dusen equations for seven of these thermometers ranged from 1.49375 to 1.49862. The δ for the eighth thermometer, *E*, was 1.51155. After minimizing the effects of relative calibration errors, the maximum difference between the readings of the group of seven thermometers was 7 millidegrees between −190° and 0° C and 1.$_3$ millidegrees between 0° and 100° C. Maximum differences from the mean were 4.$_0$ and 0.$_8$ millidegree, respectively. For thermometer *E*, deviations from the mean of the other seven were 15 millidegrees at −110° C and 3 millidegrees at +50° C. Tables of differences between readings of platinum resistance thermometers arising from assumed calibration errors at the fixed points were calculated and are included.

CONTENTS

I. INTRODUCTION

The International Temperature Scale below 660° C is defined in terms of a few fixed points and by interpolation formulas for strain-free, high-purity platinum resistance thermometers [1].[1] The platinum resistance thermometer was chosen as the basic instrument because of its precision, convenience, and reproducibility. The scale was made to agree, as closely as was possible at the time of its adoption, with the thermodynamic scale by selection of the fixed points and the temperatures assigned to them and by selection of the interpolation instrument and formulas. Exact agreement of the two scales, though desirable, is not necessary, provided the International scale is accurately reproducible, because then the relation between the two scales can be determined at any time within the limits of reproducibility.

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[1] Figures in brackets indicate the literature references at the end of this paper.

The reproducibility of a temperature scale depends upon the agreement of the values obtained when different thermometers are used to measure the same temperature. If groups of thermometers are used, and if the mean reading of one group is compared with those of other groups, greater reproducibility is to be expected than when individual thermometers are compared. There are a number of factors that may influence the reading of a thermometer. Besides those factors associated with the calibrating and measuring apparatus and technique, there are others associated with the intrinsic properties of the thermometers themselves. The intrinsic differences in thermometers that result from variations in materials, construction, or annealing are limited by specifications for the thermometers.

The accumulated experience with resistance thermometers indicates that the reproducibility attainable in repeated measurements with an individual instrument is very satisfactory. A considerable amount of evidence in the literature indicates that the differences between various thermometers, while small, are consistent enough to show that they are real. The information on this point in the range below 0° C is somewhat scanty, consisting of that obtained in comparisons of resistance thermometers with gas thermometers [2, 3]. There is little direct evidence concerning the differences which may exist between groups of thermometers.

Direct evidence concerning the reproducibility of the International scale can be obtained by a systematic intercomparison of a group of thermometers used to maintain the scale. If such a comparison should show satisfactory agreement among all the thermometers, nothing more definite concerning reproducibility would be established than the probability that other similar thermometers would also show satisfactory agreement. If the agreement should not be as good as desired, it would indicate that a stricter definition of the scale might be advantageous, and the data obtained might or might not indicate in what respects the definition could be advantageously revised.

Eight thermometers, each satisfying the present requirements for the International Temperature Scale, were used for the intercomparisons reported here. The thermometers were made in three institutions, using wire from at least three manufacturers. All the thermometers were of the same type, consisting of a coiled helix supported on a mica frame. Obviously, any conclusions concerning reproducibility will be more limited than those which might have been drawn if thermometers of other types had been included. However, if the thermometers were really strain-free, as is indicated by their constancy of calibration after exposure to temperatures from 445° to −190° C, it is not to be expected that the behavior of other strain-free thermometers would differ appreciably from these, even though the manner of supporting the wire were different. Moreover, the thermometers used included not only a number which might be considered typical but also some selected from the available group so as to include the widest range of characteristics consistent with the requirements of the International Temperature Scale adopted in 1927.

The treatment of the data differs in one important respect from that followed in most of the previous work in resistance thermometry at this Bureau. It was found that the constancy of the thermometers and the accuracy of the resistance measurements were such that the resistance of each thermometer could be considered as depending only on its temperature, at least over the limited period (less than 1 year) during which the calibrations and intercomparisons were made. With this procedure, the resistance at 0° C, R_0, for example, was treated as a constant rather than as a quantity subject to small variations from time to time. If the measurements had extended over a longer period of time or had been made in the summer months, when the humidity is likely to be high, this procedure might not have been possible, but in the present instance it appears to be justified by the results obtained. With this method, a further refinement in handling the data was possible. Making use of the intercomparisons at or near the calibration points, the calibration data were adjusted so as to bring the thermometers into as good agreement as possible at the calibration points. Using these adjusted calibration data for calculating temperatures at intermediate points, the differences found between the various thermometers would be those inherent in the instruments themselves, and would be almost independent of calibration errors.

II. APPARATUS AND PROCEDURE

The experimental work consisted of two parts: (a) The calibration of each individual thermometer on the International Temperature Scale, and (b) the intercomparison of the calibrated thermometers. Part (a) was carried out in the prescribed manner [1]. Part (b) was carried out in two different comparators, one covering the range from the sulfur point down to room temperature, and the other covering the range below 100° C. In the region of overlap, data were taken with both comparators, which served as a check on their satisfactory operation.

The high-temperature comparator consisted of a vertical furnace containing an aluminum block with wells in which the thermometers were placed. Outside this block were alternate layers of insulating material and aluminum, to promote temperature uniformity in the block. Above the comparison block and separated from it was a guard block through which the thermometers passed, so that their total depth of immersion was considerably greater than the minimum specified by the International Temperature Scale for sulfur-point calibrations. The furnace was controlled to better than 1° C by an automatic regulator, the precise regulation of the block being accomplished manually by means of an auxiliary heater on the comparison block. The guard block also carried a heater, by means of which its temperature was maintained approximately equal to that of the comparison block.

The low-temperature comparator, in which the larger part of the measurements were made, was designed for enclosure in a Dewar flask. It consisted of a copper comparison block with wells in which the thermometers were placed, surrounded by a heavy copper thermal shield. The block carried one heater and the shield two, by means of which their temperatures could be held constant through manual regulation of the heating currents. The block and shield were supported by a copper-nickel tube, which passed upward axially through the source of refrigeration. The latter was a deep container for liquid air or liquid hydrogen, located in the upper part of the Dewar flask, several inches above the comparison block. Oxygen or hydrogen gas could be admitted to the axial copper-nickel tube. This gas condensed in the part of the tube surrounded by the refrigerant, ran down the tube, and evaporated in contact with the thermal shield of the block. By varying the pressure of the gas, the rate of removal of heat from the block could be controlled.

The resistance thermometers were of the same type as those used previously [4], each consisting of a coiled helix supported on a mica frame. Each thermometer was sealed into a helium-filled cylindrical platinum case (5 by 50 mm) by a soft-glass seal, through which the leads passed. In the low-temperature comparator the space between the platinum cases and the thermometer wells was filled with low-melting solder. For high-temperature work and for all calibrations at fixed points, these platinum-encased thermometers were placed at the bottom of long glass protecting tubes (1 by 60 cm). These tubes contained several mica baffles or plugs of glass wool to reduce convection in the section above the thermometer and were filled with helium. Constants of the resistance thermometers are given in tables 1 and 2.

In taking readings, the same general procedure was followed with both comparators. At a given temperature, approximate values of the resistances of all the thermometers were obtained while time was being allowed for the disappearance of temperature gradients and the reaching of a satisfactorily steady state. Thermometer *P*, which had been chosen as the primary thermometer, was then read alternately with the others until each of these had been read twice. The thermometer current was in all cases 1 milliampere. When the rate of drift was negligible, every third reading was made on the "primary" thermometer; otherwise every second reading was on the primary, including of course the first and last readings in either case. When the primary-thermometer readings before and after were not identical, a linear change of resistance with time was assumed in computing simultaneous resistances of a given thermometer and the primary. The rate of drift was kept as near zero as possible and seldom exceeded 2 millidegrees between successive readings of the primary thermometer. In the case of thermometers *F* and *G*, which had been previously calibrated, no measurements were made above 100° C. With the other thermometers, the high-temperature calibrations and intercomparisons were in general made first, so that any changes caused by the high temperatures would not affect the accuracy of the low-temperature work.

The four leads to each resistance thermometer enter the platinum case through a glass seal. Above 300° C this glass became slightly conducting, as evidenced by a slight initial kick of the galvanometer, followed by a drift back toward a position of equilibrium. A separate investigation of this effect showed that by taking proper precautions the error introduced by it could be made negligible.[2]

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[2] These precautions are based on the rapid reduction in conductance that results from the polarization of the glass during the first few minutes after the electromotive force is applied. A preliminary report of the investigation was given at a meeting of the American Physical Society [5] and a more complete report is being prepared for publication.

III. ADJUSTMENT OF THE DATA

The original data obtained when the various resistance thermometers were calibrated at the sulfur, steam, ice, and oxygen points are given in table 1. The values in italics were obtained by averaging the original determinations. The constants α, δ, and β were computed from the italicized values in the ordinary way, each thermometer being entirely independent of the rest. Using these calibration constants, the simultaneous resistance values obtained by intercomparison were converted to simultaneous temperature indications. Representing the temperature indication of the ith thermometer by t_i and that of the particular thermometer chosen as the primary thermometer by t_P, the differences, t_i-t_P, were computed. The results of this reduction are shown in figure 1. To avoid confusion, the experimental points were omitted in preparing this figure for publication. However, the disposition of points relative to the curves which represent them is the same for both figure 1 and figure 2, and may be consulted in the latter figure, where each curve is plotted on separate axes.

It is apparent from figure 1 that, with this method of computation, the results obtained with thermometers *F* and *G* at −110° *C* differed by about 10 millidegrees from the mean of the others, and that thermometer *E* differed by somewhat more in the opposite direction. Since the curves do not go through zero at the calibration points, it is apparent that the comparison data near these points are in some cases not in good agreement with the calibration data. The notable discrepancies are those for thermometers *F* and *G* at the ice and oxygen points, *F* at the steam point, and *P* (base line) at the sulfur point. The question is immediately raised as to how the curves would be altered if the effects of these discrepancies, which arise from the relative calibration errors of the thermometers, could be eliminated. The constants in table 1 were obtained by the independent calibration of each thermometer at the four fixed points, while as a result of the intercomparisons any one of the thermometers could be used to calibrate all the rest. Under these conditions the calibration constants given in table 1 may be improved upon by combining all the available data by the method of least squares. Thus through the medium of the intercomparisons, each individual calibration datum has a share in determining the constants of all the thermometers in the group.

The method of adjusting the calibration data will be explained briefly. We represent the reading of the ith thermometer, when the constants of table 1 are used, by t_i. This quantity has the dimensions of temperature, but because of the possibility of calibration errors it is not to be confused with the true temperature. We take the steam-point adjustment as an example, and represent by $(t_i)_{100}$ the reading of the ith thermometer when the true temperature is exactly 100° C. Then the original steam-point calibration of each thermometer yields an equation of the form

$$(t_i)_{100}=100.$$

There is no inconsistency in the eight equations of this type and no improvement in the calibrations can be made without additional data.

TABLE 1.—*Calibration data for the eight thermometers intercompared*

[Original data are in ordinary type and averages in italics. Values of α, δ, and β were computed from the italicized data in the ordinary way, each thermometer being entirely independent of the rest.]

Thermometer	R_0		R_{100}		R_S		R_{O_2}		α	δ	β
A	Nov. 2, 1940	23.24346	Oct. 24, 1940	32.32875	Oct. 23, 1940	61.55375	Nov. 7, 1940	5.74009			
	Nov. 4, 1940	46	Oct. 31, 1940	51	Oct. 29, 1940	228	do	23			
	Nov. 8, 1940	46	Nov. 4, 1940	65	Oct. 30, 1940	335	Nov. 8, 1940	46			
Average		*23.24346*		*32.32864*		*61.55313*		*5.74026*	0.003908704	1.49651	0.11183
B	Nov. 2, 1940	28.66101	Oct. 24, 1940	39.90011	Oct. 23, 1940	76.05181	Nov. 7, 1940	7.00648			
	Nov. 4, 1940	01	Nov. 1, 1940	11	Oct. 29, 1940	018	do	25			
	Nov. 8, 1940	01	Nov. 4, 1940	31	Oct. 30, 1940	176	Nov. 8, 1940	45			
	Jan. 8, 1941	10									
Average		*28.66103*		*39.90018*		*76.05125*		*7.00639*	0.003921405	1.49776	0.11232
C	Nov. 2, 1940	27.49012	Oct. 24, 1940	38.24775	Oct. 23, 1940	72.84892	Nov. 7, 1940	6.76148			
	Nov. 4, 1940	17	Nov. 1, 1940	802	Oct. 29, 1940	796	do	34			
	Nov. 8, 1940	12	Nov. 4, 1940	772	Oct. 31, 1940	818	Nov. 8, 1940	81			
	Jan. 8, 1941	21									
Average		*27.49016*		*38.24783*		*72.84835*		*6.76154*	0.003913280	1.49888	0.11287
D	Nov. 2, 1940	25.75597	Oct. 24, 1940	35.86399	Oct. 23, 1940	68.38110	Nov. 7, 1940	6.28402			
	Nov. 4, 1940	97	Nov. 1, 1940	425	Oct. 29, 1940	07	do	51			
	Nov. 8, 1940	99	Nov. 4, 1940	409	Oct. 30, 1940	43	Nov. 8, 1940	69			
	Dec. 27, 1940	98									
Average		*25.75598*		*35.86411*		*68.38120*		*6.28441*	0.003924576	1.49518	0.11086
E	Nov. 2, 1940	22.63930	Oct. 24, 1940	31.49937	Oct. 18, 1940	59.97864	Nov. 7, 1940	5.54790			
	Nov. 4, 1940	30	Oct. 31, 1940	03	Oct. 29, 1940	762	do	801			
	Nov. 8, 1940	30	Nov. 4, 1940	08	Oct. 30, 1940	814	Nov. 8, 1940	821			
	Dec. 27, 1940	31									
Average		*22.63930*		*31.49916*		*59.97813*		*5.54804*	0.003913487	1.51178	0.12170
F	Jan. 19, 1935	27.54605	Jan. 10, 1935	38.33267	Jan. 9, 1935	73.03269	May 21, 1935	6.77095			
	May 17, 1935	587	Jan. 18, 1935	262	Jan. 17, 1935	19	do	121			
			Mar. 26, 1935	301			May 22, 1935	112			
			Mar. 27, 1935	260			do	227			
Average		*27.54596*		*38.33272*		*73.03244*		*6.77139*	0.003915914	1.49547	0.10849

G	Jan. 19, 1935	27.52759	Jan. 10, 1935	38.30709	Jan. 9, 1935	72.98538	May 21, 1935	6.76698			
	May 17, 1935	46	Jan. 18, 1935	54	Jan. 17, 1935	19	do	656			
			Mar. 26, 1935	27			May 22, 1935	621			
			Mar. 27, 1935	12			do	767			
Average		*27.52752*		*38.30726*		*72.98528*		*6.76686*	0.003915987	1.49493	0.10844
P	Nov. 2, 1940	28.91754	Oct. 24, 1940	40.25732	Oct. 23, 1940	76.73164	Nov. 7, 1940	7.07050			
	Nov. 4, 1940	54	Nov. 1, 1940	20	Oct. 29, 1940	042	do	6977			
	Nov. 8, 1940	61	Nov. 4, 1940	26	Oct. 30, 1940	158	Nov. 8, 1940	6975			
	Dec. 27, 1940	66									
	Jan. 8, 1941	67									
Average		*28.91760*		*40.25726*		*76.73121*		*7.07001*	0.003921370	1.49798	0.11189

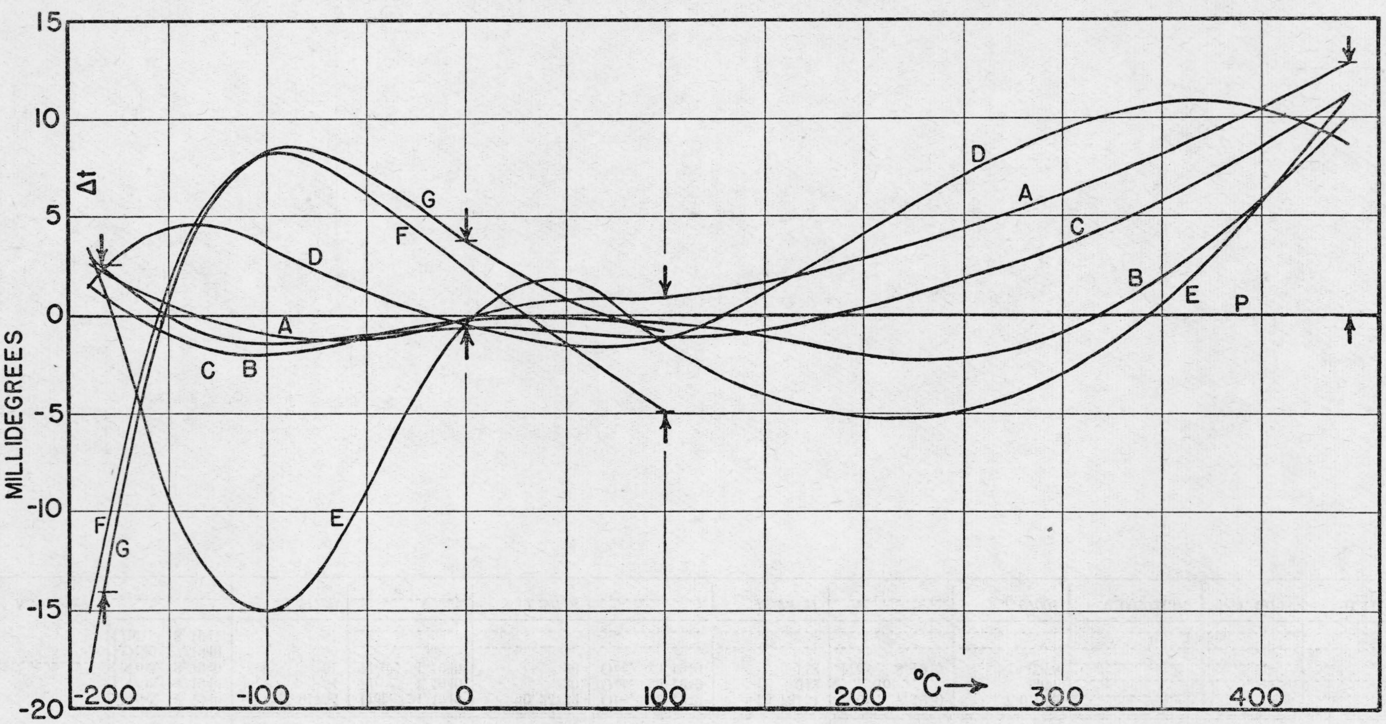

FIGURE 1.—*Deviations, $t_i - t_P$, of the readings of other thermometers from the reading of the primary thermometer.*

Temperatures computed with the unadjusted calibration constants.

We have additional data, however, in the difference curves of figure 1. By reading off the ordinates of the various curves at 100° C, we obtain seven equations of the form

$$(t_i - t_P)_{100} = a_i,$$

where a_i is the value read from the curve. The 15 equations contain 8 unknowns, and the least-squares solution yields the most probable value of the reading of each thermometer when the true temperature (as determined by the entire group) is exactly 100° C. For thermometer D, for example, it was found that $(t_D)_{100} = 99.9997$. Its calibration constants were therefore adjusted, so that in the neighborhood of the steam point the reading of the thermometer would be 0.0003 higher than before. Similar adjustments for all the thermometers were made at all the fixed points except where data were not available (F and G at the sulfur point). The adjusted calibration data are given in table 2. The adjusted value of α may of course be obtained immediately from the value of R_{100}/R_0.

TABLE 2.—*Adjusted calibration data*

[The figures in this table were obtained by utilizing both the data of table 1 and the results of the intercomparisons]

Thermometer	R_0	R_{100}/R_0	R_S/R_0	R_{O_2}/R_0	δ	β
A	23.24347	1.390875	2.648203	0.246967	1.49663	0.11160
B	28.66103	2140	53475	4454	1.49767	1240
C	27.49014	1326	49986	5965	1.49862	1297
D	25.75597	2457	54969	4001	1.49505	1087
E	22.63929	1346	49295	5063	1.51155	2183
F	27.54626	1560	51265	5760	1.49375	1080
G	27.52797	1576	51314	5749	1.49405	1042
P	28.91764	2138	53415	4480	1.49853	1183

The calibration data for thermometers F and G at the steam, ice, and oxygen points were given zero weight, so that in effect this adjustment was equivalent to recalibrating F and G at these points in terms of the calibration data for the other six thermometers. The reasons for considering the calibrations of these thermometers less accurate than the rest are given in section IV of this paper.

The original constants in table 1 had now served their purpose and were not further used. The intercomparison data were next recomputed with the adjusted constants and the differences, $t_i - t_P$, again found and plotted. The results are given in figure 2.

If the elimination of the effect of calibration errors had been the only purpose to be served by the adjustment of the data, the desired result could have been achieved by simpler methods—for example, by merely disregarding the original calibration data on all thermometers but one. However, this method of adjustment was used primarily to secure the most reliable calibration of each individual instrument for use as a thermometer, and this adjustment automatically eliminated the differences at the fixed points that are of concern in the present work.

The magnitude of the adjustments at the various calibration points is shown in table 3. In the neighborhood of each calibration point, temperatures computed with the unadjusted constants are greater

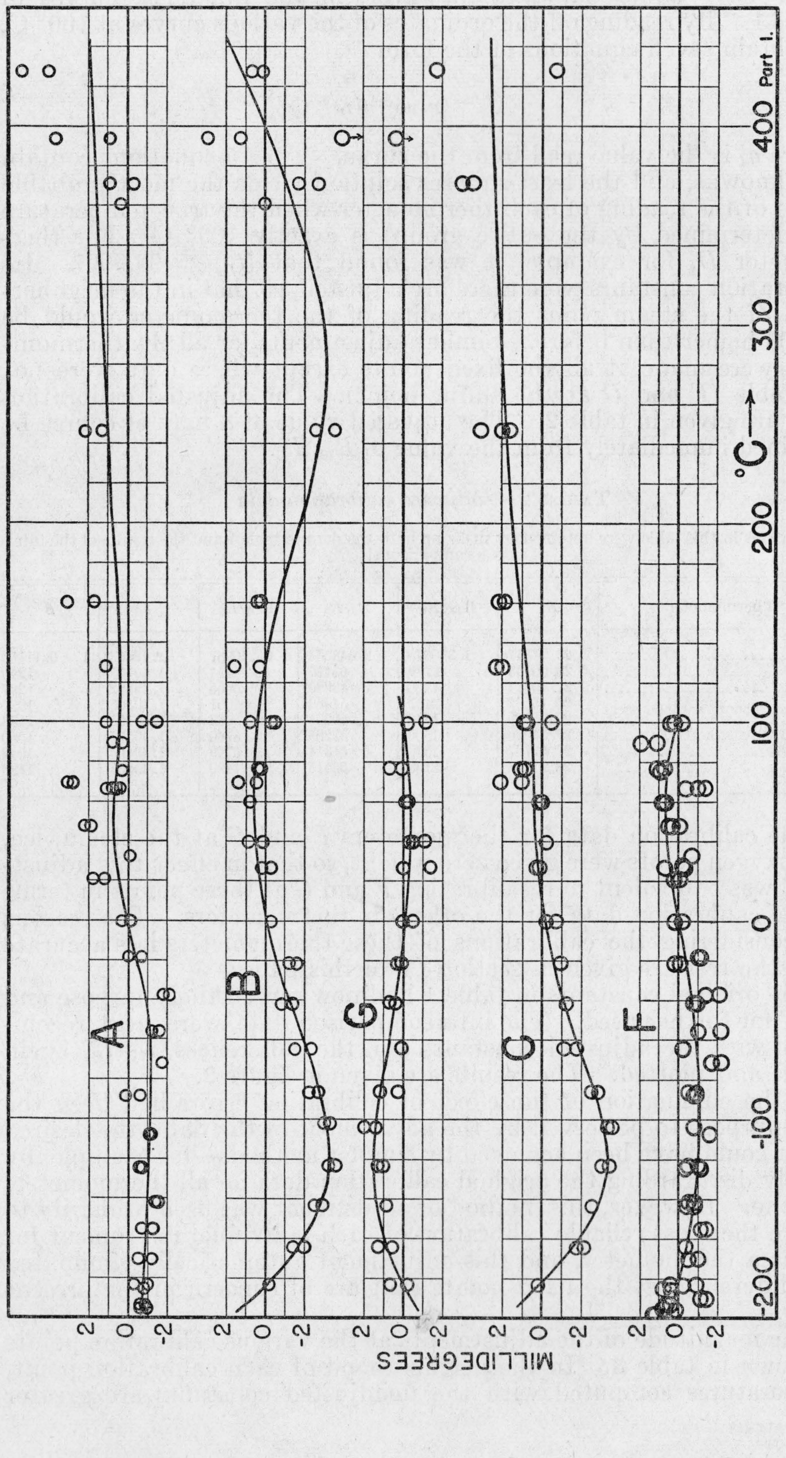

FIGURE 2.

(See also part 2, below.)

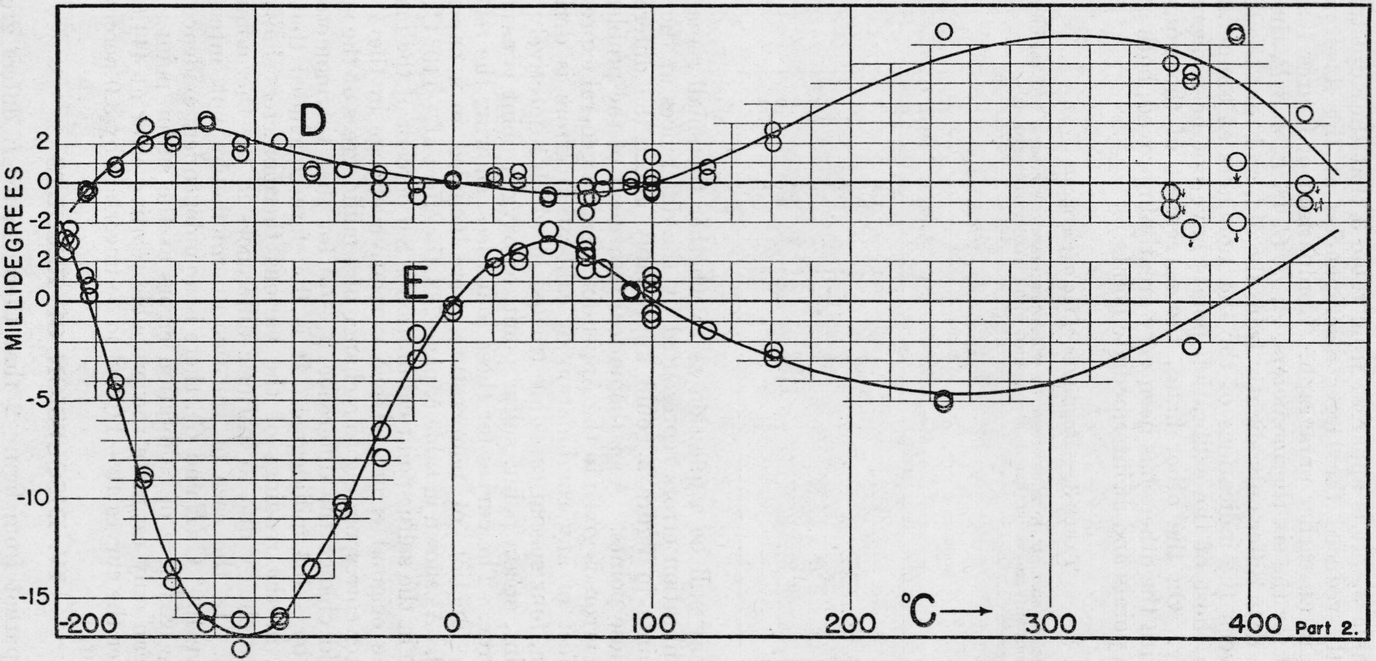

FIGURE 2 (Continued).—*Deviations, $t_i - t_P$, of the readings of other thermometers from the readings of the primary thermometer.*

Temperatures computed with the adjusted calibration constants.

than those computed with the adjusted constants by the amounts given in table 3. It is apparent from table 3 that the adjustments have greatly reduced the larger discrepancies at the fixed points mentioned in an earlier paragraph. Comparison of figures 1 and 2 shows that for the six thermometers, *A, B, C, D, E,* and *P*, the new calibration data differ only slightly from the old, and that the differences between the indications of the thermometers at temperatures other than those of the calibration points have not been decreased appreciably. On the other hand, the agreement of thermometers *F* and *G* with the others has been very greatly improved, both at the calibration points and at intermediate points.

TABLE 3.—*Magnitude of the adjustments*

[This table gives the amounts, in millidegrees, by which temperatures computed with the unadjusted constants are greater than those computed with the adjusted constants]

Thermometer	*t*(unadjusted) −*t*(adjusted) at—			
	−182.97° C	0° C	100° C	444.60° C
A	1.1	0.1	1.3	3.7
B	−0.7	.0	−0.1	0.8
C	.6	−.2	−.8	1.5
D	.6	−.1	−.3	1.1
E	.1	−.1	−.8	0.4
F	−13.6	2.7	−4.2	.0
G	−15.8	4.1	0.0	.0
P	−1.8	0.3	.8	−7.1

There may still be a question as to whether the small remaining relative calibration errors represented by the differences at the calibration points in figure 2 could appreciably affect the differences between these points. A mathematical treatment of the problem of calibration errors is given in the Appendix, where a general expression for the effect of any combination of calibration errors is derived. In addition, four special cases are treated in which respectively only the ice point, steam point, sulfur point, or oxygen point is assumed to be in error. The curves for these four cases, showing the relative error as a function of temperature, were included in a paper by Mueller [6]. As shown in table 5, page 238, the effect at −110° C of an error made in the sulfur-point calibration is 8.0 percent of the magnitude of the original error. Hence a relative change in the δ of a thermometer corresponding to a change of 4 millidegrees at the sulfur point would change the difference $(t_i - t_P)$ for that thermometer at −110° C by +0.3 millidegree. It is therefore concluded that the differences in the readings of the various thermometers shown by figure 2 are chiefly due to intrinsic differences in the thermometers themselves. In the case of the two thermometers not intercompared above 100° C (*E* and *G*), there is no independent evidence concerning the existence of calibration errors at the sulfur point. For these, we can only estimate the probable relative error at 444.60° C and consider the curves at −110° C to be uncertain by 8.0 percent of this amount.

IV. DISCUSSION OF RESULTS

It is apparent from figure 2 that thermometer *E* differs greatly from the other seven thermometers, and that conclusions as to the reproducibility of the International Temperature Scale, especially below 0° C, by means of a single thermometer will depend largely on whether or not this thermometer is included. Examination of the data shows that thermometer E has a fairly high fundamental coefficient, α, but that the value of δ is unusually large, especially for such a value of α. It has been pointed out [6] that the limitation on R_S/R_0 as given in the International Temperature Scale does not constitute an effective limitation on δ, and that this might well be replaced by the requirement that $(R_S-R_0)/(R_{100}-R_0)$ shall be between 4.216 and 4.218. Since the temperature assigned to the sulfur point is 444.60° C, this corresponds to limiting δ to the range 1.4882 to 1.5012. Such a requirement would definitely exclude thermometer E. While the existence of such a thermometer and the fact that it complies with the present requirements of the International Temperature Scale cannot be disregarded, it appears legitimate to state the conclusions that would result if thermometer E were excluded from the group, as well as those resulting if it is retained.

If thermometer E is included, it appears that two thermometers meeting the requirements of the International Temperature Scale may differ by about 20 millidegrees near −110° C, whereas if E is excluded, the maximum difference below 0° C falls to about 7 millidegrees, and the difference between the indication of any one thermometer and the mean of the group is not more than about 4 millidegrees.

The results show that if a reproducibility considerably better than 10 millidegrees is required, it is necessary to exclude thermometers like E. This could be done very simply by setting limits such as those mentioned above for the value of δ. From the nature of the case, it is not possible to prove that such a restriction would also be sufficient.

The comparison of the thermometers, after minimizing relative calibration errors (fig. 2), reveals differences in the temperature scales arising from inherent differences of the thermometers. Comparison of the curves of thermometers F and G in figures 1 and 2 shows that relative calibration errors can be of greater importance than the inherent differences of the thermometers. These thermometers were calibrated 5 years earlier than the other six thermometers of the group investigated, and by a different person using a different Mueller bridge. Also, the apparatus used in making the earlier oxygen-point calibrations was not of the same type as that now in use [7]. It was to be expected that the relative calibration errors of these thermometers, on the one hand, and the other six thermometers of the group, on the other, would be larger than the relative calibration errors of the members of the group of six thermometers. We do not believe that the observed differences are due to changes in the thermometers, but attribute them to the factors above mentioned. It is thought, however, that the differences are too large to be typical of calibrations made at different times and with different equipment.

It is instructive to consider the differences that may arise from assumed calibration errors. As reasonable limits for calibration errors at the ice and steam points, we may assume 1 and 3 millidegrees, respectively. Table 5, page 238, shows that the differences between the scales arising from these assumed errors are 5.4 millidegrees at −110° C and 1.2 millidegrees at +50° if the errors are of opposite sign, and 1.6 millidegrees at −110° C and 2.1 millidegrees at +50°

if they are of the same sign. These do not include the smaller differences that arise from calibration errors at the oxygen and sulfur points. The figures indicate that with present technique and apparatus relative calibration errors may give rise to differences in scales nearly as large as those arising from inherent differences in the seven thermometers investigated, whose δ's lie between 1.4936 and 1.4986. Differences of 0.01° at $-110°$ and a few thousandths at $+50°$ may be expected occasionally from not unreasonably large calibration errors.

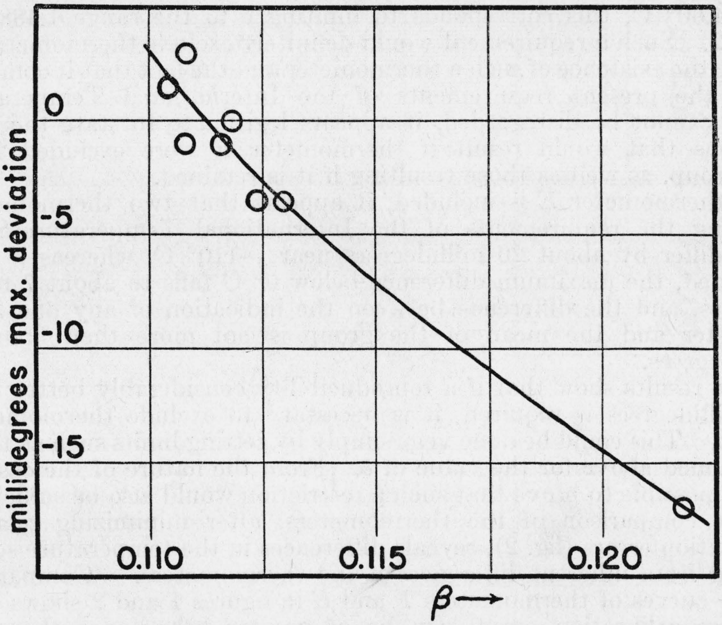

FIGURE 3.—*Maximum deviation* ($t_i - t_P$) *between* $-182.97°$ *and* 0° C, *versus the calibration constant,* β.

Each resistance thermometer gives one experimental point.

The use of the same bridge for the calibration of a thermometer and for temperature measurements is recommended for obtaining the best reproducibility of a temperature scale. However, the use of the same bridge is not always feasible. In this connection, it should be realized that resistance coils of bridges and potentiometers are subject to seasonal as well as secular changes, and should be calibrated as frequently as is necessary to obtain the desired accuracy of resistance measurement.

The maximum values of $t_i - t_P$ were read from the curves in figure 2 and recorded in table 4. To exhibit any relationships that might be present in the data, figures 3 and 4 were prepared. In figure 3 the values in the second column of table 4 are plotted against the values of β. Even without the point representing thermometer E there is some indication of a relationship, but the differences involved are small.

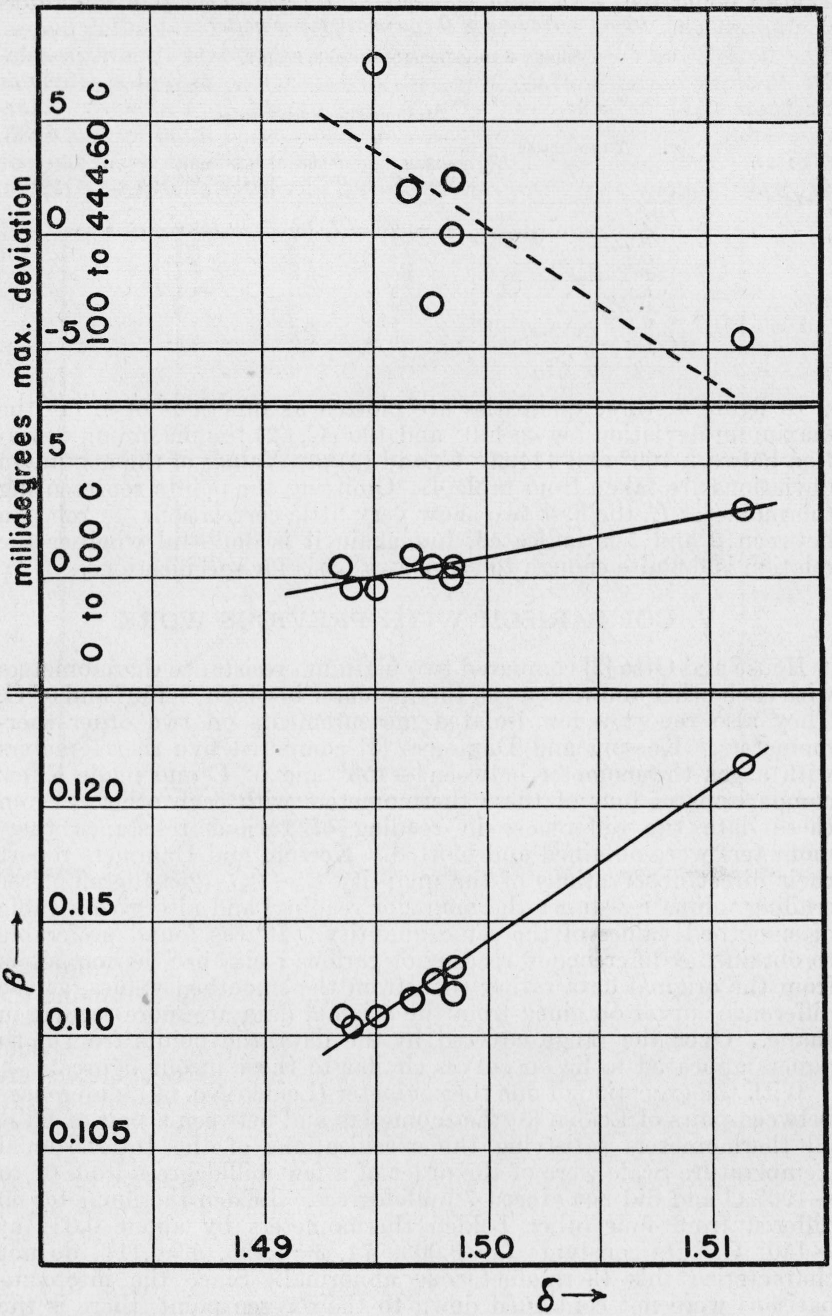

FIGURE 4.—*Top, maximum deviation* ($t_i - t_P$) *between 100° and 444.60° C; middle, maximum deviation,* $t_i - t_P$, *between 0° and 100° C; bottom, calibration constant,* β.

The abscissa for the three curves is the calibration constant, δ.

TABLE 4.—*Maximum deviation in millidegrees of the reading of a given thermometer from the reading of the primary thermometer*

Values were read from the curves in figure 2.

Thermometer	Maximum deviation between—		
	−182.97° and 0° C	0° and 100° C	100° and 444.60° C
A	−1.0	1.0	2.0
B	−3.5	0.5	−3.0
C	−3.5	.5	2.5
D	3.0	−0.5	7.5
E	−17.0	3.0	−4.5
F	−1.0	.5	------------
G	1.5	−0.5	------------
P	0.0	0.0	0.0

In figure 4, three quantities are plotted as functions of δ: (1) the maximum deviation between 0° and 100° C, (2) the maximum deviation between 100° and 444.60° C, and (3) β. Values of the maximum deviations are taken from table 4. Omitting the points representing thermometer *E*, the first two show very little correlation. A relation between β and δ is indicated, but again it is doubtful whether the relation is definite enough to serve as a basis for specifications.

V. COMPARISON WITH PREVIOUS WORK

Heuse and Otto [2] compared two platinum resistance thermometers with each other and with a gas thermometer between −130° and 0° C. They also report a few isolated measurements on two other thermometers. Keesom and Dammers [3] compared five thermometers with a gas thermometer between −153° and 0° C and made a few comparisons of four of these thermometers with each other. From these data the differences in reading of various resistance thermometers were obtained and plotted. Keesom and Dammers report their direct observations of the quantity $t_{A\alpha}-t_{Int.}$ (gas thermometer reading minus resistance-thermometer reading) and also give a table of smoothed values of the same quantity. It was found preferable to obtain the difference in reading of various resistance thermometers from the original data rather than from the smoothed values, as the difference curves obtained from the original data are more regular in shape. Over the range covered by the data, the points from both papers appeared to lie on curves similar to those in our figure 1.

With the exception of one thermometer (Leiden No. 69), differences between pairs of Leiden [3] thermometers and between a pair of PTR [2] thermometers satisfying the specifications of the International Temperature Scale were of the order of a few millidegrees from 0° to −190° C and did not exceed 7 millidegrees. Leiden thermometer 69 differed from four other Leiden thermometers by about 0.03° at −140° C. Its constants ($\alpha=0.0039074$, $\delta=1.495$, $\beta=0.111$) do not characterize this thermometer as abnormal. Since the intercomparisons were not continued down to the oxygen point, there is the possibility that if they had been continued, an appreciable difference at this point might have been found.

Confirmation for the relation between β and the maximum deviation found for the eight thermometers of the present investigation was looked for in the results of the comparisons made by Keesom and Dammers and by Heuse and Otto. Although there was some confirmation, there were almost as many exceptions as agreements. How many of the exceptions it would be possible to attribute to calibration errors, it is difficult to say.

Blaisdell and Kaye [8] determined the temperature of the mercury boiling point (356.57° C.) with 12 platinum resistance thermometers, with δ ranging from 1.492 to 1.496. The temperatures obtained had a spread of 10.7 millidegrees, the thermometers with the highest δ's, in general yielding the lowest temperatures. They state, however, that not all of the variation in reading can be attributed to differences in δ. This dependence of the thermometer reading on δ above 0° is consistent with the dependence on β at low temperatures found in the present work and indicates that thermometers with large values of δ give values that are too low between $-182.97°$ and 0° C, too high between 0° and 100°, too low again between 100° and 444.60°, and presumably too high between 444.60° and 660°.

In addition to the above-mentioned papers, there are others which give some relevant information, often limited to a single fixed point whose value has been determined with more than one thermometer. A number of these were investigated by plotting for each set of observations deviations from the mean temperature indication against deviations from the mean δ or β, a process which takes full advantage of the internal consistency of the data. The scattering of these data was in general so great that they could not be said to confirm or disprove the results in the present work. This may mean, as Blaisdell and Kaye have already pointed out for the dependence of the mercury boiling temperature on δ, that the relation between thermometer readings and δ or β depends upon the kinds of impurities in the platinum wire and the construction of the thermometer.

VI. CONCLUSIONS

1. After minimizing relative calibration errors, differences between the temperatures determined with seven of the eight platinum resistance thermometers investigated, having δ's ranging from 1.49375 to 1.49862, were of the order of a few thousandths of a degree between 0° and $-190°$ C. and were in no case greater than 7 millidegrees. Maximum differences from the mean of the group were not over 4 millidegrees. The eighth thermometer, E, though it satisfied the requirements of the International Temperature Scale, had an unusually large δ (for its α) and deviated 15 millidegrees at $-110°$ C. from the mean of the other seven thermometers. These results indicate that the reproducibility of the International Temperature Scale below 0° C. can be considerably improved by setting limits to the admissible values of δ, which is equivalent to setting limits on $(R_S-R_0)/(R_{100}-R_0)$.

2. Between 0° and 100° C. deviations from the mean were less than 1 millidegree except for thermometer E, which, at 50° C. deviated by about 3 millidegrees from the mean of the other seven.

3. It might be said that for the seven normal thermometers of the group investigated, differences between the scales of thermometers arising from errors in calibrations carefully made are a little smaller

than the differences between the scales attributable to inherent differences in thermometers themselves. Not unreasonably large calibration errors, however, may lead to differences between the scales of these thermometers that are larger than differences arising from inherent differences in the thermometers.

4. In general, it does not seem that the agreement between thermometers can be substantially improved by setting limits for β as well as δ. Such limits might be useful in excluding unusually large calibration errors or thermometers having unusual characteristics not found in those used in this intercomparison.

5. Since there are good reasons for excluding thermometer E, there is only one thermometer, Leiden 69, which differs by more than 0.01° from others with which it was compared. On the whole, it seems a reasonable expectation that very few thermometers, if carefully calibrated according to the International Temperature Scale, will differ from each other by more than 0.01° C below 0° C.

The authors are pleased to acknowledge the valuable assistance received from E. F. Mueller in the preparation of this paper.

VII. APPENDIX. CALIBRATION ERRORS

Temperatures on the International Temperature Scale are computed from the Callender-Van Dusen equation, which may be written in either of the forms:

$$R/R_0=1+At+Bt^2+C(t-100)t^3 \tag{1}$$

or

$$t=\frac{1}{\alpha}\left(\frac{R}{R_0}-1\right)+\delta\left(\frac{t}{100}-1\right)\frac{t}{100}+\beta\left(\frac{t}{100}-1\right)\left(\frac{t}{100}\right)^3 \tag{2}$$

the terms containing C and β being retained only below the ice point. Although the discussion which follows is based on the latter form, the final results apply equally well to both, since they are mathematically equivalent.[3]

If we write eq 2 in the form

$$\phi=0=t-\frac{1}{\alpha}\left(\frac{R}{R_0}-1\right)-\delta\left(\frac{t}{100}-1\right)\frac{t}{100}-\beta\left(\frac{t}{100}-1\right)\left(\frac{t}{100}\right)^3 \tag{3}$$

and compute $\Delta\phi$ by partial differentiation, we obtain, since $\Delta\phi=0$, a relation which may be used to compute the error Δt resulting from the errors $\Delta\alpha$, ΔR, ΔR_0, $\Delta\delta$, and $\Delta\beta$. The usefulness of this expression is strictly limited, however, by the fact that the errors involved are not all independent errors of observation.[4]

The directly observed and independent quantities from which the calibration constants are computed are R_0, R_{100}, R_S, and R_{O_2}, and an error equation containing these quantities rather than α, δ, and β is required. The subscripts 0, 100, S, and O_2 refer to values at exactly 0°, 100°, 444.60°, and −182.97° C, respectively. In the discussion which follows, it will be assumed, as may be done without loss of generality, that the temperature is always known exactly and that only meas-

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[3] It is easily shown that the constants of the two equations are related as follows:

$$\begin{array}{ll} A=\alpha(1+10^{-2}\delta) & \alpha=A+100B \\ B=-10^{-4}\alpha\delta & \delta=-\dfrac{10^4B}{A+100B}=-\dfrac{10^4B}{\alpha} \\ C=-10^{-8}\alpha\beta & \beta=-\dfrac{10^8C}{A+100B}=-\dfrac{10^8C}{\alpha} \end{array}$$

[4] Beattie, Benedict, Blaisdell, and Kaminsky (reference [9] p. 358–360, [10] p. 373–374, and [11] p. 385) have published tables of values of the errors in temperature caused by errors in α, δ, etc. Because they do not take into account the relationship between the various calibration constants, these tables are inapplicable to the types of error which are most likely to occur in practice. For example, suppose that the sulfur-point resistance is correctly determined but is used with an incorrect α to obtain δ. Then δ will be in error by an amount just sufficient to compensate at 444.60° C for the error in α, so that in the neighborhood of the sulfur point the total calibration error will be zero.

ured resistance is subject to error. It is convenient to make the replacement of α, δ, and β in two steps, introducing first the "platinum temperature" defined by

$$p \equiv \frac{1}{\alpha}\left(\frac{R}{R_0}-1\right) \tag{4}$$

and finally expressing all platinum temperatures in terms of resistances. Substituting eq. 4 in eq. 3,

$$\phi = 0 = t - p - \delta(\tau-1)\tau - \beta(\tau-1)\tau^3, \tag{5}$$

where, for convenience, $t/100 \equiv \tau$ has been written.

The conditions by which δ and β are determined are $\phi_S = 0$ and $\phi_{O_2} = 0$, or

$$\begin{aligned} 0 &= t_S - p_S - \delta(\tau_S - 1)\tau_S \\ 0 &= t_{O_2} - p_{O_2} - \delta(\tau_{O_2} - 1)\tau_{O_2} - \beta(\tau_{O_2} - 1)\tau_{O_2}^3. \end{aligned}$$

Eliminating δ and β between these equations and eq 5, ϕ may be put in the symmetrical form

$$\phi = 0 = t - p - (t_S - p_S)\frac{(\tau-1)\tau}{(\tau_S-1)\tau_S}\left(1 - \frac{\tau^2}{\tau_{O_2}^2}\right) - (t_{O_2} - p_{O_2})\frac{(\tau-1)\tau^3}{(\tau_{O_2}-1)\tau_{O_2}^3}. \tag{6}$$

In terms of these variables, we have, since $\Delta\phi = 0$,

$$\frac{\partial\phi}{\partial t}\Delta t + \frac{\partial\phi}{\partial p}\Delta p + \frac{\partial\phi}{\partial p_S}\Delta p_S + \frac{\partial\phi}{\partial p_{O_2}}\Delta p_{O_2} = 0,$$

or

$$\frac{\partial\phi}{\partial t}\Delta t = \Delta p - \frac{(\tau-1)\tau}{(\tau_S-1)\tau_S}\left(1 - \frac{\tau^2}{\tau_{O_2}^2}\right)\Delta p_S - \frac{(\tau-1)\tau^3}{(\tau_{O_2}-1)\tau_{O_2}^3}\Delta p_{O_2}. \tag{7}$$

In eq 4 we replace $1/\alpha$ by its value in terms of resistances, obtaining

$$\begin{aligned} p &\equiv \frac{100R_0}{R_{100}-R_0}\,\frac{R-R_0}{R_0} \\ &\equiv 100\frac{R-R_0}{R_{100}-R_0} \end{aligned}$$

In terms of resistance intervals, writing $R - R_0 \equiv I$ and $R_{100} - R_0 \equiv F$ (the fundamental interval)

$$p = 100\frac{I}{F}$$

whence

$$\Delta p = 100\frac{\Delta I}{F} - p\frac{\Delta F}{F}.$$

Substituting into eq 7, we obtain

$$\begin{aligned} \frac{\partial\phi}{\partial t}\Delta t = {} & 100\frac{\Delta I}{F} - p\frac{\Delta F}{F} \\ & - \left(100\frac{\Delta I_S}{F} - p_S\frac{\Delta F}{F}\right)\frac{(\tau-1)\tau}{(\tau_S-1)\tau_S}\left(1-\frac{\tau^2}{\tau_{O_2}^2}\right) \\ & - \left(100\frac{\Delta I_{O_2}}{F} - p_{O_2}\frac{\Delta F}{F}\right)\frac{(\tau-1)\tau^3}{{}_2-1)\tau_{O_2}^3}. \end{aligned}$$

The terms in p, p_S, and p_{O_2} may be replaced by similar terms in t, utilizing the symmetrical equation, 6, giving

$$\frac{\partial\phi}{\partial t}\Delta t=100\frac{\Delta I}{F}-t\frac{\Delta F}{F}$$

$$-\left(100\frac{\Delta I_S}{F}-t_S\frac{\Delta F}{F}\right)\frac{(\tau-1)\tau}{(\tau_S-1)\tau_S}\left(1-\frac{\tau^2}{\tau_{O_2}^2}\right)$$

$$-\left(100\frac{\Delta I_{O_2}}{F}-t_{O_2}\frac{\Delta F}{F}\right)\frac{(\tau-1)\tau^3}{(\tau_{O_2}-1)\tau_{O_2}^3}. \tag{8}$$

The partial derivative, $\partial\phi/\partial t$, has a simple physical significance. Written in full to show what variables are held constant, it is

$$\left(\frac{\partial\phi}{\partial t}\right)_{p,\ p_S,\ p_{O2}}.$$

If we solve eq 6 for p and consider p as a function of t, p_S, and p_{O_2}, and calculate $\partial p/\partial t$, we find that

$$\left(\frac{\partial\phi}{\partial t}\right)_{p,\ p_S,\ p_{O2}}=\left(\frac{\partial p}{\partial t}\right)_{p_S,\ p_{O2}}.$$

It is also easily shown, by using eq. 5 and 6, that

$$\left(\frac{\partial p}{\partial t}\right)_{p_S,\ p_{O_2}}=\left(\frac{\partial p}{\partial t}\right)_{\delta,\beta}$$

which is simply the rate of change of platinum temperature with ordinary temperature. Substituting this, eq 8 becomes

$$\left(\frac{\partial p}{\partial t}\right)_{\delta,\beta}\Delta t=\frac{100}{F}\left[\Delta I-\tau\Delta F-(\Delta I_S-\tau_S\Delta F)\frac{(\tau-1)\tau}{(\tau_S-1)\tau_S}\left(1-\frac{\tau^2}{\tau_{O_2}^2}\right)-\right.$$

$$\left.(\Delta I_{O_2}-\tau_{O_2}\Delta F)\frac{(\tau-1)\tau^3}{(\tau_{O_2}-1)\tau_{O_2}^3}\right]. \tag{9}$$

This is the general expression and may be used to compute the total effect of any combination of errors that is likely to occur. It should not be forgotten that *all terms originating from the oxygen-point calibration are to be dropped above 0° C.* Some care is required in assigning values to the various errors, particularly when R_0, R_{100}, R_S, R_{O2}, and R have been measured at widely different times or with different equipment.

The types of error to be expected under various experimental conditions have been discussed fully by Mueller [*6*] and will not be treated here.

In many cases it is reasonable to assume that the same pair of values should be used for R_{100} and R_0 wherever they occur in eq 9. Making this assumption, it is of interest to treat five special cases of error, in which, respectively, only one of the five resistances, R_0, R_{100}, R_S, R_{O2}, and R, is incorrectly determined.

Case 1.—$\Delta R_0\neq 0$, no other errors

In this case, eq 9 reduces to

$$\Delta t\frac{\partial p}{\partial t}=\frac{100\Delta R_0}{F}(\tau-1)\left[1-\frac{\tau}{\tau_S}\left(1-\frac{\tau^2}{\tau_{O_2}{}^2}\right)-\frac{\tau^3}{\tau_O{}^3}\right]. \tag{10}$$

This equation is relatively simple in form because both sides are in terms of platinum temperature, in which the Callendar-Van Dusen equation is explicit. The error ΔR_0 is equivalent to an error $100\Delta R_0/F$ in platinum temperature at the ice point. The function of τ on the right gives the relative importance of the error at any temperature. As would be expected, it vanishes at all calibration points except the ice point, at which it has the value -1. The negative sign signifies that if in calibration the value assigned to R_0 is too high, subsequent measurements of temperature near the ice point will yield values which are too low.

Because different thermometers have different values of R_0, values of ΔR_0 may not be directly comparable. Hence it is more convenient to estimate the calibration error in terms of temperature. To permit this, eq 10 may be evaluated for $\tau=0$, giving

$$\Delta t_0\left(\frac{\partial p}{\partial t}\right)_0=\frac{100\Delta R_0}{F}(-1),$$

where Δt_0 is simply the error at the ice point resulting from the calibration error ΔR_0. Dividing eq 10 by this, we obtain

$$\frac{\Delta t}{\Delta t_0}=\frac{\left(\frac{\partial p}{\partial t}\right)_0}{\left(\frac{\partial p}{\partial t}\right)}(1-\tau)\left[1-\frac{\tau}{\tau_S}\left(1-\frac{\tau^2}{\tau_{O_2}{}^2}\right)-\frac{\tau^3}{\tau_{O_2}{}^3}\right], \tag{11a}$$

which, above the ice point becomes

$$\frac{\Delta t}{\Delta t_0}=\frac{\left(\frac{\partial p}{\partial t}\right)_0}{\left(\frac{\partial p}{\partial t}\right)}(1-\tau)\left(1-\frac{\tau}{\tau_S}\right). \tag{11b}$$

The quantities $(\partial p/\partial t)_0$ and $\partial p/\partial t$ contain the constants δ and β. Assuming δ and β to be 1.495 and 0.111, respectively, values of eq 11 were computed at 10-degree intervals. The values are given in column 2 of table 5.

Case 2.—$\Delta R_{100}\neq 0$, no other errors.

In this case, eq 9 reduces to

$$\Delta t\frac{\partial p}{\partial t}=\frac{100\Delta R_{100}}{F}\tau\left[-1+\frac{\tau-1}{\tau_S-1}\left(1-\frac{\tau^2}{\tau_{O_2}{}^2}\right)+\frac{(\tau-1)\tau^2}{(\tau_{O_2}-1)\tau_{O_2}{}^2}\right]. \tag{12}$$

Most of the discussion of case 1 is applicable. Following the procedure of that section, we obtain

$$\frac{\Delta t}{\Delta t_{100}}=\frac{\left(\frac{\partial p}{\partial t}\right)_{100}}{\frac{\partial p}{\partial t}}\tau\left[1-\frac{\tau-1}{\tau_S-1}\left(1-\frac{\tau^2}{\tau_{O_2}{}^2}\right)-\frac{(\tau-1)\tau^2}{(\tau_{O_2}-1)\tau_{O_2}{}^2}\right], \tag{13a}$$

which above the ice point becomes

$$\frac{\Delta t}{\Delta t_{100}}=\frac{\left(\frac{\partial p}{\partial t}\right)_{100}}{\frac{\partial p}{\partial t}}\tau\,\frac{\tau_S-\tau}{\tau_S-1}. \tag{13b}$$

Function 13 has the value 1 at 100° C and vanishes at all other calibration points. Values computed at 10-degree intervals are given in table 5.

Case 3.—$\Delta R_S\neq 0$, no other errors.

In this case, we obtain

$$\frac{\Delta t}{\Delta t_S}=\frac{\left(\frac{\partial p}{\partial t}\right)_S}{\frac{\partial p}{\partial t}}\,\frac{(\tau-1)\tau}{(\tau_S-1)\tau_S}\left(1-\frac{\tau^2}{\tau_{O_2}{}^2}\right) \tag{14a}$$

below the ice point, and above the ice point

$$\frac{\Delta t}{\Delta t_S} = \frac{\left(\frac{\partial p}{\partial t}\right)_S}{\frac{\partial p}{\partial t}} \frac{(\tau-1)\tau}{(\tau_S-1)\tau_S} \tag{14b}$$

This function has the value 1 at 444.60° C, and vanishes at all other calibration points. Values computed at 10-degree intervals are given in table 5.

Case 4.—$\Delta R_{O_2} \neq 0$, no other errors.

With this type of error we have below the ice point

$$\frac{\Delta t}{\Delta t_{O_2}} = \frac{\left(\frac{\partial p}{\partial t}\right)_{O_2}}{\frac{\partial p}{\partial t}} \frac{(\tau-1)\tau^3}{(\tau_{O_2}-1)\tau_{O_2}^3}. \tag{15}$$

This function has the value 1 at − 182.97° C, vanishes at 0° C, and is of course not to be used at higher temperatures. Values computed at 10-degree intervals are given in table 5.

Case 5.—$\Delta R \neq 0$, no other errors.

In this case,

$$\Delta t = \frac{1}{\frac{\partial p}{\partial t}} \frac{100 \Delta R}{F}. \tag{16}$$

This is included for completeness. The error is not connected with the calibration but with the subsequent use of the thermometer.

TABLE 5.—*Numerical values of functions 11, 13, 14, and 15*

For use in finding the errors at temperature *t* when the errors Δt_0, Δt_{100}, Δt_S, and Δt_{O_2} resulting from calibration are known]

t	$\Delta t/\Delta t_0$	$\Delta t/\Delta t_{100}$	$\Delta t/\Delta t_S$	$\Delta t/\Delta t_{O_2}$
°*C*				
−200	−1.067	0.765	−0.060	1.365
−190	−.405	.287	−.022	1.141
−182.97	*0*	*0*	*0*	*1*
−180	.157	−.110	.008	.944
−170	.625	−.433	.033	.773
−160	1.008	−.687	.052	.625
−150	1.312	−.879	.065	.498
−140	1.546	−1.016	.075	.391
−130	1.716	−1.102	.080	.302
−120	1.831	−1.144	.081	.228
−110	1.896	−1.148	.080	.169
−100	1.918	−1.119	.077	.121
−90	1.904	−1.062	.071	.084
−80	1.860	−.983	.064	.056
−70	1.791	−.885	.056	.036
−60	1.703	−.773	.048	.021

TABLE 5.—*Numerical values of functions 11, 13, 14, and 15*—Continued

t	$\Delta t/\Delta t_0$	$\Delta t/\Delta t_{100}$	$\Delta t/\Delta t_S$	$\Delta t/\Delta t_{O_2}$
°*C*				
−50	1.600	−.651	.039	.012
−40	1.487	−.523	.030	.006
−30	1.367	−.391	.021	.002
−20	1.244	−.258	.013	.001
−10	1.121	−.127	.006	.000
0	*1*	*0*	*0*	*0*
10	.882	.123	−.005	
20	.769	.240	−.009	
30	.659	.353	−.012	
40	.552	.461	−.014	
50	.450	.564	−.014	
60	.352	.662	−.014	
70	.258	.755	−.012	
80	.168	.841	−.009	
90	.082	.923	−.005	
100	*0*	*1*	*0*	
110	−.078	1.071	.006	
120	−.151	1.137	.014	
130	−.221	1.198	.023	
140	−.286	1.252	.033	
150	−.347	1.302	.044	
160	−.403	1.346	.057	
170	−.455	1.384	.071	
180	−.503	1.416	.086	
190	−.546	1.443	.103	
200	−.585	1.464	.120	
210	−.619	1.479	.140	
220	−.648	1.488	.160	
230	−.673	1.491	.182	
240	−.693	1.488	.205	
250	−.709	1.479	.230	
260	−.720	1.464	.256	
270	−.725	1.442	.283	
280	−.726	1.415	.312	
290	−.722	1.381	.342	
300	−.714	1.340	.373	
310	−.700	1.293	.406	
320	−.681	1.240	.441	
330	−.657	1.180	.477	
340	−.628	1.113	.514	
350	−.593	1.040	.553	
360	−.554	.960	.594	
370	−.508	.872	.636	
380	−.458	.778	.680	
390	−.402	.678	.725	
400	−.341	.570	.772	
410	−.274	.454	.820	
420	−.202	.332	.870	
430	−.124	.202	.922	
440	−.040	.065	.975	
444.60	*0*	*0*	*1*	
450	.049	−.079	1.030	
460	.144	−.231	1.086	
470	.245	−.390	1.145	
480	.352	−.557	1.205	
490	.465	−.732	1.267	
500	.585	−.915	1.330	
510	.710	−1.105	1.396	
520	.841	−1.304	1.463	
530	.979	−1.510	1.532	
540	1.123	−1.725	1.602	
550	1.273	−1.948	1.675	
575	1.677	−2.542	1.865	
600	2.123	−3.190	2.067	
625	2.611	−3.892	2.281	
650	3.143	−4.650	2.508	
660	3.368	−4.970	2.602	

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1.0000 to 2.5000	.0001	15
2.500 to 5.000	.001	15
5.00 to 10.00	.01	12

(1939) XV+535 pages; bound in buckram, $2.00.

MT3. Tables of Circular and Hyperbolic Sines and Cosines for Radian Arguments:

Contains 9 decimal place values of sin x, cos x, sinh x and cosh x for x (in radians) ranging from 0 to 2 at intervals of 0.0001.
(1939) XVII+405 pages; bound in buckram, $2.00.

MT4. Tables of Sines and Cosines for Radian Arguments:

Contains 8 decimal place values of sines and cosines for radian arguments ranging from 0 to 25 at intervals of 0.001.
(1940) XXIX+275 pages; bound in buckram, $2.00.

MT5. Tables of Sine, Cosine, and Exponential Integrals, Volume I:

Values of these functions to 9 places of decimals from 0 to 2 at intervals of 0.0001.
(1940) XXVI+444 pages; bound in buckram, $2.00.

MT6. Tables of Sine, Cosine, and Exponential Integrals, Volume II:

Values of these functions to 9, 10, or 11 significant figures from 0 to 10 at intervals of 0.001. with auxiliary tables.
(1940) XXXVII+225 pages; bound in buckram, $2.00.

MT7. Table of Natural Logarithms, Volume I:

Logarithms of the integers from 1 to 50,000 to 16 places of decimals.
(1941) XVIII+501 pages; bound in buckram, $2.00.

MT8. Tables of Probability Functions, Volume I:

Values of these functions to 15 places of decimals from 0 to 1 at intervals of 0.0001 and 1 to 5.6 at intervals of 0.001.
(1941) XXVIII+302 pages; bound in buckram, $2.00.

MT9. Table of Natural Logarithms, Volume II:

Logarithms of the integers from 50,000 to 100,000 to 16 places of decimals.
(1941) XVI II+501 pages; bound in buckram, $2.00.

[Continued on p. 4 of cover]

MT10. TABLE OF NATURAL LOGARITHMS, VOLUME III:
(Logarithms of the decimal numbers from 0.0001 to 5.0000, to 16 places of decimals.)
(1941) XVIII+501 pages; bound in buckram, $2.00.

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Copies of these publications have been sent to various Government depositories throughout the country, such as public libraries in large cities, and colleges and universities, where they may be consulted.

A mailing list is maintained for those who desire to receive announcements regarding new tables as they become available. A list of the tables it is planned to publish will be sent on request.