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A DETERMINATION OF THE MAGNETIC SATURATION INDUCTION OF IRON AT ROOM TEMPERATURE

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ABSTRACT

A determination of the magnetic saturation induction of iron at room temperature has been made, using a modification of the Ewing isthmus method. Specimens were taken from several ingots of iron of exceptional purity prepared at the National Bureau of Standards. Corrections were made for the effect of the small amounts of impurities present, and care was taken to minimize systematic and accidental errors. The value found for pure iron at 25° C, assuming a density of 7.874 g/cm^3, is 21.58 $\pm$0.01 kilogauss.

CONTENTS

I. INTRODUCTION

The determination of the magnetic-saturation value of iron at room temperature reported in the present paper was undertaken as part of a more extensive program initiated some time ago by the Division of Metallurgy of the National Bureau of Standards for the purpose of investigating the fundamental properties of pure iron. To this end, Thompson and Cleaves [1] [1] prepared several ingots of exceptionally pure iron. This material compares favorably with any previously produced, not only with respect to purity but also with respect to the accuracy to which the amounts of the impurities have been determined. The preparation and analysis of the material are fully described in the paper cited above. The total amount of impurities out of 55 elements sought was of the order of 0.01 percent, of which carbon did not exceed 0.001 percent.

Ten of the ingots were selected as being representative of the maximum range of impurities, both with respect to the total amount and with respect to individual impurities, and magnetic test specimens were prepared from these ingots. Data on the compositions of the individual ingots are given in table 1.

[1] Numbers in brackets indicate the references at the end of the paper.

TABLE 1.—*Composition of ingots*

Ingot number	Impurities								
	Be	C	Cu	H_2	N_2	O_2	S	Si	Total [1]
	%	%	%	%	%	%	%	%	%
3	<0.001	0.001	<0.002	0.000_1	0.000_4	0.000_5	0.002_6	0.003	$<0.011_1$
6	<.001	<.001	<.002	$.000_2$	$.000_2$	$.004_5$	$.001_3$	.001	$<.011_7$
7	nil	<.001	<.002	$.000_2$	$.000_1$	$.002_0$	$.002_1$	.001	$<.008_9$
8	nil	----------	<.002	$.000_2$	$.000_0$	$.003_2$	$.001_0$	.001	$<.008_9$
12	nil	<.001	<.002	$.000_2$	$.000_5$	$.001_4$	$.001_2$	.002	$<.008_8$
13	nil	<.001	<.002	$.000_1$	$.000_3$	$.004_0$	$.002_4$	nil	$<.010_3$
14	nil	.001	<.002	$.000_1$	$.000_2$	$.002_0$	$.001_1$	0.002	$<.008_9$
18	nil	.001	<.002	$.000_2$	$.000_1$	$.000_4$	$.001_1$	nil	$<.008_3$

[1] Including allowances for elements not determined in the individual ingots, based on average amounts found in other ingots.

II. PREVIOUS DETERMINATIONS

The first magnetic measurements with very strong fields were made in 1889 by Ewing and Low [2], who reported a saturation induction of 21.36 kilogausses for a sample of wrought iron of unspecified purity. They used their now well-known isthmus method. Since that time several values have been reported, ranging from 21.26 to 22.58 kilogausses. This range of about 6 percent may be attributed partly to inaccuracies in measurement and partly to the effect of impurities in the materials. Apparatus and experimental technique have been improved, and the effects of various common impurities have been determined, so that recent values are in much better agreement.

In 1918 Gumlich [3] reported a value of 21.62 kilogausses, which for several years was the most generally accepted figure. He used a "yoke-isthmus" method and corrected for the effect of certain impurities. Three later investigations carried out in different countries and by different experimental methods yielded results differing very little from that given by Gumlich. In 1926 Honda and Kaya [4] in Japan, working with single crystals of iron, obtained a value of 21.58 kilogausses. In 1929 Weiss and Forrer [5] in France, using small ellipsoids magnetized between the poles of an electromagnet, obtained a value of 21.56 kilogausses, assuming a density of 7.878 g/cm^3. The latest report comes from the Physikalisch-Technische Reichsanstalt, in Germany, where, in 1937, Steinhaus, Kussmann, and Schoen [6] obtained 21.58 $\pm$0.01 kilogausses. They used a somewhat modified form of Gumlich's yoke-isthmus method and corrected for the effect of impurities. Their value also is based on a density for pure iron of 7.878 g/cm^3.

III. METHOD AND APPARATUS

In the present investigation, a method based upon the isthmus method of Ewing and Low [2] was adopted, because by this method magnetizing forces up to 10,000 oersteds can be applied with a moderate expenditure of power and without undue heating of the specimen. Furthermore, the space within which the magnetizing field is measured is easily accessible, so that its uniformity can readily be investigated.

The apparatus employed was the High-H permeameter [7] with special pole pieces and test coils. The arrangement of the pole pieces

and test coils is shown in figure 1. Extension pieces, E, on which are screwed the pole pieces, *P*, are inserted in the poles, *C*, of the permeameter. The pole pieces and extensions are drilled axially to permit the insertion of the specimen, *S*.

The magnetic circuit is completed by two large laminated U-shaped yokes (not shown) The main magnetizing coils, *M*, surround the poles, but there are some additional windings on the yokes.

The test-coil system, *T*, consists of five concentric and coaxial coils of 200 turns each of No. 44 AWG enamel-covered copper wire

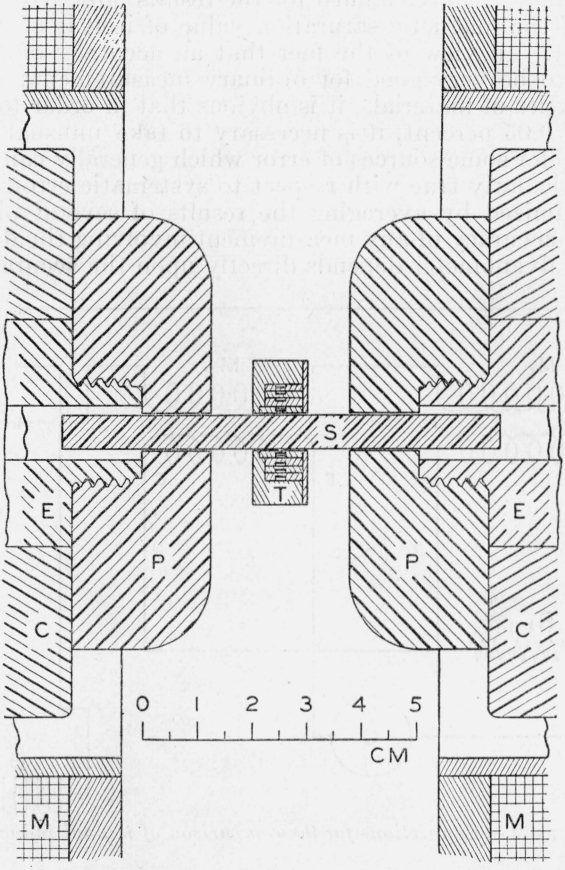

FIGURE 1.—*Arrangement of pole pieces and test coils.*

wound on hard-rubber forms. The coils are 2 mm long. Their effective diameters, as determined from values of area-turns, are 7.22, 7.84, 10.86, 13.52 and 15.78 mm, respectively. These diameters correspond to distances of 0.61, 0.92, 2.43, 3.76 and 4.89 mm, respectively, from the surface of a specimen 6 mm in diameter. By means of a selector switch, it is possible to connect these coils to a ballistic galvanometer in any one of several combinations.

The ballistic galvanometer has a natural period of approximately 30 seconds. The external resistance for critical damping is about

40,000 ohms. However, since the external resistance is generally less than 200 ohms, the galvanometer performance closely approximates that of a fluxmeter. The sensitivity is adjusted in the usual way, using a mutual inductor, standard resistor, and potentiometer. The precision with which the galvanometer deflections can be read is greatly improved by lenses mounted in front of the galvanometer scale.

IV. CALIBRATION AND ADJUSTMENT

The accuracy usually claimed for the results obtained in the determination of the magnetic saturation value of iron is of the order of 0.05 percent. In view of the fact that an accuracy of 1 percent is considered to be very good for ordinary measurements of the magnetic properties of materials, it is obvious that in order to achieve an accuracy of 0.05 percent, it is necessary to take unusual precautions and to consider some sources of error which generally can be ignored. This is particularly true with respect to systematic errors, which can not be minimized by averaging the results of several observations.

The final accuracy of any measurement involving the deflections of a ballistic galvanometer depends directly upon the accuracy to which

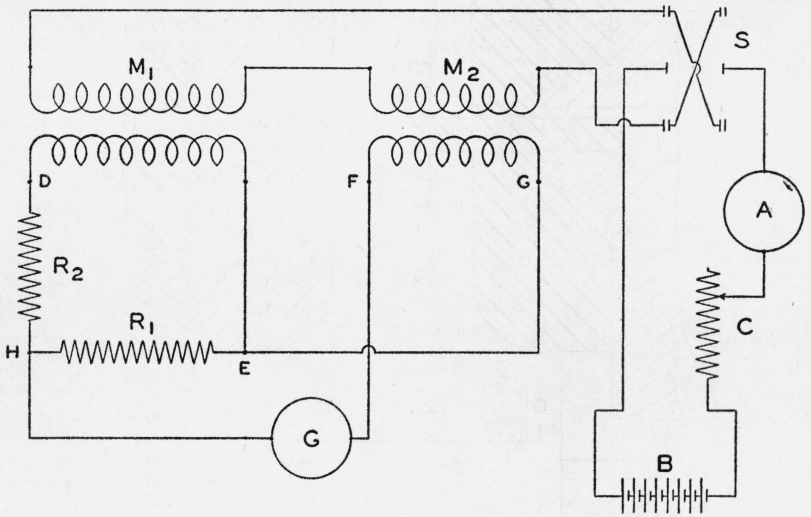

FIGURE 2.—*Connections for the comparison of mutual inductors.*

the value of the mutual inductor used for calibration is known. It is difficult by the usual methods of measuring mutual inductance to obtain an accuracy much better than 0.1 percent. This obviously would be inadequate for the present purpose. Fortunately, there was available a mutual inductor constructed by the Resistance Measurements Section of this Bureau for use in connection with the determination of electrical resistance in absolute measure. Its value is known to 20 parts in a million, or 0.002 percent. The mutual inductors used for the calibration of the ballistic galvanometer were measured by comparison with this standard by the method indicated in figure 2. M_1 and M_2 are the mutual inductors to be compared, of which M_1 must have the greater value. The primary coils are connected in series to

the battery through a reversing switch, rheostat, and ammeter. The secondary coils are connected in opposition, as shown. R_1 and R_2 are calibrated precision rheostats and are adjusted to such values that there is no residual deflection of the galvanometer upon reversal of the primary current. When balance is obtained, the value of M_2, in terms of M_1, is

$$M_2 = M_1 \frac{R_1}{R_1 + R_2}.$$

The resistance of M_1 and its connections must be included in the value of R_2. The usual precautions against the effect of stray fields were taken by separating the two inductors by some distance and so orienting them as to avoid interaction between the primary of one inductor and the secondary of the other.

The accuracy attainable by this method depends upon the magnitude of the inductances, their ratio, the accuracy to which the resistances R_1 and R_2 are known, and the sensitivity of the galvanometer. In the present work, calibrated precision rheostats were used, and settings were made, using different combinations, so as to average out any residual errors in the values of resistance. It is estimated that in this way the values of the inductors used in calibrating the ballistic galvanometer were determined to within 0.01 percent. These values are based on the present international electrical units.

The values of area-turns of the test coils were determined in a similar manner by placing the coils in a solenoid whose constant had been determined by comparison with a standard single-layer solenoid and measuring the mutual inductance. The accuracy of this determination was not so good as that obtained in the measurement of the mutual inductors, because the values of mutual inductance were much smaller. However, this had no important effect on the final result because a systematic error in the observed values of magnetizing force does not affect the saturation value obtained by extrapolation.

One factor which has an important bearing on the accuracy of the measurements is the degree of uniformity of the field in the space immediately adjacent to the specimen, which is occupied by the test coils. This is a function of the distance between the pole pieces, referred to as the gap length. Preliminary investigation of the distribution corresponding to various values of the gap length revealed the fact that the area-turns of the various combinations of test coils were not known to a sufficient degree of precision for this purpose. For this reason, explorations were carried out by means of a single coil so mounted that it could be moved in a radial direction from the surface of the specimen, with its axis always kept parallel to the direction of the field. By using the galvanometer at its maximum sensitivity and moving the coil from various points at measured distances from the surface of the specimen to a point taken as a reference, it was possible to determine the distribution to a satisfactory degree of precision. It was found, as shown in figure 3, that if the gap is too short, the intensity of the field increases with distance from the surface of the specimen. If the gap is too long, the field decreases with radial distance. A gap length of 2.5 cm was found to give a substantially uniform field within a distance of 3 mm from the surface of the speci-

men. Since the inner three coils which were used for the final measurements are all within this distance, the gap length was set at 2.5 cm.

The substantially uniform field in the space occupied by the test coils made it possible to eliminate experimentally the magnetizing

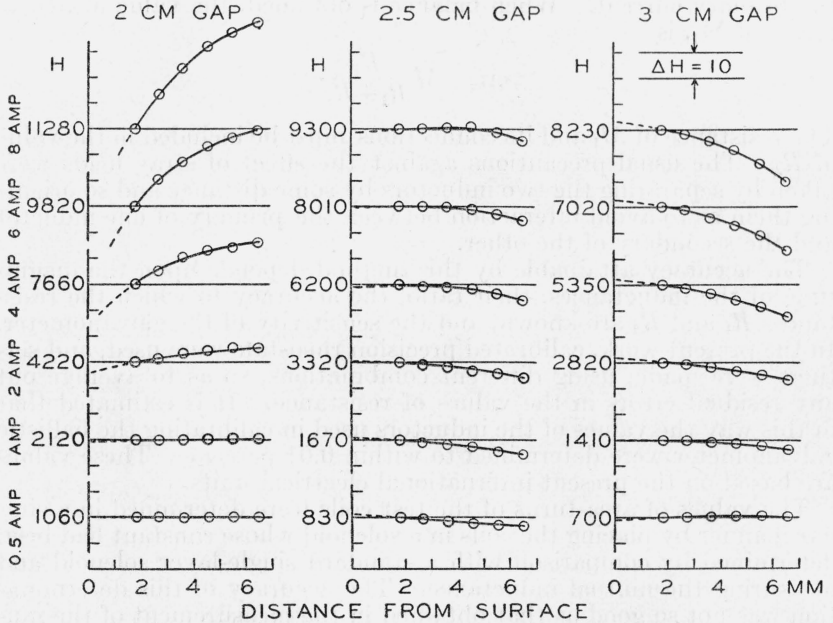

FIGURE 3.—*Influence of gap length on the distribution of the field in a radial direction from the surface of the specimen.*

Note that the observed deviations within 3 mm of the surface for the 2.5-cm gap are not greater than 2 oersteds.

field from the observations and thus avoid errors which might result from subtracting independently observed values of total induction

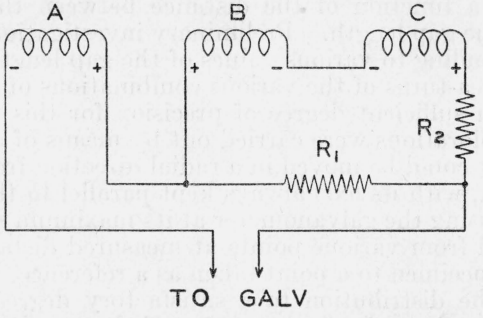

FIGURE 4.—*Connections for adjusting the effective area-turns of the B- and H-coils to equality.*

and magnetizing force. This was done in the manner indicated in figure 4. In the figure, *A*, *B*, and *C* represent the inner three test coils. Coils *B* and *C* are connected in series opposition and so give a measure of the field in the zone between them. The effective area-

turns of this combination can be made equal to the area-turns of coil A by means of resistances R_1 and R_2 connected in the same manner as that used for the comparison of mutual inductances. Preliminary adjustment was made by setting R_1 and R_2 so that

$$\frac{R_1}{R_1+R_2}=\frac{(AN)_A}{(AN)_{BC}},$$

where

$(AN)_A$=area-turns of coil A,
$(AN)_{BC}$=area-turns of zone between coils B and C.

The resistances of coils B and C and their leads are included in the value of R_2. Since the coils and leads are of copper, so that their resistance might vary enough with changes in temperature during a run to cause an error in the compensation, the resistance of R_2, including the coils and leads, was measured frequently during the observations, and R_2 was adjusted to compensate for any change which may have occurred.

The compensation was checked experimentally by placing the coils in a uniform field between flat pole pieces in the apparatus. There was no observable deflection of the galvanometer upon reversal of a field of 10,000 oersteds. The galvanometer sensitivity was such that a difference corresponding to 1 oersted, or 0.01 percent, would have been detected.

Since the test coils were only 2 mm long, possible difficulties due to nonuniform distribution of the field in a longitudinal direction were avoided.

The performance of the galvanometer approximates that of a fluxmeter nearly enough for ordinary conditions. However, since the time required to complete the reversal of the magnetizing current in the present apparatus is of the order of 3 seconds, it was found necessary to correct for the effect of the delayed impulse. The amount of the correction was determined by observing the amount by which the deflection due to the reversal of a given current in the primary of the mutual inductor was reduced by connecting the magnetizing coils in series with the inductor primary. This was done for a series of currents in the same range as that used in the final measurements. The correction was found to vary linearly with the reciprocal of the magnetizing force, and this relation was used to calculate the correction to be applied to the saturation value. The correction thus determined is 0.044 kilogauss, or approximately 0.2 percent of the saturation value.

V. TEST SPECIMENS

A specimen was cut from each of 10 ingots and were numbered to correspond with the ingot numbers. One bar was subsequently found to be cracked and one was considered to be excessively porous, so that these specimens were discarded. The bars had been swaged to a diameter of approximately 6.3 mm and were originally about 12 cm long. They were later ground to a diameter of 6.0 mm and cut to a length of 8 cm.

The determination of the cross-sectional areas of the specimens is of considerable importance, because the accuracy of the results depends directly upon the accuracy to which these areas are known.

Mechanical measurements of diameter can be made to the requisite accuracy, but if the specimens are porous there can be no assurance that the area determined from the outside diameter at a given section represents the true area of solid metal at that section. In the light of recent data calculated from lattice constants and of direct measurements on drawn wires presumably free from appreciable porosity, the most probable value for the density of pure iron appears to be 7.874 ± 0.001 g/cm^3. This value was therefore adopted. The observed values for most of the bars are somewhat lower than 7.874 g/cm^3, and this was taken to indicate a slight porosity. The values are given in table 2.

TABLE 2.—*Density*[a] *of specimens (in grams per square centimeter)*

	Specimen number—							
	3	6	7	8	12	13	14	18
12 cm, swaged	7.875	7.869	7.873	7.872	7.875	7.867	7.860	7.869
8 cm, ground	[b] 7.869	7.871	7.870	[b] 7.873	7.872	7.867	7.862	7.872

[a] Determined by the Capacity and Density Section of this Bureau, and estimated to be accurate within ± 0.002.
[b] After annealing.

The average cross-sectional area of solid iron can be found from its length, mass, and the true density. However, if the specimen is porous, this gives no indication of the variation of area along the length nor of the exact area at any one place. This variation was investigated by exploring the bars magnetically along their length. By means of small brass rods inserted in the axial holes in the pole pieces, the specimens could be moved along longitudinally by measured amounts as indicated on a scale. By observing the galvanometer deflections when a bar was moved from different positions to a fixed position taken as a reference, it was possible to determine the variation of apparent induction corresponding to a fixed value of magnetizing force. By using the galvanometer at maximum sensitivity, it was possible to observe differences as small as 0.001 kilogauss. The galvanometer was calibrated in terms of the average area, so that the observed variations were proportional to the variations in area.

Since the swaged bars showed large variations in induction, sometimes amounting to as much as 0.5 percent, it was decided to grind the bars in order to have more nearly uniform areas. After the bars were ground, the variations were found to be very much smaller. The curves shown in figure 5 are typical of the results obtained. The magnetic variations after grinding the bars correspond quite closely to the variations in area as determined from measurements of diameter. The relative differences may be attributed for the most part to nonuniformity in distribution of the material due to porosity. Bar 10 still showed so much variation, even after grinding, that it was discarded.

Since it was feasible to explore only 8 cm of the length of the bars, they were cut to that length, and for the calculation of average area only that part was used for which distribution had been determined. The final measurements were made at the middle point of the bars

with the galvanometer calibrated for the average area. The results were then corrected by the percentage that the actual area at the point of observation differed from the average, as indicated by the magnetic exploration data.

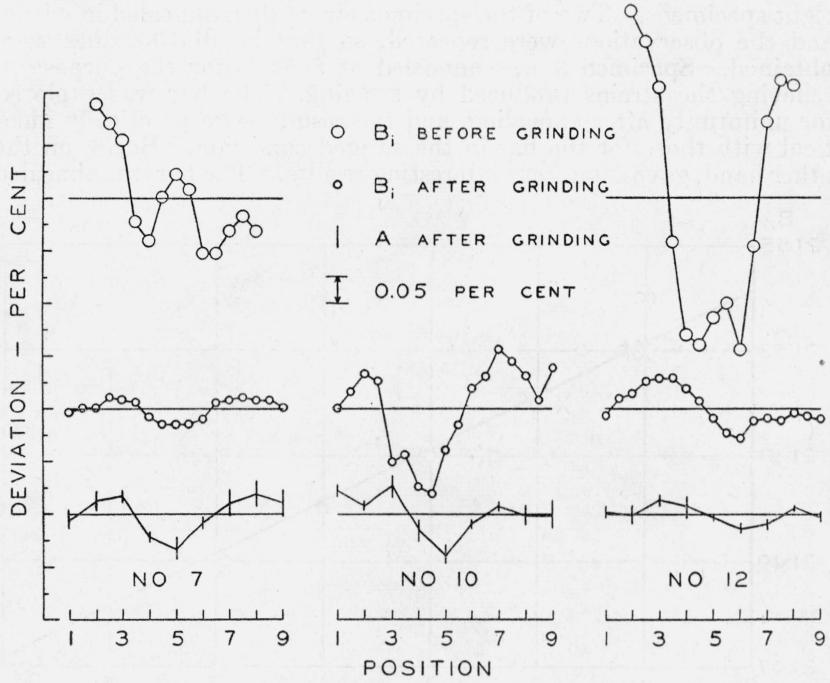

FIGURE 5.—*Typical results of magnetic exploration.*

The lower curves represent the variations in area as determined by measurements of diameter. The upper and middle curves represent the variations in intrinsic induction with position on the bar at a magnetizing force of 5,000 oersteds. The positions indicated are 1 cm apart. Position 5 is the middle of the bar.

VI. FINAL OBSERVATIONS AND RESULTS

Final observations were made at magnetizing forces ranging from 1,500 to 10,000 oersteds. Previous investigators [5, 6] have found that for values of intrinsic induction, B_i, within 1 percent of the saturation value, B_s, the law of approach to saturation is represented by the equation

$$B_i = B_s - b/H.$$

By plotting B_i against $10^4/H$, this law was found to hold for the present results within the experimental error. A typical set of observations plotted in this way is shown in figure 6. In determining the best value of B_s and the slope, b, all of the observed values were utilized by solving the simultaneous equations.

$$\Sigma B_i = NB_s - \Sigma(10^4/H) \times b$$
$$\Sigma(B_i \times 10^4/H) = \Sigma(10^4/H)B_s - \Sigma[(10^4/H)^2] \times b$$

where

ΣB_i=sum of observed values of B_i (kilogausses)
N=number of observed points
B_s=saturation value (kilogausses)
$\Sigma(10^4/H)$=sum of values of $10^4/H$ (H in oersteds)
b=slope of line

It may be noted that none of the observed points departs from the calculated line by as much as 0.01 kilogauss, which was the least count in the observations.

Two independent sets of observations were made on each of the eight specimens. Two of the specimens were then annealed in vacuo, and the observations were repeated, so that in all 10 values were obtained. Specimen 3 was annealed at 865° C for the purpose of relieving the strains produced by swaging. The bar was explored for uniformity after annealing, and the results were practically identical with those for the bar in the swaged condition. Bar 8, on the other hand, gave some very interesting results. The bar was annealed

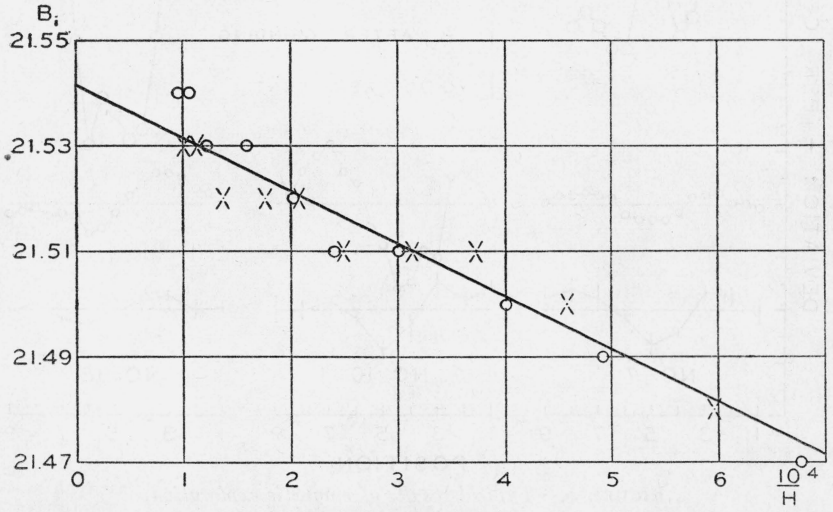

FIGURE 6.—*Method of plotting the observations and extrapolating to the saturation value.*

The points marked by circles are for the first set of observations. Those marked by crosses are for the second set. The position of the line was located by calculation, using all of the observed points

at 1,020° C, with the idea of producing grain growth as well as relieving strains. It was hung from one end in a vertical furnace. The results of magnetic exploration before and after annealing are shown in figure 7. The magnetic exploration of bar 8 after it was annealed indicated that the bar was tapered, and this was confirmed by mechanical measurements of diameter. The high temperature evidently had softened the bar enough to permit a progressive elongation, greatest at the top, which produced a slight taper. It is of interest to note, however, that no appreciable change in the saturation value was brought about by annealing in either case.

The final data and corrected values are given in table 3. In column 1 are given the values of B_s obtained by extrapolation of the observed values of B_i in the manner indicated above. Columns 2, 3, and 4 show the corrections for nonuniformity of area, delayed impulse, and impurities, respectively. In view of the small amounts of the impurities present and the correspondingly small correction necessary, no attempt was made to determine their effects independently. The data given by other investigators were assumed to be sufficiently

accurate for the purpose. The corrections for C, Cu, N, O, and S were based on the work of Steinhaus, Kussmann, and Schoen [6], and the correction for Si from that of Gumlich [3]. The total correction was calculated to be approximately 0.02 percent, or 0.004 kilogauss, which is less than the probable error of the magnetic measurements.

Column 5 gives the final corrected values, showing a mean of 21.578 kilogausses. The percentage deviations from the mean are

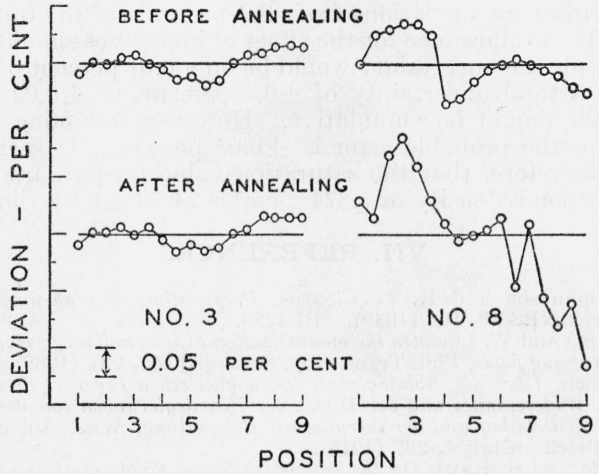

FIGURE 7.—*Effect of annealing on the uniformity of area*

The bars were hung from one end in a vertical furnace and annealed in vacuo. Annealing at 865° C had no appreciable effect on the uniformity of bar 3. Bar 8 was annealed at 1,020° C, which softened the bar enough to allow it to stretch slightly and produce a definite taper. The positions indicated are 1 cm apart on the bars. Position 5 is at the middle.

given in column 6, the numerical average being 0.042 percent and the maximum 0.074 percent.

TABLE 3.—*Final results*

[Column 1. Value by extrapolation of observed points. Column 2. Correction for nonuniformity of area. Column 3. Correction for delayed impulse. Column 4. Correction for effect of impurities. Column 5. Final value. Column 6. Deviation from the mean, percent]

Specimen number	Intrinsic induction (kilogausses)					
	1	2	3	4	5	6
3	21.523	+0.005	+0.044	+0.004	21.576	−0.009
3 [a]	21.520	.005	+0.044	+0.004	21.573	−.023
6	21.535	.001	+0.044	+0.004	21.584	+.028
7	21.520	.007	+0.044	+0.004	21.575	−.014
8	21.537	.006	+0.044	+0.004	21.591	+.060
8 [a]	21.543	−.001	+0.044	+0.004	21.590	+.056
12	21.541	+.004	+0.044	+0.004	21.593	+.069
13	21.512	.008	+0.044	+0.004	21.568	−.046
14	21.490	.024	+0.044	+0.004	21.562	−.074
18	21.520	.001	+0.044	+0.004	21.569	−.042
Mean					21.578	0.042
Maximum						0.074

[a] Annealed.

In considering the probable accuracy of the final result, the greatest source of uncertainty is the effect of the delayed impulse, for which a correction of 0.2 percent was made. The error in determining this correction is probably not greater than 25 percent of the correction, which leaves an uncertainty, then, of 0.05 percent. The mutual inductance of the inductor was known to 0.01 percent; the measurement of current for calibrating the galvanometer was good to 0.01 percent; the error due to imperfect compensation of the test coils did not exceed 0.01 percent; and the probable experimental error, as calculated from the deviations from the mean, is of the order of 0.01 percent. If the allowance for the effect of impurities should be wrong by 50 percent, the uncertainty would be only 0.01 percent of the total. This gives a total uncertainty of ± 0.1 percent, or ± 0.02 kilogauss, if the errors should be cumulative. However, assuming a random distribution, the probable error is ± 0.05 percent. It seems safe to conclude, therefore, that the saturation value for pure iron at 25° C and based upon a density of 7.874 g/cm^3 is 21.58 ± 0.01 kilogauss.

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