U. S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS

RESEARCH PAPER RP1379

Part of Journal of Research of the National Bureau of Standards, Volume 26, April 1941

DAYTIME PHOTOELECTRIC MEASUREMENT OF CLOUD HEIGHTS

By Maurice K. Laufer and Laurence W. Foskett

ABSTRACT

A photoelectric detector is used in conjunction with a modulated beam of light for the measurement by triangulation of the height of clouds during the daytime. An a-c operated mercury-arc lamp is used to obtain the modulated beam. An electronic "synchronous switch" is used to eliminate the effect of the varying background brightness of the clouds. The shot noise of the phototube, resulting from the relatively high brightness of clouds during the daytime, limits the detection. Dark overcast clouds at an elevation of 9,000 feet have been detected.

CONTENTS

I. INTRODUCTION

A beam of light incident on the base of a cloud is scattered to such an extent that at night the spot produced by a searchlight is generally visible. This phenomenon of scattering of light by water drops has made possible the employment of optical methods for the measurement of cloud heights. Middleton [1] has discussed the use of the ceiling projector [1][2] for night measurements, at which time the angular elevation of the spot can be determined by means of a visual instrument such as an alidade. The height of the cloud is determined from the solution of the vertical triangle formed by the line of sight on the spot, the beam of the projector, and the base line connecting the projector and alidade.

Several years ago Middleton initiated the first work on a photoelectric detector which would extend the use of the ceiling projector to the daytime measurement of cloud heights through the utilization of a modulated beam. Meanwhile F. V. Jones and A. H. Mears, of the United States Weather Bureau, had been considering the same

[1] Middleton also calculated the amount of signal light which is reflected from the spot, and section 3 of his paper applies equally well to daytime and nighttime measurements.
[2] Figures in brackets indicate the literature references at the end of this paper.

solution of the problem when Middleton's work became known to them. The Weather Bureau requested the National Bureau of Standards to develop the necessary equipment for measuring cloud heights in this way. The present paper describes a projector equipped with a mercury-arc lamp operating on alternating current to obtain a modulated beam, and a phototube and amplifier which have been used to detect daytime clouds up to 9,000 feet.

II. MEASURING EQUIPMENT

1. THEORY

Modulating a projected beam so that, after scattering, the light retains its original "identification-tag" was suggested by Tuve [2] as a possible means of studying the upper atmosphere. This suggestion would seem to make it possible to use a phototube and tuned amplifier to sort out completely the identified light from the background light. But the shot noise of the phototube, resulting from the background light, and the other noises associated with the phototube and the amplifier limit the minimum modulated light signal that can be detected [3, 4]. When the background is a daytime cloud, the resulting phototube current may be greater than 10^6 times the signal current which can be obtained from a projector of reasonable size and of moderate power consumption.

Because of the relatively high brightness of daytime clouds, it is possible, if the phototube has very little or no granular noise [3, 5], to design the phototube and amplifier circuit so that all other associated noises are negligible compared with the shot noise of the phototube. This may be done without reducing the ratio of signal to shot noise, and hence the ratio of signal to total noise is increased. The reason is as follows: For any infinitesimal frequency interval, the theoretical ratio of the mean-square shot noise of the phototube to the mean-square thermal noise of the circuit coupling the phototube to the first stage of the amplifier is given by [4]

$$\frac{eI_gZ^2}{2KTR}, \tag{1}$$

where e=charge on the electron, I_g=phototube current, K=Boltzmann's constant, T=absolute temperature, Z=impedance of the coupling circuit for the frequency of the interval, and R=real part of the impedance. (Neglecting tube capacities, $Z=R$ for direct coupling.) Thus, if Z^2/R is made sufficiently large, the shot noise becomes predominant for a given phototube current; and if R is large enough, the thermal noise will exceed the tube noises of the first stage. If, for a narrow band amplifier, Z is considered to be independent of frequency, the signal and the shot noise are proportional to Z. Hence it follows that no sacrifice is made in the ratio of signal to shot noise by increasing Z until the shot noise predominates above the other noises.

The problem of detection is further complicated by the large variations in the background brightness which arise from the motion of the clouds across the beam of the projector, changes in the formation of the clouds, and fluctuations in the amount of daylight incident on the clouds. In order that the total light received will have a detectable modulated component resulting from the projector, the frequency of modulation of the beam must not be too low; otherwise the random variations in the background light will obliterate the signal.

In continuation of Middleton's initial work, the Canadian Westinghouse Co. used mechanical modulation at a frequency of 400 cycles per second. The precision with which the parts had to be made, and the difficulties encountered with bearing failures because of the high speeds required, indicated that the recently developed high-intensity mercury-arc lamps might prove to be a better means of obtaining a modulated beam. When these lamps are operated on 60-cycle current, the modulation is approximately 95 percent [6] and has a frequency of 120 cycles per second. Although this frequency is less than that used by the Westinghouse Co., it is sufficiently high to overcome the effect of the relatively slow variations of the background light. Recent tests show, however, that this is about the minimum frequency that can be used.

2. PROJECTOR

The projector consists of a 1,000-watt, water-cooled, A–H6 lamp [6], located at the focus of a 24-inch parabolic mirror having a focal length of 10 inches. The dimensions of the mercury arc are approximately 3/50 by 1 inch. The optical axis of the mirror is vertical, and the lamp is mounted horizontally so that the longest dimension of the spot on the base of the cloud is perpendicular to the geometrical plane determined by the beam and the detector. Since the detector must scan the base of the cloud to locate the spot, this arrangement decreases slightly the accuracy with which the optical axis of the detector must coincide with the exact center of the beam during the scanning process. Likewise it is the narrow dimension of the spot that will be traversed, and a more precise determination of the angular elevation results.

3. PHOTOTUBE AND OPTICAL SYSTEM OF THE DETECTOR

Since the signal is proportional to the signal light received, and the shot noise is proportional to the square root of the background light received, the field of view of the detector should include the entire spot, but no more, in order to obtain the maximum signal to noise ratio. Practically, the entire spot is difficult to define. The brightness of the mercury arc [6] is greatest at the center and falls off rapidly at the edges. Imperfections in the mirror of the projector, aberrations, and irregularities in the base of the cloud also tend "to smear" the spot. Probably the most important consideration is the change in the projected area of the spot on a plane perpendicular to the optical axis of the detector as the angular elevation of the spot varies with cloud height.

A diaphragm located at the focus of a lens provides a simple optical system having light-gathering properties and a restricted field of view. For this arrangement the phototube is placed behind the diaphragm. The light rays after passing through the diaphragm are divergent, and if all these rays are to be intercepted by the cathode of the phototube, the focal length of the lens that can be used is determined by the cathode dimensions, the distance between the diaphragm and the cathode, and the aperture of the lens. To a first approximation, the diaphragm opening required to limit the field of view to the spot is equal to the size of the source multiplied by the ratio of the focal length of the lens of the detector to the focal length of the mirror in the projector.

In choosing a phototube, one is limited to the vacuum type because of the greater shot noise associated with the gas-filled types [7] for the same phototube current. Barrier-layer photocells cannot be used, because their voltage sensitivity is reduced to practically zero for high flux density. If the phototube is sensitive only to the wavelength interval of the signal light, the effective background light is reduced and hence the signal to noise ratio is increased. The spectral response of the RCA 929 phototube [8] matches the spectral distribution of the light from the A–H6 mercury-arc lamp exceedingly well. Its sensitivity is greater than that of the average gas-filled phototube, and tests have failed to indicate the presence of any granular noise. The dimensions of the cathode are approximately ⅞ by 1¹⁄₁₆ inch.

The final optical arrangement used consisted of an 8-inch plano-convex glass condensing lens, a diaphragm having a slit opening approximately ³⁄₂₅ by 1¹⁄₁₆ inch, with the longest dimension horizontal and the 929 phototube amounted horizontally as close behind the diaphragm as possible. The specified focal length of the lens is 19 inches but, because of the severe chromatic aberration, the diaphragm was located about 17 inches from the convex surface, which corresponds approximately to the focal length for blue light. The long dimension of the diaphragm is equal to the maximum dimension of the cathode surface, and the other dimension was chosen in accordance with the previous discussion. The optical system is mounted so that it may be rotated about a horizontal axis in order to scan the base of the cloud for locating the spot. Phototube currents as high as 25 microamperes have been obtained when the detector is directed at clouds illuminated by direct sunlight, and currents as low as 2 microamperes result from dark overcast daytime clouds. The detector was located about 1,000 feet from the projector.

4. AMPLIFIER

The phototube and 120-cycle, resistance-capacitance tuned amplifier circuit [9, 10] are shown in figure 1. A 5-foot, flexible, shielded cable having a capacitance of 26 $\mu\mu$f per foot connects the phototube to the amplifier. This cable, the blocking capacitor, C_1, the phototube-load resistor, R_1, and the grid-leak resistor, R_2, make up the coupling circuit between the phototube and the first stage. The blocking capacitor is required to prevent the variations in the background light from changing the bias on the first stage. The maximum impedance of the coupling circuit that can be used practically, and consequently the maximum initial signal that can be obtained, is limited by the dynamic grid impedance of the tube of the first stage. The 38-type tube has a high dynamic grid impedance, since the grid current for the voltages used is less than 10^{-11} ampere [3]. The choice of this tube permits the use of a 10^8-ohm grid leak, R_2. Under these operating conditions the grid current shot noise is negligible. The values of the component parts of the coupling circuit fulfill the conditions required by eq 1 for shot noise predominance and also sharpen the tuning of the amplifier. It was found necessary to use for C_1 a capacitor having a leakage resistance greater than 10^{12} ohms to keep the grid bias constant. At 79 and 180 cycles the amplification of the amplifier is 0.707 times the maximum amplification at 120 cycles.

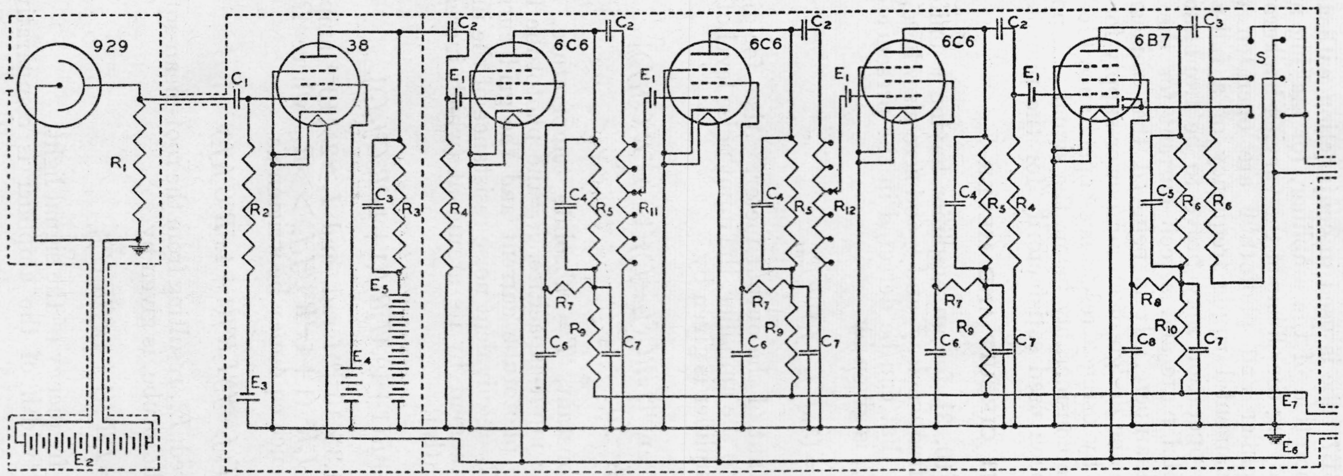

FIGURE 1.—*Wiring diagram of phototube and 120-cycle, resistance, capacitance tuned amplifier having the following circuit constants:*

	Microfarads		*Volts*		*Megohms*
C_1	0.0000125	E_1	1.0	R_1	10
C_2	.00025	E_2	270	R_2	100
C_3	.002	E_3	1.5	R_3	1
C_4	.0055	E_4	4.5	R_4	5
C_5	.004	E_5	45	R_5	0.25
C_6	.1	E_6	6	R_6	.5
C_7	2.0	E_7	135	R_7	4.3
C_8	0.25			R_8	6
				R_9	1.4
				R_{10}	0.08
				R_{11}	5.1
				R_{12}	5.05

S=DP–DT switch

297660—41——5

The diode part of the 6B7 tube is used in conjunction with a d-c meter for checking the operation of the amplifier, for making noise measurements, and for studying the effects of the varying background light. As used, the amplifier and phototube are operated with batteries. Since the fundamental ripple frequency of a full-wave rectifier is 120 cycles, a-c operation may prove to be troublesome. In this connection, the total plate and screen current for the five stages is less than 1 milliampere. It is believed that this small current will make possible the design of an a-c power supply for the plate and screen voltages by using a resistance-capacitance filter to reduce the 120-cycle ripple considerably below the signal. A power supply of this type has been used satisfactorily for the phototube.

5. OUTPUT CIRCUIT AND METER

According to Johnson [10, 3], for an amplifier having a uniform amplification between the frequencies f_1 and f_2 and zero elsewhere, the smallest steady signal, E_s, that can be detected in the presence of a statistical noise voltage, E_n, is given by

$$E_s=(5\lambda/(f_2-f_1))^{1/2}E_n, \tag{2}$$

where λ is the decay constant of the output meter. Also, for direct coupling of the phototube to the amplifier, the input noise level of the coupling circuit and the amplifier is given by

$$\overline{E_n^2}=4KT\{(1/2\pi C)(\tan^{-1}[2\pi RC(f_2-f_1)/(1+4\pi^2R^2C^2f_1f_2)]\\ (1+19.4I_gR)+\mathrm{R_t}(f_2-f_1)\} \tag{3}$$

where K=Boltzmann's constant, T= absolute temperature, C= sum of static and dynamic input-capacities, R=the phototube load resistance, I_g=sum of the phototube current and the grid current from the input tube, and R_t=equivalent noise resistance of the input tube. Replacing the arc-tangent by its argument (which increases the value for the noise slightly),

$$\overline{E_n^2}=4KT(f_2-f_1)[R_t+R(1+19.4I_gR)/(1+4\pi^2R^2C^2f_1f_2)]. \tag{4}$$

For $R=10^7$ ohms, $C=1.3\times10^{-10}$ farad, and $I_g\geqq2\times10^{-6}$ ampere, 19.4 $I_gR>>1$ and 19.4 $I_gR^2/(1+4\pi^2R^2C^2f_1f_2)>>R_t$, where R_t is about 5×10^4 ohms [3]. Hence eq 4 may be written as

$$\overline{E_n^2}=77.6KT(f_2-f_1)I_gR^2/(1+4\pi^2R^2C^2f_1f_2) \tag{5}$$

The signal-voltage squared, E_s^2, resulting from the root-mean-square signal-current, I_s, in the phototube, is given by

$$E_s^2=I_s^2R^2/(1+4\pi^2f_0^2R^2C^2), \tag{6}$$

where f_0 is the modulation frequency of the signal light.

If the percentage band width of the amplifier is comparatively small, that is, if

$$(f_2-f_1)/(f_1f_2)^{1/2}\lesseqqgtr 1 \tag{7}$$

then

$$f_0^2\doteq f_1f_2 \tag{8}$$

and the ratio of signal to noise from eq 5 and 6 is given by

$$E_s/E_n = I_s/[(f_2 - f_1)(77.6KTI_g)]^{1/2}. \tag{9}$$

Equating E_s/E_n from eq 2 and 9, the minimum root-mean-square signal-current that can be detected is given by

$$I_{s_{\min}} = (388\lambda KTI_g)^{1/2}. \tag{10}$$

This equation, based on Johnson's original eq 2, 7, and 8 is contrary to the accepted idea, namely, that the minimum detectable signal current is proportional to the square root of the band width [4, 7, 11, 12]. As pointed out by Johnson, eq 10 shows that $I_{s_{\min}}$ does not depend on the band width, if the band width is sufficiently narrow to satisfy eq 7, but depends on the decay constant of the output meter; that is, the minimum detectable signal is inversely proportional to the square root of the period of the output meter.

If it is assumed that the band width of the amplifier of figure 1 is 101 cycles (that is, $f_1 = 79$ c/s and $f_2 = 180$ c/s, the frequencies at which the amplification is 0.707 times the amplification at 120 c/s), eq 7 is satisfied. Furthermore, the approximations that must be made to obtain eq 5 when using the complex impedance for the coupling circuit introduce an error which is the same order of magnitude as that used above. The case for direct coupling is given for simplicity.

The above discussion is valid for noises which are constant except for statistical fluctuations. But during the time the base of the cloud is being scanned to find the spot, the output current will be varying in general by an amount greater than that which will result when the spot is located even if a long-period meter is used to damp out almost completely the statistical fluctuations. This results from the variations in the background brightness of the clouds as stated before, and even more so from the change in brightness from cloud to cloud as the scanning is made. If, however, a 120-cycle synchronous commutator, which is phased with the signal, is introduced beween the amplifier and the meter, the average output current in the absence of the signal will be zero because of the random phase of the shot noise. Thus the variations in brightness will not give a false indication of the signal, since the same average zero reading will be obtained for the shot noise whether the average phototube current is 25 microamperes or 2 microamperes. A second advantage results because effectively an increased scale length can be used. Normally the shot noise would produce an average reading 10 to 100 or even 1,000 times that resulting from the signal, but by reducing the average noise-output current to zero the entire scale of the meter can be used for indicating the signal.

There is a practical limit for the maximum period of the meter that can be used. If the scanning is made at a reasonable rate and if the period of the meter is too long, the spot will be passed without obtaining a noticeable indication. A period greater than 1 minute has been found to be unsatisfactory.

An "electronic switching-circuit," which performs the same function as a synchronous commutator, is shown in figure 2. The rectified, but nonfiltered, output of the transformer, T_1, is the plate supply for the 6J7 tube. If the resistance, R_4, is correctly adjusted, the plate current consists of square-topped pulses which are maintained for approximately 1/240 second. The time-average of the *IR* drop across the plate resistor, R_3, can be obtained from the potential across the capacitor, C_2, and this potential is reasonably steady because of the long period of the series circuit consisting of R_2 and C_2. Furthermore, the magnitude of this potential is almost completely independent of the amplified shot noise from the amplifier because of the random phase as in the case of a synchronous commutator. But when a sustained and phased 120-cycle signal is added, the potential across C_2 increases or decreases depending on whether the positive or negative

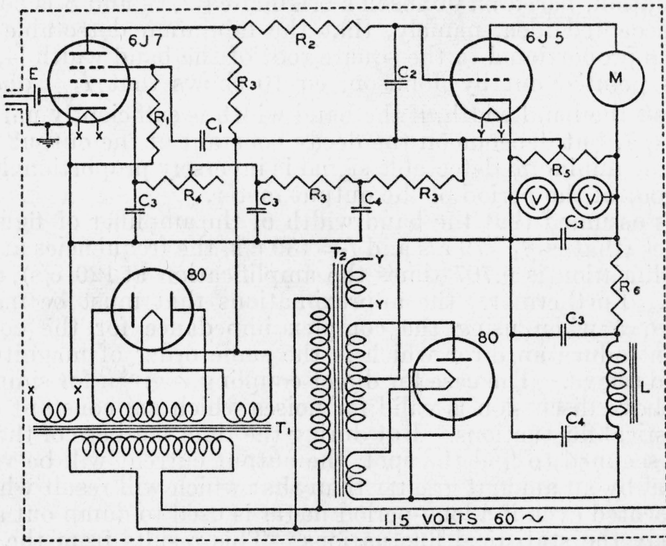

FIGURE 2.—*Wiring diagram of output circuit and meter having the following circuit constants:*

	Microfarads		Megohms
C_1	0.0001	R_1	20
C_2	4	R_2	3
C_3	40	R_3	0.1
C_4	4	R_4	.1
		R_5	.2
		R_6	.003

T_1=830 volts a-c, CT.
T_2=700 volts a-c, CT.
E=1.25 volts (Mallory grid bias cell).
L=15 henries at 85 milliamperes.
M=0 to 10, d-c milliammeter.
V=VR-150 voltage regulator tube.

portion of the signal is present during the time the plate current is flowing. A previous switching circuit contained a phase-shifting element in front of the primary of the transformer, T_1; but it so happens that it is not needed, because the positive portion of the signal is correctly phased to within a few degrees when the transformer for the mercury-arc lamp in the projector is connected to the same alternating-current supply. The remainder of the circuit shown in figure 2 is essentially a vacuum-tube voltmeter, which is used to indicate the change in the potential across C_2.

In practice, the potentiometer, R_5, is adjusted to give a plate current of about 7 milliamperes. In general, this current will be fluctuating by about 0.5 milliampere. If, during the scanning process to find the spot, the current drops to less than 6 milliamperes, one can be sure that the spot has been located. When the signal is so weak that the decreased reading fluctuates about this point (6 milliamperes), the detection can be verified by repeating the traverse to make sure that a spurious indication had not been obtained.

III. RESULTS

During the daytime, dark overcast clouds at an elevation of 9,000 feet have been readily detected with the equipment described. For cumulus clouds illuminated by direct sunlight and having elevations up to 4,000 feet, the detection is positive.

In the laboratory, measurements have been made to determine the minimum signal which can be detected with certainty in the presence of shot noise. Battery-operated incandescent lamps were used to simulate the background light and were arranged to give a phototube current of 5 microamperes. A sector disk driven by a synchronous motor was placed between a small flashlight lamp and the phototube to obtain a 120-cycle chopped signal. The flashlight lamp was located finally about 80 feet from the phototube. The resulting signal light was about 5×10^{-7} times the background light and according to eq 10 the signal current was less than 2 times the theoretical minimum.

IV. DISCUSSION

There are several possibilities for increasing the sensitivity of the equipment. Among the most obvious are the use of a larger mirror in the projector to give a more intense beam of light and the use of a larger lens in the detector. The latter possibility would increase the flux density on the cathode of the phototube to such an extent that it might be excessive and destroy the photosensitive surface. If the signal could be restricted to a single wavelength, it is conceivable that the background light could be reduced to a negligible factor by using filters which transmit the signal light and only a very small part of the background spectrum. The high-intensity mercury-arc lamp is the only practical source of high brightness having a considerable percentage of its energy emitted in narrow wavelength intervals. But to take advantage of this, one requires filters which have narrower transmission bands than are now available. Many filter combinations have been tried, but a net loss was obtained rather than a gain in the signal to noise ratio, because the signal is proportional to the transmission of the filter whereas the shot noise resulting from the background light is proportional to the square root of the transmission.[2]

The authors thank E. A. Johnson, of the Department of Terrestrial Magnetism, Carnegie Institution of Washington, for several helpful discussions and for granting them the permission to read reference [3] in manuscript form.

[2] Let I_s and I_g be the phototube currents resulting from the signal light and the background light, respectively. Then the ratio of signal to noise, E_s/E_n, may be written as $kI_s/I_g^{1/2}$. If a filter is introduced, the ratio of signal to noise becomes $kI_sT_s/(I_gT_g)^{1/2}$, where T_s and T_g are the respective transmission factors based on the spectral response of the phototube for the signal light and for the background light. Hence $T_s/T_g^{1/2}$ must be greater than unity in order to increase the signal to noise ratio.

V. REFERENCES

[1] W. E. Knowles Middleton, *On the theory of the ceiling projector,* J. Opt. Soc. Am. **29,** 340 (1939).

[2] M. A. Tuve, E. A. Johnson, and O. R. Wolf, *A new experimental method for study of the upper atmosphere,* Terr. Mag. **40,** 452 (1935).

[3] E. A. Johnson, W. H. Mock, and R. E. Hopkins, *The limiting sensitivity of the alternating-current method of photo-cell-current amplification,* J. Opt. Soc. Am. **29,** 506 (1939).

[4] J. B. Johnson and F. B. Llewellyn, *Limits to amplification,* Elec. Eng. **53,** 1449 (1934).

[5] G. L. Pearson and C. J. Christensen, *Spontaneous resistance fluctuations in carbon microphones and other granular resistances,* Bell Sys. Tech. J. **15,** 197 (1936).

[6] E. B. Noel, *The development of water-cooled quartz mercury lamps,* J. Applied Phys. **11,** 325 (1940).

[7] B. A. Kingsbury, *The shot effect in photoelectric currents,* Phys. Rev. **38,** 1458 (1931).

[8] A. M. Glover and R. B. Janes, *A new high-sensitivity photosurface,* Electronics, 26 (Aug. 1940).

[9] E. A. Johnson, *Design of tuned resistance-capacity coupled amplifiers,* Rev. Sci. Inst. **7,** 130 (1936).

[10] E. A. Johnson, *The limiting sensitivity of an alternating-current method of measuring small magnetic moments,* Rev. Sci. Inst. **9,** 263 (1938).

[11] F. Preisach, *Noise reduction by means of photo-electric multipliers,* Wireless Eng. **16,** 169 (1939).

[12] E. A. Johnson and C. Neitzert, *The measurement of small alternating voltages at audiofrequencies,* Rev. Sci. Inst. **5,** 196 (1934).

WASHINGTON, November 5, 1940.