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ABSOLUTE PRESSURE CALIBRATIONS OF MICROPHONES

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ABSTRACT

A tourmaline-crystal disk was used both as a microphone (direct piezoelectric effect) and as a sound source (converse piezoelectric effect). Application of a principle of reciprocity to the acoustic measurements gave an absolute determination of the piezoelectric modulus $d_{33} + 2d_{31}$ of tourmaline under hydrostatic pressure. A condenser microphone was calibrated by the tourmaline disk. The same principle was applied to data obtained by using a condenser microphone as both source and microphone to secure an absolute calibration of another condenser microphone. It was proved experimentally that the tourmaline disk and the condenser microphones satisfied the principle of reciprocity. The absolute acoustic determination of the piezoelectric modulus gave $d_{33} + 2d_{31} = 2.22 \times 10^{-17}$ coul/dyne. The "reciprocity" calibrations agreed with the results of electrostatic actuator, pistonphone, and "smoke particle" calibrations, but disagreed with thermophone calibrations of the condenser microphones.

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I. INTRODUCTION

Absolute measurements of sound intensity are of basic importance in acoustics. Such measurements are most conveniently made by means of calibrated microphones, and it is important to have accurate methods of making both absolute pressure and absolute free-field calibrations of microphones. Absolute pressure calibrations are needed at low frequencies to supplement free-field Rayleigh-disk measurements at higher frequencies, and are needed to check Rayleigh-disk measurements in the frequency region in which both pressure and free-field calibrations should be the same. Absolute pressure calibrations are also needed in audiometric work.

First calibrations of a condenser microphone made with an electrostatic actuator and with a thermophone, both of conventional design, showed a difference much greater than the probable error of the determination. The difference between the thermophone and the free-field Rayleigh-disk calibrations of a condenser microphone, in the

frequency region in which both calibrations should be the same, was also greater than the probable error. It was decided to use piezoelectric crystals to throw some light on the discrepancies between the results obtained with the thermophone, the electrostatic actuator, and the Rayleigh disk.

II. PRINCIPLE OF RECIPROCIITY

1. GENERAL

Shortly after the discovery of the direct piezoelectric effect by the brothers P. and J. Curie, M. G. Lippmann predicted the existence of a converse piezoelectric effect, and showed quantitatively how large it should be.

Lippmann's proof, generalized by W. Voigt [1]¹, is in reality a special case of a principle of reciprocity. Other special cases of the principle are well known in applied mechanics, electrical engineering, and acoustics. This principle was discussed by Lord Rayleigh [2] for mechanical systems and electrical systems, and was discussed by S. Ballantine [3] for certain mixed mechanical and electrical systems. The same principle of reciprocity can be applied not only to piezoelectric crystals but also to condenser microphones, moving-coil microphones, and to the thermophone. By means of this principle, the output of a transducer (i. e., a device which is used as both sound source and microphone) as a sound source can be computed from its response as a microphone, and vice versa.

Suppose that a pressure sinusoidal in time

$$p = p_0 e^{i\omega t}$$

is applied to the diaphragm of a transducer (e. g., a condenser microphone); suppose electric charge

$$q = q_0 e^{i\omega t}$$

is released from the transducer. The release of this charge, q , by the pressure, p , is called the direct effect (see fig. 1). It is assumed that the charge released is directly proportional to the pressure. This assumption of linearity is sufficient for the application of the reciprocity principle.

If the proportionality between p and q is given by

$$q = \tau p, \tag{1}$$

the principle of reciprocity asserts that

$$v = \tau u, \tag{2}$$

where τ is numerically the same in both eq 1 and 2, if q , p , v , and u are all in the same system of units, and where

$$v = v_0 e^{i\omega t}$$

¹ Figures in brackets indicate the literature references at the end of this paper.

is the change of volume caused by the motion of the diaphragm due to an applied voltage

$$u = u_0 e^{i\omega t}.$$

The volume change, v , due to the voltage, u , is called the converse effect (see fig. 2). There might be differences in phase among the quantities q , p , v , and u . To account for phase differences, the

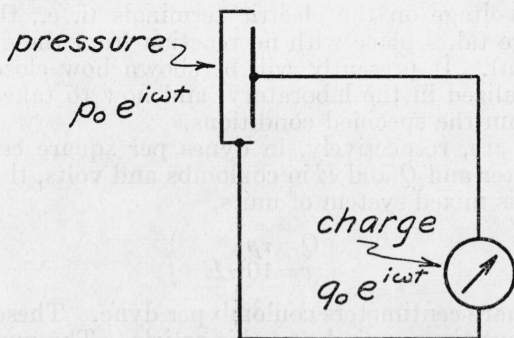


FIGURE 1.—Direct effect.
Transducer used as microphone.

coefficients q_0 , p_0 , v_0 , and u_0 , and the transduction coefficient, τ , are in general complex numbers, and τ might also be a function of frequency.

The principle of reciprocity applies without regard to the mechanism of operation of the transducer, provided only it is linear. The principle applies even if there are frictional forces or thermal actions in the

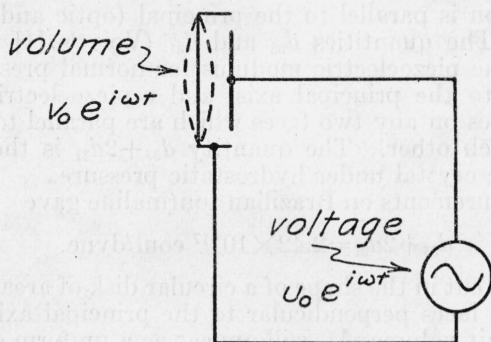


FIGURE 2.—Converse effect.
Transducer used as sound source.

transducer, provided these are linear (i. e., $\tau = q/p$ is independent of p). Thus a thermophone without a polarizing current is not linear when used as a sound source and the principle does not apply. Indeed, the thermophone used as a microphone under such circumstances has no response. But with a polarizing current the thermophone becomes a linear sound source (within certain limits), and the reciprocity theorem can be applied within these limits to compute the response as

a microphone from the output of the thermophone as a source of sound. Similarly, the principle can be applied to a condenser microphone having a polarizing voltage, but the principle is not applicable to an unpolarized condenser microphone.

Besides the assumption of linearity, it is also assumed that the only force applied in the direct effect is the pressure on the diaphragm (i. e., the electric terminals of the transducer are shorted, so that no electric forces are applied), and that the only force applied in the converse effect is the voltage on the electric terminals (i. e., the diaphragm volume change takes place with no reaction from the gas in front of the diaphragm). It presently will be shown how closely these conditions are realized in the laboratory, and how to take into account departures from the specified conditions.

If p and v are, respectively, in dynes per square centimeter and cubic centimeter and Q and E in coulombs and volts, then eq 1 and 2 become, in this mixed system of units,

$$\left. \begin{aligned} Q &= \tau p \\ v &= 10^7 \tau E \end{aligned} \right\} \quad (3)$$

and τ is in square centimeters coulomb per dyne. These units will be used throughout the remainder of this article. The quantities representing voltage, charge, pressure, and volume changes will henceforth have root-mean-square (rms) values.

2. APPLICATION TO PIEZOELECTRIC CRYSTALS

Tourmaline, a polar crystal of the trigonal system, acquires a uniform electric moment per unit volume under hydrostatic pressure. The magnitude of the moment per cubic centimeter is

$$M = (d_{33} + 2d_{31})p \text{ coul/cm}^2,$$

and its direction is parallel to the principal (optic and piezoelectric) crystal axis. The quantities d_{33} and d_{31} (Voigt's [1] notation) are, respectively, the piezoelectric modulus for normal pressures on faces perpendicular to the principal axis, and a piezoelectric modulus for normal pressures on any two faces which are parallel to the principal axis and to each other. The quantity $d_{33} + 2d_{31}$ is the piezoelectric modulus of the crystal under hydrostatic pressure.

Voigt's measurements on Brazilian tourmaline gave

$$d_{33} + 2d_{31} = 2.42 \times 10^{-17} \text{ coul/dyne.}$$

For a crystal cut in the shape of a circular disk of area A and having its parallel flat faces perpendicular to the principal axis, the electric moment per unit volume, M , will appear as a uniform electric charge per unit area, M , on the flat faces, or the total charge on each flat face will be

$$Q = AM = A(d_{33} + 2d_{31})p \text{ coul.}$$

Then, by the principle of reciprocity, a potential difference of e'_s (volts) applied between the flat faces will cause a volume change of the crystal of amount

$$v = 10^7 A (d_{33} + 2d_{31}) e'_s \text{ cm}^3. \quad (4)$$

A tourmaline crystal is then used in the following way as both sound source and microphone to secure an absolute pressure calibra-

tion of (for example) a condenser microphone, and to secure also the hydrostatic piezoelectric modulus of the crystal.

Let the response of the condenser microphone be

$$\rho = \frac{e_m}{p} \text{ cm}^2 \text{ volts/dyne,}$$

where e_m is the voltage output of the microphone due to the sound pressure, p . The response of the crystal used as a microphone (direct effect) is

$$\frac{A(d_{33} + 2d_{31})}{C} = \frac{Q}{Cp} \text{ cm}^2 \text{ volts/dyne,}$$

where C (farads) is the capacity of the crystal microphone. The crystal response is expressed in this way because Q/C (the voltage output of the crystal microphone) can be readily measured, whereas it is not feasible to measure Q directly.

The first step is to measure the ratio of the condenser-microphone response to the crystal-microphone response. The ratio is

$$\frac{\rho C}{A(d_{33} + 2d_{31})} = \frac{e_m}{e_s}, \quad (5)$$

and is got experimentally by applying the same pressure (or pressures in a known ratio) to both microphones and by measuring the ratio of the output voltages, e_m/e_s , by means of an attenuation box or known resistances. The voltage output of the condenser microphone is e_m , and that of the crystal for the same sound pressure is e_s .

The second step is to use the crystal as a source, acting on the condenser microphone as a microphone through a gas-filled cavity of volume, V (cm^3). If B (dynes/ cm^2) is the barometric pressure in the gas and γ is the ratio of specific heats of the gas, then the piezoelectric volume change of the crystal given by eq 4 will give rise to a pressure

$$p = \left(\frac{10^7 \gamma B}{V} \right) A(d_{33} + 2d_{31}) e'_s \text{ dynes/cm}^2,$$

provided v is small in comparison with V . The condenser microphone voltage output due to this pressure will be

$$e'_m = \left(\frac{10^7 \gamma B}{V} \right) A(d_{33} + 2d_{31}) \rho e'_s \text{ volts.} \quad (6)$$

Simultaneous solution of eq 5 and 6 yields

$$\left. \begin{aligned} \rho &= \sqrt{\frac{10^{-7} V}{\gamma B C} \left(\frac{e_m}{e_s} \right) \left(\frac{e'_m}{e'_s} \right)} \text{ cm}^2 \text{ volts/dyne} \\ A(d_{33} + 2d_{31}) &= \sqrt{\frac{10^{-7} C V}{\gamma B} \left(\frac{e_s}{e_m} \right) \left(\frac{e'_m}{e'_s} \right)} \text{ cm}^2 \text{ coul/dyne} \end{aligned} \right\} \quad (7)$$

Thus an absolute pressure calibration of a condenser microphone is obtained by absolute measurements of a volume, the barometric pressure, and a capacitance, and by measurements of two voltage ratios. An absolute determination of the adiabatic piezoelectric modulus of the tourmaline crystal under hydrostatic pressure is obtained by measurement of the crystal area in addition to the quanti-

ties enumerated above. The only quantity which is not measured directly is γ , whose values for air, hydrogen, and helium are accurately known.

Superimposed on the piezoelectric action in the direct effect will be the pyroelectric effect (since temperature changes in the gas which accompany pressure changes will penetrate into the crystal), and superimposed on the piezoelectric action in the converse effect will be the electrocaloric effect (since temperature changes in the crystal which accompany voltage changes will penetrate into the gas). But the principle of reciprocity will hold nevertheless, and eq 7 will give a quantity which represents a mixed piezoelectric and thermoelectric modulus at low frequencies, and which will approach asymptotically the adiabatic piezoelectric modulus at high frequencies.

In cavities of small volume, the elastic volume changes of the crystal (used as a microphone) due to an applied pressure, will in themselves give rise to a pressure superimposed on the applied pressure calculated on the assumption of no yielding of the crystal. The crystal suffers a volume change also if the faces are not at the same potential, because the potential difference, Q/C , acquired by the crystal causes a piezoelectric volume change by virtue of the converse effect, and likewise will give rise to a pressure superimposed on the calculated applied pressure. In effect, the piezoelectric modulus will appear to be a function of cavity volume, if the crystal volume changes are appreciable and due allowance is not made for them. This will be called the volume effect. For tourmaline the adiabatic compressibility is about 10^{-12} cm²/dyne. Therefore, the ratio of the pressure caused by elastic volume change to the applied pressure is less than 10^{-6} for cavity volumes greater than 10 cm³ and crystal volumes less than 4 cm³. For tourmaline the adiabatic piezoelectric modulus is about 2.4×10^{-10} cm/volt. Therefore, the ratio of the pressure caused by piezoelectric volume change to the applied pressure is less than 10^{-8} for cavity volumes greater than 10 cm³, crystal areas less than 15 cm², and crystal-microphone capacitance greater than 30×10^{-12} farad. With these restrictions, the volume effect for a tourmaline crystal used as a microphone is completely negligible, and by the principle of reciprocity the volume effect for a tourmaline crystal used as a source is also negligible, with the same restrictions. The volume effect for condenser microphones will be discussed in section 3.

Quartz and Rochelle salt have no electric moment under hydrostatic pressure. This conclusion is reached from considerations of the crystal symmetry of these materials. Consequently, such crystals cannot be used in the same way as tourmaline is used to obtain an absolute calibration of a microphone.

3. APPLICATION TO CONDENSER MICROPHONES

Let ρ_1 (cm² volts/dyne) be the response and C_1 (farads) the capacitance of a condenser microphone, I , which is to be used as both source and microphone to calibrate a second condenser microphone, II , whose response is ρ_2 (cm² volts/dyne). The equations of the direct and converse effects for the condenser microphone are

$$\left. \begin{aligned} Q &= \rho_1 C_1 p \\ v &= 10^7 \rho_1 C_1 E \end{aligned} \right\} \quad (8)$$

These correspond to eq 3. Microphone *I* is then used in exactly the same way as was the tourmaline crystal to secure an absolute pressure calibration of microphone *II*. That is to say, the first step is to obtain the ratio of the responses, ρ_1/ρ_2 , by applying the same pressure to both microphones, and the second step is to couple microphone *I* (used as a sound source) to microphone *II* by means of a gas-filled cavity of known volume, from which is obtained the product of the responses, $\rho_1 \rho_2$. The final equations for the responses, ρ_1 and ρ_2 , are

$$\left. \begin{aligned} \rho_1 &= \sqrt{\frac{10^{-7}V}{\gamma BC_1} \left(\frac{e_1}{e_2}\right) \left(\frac{e'_2}{e'_1}\right)} \text{ cm}^2 \text{ volts/dyne} \\ \rho_2 &= \sqrt{\frac{10^{-7}V}{\gamma BC_1} \left(\frac{e_2}{e_1}\right) \left(\frac{e'_2}{e'_1}\right)} \text{ cm}^2 \text{ volts/dyne} \end{aligned} \right\} \quad (9)$$

These correspond to eq 7. In these equations V (cm^3) is the volume of the cavity in which microphone *I* is used as a sound source, B (dynes/cm^2) is the barometric pressure in the cavity, γ is the ratio of the specific heats of the gas in the cavity, e_1 and e_2 are, respectively, the voltage outputs of microphones *I* and *II* for the same applied pressure, and e'_2 is the voltage output of microphone *II* for a voltage e'_1 applied to microphone *I* when used as a sound source (the two microphones being coupled by the cavity of volume V).

The condenser microphone diaphragm suffers a volume change if the potential difference between the diaphragm and back plate is not steady, because the alternating potential difference, Q/C , causes a volume change by virtue of the converse effect, and will therefore give rise to a pressure superimposed on the applied pressure calculated on the assumption of no volume change of the microphone. The ratio of the pressure caused by this volume change to the applied pressure is less than 5×10^{-3} for a condenser microphone having a response less than 3×10^{-3} cm^2 volt/dyne and a capacitance less than 3.5×10^{-10} farad, and for cavity volumes greater than 10 cm^3 . This part of the volume effect was therefore neglected.

The compressibility (admittance) of the diaphragm gives rise to a volume effect which is not negligible. An absolute calculation of the amount would require an exact knowledge of the construction of the microphone. In the absence of such knowledge, the amount of the volume effect must be determined experimentally by making response measurements at several different cavity volumes. Unfortunately, at high-frequencies volume effects due to cavity resonances swamp those due to diaphragm compressibility, and there is no way of separating the two effects. This means that a precise determination of the volume effect can be made only at low frequencies. Let the compressibility of the diaphragm *II* be c_2 (cm^5/dyne); that is, a pressure of p (dynes/cm^2) on the diaphragm causes a volume change of $c_2 p$ (cm^3). Then at frequencies less than the lowest natural frequency of diaphragm *II* the compressibility will have the effect of increasing the cavity volume which appears in eq 9 from V to $V + \gamma B c_2$. It is clear from the principle of reciprocity that the same volume effect will be present when the microphone is used as a sound source. If c_1 is the compressibility of diaphragm *I*, then the total effect will be to increase

the volume which appears in eq (9) from V to $V + \gamma Bc_1 + \gamma Bc_2$. The quantity γBc (cm^3) will hereafter be denoted by κ (cm^3), and will be called the volume constant of the microphone.

III. EXPERIMENTAL RESULTS

1. REVERSIBILITY OF TRANSDUCERS

A "reversible" transducer is one which satisfies the principle of reciprocity. A simple experimental way of ascertaining whether or not two transducers are reversible is the following. The transducers are coupled by a cavity of fixed volume. A voltage e_1 is applied to transducer *I* (source), and the corresponding voltage output e'_2 of transducer *II* (microphone) is measured. Then a voltage e_2 is applied to transducer *II* (source), and the corresponding voltage output e'_1 , of transducer *I* (microphone) is measured. Then the over-all coupled system is reversible, if the equation

$$\frac{C_2}{C_1} = \left(\frac{e_1}{e'_2} \right) \left(\frac{e'_1}{e_2} \right) \quad (10)$$

is satisfied, where C_2 and C_1 are the capacitances of transducers *II* and *I*, respectively. Thus eq10 is a necessary condition for the reversibility of each transducer. If a third transducer is used and eq10 holds for each pair of transducers, this will be a sufficient condition for the reversibility of each of the three. Actually, it will be entirely safe to assume each of two transducers is reversible if eq10 holds when the two transducers are coupled. Thus the reversibility of a transducer is determined experimentally by purely electrical measurements of two voltage ratios and a capacitance ratio.

Such measurements were made on the following: A Western Electric condenser microphone type 394, No. 4829 (denoted hereafter as microphone 2); a Western Electric condenser microphone type D96436, No. 70 (denoted hereafter as microphone 70); a tourmaline crystal transducer (denoted hereafter as transducer *CD*). Transducer *CD* was made of black California tourmaline, 3.8 cm in diameter and 0.3 cm in thickness. The front face was coated with tin foil affixed with wax, and the tourmaline was wrung on to a brass block. The block was insulated from the outside brass block (at ground potential) which was connected to the tin foil.

The voltage ratios appearing in eq10 were measured at various frequencies by means of an attenuation box to the nearest decibel and by means of an output meter to the nearest one-tenth decibel. The capacitances were separately measured at 1,000 c/s (by means of a strict substitution method) with a variable capacitance calibrated in the capacitance section of this Bureau. The results are shown in figure 3. The gases which were used with each pair of transducers are indicated in the legend. The sound pressure was about 2×10^{-2} dyne/cm² with the tourmaline transducer *CD* as source, and about 10 dynes/cm² with transducer *CD* as microphone. In the other two cases plotted in figure 3, the pressure was about 1 dyne/cm².

The estimated probable error in $20 \log_{10} (C_2/C_1)$ is 0.1 db, and in $20 \log_{10} (e_1/e'_2) (e'_1/e_2)$ is less than 0.2 db. The conclusion from the results plotted in figure 3 is, that within the experimental uncertainty

eq10 is satisfied. Consequently, condenser microphones 2 and 70 and tourmaline transducer *CD* are reversible. This is a direct experimental proof of the equality of the direct and converse piezoelectric moduli in tourmaline under hydrostatic pressure.

After Lippmann's theoretical proof of the equality of the direct and converse piezoelectric moduli, J. and P. Curie made an attempt to verify the equality for the modulus, d_{11} in quartz, but experimental difficulties enabled them to obtain only a very rough agreement between the two moduli. Since then, H. Osterberg and J. W. Cookson [4] have concluded that measurements of d_{11} by some observers who used the direct effect only have agreed on the whole with measurements of d_{11} by other observers who used the converse effect only.

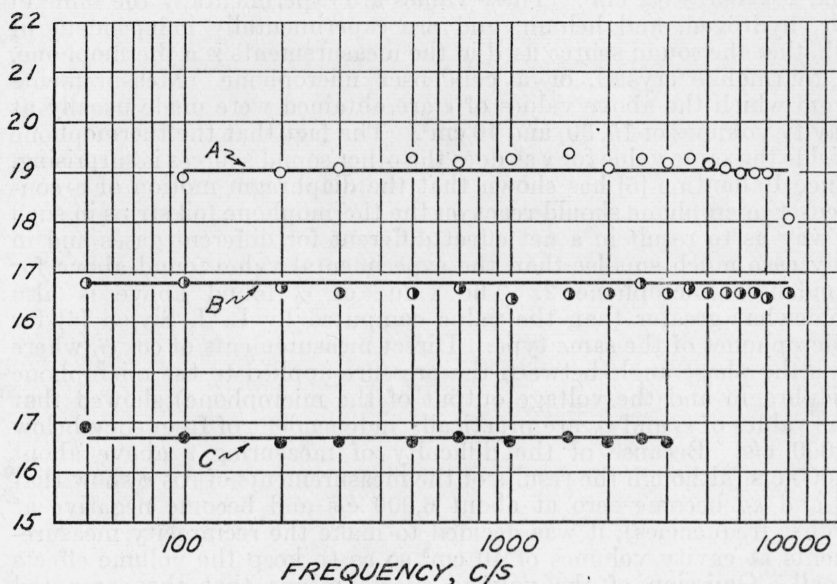


FIGURE 3.—Graph showing reversibility of tourmaline transducer and condenser microphones.

○ = $20 \log_{10} (e_1/e'_2) (e'_1/e_2)$, and curve A = $20 \log_{10} (C_2/C_1)$ at 1,000 c/s, for condenser microphone 2 and tourmaline transducer *CD*, in helium.

● = $20 \log_{10} (e_1/e'_2) (e'_1/e_2)$, and curve B = $20 \log_{10} (C_2/C_1)$ at 1,000 c/s, for condenser microphones 2 and 70, in helium.

● = $20 \log_{10} (e_1/e'_2) (e'_1/e_2)$, and curve C = $20 \log_{10} (C_2/C_1)$ at 1,000 c/s, for condenser microphones 2 and 102, in air.

Because of variations in the composition of tourmaline from different sources, it is not possible, in order to decide the equality of the two moduli for tourmaline, to compare the results of some observers who used the direct effect only with the results of other observers who used the converse effect only.

2. MICROPHONE CALIBRATIONS

The reversibility of condenser microphones 2 and 70 and of transducer *CD* having been established, the principle of reciprocity was applied to data obtained with these transducers to secure absolute pressure calibrations of microphones 2 and 70.

The results are shown in figures 4 and 5. The reciprocity calibrations are those obtained with condenser microphone 2 used as both source and microphone. The cavity volumes and gases which were used are indicated in the legends. The estimated probable error of measurement is 0.2 db. The volume effects of the microphone diaphragms and the cooling effect of the walls of the cavity have not been included in the responses, which were computed directly from eq 9 (the geometrical volume of the cavity being used for V). These omissions introduce systematic errors not much greater than the probable error stated above. The amounts of these are deduced in the following paragraph.

Measurements of the volume constant, κ , gave $\kappa_2=1.3 \pm 0.2 \text{ cm}^3$, and $\kappa_{70}=0.3 \pm 0.2 \text{ cm}^3$. These values are experimentally the same in air, hydrogen, and helium, and are experimentally independent of whether the sound source used in the measurements is a thermophone, a tourmaline crystal, or a condenser microphone. Measurements from which the above values of κ are obtained were made usually at cavity volumes of 15, 30, and 40 cm^3 . The fact that the thermophone yields the same value for κ as does the other sound sources is surprising, since Ballantine [5] has shown that the diaphragm motion of a condenser microphone should react on the thermophone foil strips in such a way as to result in a net effect different for different gases and in any case much smaller than the experimental value found above for condenser microphone 2. The value of κ_2 found above is also somewhat greater than the value computed by L. J. Sivian [6] for microphones of the same type. Direct measurements of $\cos \phi$ (where ϕ is the phase angle between the pressure applied to the microphone diaphragm and the voltage output of the microphone) showed that the values of κ_2 and κ_{70} are practically independent of frequency below 3,000 c/s. Because of the difficulty of measuring κ above about 3,000 c/s (although the results of the measurements of $\cos \phi$ show that κ_2 and κ_{70} become zero at about 6,000 c/s and become negative at higher frequencies), it was decided to make the reciprocity measurements at cavity volumes of 40 cm^3 so as to keep the volume effects small. Omission of the volume effects means that the computed response of microphone 2 is low by 0.3 db and of microphone 70 is low by 0.1 db below 3,000 c/s. The cooling effect of the walls of the cavity must also be considered. It is usually incorrectly assumed that the temperature variation of the gas at the walls of the cavity is zero, on the grounds that the thermal conductivity of the metal walls is much greater than that of the gas. However, in a quasi-stationary temperature distribution (all temperatures sinusoidal functions of the time) the quantity of importance is the thermal diffusivity, which is the thermal conductivity divided by the product of density and specific heat. The thermal diffusivities of brass, air, hydrogen, and helium are, respectively, at room temperature and barometric pressure 0.34, 0.28, 1.67, and 1.78 $\text{cm}^2 \text{ sec}$. From these values it can be deduced that a better boundary condition at the walls of the cavity is that the rms temperature at the walls is one-half the rms temperature in the gas at a distance from the walls. The inclusion of this boundary condition in the reciprocity calibrations raises the computed responses of microphones 2 and 70 by less than 0.1 db, for frequencies greater than 50 c/s and a cavity volume of 40 cm^3 , and for any of the three gases mentioned above. The upshot of all the fore-

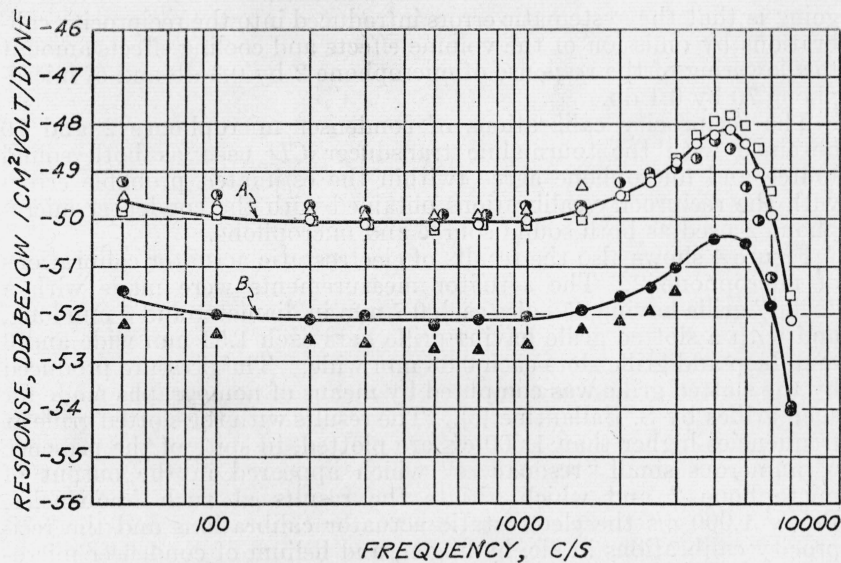


FIGURE 4.—Calibration graph of condenser microphone 2.

- and curve A = reciprocity in hydrogen, cavity volume 40 cm³.
- △ = reciprocity in air, cavity volume 40 cm³.
- = reciprocity in helium, cavity volume 40 cm³.
- = electrostatic actuator.
- and curve B = thermophone in hydrogen, cavity volume 8.5 cm³.
- ▲ = thermophone in air, cavity volume 8.5 cm³.

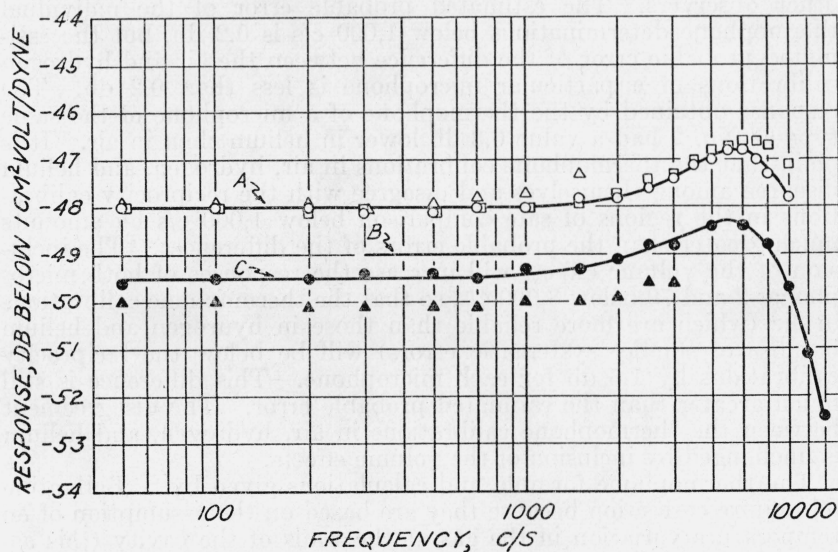


FIGURE 5.—Calibration graph of condenser microphone 70.

- and curve A = reciprocity in hydrogen, cavity volume 40 cm³.
- △ = reciprocity in air, cavity volume 40 cm³.
- = reciprocity in helium, cavity volume 40 cm³.
- Curve B = Rayleigh disk.
- and curve C = thermophone in hydrogen, cavity volume 10.8 cm³.
- ▲ = thermophone in air, cavity volume 10.8 cm³.

going is that the systematic errors introduced into the reciprocity calibrations by omission of the volume effects and cooling effects amount to a lowering of the response of microphone 2 by 0.3 db and of microphone 70 by 0.1 db.

The reciprocity calibrations of condenser microphones 2 and 70 obtained with the tourmaline transducer *CD* used as both sound source and microphone agree (within the estimated probable error) with the reciprocity calibrations obtained with the condenser microphone 2 used as both sound source and microphone.

Figure 4 shows also the results of electrostatic actuator calibrations of microphone 2. The actuator measurements were made with a "solid" grille having 25 holes each 0.7 mm in diameter and 3 mm long, and with a slotted grille having grille bars each 1.53 mm wide and 3 mm deep and grille slots each 0.88 mm wide. The pressure produced by the slotted grille was computed by means of nomographs made for such grilles by S. Ballantine [5]. The results with the slotted grille at frequencies higher than 1,000 c/s are plotted, in spite of the presence of numerous small "resonances" which appeared in the output of microphone 2 and which vitiate the results at such frequencies. Below 1,000 c/s the electrostatic actuator calibrations and the reciprocity calibrations in air, hydrogen, and helium of condenser microphone 2 all agree within the estimated probable error.

Figures 4 and 5 also show the results of gold foil thermophone calibrations of condenser microphones 2 and 70. The thermophone results were computed with the formula given by S. Ballantine [5], except for minor corrections which will be presently discussed. The thermophone cavity volumes are about the sizes used by several other observers. The estimated probable error of the individual thermophone determinations below 1,000 c/s is 0.2 db, but the estimated probable error of the difference between the air and hydrogen calibrations of a particular microphone is less than 0.2 db. The response obtained by the thermophone of a microphone of the same type as No. 2 had a value 0.3 db lower in helium than in air. It is plain that the thermophone calibrations in air, hydrogen, and helium disagree among themselves and disagree with the reciprocity calibrations in the regions of safe comparison below 1,000 c/s by amounts much greater than the probable errors of the differences. The inclusion of the volume effects will increase the responses of both microphones 2 and 70 below 3,000 c/s, so that the thermophone calibrations in air (which are more reliable than those in hydrogen and helium because of smaller systematic errors) will be below the reciprocity calibrations by 1.5 db for each microphone. This difference is still much greater than the estimated probable error. The disagreement between the thermophone calibrations in air, hydrogen, and helium is unchanged by inclusion of the volume effects.

The thermophone formula and calculations given by S. Ballantine [5] require correction because they are based on the assumption of no temperature variation in the gas at the walls of the cavity (this assumption was discussed above) and on the assumption that commercial gold foil has the same specific heat as pure gold. The specific heat of the gold foil, calculated from the results of a chemical assay, was 0.0328 cal/g° C, whereas that of pure gold is 0.0313 cal/g° C. The two corrections are in opposite directions and almost nullify one

another, thus leaving calculations made from Ballantine's thermophone formula practically unchanged.

Pistonphone measurements were made by Guy Cook and the author on a condenser microphone of the same type as microphone 2. The measurements were made between 50 and 200 c/s in a cavity of 15 cm³ at a sound pressure of about 5 dynes/cm². Air, hydrogen, and helium were used. The piston was driven by a loudspeaker coil, and its amplitude of motion was measured by means of an optical lever and autocollimator. The amplitude measurements were checked by a microscope having a calibrated graticule. The computed probable error of the pistonphone results was 0.3 db, and the results agreed with an electrostatic actuator calibration of the same microphone within the probable error.

Resonant-tube measurements were made by Guy Cook and the author on the same condenser microphone. The microphone was placed at one end of the resonant tube. The amplitude of motion of illuminated tobacco-smoke particles was observed (with a microscope having a calibrated graticule) at the central displacement loop in the gas in the resonant tube. Observations were made at 95 and 285 c/s at a sound pressure of about 75 dynes/cm². The estimated probable error of the resonant tube results was 0.5 db, and the results agreed with the electrostatic actuator calibration of the same microphone within the estimated probable error.

The disk curve *B* in figure 5 is placed 0.5 db above the thermophone calibration in hydrogen, and is the result of Rayleigh-disk and thermophone comparisons between 300 and 1,000 c/s on several other microphones of the same type as microphone 70. The Rayleigh-disk measurements were made in the free field by Guy Cook.

The conclusion is, that in the usually accepted thermophone formula there are systematic errors which are different for different gases, and which are much greater than the error of measurement. The trouble probably lies in the assumed modus operandi of the thermophone. It has been suggested that the gold foil possibly adsorbs and emits gas during each thermal cycle. Another suggestion is, that viscosity forces have an effect on the pressure produced by the thermophone. The computed responses in air, hydrogen, and helium are in the same order as the viscosity coefficients of these gases.

3. PIEZOELECTRIC MODULI

The results of acoustic measurements, based on the principle of reciprocity, of the adiabatic piezoelectric modulus $d_{33} + 2d_{31}$ of California tourmaline are shown in figure 6. The tourmaline was crystal *CD*, which was used as both source and microphone.

The results of static testing-machine measurements (by use of the direct effect) of the modulus d_{33} and Voigt's [1] static measurements (also by use of the direct effect) on Brazilian tourmaline are also given in figure 6. The frequency scale of the abscissas does not apply to the horizontal lines representing the static tests; these lines were drawn merely to facilitate comparisons with the acoustic measurements. Static measurements yield isothermal moduli, whereas the acoustic measurements yield adiabatic moduli. However, D. A. Keys [7] has shown from thermodynamic considerations that the difference between the isothermal and adiabatic moduli is less than

0.3 percent. Below 500 c/s the acoustic measurements do not yield the adiabatic modulus, since the droop in the measured modulus $d_{33}+2d_{31}$ at low frequencies is due to the superposition of thermo-electric effects on the piezoelectric action. The droop above 3,000 c/s is caused by cavity resonances.

The testing-machine measurements of the modulus d_{33} were made at loads between 1,000 and 3,000 lb in a hydraulic testing machine. A preassigned load was applied to a crystal disk 3.8 cm in diameter and 0.15 cm thick, and the electric charge released from the crystal on sudden release of the testing-machine load was measured by a calibrated ballistic galvanometer. A check of the experimental setup

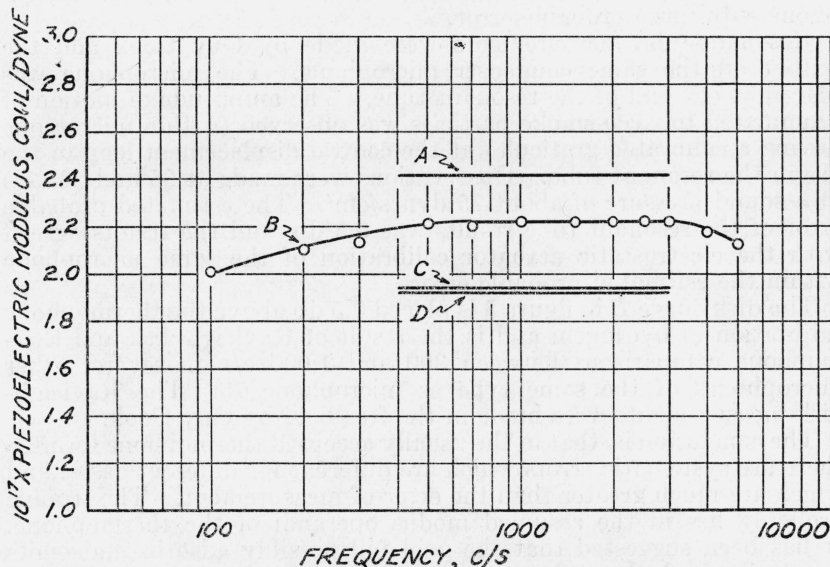


FIGURE 6.—Graph of piezoelectric moduli of tourmaline.

- and curve B=acoustic measurements of $d_{33}+2d_{31}$ for California tourmaline.
 Curve A=static measurements by Voigt of $d_{33}+2d_{31}$ for Brazilian tourmaline.
 Curve C=static testing-machine measurements of d_{33} for California tourmaline.
 Curve D=static measurements by Voigt of d_{33} for Brazilian tourmaline.

was made by measuring the piezoelectric modulus d_{11} of quartz, which is well known. The estimated probable error in the measured value of the modulus d_{33} for black California tourmaline is 2 percent.

The curious discovery was made that it is absolutely essential to load the crystal through rigid (e. g., steel) bearing blocks. Erroneous results were obtained when the tourmaline and quartz were loaded through Plexiglass (a methacrylate resin of low modulus of elasticity) bearing disks. It was established that these anomalies were not due to friction. Evidently the crystal is subjected to thermal actions and to large shearing stresses (in addition to the compressive stresses) when loaded through Plexiglass disks in contact with the crystal, and these thermal actions and shearing-stresses cause the anomalous piezoelectric effects.

The following measurements were made on California tourmaline, used as a sound source, in addition to the measurements of $d_{33}+2d_{31}$ given in figure 6. Three crystals (each 3.8 cm in diameter and 0.15

cm thick) cut from the same batch of tourmaline had identical piezoelectric moduli. When two of these disks were wrung together with vaseline so that the analogous ends of their crystal axes pointed in the same direction, the piezoelectric modulus was the same as that of a single disk, which shows that the acoustically measured modulus is independent of the crystal thickness. When the same two disks were wrung together so that the analogous ends of their axes pointed in opposite directions, the piezoelectric modulus was zero. Each disk suffered a volume change under the applied voltage, but the net volume change was zero, since the individual volume changes differed in phase by π . The piezoelectric modulus was independent of applied voltage in the range from 130 to 1,300 volts. The foregoing measurements were made in air, between 200 and 1,000 c/s, in a cavity of about 20-cm³ volume. It is understood that the conclusions are valid only within the experimental uncertainty, which is about 2 percent of the modulus $d_{33} + 2d_{31}$. In addition, a determination of the volume constant, κ , for a crystal yielded $\kappa = 0$, within the experimental uncertainty of about 0.3 cm³, which confirms the theoretical conclusion that a crystal used as a transducer should have no detectable volume effect.

A quartz-crystal disk, 3.6 cm in diameter and 0.46 cm thick, having its flat faces perpendicular to one of the three diagonal axes of the quartz, was used as a sound source in a cavity of about 20 cm³. The measured piezoelectric modulus was about 1 percent of that of tourmaline at 500 c/s. At lower frequencies the disk had a small sound output, evidently due to electrocaloric effects superimposed on the piezoelectric. The conclusion is, that within the experimental uncertainty, the piezoelectric modulus of quartz under hydrostatic pressure is zero. This confirms experimentally the theoretical conclusion reached from considerations of crystal symmetry.

IV. INSTRUMENTS AND TECHNIQUE

The cavities were made of simple geometrical shapes (e. g., right circular cylinders), and the volumes were computed from measurements of the linear dimensions. The scale and micrometer calipers used in these measurements were compared with secondary standards kept in this Bureau. The estimated probable error in the measurement of volume is 0.4 percent.

The barometric pressure was measured by an aneroid barometer which from time to time was compared with a mercury barometer. The estimated probable error in the measurement of pressure by the aneroid barometer is 0.1 percent.

Microphone capacitances were measured by a strict substitution method. A variable air condenser, calibrated in the capacitance section of this Bureau, was used as a secondary standard. As a check on the technique, a particular condenser was measured in both the sound chamber and in the capacitance section. The estimated probable error in capacitance measurement is about 1 percent.

Voltage ratios were measured by means of an attenuation box to the nearest decibel, and by a copper oxide rectifier output meter to the nearest 0.1 db. The attenuation box was calibrated by comparison with a calibrated potentiometer, and had errors less than 0.1 db. The estimated probable error in the measurement of voltage ratio is

0.1 db. The output meter was used over a range not exceeding 0.5 db.

Nearly all the experimental measurements were made at room temperatures between 20° and 25° C, and at barometric pressures between 742 and 758 mm Hg. Within these limits there was no dependence of the responses of the condenser microphones on temperature or barometric pressure. Small variations in the responses (about 0.5 db in range) seemed to be entirely random in character.

Various tricks of technique were used to detect and eliminate spurious voltages in the microphone circuit. Such voltages are the commonest cause of trouble. For example, electrostatic coupling between the gold foil of the thermophone and the high-potential side of the tourmaline microphone causes a spurious voltage in the tourmaline-microphone voltage output. This will easily occur because the voltage applied to the thermophone is about 10^4 times greater than the voltage output of the tourmaline microphone. In another case, resistance coupling between the tourmaline-crystal sound-source circuit and the low-potential (ground) side of the condenser microphone causes a spurious voltage in the condenser-microphone voltage output. This might happen because the voltage applied to the crystal source is about 10^6 times greater than the voltage output of the condenser microphone. Reversal of the polarizing voltage on the condenser microphone (or, in the case of the thermophone, reversal of the polarizing current) will change the phase of the spurious voltage with respect to the voltage generated in the microphone by π , and the voltage output of the microphone will not be independent of polarity (as it should be), if the spurious voltage is appreciable. In crystal measurements, substitution of a dummy glass (or any nonpiezoelectric material) transducer for the crystal transducer will readily reveal spurious voltages.

V. SUMMARY

Application of a general principle of reciprocity to the action of a piezoelectric crystal or to a microphone yields absolute pressure calibrations of another microphone, and also yields absolute values of the adiabatic piezoelectric modulus under hydrostatic pressure. Absolute measurements of a volume, the barometric pressure and a capacitance are needed in addition to measurements of two voltage ratios; also, the ratio of specific heats of the gas used in the cavity is required.

Calibrations secured in this way are in good agreement with electrostatic actuator, pistonphone, and "smoke particle" calibrations of a condenser microphone. Thermophone calibrations in air, hydrogen, and helium disagreed among themselves and with the reciprocity calibrations of the same microphones by amounts much greater than the experimental uncertainty.

Absolute acoustic measurements of the adiabatic piezoelectric modulus of black California tourmaline under hydrostatic pressure gave $d_{33} + 2d_{31} = (2.22 \pm 0.06) \times 10^{-17}$ coul/dyne. It was experimentally proved that the direct and converse piezoelectric moduli $d_{33} + 2d_{31}$ are equal, within the experimental uncertainty.

The following is a tentative set-up for securing absolute pressure calibrations of condenser microphones. A tourmaline-crystal source, calibrated by application of the principle of reciprocity, will be the

primary standard. A condenser microphone will serve as a secondary-standard source, which will be calibrated from time to time by the primary tourmaline standard.

There are common crystalline household substances which are piezoelectric under hydrostatic pressure, and which might be used as sound sources. Such are sucrose (cane sugar) and tartaric acid (sour salt). The latter at room temperatures has a piezoelectric modulus about double that of tourmaline.

The theoretical conclusions and experimental results contained in this article were reported at the meeting of the Acoustical Society of America in Washington, D. C., in April 1940. Similar theoretical conclusions were reached by W. R. MacLean [8] and published in the July 1940 issue of the *Journal of the Acoustical Society of America*.

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VI. REFERENCES

- [1] W. Voigt, *Lehrbuch der Kristallphysik* (B. G. Teubner, Leipzig, 1910).
- [2] Rayleigh, *The Theory of Sound* (Macmillan & Co., London, 1896).
- [3] S. Ballantine, *Proc. Inst. Radio Engrs.* **17**, 929 (1929).
- [4] H. Osterberg and J. W. Cookson, *Rev. Sci. Instr.* **6**, 347 (1935).
- [5] S. Ballantine, *J. Acous. Soc. Am.* **3**, 319 (1932).
- [6] L. J. Sivian, *Bell System Tech. J.* **10**, 96 (1931).
- [7] D. A. Keys, *Phil. Mag.* [6] **46**, 999 (1923).
- [8] W. R. MacLean, *J. Acous. Soc. Am.* **12**, 140 (1940).

WASHINGTON, August 14, 1940.