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HUE, SATURATION, AND LIGHTNESS OF SURFACE COLORS WITH CHROMATIC ILLUMINATION

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ABSTRACT

The visual mechanism of a normal observer is so constructed that objects keep nearly their daylight colors even when the illuminant departs markedly from average daylight. The processes by means of which the observer becomes adapted to the illuminant or discounts most of the effect of a nondaylight illuminant are complicated; they are known to be partly retinal and partly cortical. By taking into account the various fragments of both qualitative and quantitative information to be found in the literature, relations have been formulated by means of which it is possible to compute approximately the hue, saturation, and lightness (tint, value) of a surface color from the tristimulus specifications of the light reflected from the surface and of the light reflected from the background against which it is viewed. Preliminary observations of 15 surfaces under each of 5 different illuminants have demonstrated the adequacy of the formulation, and have led to an approximate evaluation of the constants appearing in it. More detailed and extensive observations have been carried out in the psychological laboratories of Bryn Mawr College, and these observations have resulted in an improved formulation.

CONTENTS

I. INTRODUCTION

For many years it was the view of psychologists that the chromatic-adapting power of the eye is unlimited. Constant color stimuli, they said, when acting for a long period on the retina, gradually become less effective and finally fail to produce sensations of chromatic color. When, for example, chromatically tinted glasses are continuously worn, external objects are seen sooner or later nearly in their natural or daylight colors.

This view is a correct one only when applied to moderate departures from daylight in the spectral distribution of energy of the illuminant, but it will readily be understood that there are illuminants (for example, those having all of their energy confined to a narrow wavelength region) by which there is no basis, except memory, through which all objects can possibly assume their daylight colors. For such illuminants, or for vision by an observer wearing highly selective glasses, the old view is, therefore, quite wrong [27],[1] and it becomes a matter of some interest, since we know that the eye cannot possibly detect color differences in the absence of a corresponding stimulus difference, to discover what degree of departure from daylight can be compensated for by chromatic adaptation.

The problem stated in this way was proposed by Harry Helson, Bryn Mawr College, in 1932, and in the course of planning experimental work for its solution, to be carried out jointly at the National Bureau of Standards and at Bryn Mawr College, it was found expedient to restate the problem in the following somewhat broader way: Given the radiant energy of the illuminant as a function of wavelength and the spectral apparent reflectances of the surfaces in the field of view, it is required to compute the hue, saturation, and lightness of the surface colors.

It may be noted that the latter problem includes the former and is more easily susceptible of explicit solution. Thus a slight change in illuminant changes the appearance of objects slightly, and the question of how much selectivity of illuminant can be tolerated must be answered in terms of how much change in surface color is to be tolerated. There also would be some commercial interest in the solution of this problem, if it could be done cheaply enough for a given sample; thus a frequent question to be answered by the textile salesman is, "Does this piece of goods change color in artificial light?" Another commercially important question is, "These two color samples harmonize in daylight, but do they go well together in artificial light?" The general problem stated above includes the first of these as a subproblem, and it bears importantly on the other two. The general problem, furthermore, includes that of description of chromaticity differences between large chromatic samples viewed in daylight, as by an inspector.

It is the purpose of the present paper to summarize the general principles which have been found in the literature; to present formulas for hue, saturation, and lightness which embody these principles; to describe experiments designed to test in a preliminary way the adequacy of the formulas; and to show the agreement between the formulas and preliminary experiment.

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[1] Figures in brackets (sometimes followed by a page number) indicate the literature references at the end of this paper.

II. DEFINITIONS

1. PSYCHOLOGICAL TERMS

Aperture color (*Lochfarbe* [7, 48], *reduced color* [60, p. 42], *film color* [58, 60], *Flächenfarbe* [60, p. 23; 48])—color such as that experienced as filling a hole in a screen (hence, usually as a small portion of the visual field); it may be either near the plane of the screen or indefinitely far behind it; it is neither concentrated in a single plane nor spread throughout a definite volume; it is nonlocated in depth. Aperture color has the following attributes: hue, saturation, and brightness.

Volume color (*bulky color* [58], *body color* [92])—color experienced as a property of a volume or bulk.

Surface color [58] (*Oberflächenfarbe* [7, p. 419; 16; 48; 92])[2]—color experienced as a property of a surface. Surface colors are hard, resist the gaze, take the plane of the object surface, and have its texture [60, p. 23]. We deal here with three of the attributes of surface color—hue, saturation, and lightness.

Hue [87, p. 534]—that attribute of certain colors which permits them to be classed as red, yellow, green, blue, and their intermediates.

Chromatic colors [87, p. 535]—colors possessing hue.

Achromatic colors [87, p. 535]—colors not possessing hue. There is, for example, a series of achromatic aperture colors ranging between very dim and very bright, and a series of achromatic surface colors (grays) ranging between black and white.

Saturation.—The attribute of any chromatic color which determines the degree of its difference from the achromatic color most closely resembling it.

Brightness (*insistence* [48, p. 462; 60, p. 38], *Helligkeit* [7])—the attribute of any aperture color which permits it to be classed as equivalent to some member of the series of achromatic aperture colors ranging between *very dim* and *very bright*.

Lightness [37, p. 213] (*brilliance* [87, p. 534], *brightness* [60], *whiteness* [52, p. 243], *Weisslichkeit* [16, p. 612; 7])[3]—the attribute of any surface color which permits it to be classed as equivalent to some member of the series of grays ranging between *black* and *white*.

2. PSYCHOPHYSICAL TERMS

Luminous reflectance (hereinafter simply *reflectance*)—the ratio of the quantity of light reflected from a surface to that incident on it.

Luminance [37] (*brightness* [33, p. 6])—the quotient of the luminous intensity of a surface measured in a given direction by the area of this surface projected on a plane perpendicular to the direction considered.

Apparent reflectance, *A*, [61, p. 33]—the reflectance which a perfectly diffusing surface, under the same illuminating and viewing conditions, would have to possess in order to yield the same luminance as the surface in question. In the present work, Munsell samples

[2] Wood [92] uses the term "*color*," only in the restricted sense of chromatic color.

[3] Bocksch [7, p. 430] uses the terms *Helligkeit* and *Weisslichkeit*, in accord with our terms *brightness* and *lightness*, respectively; he makes a special point, however, of his belief that the distinction is not connected with that between surface color and aperture color, but applies to all color impressions regardless of mode of appearance. We have adopted the view given by Helmholtz [30, p. 130] as characteristic of ordinary speech; this view is supported by Fiedler's experiments [15], and also agrees with Mintz [63], who considers Fiedler's experiments to be inconclusive.

were used, and these are so nearly perfect diffusers that the illuminating and viewing conditions need not be carefully controlled, provided unidirectional illumination, with viewing at the angle of regular reflection, be avoided, as has always been done in these experiments.

Tristimulus specification [39, p. 359]—specification of the stimulus for an aperture color consisting of the amounts of three primary stimuli required to produce a color match; this specification may be computed from the spectral-energy distribution of the stimulus [23, 39], provided the visual characteristics of the observer be known. In the present work the OSA standard observer [18; 87, p. 549] was used, and the coordinate system was the uniform-chromaticity-scale (UCS) triangle [40].

Trilinear coordinates (r, g, b)—the amounts of the three primary stimuli expressed as fractions of their total. In the present work a tristimulus specification is given in the form of the relative luminance of the stimulus (proportional to the apparent reflectance of the sample since we deal here only with groups of equally illuminated samples in the same plane viewed by reflected light) combined with two of the three trilinear coordinates, it being unnecessary to give the third because, taken together, the three sum to unity by definition.

3. SPECIAL SYMBOLS

$\bar{A}\equiv$ arithmetical average apparent reflectance of the samples in the field of view.

$\bar{A}_{\log}\equiv$ logarithmic average of the same quantities.

$A_0\equiv$ apparent reflectance of the background against which the samples are viewed.

$A_f\equiv$ apparent reflectance of field—the average apparent reflectance of the samples and background weighted according to proximity in past time [30, p. 268] and in space [56, p. 238] to the central part of the visual field.[4]

$A'\equiv$ adaptation reflectance—the apparent reflectance at which a sample of trilinear coordinates equal to those of the field (r_f, g_f, b_f) appears most nearly achromatic, a function of the whole viewing situation.

$I\equiv$ illuminance [37, p. 208] of sample plane in foot candles.

$(\mathrm{r}_0, \mathrm{g}_0, \mathrm{b}_0)\equiv$ trilinear coordinates of the illuminant.

$(r_f, g_f, b_f)\equiv$ trilinear coordinates of the field, the parts of the field being weighted in taking an average as in the definition of A_f.

$(r_n, g_n, b_n)\equiv$ trilinear coordinates of the stimulus for an achromatic color; these coordinates are found to be functions of the illuminant, of the particular sample fixated, and of the samples near the fixation point in the immediate past.

$D\equiv[(r-r_n)^2+(g-g_n)^2+(b-b_n)^2]^{1/2}$—distance on the UCS triangle between sample point (r,g,b) and achromatic point (r_n,g_n,b_n).

[4] Wright [94, p. 7] makes use of a parameter, the integrated light from the scene, in his discussion of adaptation; this variable is proportional to IA_f.

$D_f \equiv [(r_f - 0.44)^2 + (g_f - 0.47)^2 + (b_f - 0.09)^2]^{1/2}$—distance from daylight point (0.44, 0.47, 0.09) on the UCS triangle to the field point (r_f, g_f, b_f). This distance has been found to be a fairly good index of the amount of chromatic adaptation induced by a 5-minute exposure of the observer to the field; it enters into the formulas for the trilinear coordinates of the achromatic point.

$H \equiv$ hue estimated by the observer on the following eight-point scale given to each observer as a part of his instructions: red, yellow-red, yellow, yellow-green, green, blue-green, blue, red-blue.[5] Some of the observers on their own initiative developed a 16-point hue scale formed by the introduction of intermediate hue names, such as bluish red, yellowish red, reddish yellow, and so on. Computed values of hue are given on the 8-point scale; summaries of observed hues, on the 16-point scale. The hue notation for an achromatic color is N, meaning "no hue."

$S \equiv$ saturation estimated by the observer on an 11-point scale running from zero for an achromatic color (black, gray, or white) up to 10 for the most saturated of the 15 daylight surface colors presented.

$L \equiv$ lightness estimated by the observer on an 11-point scale running from zero for black up to 10 for white, the intermediate values being used to designate a series of grays with uniform visual spacing.

$L' \equiv$ lightness expressed according to a formula derived from one deduced on theoretical ground and experimentally verified by Adams and Cobb [2], with constants adjusted to make the zero point of the scale correspond to adaptation reflectance, A', thus: $L' \equiv 10A/(A + A_f) - 3.0$, from which it follows that $A' = (3/7)A_f$. As may be seen from the definition of adaptation reflectance, this choice of constants has to do with specification of the sample of trilinear coordinates (r_f, g_f, b_f) which appears most nearly achromatic, and it states that any such sample of apparent reflectance 3/7 that of the field will appear more nearly achromatic than any other such sample.

III. STATEMENT AND EXPLANATION OF FORMULAS

1. ASSUMPTIONS

The formulation is intended to apply to the colors of a group of matte, opaque surfaces viewed against a background in strong uniform illumination; it is based upon the following assumptions.

(a) COLOR MATCHING

If the aperture colors derived from two matte, opaque surfaces are identical (in hue, brightness, and saturation) and the surfaces are compared under the same illuminating and viewing conditions,

[5] This scale has also been suggested by Tschermak [91, p. 335].

it is assumed that the surface colors will be identical; and, conversely, if the aperture colors are different, the surface colors will be different. This assumption underlies all colorimetry of reflecting surfaces.

(b) HUE

It is assumed that the hue of an aperture color is determined by the direction of the straight line in the triangular mixture diagram connecting the point (r,g,b) representing the aperture color and the point (r_n,g_n,b_n) representing an achromatic aperture color of the same brightness. This is the usual interpretation of trilinear-coordinate differences; and by a further assumption, often unwarranted, that a single point can be chosen to represent achromatic colors, this assumption has led to the specification of hue by means of dominant wavelength [21; 23, p. 11; 73].

(c) SATURATION

It is assumed that the length, D, of this line indicates the saturation of the aperture color. This assumption is based upon the spacing of color stimuli yielded by the uniform-chromaticity-scale Maxwell triangle; it is regarded as a first approximation to the truth. The validity of the assumption is limited by possible failure of the UCS system to yield, as intended, substantially a uniform spacing for colors separated by small chromaticity differences, by possible failure of chromaticity differences to be arithmetically additive, and by possible important dependence of saturation upon brightness.

(d) CHROMATIC ADAPTATION

It is assumed that hue and saturation of a surface color are equal, respectively, to the hue and saturation of the aperture color derived from it for the same adaptive condition (retinal and cortical) of the observer. This assumption is a usual one among colorimetrists. If the adaptive state is not held constant, this assumption is known to be inadmissible.

(e) LIGHTNESS

It is assumed that the lightness of a surface color is a function of the apparent reflectance of the surface, A, and of the apparent reflectance of the field, A_f. (A possible slight dependence of lightness upon illuminance is neglected [7, p. 439; 86].) This assumption follows directly from the work of Adams and Cobb [2] by virtue of assumption (a); it has been stated explicitly also by Kirschmann [49, p. 363], Von Kries [56, p. 239], Bocksch [7, p. 343], A. Kohlrausch [53, p. 1501], and Kardos [44, p. 188], and is a corollary of the "approximate color constancy of visual objects" [16, 29, 31, 35, 47, 57, 60].

Further references to the principle that object color depends upon the ratios of light reflected from the various parts of the field rather than on the absolute amounts are cited by Helson [24, p. 450]. Bühler [10] stated essentially the same principle by making use of the concept, brightness of illuminated space (Luftlicht) instead of apparent reflectance of the field. Katz [48, p. 462] considered that the total brightness (insistence) of the visual field provides the crucial clue to illumination. Pikler [72] concluded from a series of experiments that light penetrating the translucent parts of the eye (iris, cornea) and falling diffusely on the retina supplies crucial clues both to amount and to direction of the illumination relative to which lightness of visual objects is determined. Both of these views [48, 72] are consistent with this assumption (e).

2. FORMULAS

(a) HUE

Hue in accord with assumption (b) is indicated by the direction of the line on the UCS mixture diagram connecting the point representing the aperture color (r, g, b) with that representing an achromatic aperture color of the same brightness (r_n, g_n, b_n), and in accord with assumption (*d*) this hue applies also to the surface color corresponding to the aperture color. The direction of this line is expressed by giving its slope $(r-r_n)/(g-g_n)$, and the algebraic sign of the difference $(r-r_n)$. The relation is given symbolically in eq 1, and the table of values following it supplies the numerical data.

$$H=f[(r-r_n)/(g-g_n),\ sgn(r-r_n)] \tag{1}$$

H for $r-r_n$ greater than zero	$(r-r_n)/(g-g_n)$	H for $r-r_n$ less than zero
Red-blue	0.0 to −0.38	Green-yellow.
Red	−.38 to −.98	Green.
Yellow-red	−.98 to −2.3	Blue-green.
Yellow	{−2.3 to −∞ ∞ to 2.0}	Blue.
Green-yellow	2.0 to 0.0	Red-blue.

The adjustment of these limiting values of slopes is based somewhat upon an attempt to fit our preliminary data. The angular extent assigned to each hue name is nearly a constant as would be expected on a mixture diagram of uniform chromaticity spacing, and it will be noted that the direction on the diagram assigned to red is the exact opposite of that assigned to green; similarly yellow is taken as the exact opposite of blue. It is of interest to discover how this definition of hue agrees with determinations of the stimuli for unitary and intermediate hues. Table 1 gives the wave lengths in millimicrons for the spectrum stimuli for colors of these hues found by various investigators or reported by various authorities [11, 12, 14, 80].[6] The corresponding values by eq 1 were found by taking standard ICI illuminant *C* [23, 39], representative of average daylight, as the stimulus for an achromatic color; that is, we have taken for this computation $r_n=0.44$ and $g_n=0.47$. It will be noted that eq 1 agrees well with experimentally found stimuli for the unitary hues, yellow, green, and blue, and for their intermediates, but specifies for the boundary between red and yellow-red a stimulus which experiment indicates actually to yield a somewhat bluer hue. This is a result of the simplifying assumption that red and green are exact opposites, and suggests that an assignment of hues without this simplifying restriction might give better agreement.

[6] For other references to this literature consult [67] and Dimmick and Hubbard [11].

TABLE 1.—*Spectrum stimuli for various hues*

Authority	Wave length in millimicrons									
	Red	Red to yellow-red	Yellow-red	Yellow-red to yellow	Yellow	Green-yellow	Green	Blue-green	Blue	Blue to red-blue
Helmholtz	687	656					527			431
Bezold		656		589	578	558	532	502	468	432
Donders					582		535		485	
Hess	PR[a]				574		495		471	
Hering	PR[a]				577		505		470	
Rood	700	621	597	588	581		527	502	473	438
Voeste					577		498		476	
Ladd-Franklin	PR[a]				576		505		470	
Von Kries					574		503			
Westphal	PR[a]				575		506		479	
Dreher					575		509		477	
Ridgway	644		598		577		520		473	
Goldytsch	PR[a]								468	
Bradley	656		608		579		514		469	
Goldmann					568		504		468	
Priest	680				583		515		475	
Brückner					578		498		471	
Schubert					574		500		467	
Maerz and Paul	644				585		521		452	
Purdy					576		504		476	
Drever	650				568		508		471	
Weld		622		597		577		492		456
Ornstein			596		575	562	515	492	475	
Terpstra		604	592	585	577	563	510	487	470	440
Verbeek		605	598	587	580	569	530	496		
Dimmick	494C[b]				582		515		475	
Experimental mean			598	589	577	566	512	495	472	439
Equation 1	521C[b]	499C[b]	601	583	575	562	521	490	476	461

[a] PR means that the indicated hue is more purple than extreme spectrum red.
[b] Dominant wavelength of the complementary is indicated by the letter C.

(b) SATURATION

In accord with assumption (c) saturation, S, is taken proportional to the length, D, of the straight line connecting the point representing the aperture color (r,g,b) with that representing an achromatic aperture color of the same brightness (r_n, g_n, b_n), and in accord with assumption (d) this saturation applies also to the surface color corresponding to the aperture color.

$$S=50D=50[(r-r_n)^2+(g-g_n)^2+(b-b_n)^2]^{1/2}. \tag{2}$$

The constant, 50, is adjusted so as to accord approximately with the scale used by the observers for expressing their estimates of saturation.

(c) ACHROMATIC POINT

It will be noted that the formulations for both hue, H, and saturation, S, (eq 1 and 2) are incomplete because the trilinear coordinates of the stimulus for an achromatic, or neutral, color (r_n, g_n, b_n) are as yet unrelated to the observing situation. By means of these trilinear coordinates, the chromatic adaptation (retinal and cortical) of the observer is to be defined. When a daylight-adapted observer enters a room illuminated by light differing chromatically from daylight to an important extent, a rapid chromatic adaptation takes place which is fairly complete in about 5 minutes. A large part of this chromatic adaptation may occur immediately; that is, the observer may immediately react to the surfaces as if they were illuminated by chromatic light, and there is an immediate reverse change either if the observer begins to react again to the surfaces as if illuminated by daylight, or if the samples lose their surface character and are seen as aperture colors [60, p. 10].[7] This fluctuation occurs with a facility and time character roughly equal to those with which ambiguous pictures (such as the reversible staircase) fluctuate; and, indeed, the situation with which the observer is presented is more than ambiguous, because there are usually not two but many equally valid responses possible depending on the allocation of the chromatic component of the aperture color partly to the surface color and partly to illumination color [60, p. 39] in various proportions.[8] The instantaneous changes are usually considered to be cortical [7, p. 433; 10, p. 115; 35]; some have considered the initial rapid change also to be wholly cortical, but there is considerable evidence for instantaneous retinal adaptation of electrical origin [82, 94, 95]; so it is probable that neural activity, both retinal and cortical, is involved in an essential way [4, 55, 64, p. 46]. The slower progressive changes usually coming to completion in about 5 minutes are thought to be retinal. During this time interval, the values of the trilinear coordinates (r_n, g_n, b_n) specifying the chromatic adaptation of the observer undergo corresponding changes, and no attempt has been made in the present work to separate the retinal from the cortical components, or to trace quantitatively either the course of retinal adaptation in time, as has been done by Wright [93], or its definition in terms of these trilinear coordinates (r_n, g_n, b_n). During the period following that of rapid change, an approach to equilibrium often occurs, and it has been found successful to a considerable degree to define this equilibrium state by means of eq 3, as follows:

$$\left.\begin{aligned} r_n &= r_f - D_f[0.1\ L'(r_f - 0.360) - 0.018\ b_f A_f (L')^2 \log_{10} 2000I] \\ g_n &= g_f - D_f[0.1\ L'(g_f - 0.300) - 0.030]. \end{aligned}\right\} \qquad (3)$$

Formula 3 embodies several ideas that have been widely accepted for many years. When, as was the case in our preliminary experiments, the visual field is filled with objects which, taken together, are spectrally nonselective, so that we may write $r_f = r_0$ and $g_f = g_0$, these formulas could yield strict "color constancy of visual objects" [31] only by the first terms taken alone; the longer second terms (commencing with D_f) may therefore be considered as correction terms indicating necessary departure from strict "color constancy." These correction terms describe variations of the achromatic point (r_n, g_n, b_n)

[7] Many observers can shift colors from the surface mode of appearance to the aperture mode at will [60, p. 45]. Thus A. Gelb [16, p. 599] writes, "In this respect it must be noted particularly that under certain conditions *even without the reduction screen* the dark gray paper near the window and the white wall further back can be seen in the way that corresponds with the relations between the physical stimuli. (The same thing holds for other examples.) Of course, to do this requires a characteristic inhibition, more foreign to reality, which we may designate as the 'attitude of pure optics'; we must consider the color of the paper and that of the wall 'critically' (Katz)—contrary to the natural objective way, that is, as Köhler has opportunely remarked, we must detach ourselves from the objectivity of the surface of the objects and degrade them to a kind of light area. Many people achieve this attitude relatively easily, particularly those who have an aptitude for painting; indeed they must use a dark gray pigment to represent a weakly illuminated white (such as the white-painted wall far back from the window). Whoever achieves the mode of seeing referred to, sees, to be sure, the color of the paper and that of the wall in accord with the relative illuminations; however, he does not see 'true objects' (paper, wall) of particular definite colors in different illuminations, but 'color areas' . . ."

[8] For example, a surface in red light may appear sometimes red and sometimes white or pink according to Bocksch [7, p. 371].

from the field point (r_f,g_f,b_f). Such variations were known as early as 1860, for we read in Helmholtz [30, p. 274], "When a particular color is made dominant in the visual field, a paler shade of the same hue will look white to us, and real white will seem to be the complementary color. Thus the idea of what we mean by white is altered in this case." This is an approximate statement of the facts and would lead to formulas for the achromatic point of the form

$$\left.\begin{aligned} r_n &= r_f - k(r_f - r_w) \\ g_n &= g_f - k(g_f - g_w) \end{aligned}\right\}, \tag{3a}$$

where (r_w,g_w,b_w) are trilinear coordinates of the original "white" point. Formula 3a describes excursions of the achromatic point along the straight line connecting the field point with the original "white" point, depending on the value of the constant, k, indicating degree of departure from adaptation to the field. These formulas also cover a conclusion by Kravkov [54] concerning the direction of surface-color transformation from a chromatic illuminant. Equation 3 includes four separate elaborations or corrections of this idea.

The first elaboration is the introduction of the term L', which extends eq 3a to light and dark parts of the visual field according to the principle that nonselective samples of apparent reflectance considerably greater than the adaptation reflectance, A', take the hue of the illuminant, while those of apparent reflectance considerably less take the hue of the after-image complementary [24]. Helmholtz was familiar with these facts also; he said [30, p. 275], "When we look steadily through a red glass, soon all perfectly dark objects appear to be a vivid green. Thus, alongside the red its complementary color becomes visible, . . ." This account accords with our results, except that the after-image complementary is greenish blue instead of green. Troland [88] and Karwoski [45] also called attention to the same phenomenon for stationary fixation point under the name "dimming effect." Hering [13] had previously proposed the dimming technique as a means of differentiating achromatic aperture colors from those of low saturation.

The discrepancy between mixture complementary and after-image complementary gives rise to the second elaboration of eq 3a. The after-image complementary departs from the mixture complementary by having more red-blue [19; 90; 91, p. 474; 93; 65][9]; this departure is indicated first by the introduction of the term, $0.030\ D_f$, as an addition to g_n, second by the use of the trilinear coordinates (0.360, 0.300) different from those (r_w,g_w) suggested by eq 3a, and third by the $(L')^2$ term in the expression for r_n. It was found necessary to introduce all three changes to obtain agreement with our preliminary data.

A third elaboration was introduced to take account of the greater prevalence of chromatic after-image or adaptation effects for certain regions of the Maxwell triangle. Thus for the field point (r_f, g_f, b_f) near the daylight point (0.44, 0.47, 0.09), no chromatic responses not accounted for by the spectral selectivity of the samples were found;

[9] Tschermak [91, p. 474] gives eight other references.

hence, the term D_f was introduced. Failure of Dittler and Satake [13, p. 245] to find a stimulus for an achromatic color which would also yield on dimming an achromatic color suggests that the correct term is $(k+D_f)$; but our preliminary experiments did not seem to require the introduction of the small constant term, k.

It is of interest to note that Katz [47; 60, p. 37] originally took "normal illumination" to be that found "in the open air under a lightly clouded sky," which has closely the trilinear coordinates (0.44, 0.47, 0.09) relative to which D_f is defined.

The fourth elaboration has to do with an intensification of after-image effects for the blue illuminant for high illuminances, I, and for high field reflectances, A_f. For this reason the term involving the product, $b_f A_f \log_{10} 2000\, I$, was introduced. This term depends upon information obtained from the extensive data collected by Helson and Jeffers [25, 26] which were made available in advance of their publication through the courtesy of Professor Helson. Our preliminary results refer to a single illuminance for each chromatic illuminant; so they do not afford a check upon this term, although they are consistent with it. If this term were omitted, eq 3 would define two field points on the Maxwell triangle for which hue and saturation are independent of lightness, L', instead of one field point (0.44, 0.47, 0.09). The second field point (0.36, 0.30, 0.34) was tentatively named the "blue, hueless point;" blue, because it undoubtedly refers to a blue illuminant, and hueless, because the equations suggested that a nonselective sample under an illuminant plotting at that point would generally, independent of lightness, appear gray or hueless. Attempts by Helson to set up a "blue, hueless" illuminant, however, were unsuccessful; the illuminants tried gave afterimages of feeble rather than zero chromatic component, and led to the $(L')^2$ term in eq 3. This term adds a component of greenish blue to the computed color both for bright and for dim parts of the visual field, the amount of which is proportional to $D_f b_f A_f$. The effect of this term combined with the term 0.030 D_f in the expression for g_n is to make surface colors by computation tend toward red-blue for bright visual fields having D_f and b_f greater than zero.

Equation 3 is therefore consistent with many of the data in the literature of visual psychophysics, with the preliminary data to be discussed presently, and with recent experimental data by Helson particularly adapted for testing out certain aspects of the formulation. But there is no basis for believing that it is the only formula meeting these conditions.

(d) LIGHTNESS

Lightness in accord with assumption (e) is computed from the apparent reflectance, A, of the surface in question and from the apparent reflectance of the field, A_f. The Adams and Cobb [2] formula has already been used to define lightness, L', and this is also used for comparison with estimates of our observers after adjustment of the end points of the scale to make zero correspond to the lower limit of apparent reflectance shown by our samples (A=0.03) and to make 10 correspond to the upper limit (A=1.00); these choices were made in order to give agreement with our observers' estimates of lightness near the ends of the lightness scale, black and white, and they lead to the following formula:

$$L = 10(L' - L'_{A=0.03})/(L'_{A=1.00} - L'_{A=0.03}). \tag{4}$$

By substituting the definition of L' in eq 4, the following explicit function of A and A_f, quoted by Helson [24, p. 453], is found:

$$L = \frac{10(A-0.03)\ (A_f+1.00)}{(1.00-0.03)\ (A_f+A)}. \tag{4a}$$

(c) OBSERVING SITUATIONS

Case 1.—If an observer adapted to daylight enters an enclosure filled with an illumination chromatically different from daylight, and is confronted with an array of samples, with instructions to report immediately on the aperture color derived from one of them, we should expect to compute the resulting estimates of hue and saturation from eq 1 and 2, in which the trilinear coordinates of the achromatic point (r_n, g_n, b_n) would be set equal to those for the daylight point (0.44, 0.47, 0.09). This is the classical case for which hue can be correlated with dominant wavelength computed relative to the daylight point; the observing situation to which it applies is often very transient, sometimes lasting less than a second.

Case 2.—If, however, the observer is instructed to report on a surface color as objectively as possible (that is, with an attempt to discount the illuminant color), his report, except for minor effects from simultaneous contrast, should be expected to correspond with eq 1, 2, and 3, with trilinear coordinates of the achromatic point (r_n, g_n, b_n) taken equal to those for the illuminant point (r_0, g_0, b_0). The period during which such computed hues and saturations could apply would be limited to a few seconds (5 or 10), this duration being usually too small for the development of a chromatic adaptation sufficient to influence the surface color by projection of a negative after-image of the background upon the sample [30, p. 267]. To set the achromatic point equal to the illuminant point was suggested by Priest [74], who said in support of his suggestion, "It is a well-known and significant fact that nonselective diffusely reflecting surfaces are constantly *perceived* as 'white' (or 'gray') for a wide range of variation in the spectral distribution of the illuminant." This suggestion is also implied and justified by Koffka's concept of "shift of level" [51; 52, p. 255]. By arguments based upon chromatic adaptation, which refer to field point instead of illuminant point, a similar suggestion was put forward for longer durations by Ives in 1912 [34], and by Noteboom [71] and Ströble [85] in 1934. This suggestion, intended to apply to extended durations, is allied to the "coefficient law" formulated by Von Kries [30, p. 441; 56, p. 211], and was also implied by Troland [89, p. 182] when he wrote, "* * * it appears that this process (chromatic adaptation) changes the proportions of ordinates between the color-mixture elementaries, so that we must compute the colors on the basis of a distorted triangle."

Case 3.—If the observer remains in the enclosure looking at the array of samples against a background for 5 minutes or more, and if he is asked to report upon the surface color of a sample from its appearance by momentary fixation, his report should be expected to correspond with those for the achromatic point defined by eq 3, in which the apparent reflectance of the field, A_f, is an average of the apparent reflectances of the samples and of the background, A_0, against which the samples are viewed. Since about half of the sample plane in our preliminary experiments was covered with samples, the other half consisting of the background, we have computed A_f representative of observation by momentary fixation preceded by a moving fixation point as follows:

$$A_f = (A_0 + \overline{A})/2. \tag{5a}$$

This formulation applies only to momentary observation preceded by a fast-moving fixation point, because the surface color by eq 1, 2, 3, and 4 is identified with the hue, saturation, and lightness produced by projecting an after-image of the field as a whole (A_f, r_f, g_f, b_f) upon the sample. The observer can easily tell when he fails to comply with these conditions. If his fixation point pauses too long preceding fixation of the sample, he gets a patterned after-image, which by its pattern seems not to belong to the sample [30, p. 269]. If he fixates the sample, itself, too long, the after-image of the field as a whole will die away causing him to change his report; or, perhaps, to note that the first report applies only to the edges of the sample where an after-image of the background has been renewed by small movements of the fixation point within the sample [30, pp. 268, 273]. The color of the center of the sample is dealt with in case 4.

Case 4.—If in case 3 the observer fixates the sample to be reported upon for more than a second or two,[10] we should expect the apparent reflectance, A, of the sample itself to enter into the average for A_f; the longer the fixation, the greater the weight, n, of A, thus,

$$A_f = (nA + A_0 + \overline{A})/(n+2). \tag{5}$$

It will be noted, of course, that eq 5a is the special case of eq 5 obtained by setting $n=0$. The special case corresponding to fixation of the sample, until equilibrium is reached, will be found by setting $n=\infty$; this gives

$$A_f = A. \tag{5b}$$

From the definition of L', it will be seen that eq 5b leads, except for $A=0$, to a value of L' constant at $+2$, which, in turn, through eq 1, 2, and 3, leads to hues and saturations characteristic of bright parts of the visual field, that is, to hues and saturations in accord with the principle, discussed by Helson [24], that bright parts arising from nonselective samples take the hue of the illuminant. This result accords with experiments by Helson and Judd [27] on the appearance of a uniform visual field after prolonged adaptation to it; only in a few exceptional cases did the original hue disappear after minutes-long adaptation, and then only in the sense of appearing and disappearing in rhythmic alternation.

Although our observers were instructed to report upon the appearance of the samples by momentary fixation (case 3), it is of interest

[10] It seems likely from Newhall's work [69] that blinkless fixation for as little as 5 seconds can introduce significant change.

to attempt a prediction of the colors which they would obtain by using longer than momentary fixation contrary to instructions; in this way there may be discovered an explanation for some of the individual variations which cannot be explained by the variable allocation of chromatic component between the surface and the illuminant. For this purpose a value of n between zero and infinity must be chosen, and we have taken, from a preliminary examination of Helson's data as well as our own, n equal to 2, which gives

$$A_f=(2A+A_0+\overline{A})/4. \tag{5c}$$

It may not be immediately apparent that eq 5c favors the appearance of the illuminant hue more than eq 5a; however, adaptation reflectance, A', by eq 5a, is $(3/14)(A_0+\overline{A})$, as may be seen from the definition of L', but by eq 5c it is only $(3/22)(A_0+\overline{A})$; so eq 5c gives a considerably larger range of apparent reflectance for which a nonselective sample would take on the illuminant hue.

TABLE 2.—*Adaptation reflectance, A′*

	Expression for A'	Adaptation reflectance for $\overline{A}=0.33$, $\overline{A}_{\log}=0.24$			
		$A_0=0.03$	$A_0=0.10$	$A_0=0.23$	$A_0=0.80$
Present preliminary experiment, selective samples.	$0.6(A_0\overline{A}_{\log})^{1/2}$	0.051	0.093	0.141	0.263
	$(3/14)(A_0+\overline{A})$ [eq 5a]	.077	.092	.120	.242
Helson and Jeffers [26], selective samples.	$0.32(A_0\overline{A}_{\log})^{1/2}$	.027	.050	.075	.140
	$(3/22)(A_0+\overline{A})$ [eq 5c]	.049	.059	.076	.154
Helson [24], nonselective samples.	$0.8(A_0^3\overline{A}_{\log})^{1/4}$	.041	.100	.186	.474
	$(5/28)(3A_0+\overline{A})$	.075	.112	.182	.487

Table 2 shows a comparison between values of adaptation reflectance, A', computed by eq 5a and 5c, and those found from an examination (a) of our preliminary experimental results represented by the formula $0.6(A_0\overline{A}_{\log})^{1/2}$, and (b) of the experimental results of Helson and Jeffers [26] on selective samples represented by the formula $0.32(A_0\overline{A}_{\log})^{1/2}$. It will be noted from table 2 that eq 5a yields values of A' in good agreement with the logarithmic formula previously found for our preliminary experimental results; the logarithmic formula found by Helson and Jeffers for adaptation reflectance in their experiments with the same selective samples does not, however, agree with eq 5a, but it agrees well with eq 5c. This suggests that Helson's observers used longer fixation of sample than ours did. Values of adaptation reflectance found by Helson from his results with nonselective samples [24] represented by the formula $0.8(A_0^3\overline{A}_{\log})^{1/4}$ are also given; these values are seen not to agree with either of the other two sets. Since, however, the background was more effective in determining adaptation reflectance in these experiments, as is evidenced by the weight 3 given to A_0 in the logarithmic mean, these values have been compared with $(5/28)(3A_0+\overline{A})$ and found to agree fairly well.[11] The previous view [41] that calculations of apparent reflectance of the field, A_f, should be made by logarithmic mean [24] has therefore been given up in the present treatment. The arithmetical mean is not only adequate, simpler, and applicable to fields involving completely dark areas ($A=0$), but it has also yielded improved agreement with lightness estimates of our observers, as will be seen presently. The improved values of adaptation reflectance which have resulted chiefly from Helson's work [24] have been found to permit the elimination of a factor $(1+10\ b_f)$ previously considered to be a necessary part of eq 3 [41].

These four cases describing different observing situations have been worked out for the samples used in our preliminary experiments, and it is believed that other cases are of little interest at present. One factor, however, in the observing situation which has been shown, under some circumstances, to be the controlling one has yet to be mentioned. This is the presence in the visual field of objects whose daylight colors are known to the observer. It has been shown [3] that the observer may report the "memory color" of the object instead of the color reported by an observer who is ignorant of the daylight color. No precautions were taken in our work to prevent the observers from learning the daylight colors of the samples or to prevent the subsequent identification of the samples by the shapes, which were fairly distinctive; but none of our preliminary results seem to be spurious from this cause. Although memory color may be an important factor influencing the color perceptions of an observer distracted by another task, we found that the surface color for our observing conditions was generated from the situation itself with ambiguities so limited as to exclude responses appropriate to memory color except for a few sample-illuminant-background combinations which chanced by the formulation also to yield approximately the daylight color.

IV. ILLUMINANTS AND SAMPLES USED

The samples used in the preliminary experiments were 15 Munsell papers whose apparent reflectances relative to magnesium oxide as functions of wavelength had been determined in 1926 for another purpose.[12] These samples were also used by Helson and Jeffers [25, 26]. Some measurements of apparent reflectance were made on each sample for nearly diffuse illumination and normal viewing, and some were made for 45-degree unidirectional illumination and normal viewing. There was no certain difference between the values obtained in these two ways, and the values adopted are based on both; they are given in table 3.

11 The constant, 5/28, corresponds to a redefinition of L' as $10A/(A+A_f)-4.2$. Why these data require this minor change in definition of L' is not known.

12 NBS test No. 46045. The samples are identified by Munsell notation in accord with the original Atlas of the Munsell Color System [66].

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TABLE 3.—*Spectral apparent reflectances for the 15 Munsell samples studied*

Wave-length	R 3/7	R 5/10	R 7/6	YR 4/5	Y 5/7	Y 7/8	G 3/4	G 5/7	G 7/7	B 3/5	B 5/6	B 7/4	P 3/6	P 5/6	PB 7/4
mμ															
380	0.0680	0.084	0.120	0.0502	0.0630	0.0678	0.0684	0.0457	0.080	0.090	0.123	0.098	0.115	0.102	0.113
90	.0672	.123	.341	.0504	.0651	.0710	.0685	.0481	.153	.185	.351	.400	.247	.350	.420
400	.0655	.123	.364	.0507	.0655	.0729	.0687	.0521	.200	.188	.357	.515	.262	.387	.519
10	.0630	.122	.368	.0512	.0655	.0742	.0692	.0597	.232	.187	.355	.558	.264	.405	.551
20	.0601	.120	.366	.0517	.0654	.0754	.0700	.0708	.261	.187	.355	.579	.257	.410	.574
30	.0567	.118	.363	.0524	.0654	.0765	.0711	.0895	.292	.194	.367	.597	.244	.403	.596
40	.0534	.115	.359	.0532	.0655	.0779	.0730	.119	.325	.214	.407	.615	.226	.390	.619
450	.0503	.113	.353	.0542	.0659	.0795	.0770	.152	.362	.235	.455	.633	.207	.371	.637
60	.0476	.111	.346	.0553	.0665	.0819	.0825	.191	.409	.249	.504	.652	.186	.352	.645
70	.0454	.107	.340	.0566	.0675	.0855	.0890	.233	.463	.253	.545	.666	.167	.331	.641
80	.0437	.104	.334	.0582	.0695	.0922	.0955	.278	.517	.248	.555	.675	.149	.310	.630
90	.0426	.101	.329	.0602	.0734	.105	.101	.325	.562	.235	.535	.674	.131	.289	.617
500	.0420	.0980	.328	.0631	.0803	.141	.106	.371	.600	.215	.497	.665	.115	.269	.600
10	.0417	.0955	.330	.0673	.100	.210	.111	.413	.627	.194	.450	.649	.104	.252	.573
20	.0419	.0948	.332	.0741	.148	.307	.114	.435	.639	.169	.401	.628	.0965	.239	.537
30	.0429	.0963	.338	.0850	.206	.424	.114	.432	.632	.145	.351	.604	.0935	.231	.500
40	.0448	.100	.347	.100	.250	.523	.110	.399	.610	.123	.306	.578	.0925	.228	.470
550	.0482	.106	.359	.127	.276	.570	.105	.355	.582	.105	.269	.552	.0934	.228	.448
60	.0545	.118	.377	.158	.289	.588	.0985	.307	.550	.0920	.238	.527	.0959	.231	.430
70	.0648	.143	.410	.188	.294	.595	.0908	.256	.512	.0832	.212	.502	.100	.235	.415
80	.0818	.200	.482	.219	.296	.596	.0839	.212	.475	.0769	.195	.480	.107	.241	.402
90	.114	.293	.599	.245	.295	.594	.0780	.176	.442	.0721	.184	.460	.114	.250	.395
600	.153	.433	.688	.265	.292	.591	.0734	.148	.414	.0684	.175	.440	.119	.264	.390
10	.191	.563	.741	.276	.288	.588	.0702	.127	.390	.0657	.169	.422	.124	.279	.389
20	.221	.639	.763	.285	.283	.585	.0684	.111	.370	.0641	.164	.406	.129	.297	.391
30	.244	.680	.774	.290	.277	.581	.0677	.0992	.353	.0635	.161	.394	.134	.315	.398
40	.263	.701	.778	.295	.272	.578	.0685	.0895	.338	.0634	.159	.384	.140	.336	.408
650	.280	.712	.782	.298	.269	.575	.0705	.0832	.325	.0637	.157	.377	.148	.359	.423
60	.294	.721	.786	.302	.268	.573	.0737	.0789	.315	.0645	.158	.373	.158	.383	.441
70	.305	.728	.790	.305	.268	.575	.0777	.0755	.306	.0657	.160	.370	.170	.408	.462
80	.321	.735	.793	.308	.270	.575	.0824	.0730	.299	.0673	.165	.370	.190	.433	.487
90	.342	.743	.797	.312	.274	.578	.0871	.0713	.294	.0692	.175	.371	.218	.459	.515
700	.368	.750	.800	.315	.280	.582	.0918	.0702	.289	.0712	.191	.374	.253	.484	.546
10	.398	.756	.803	.317	.287	.586	.0965	.0695	.285	.0735	.214	.379	.296	.509	.580
20	.429	.763	.806	.320	.296	.593	.101	.0692	.283	.0762	.247	.384	.351	.529	.620

The illuminants used were natural daylight from a south window and four chromatic illuminants (red, yellow, green, blue) obtained by illuminating the samples with a 500-watt, gas-filled lamp and viewing them through one or another of four Corning glass filters whose spectral transmissions were determined chiefly on a photoelectric spectrophotometer [17] and are given in table 4. A few of the lowest transmissions were checked by means of the König-Martens visual spectrophotometer [62]. The natural daylight supplied an illuminance of about 50 footcandles; the 500-watt lamp, about 700.

TABLE 4.—*Spectral transmissions of the viewing filters*

Wavelength	Lantern red	Lantern yellow	Sextant green	Lantern blue
$m\mu$				
380	------------	0.0002	------------	0.15
90	------------	.0002	------------	.28
400	------------	.0002	<0.0005	.365
10	[a]<0.001	.0002	------------	.400
20	------------	.0002	.0002	.404
30	------------	.0002	.0003	.400
40	<.00005	.0002	.0004	.384
450	------------	.0002	.0010	.360
60	<.00002	.0003	.0022	.318
70	------------	.0003	.0056	.240
80	<.00002	.0004	.0133	.143
90	------------	.0005	.0280	.070
500	<.00002	.0007	.0580	.033
10	------------	.0011	.0916	.0100
20	<.000005	.0017	.109	.0038
30	------------	.0030	.112	.0017
40	<.000005	.0055	.098	.0010
550	------------	.025	.070	.0013
60	<.000005	.278	.042	.0021
70	------------	.475	.0228	.0014
80	<.00001	.539	.0104	.0004
90	<.00001	.580	.0045	.0001
600	<.00005	.608	.0017	.0001
10	.0002	.630	.0004	.0001
20	.025	.646	<.0002	.0001
30	.273	.660	------------	------------
40	.548	.668	<.0002	<.0001
650	.645	.672	<.0001	------------
60	.681	.675	------------	<.0001
70	.699	.676	<.0001	------------
80	.707	.677	------------	.0001
90	.710	.678	<.0001	.0006
700	.713	.674	<.0001	.0015
10	.714	.669	------------	.0027
20	.714	.656	<.00005	.0025
30	.714	.650	------------	.0023
40	.714	.641	<.00005	.0022
750	.712	.636	------------	.0020
760	.711	.629	<.00005	.0018

[a] Numbers preceded by the "less-than", <, sign indicate spectral transmissions greater than those possessed by the respective filters.

The tristimulus specifications of the light reflected by each of the 15 samples when it is illuminated in turn by each of the 5 illuminants were computed from the data of tables 3 and 4. Standard ICI illuminant *A* [23, 39] was used in these computations as representative of the 500-watt lamp; and Abbot-Priest sunlight [38, p. 525] was used as representative of natural south daylight. The OSA observer and coordinate system [18; 87, p. 549] was used for the first computation of tristimulus specifications, and these were later transferred to the uniform-chromaticity-scale system [40]. Table 5 gives two (r and g) of the trilinear coordinates on this system; the third may be found, if desired, as $b=1\text{-}r\text{-}g$.[13] Figure 1 shows the uniform-chromaticity-scale triangle on which have been plotted points representing (except for the red illuminant) each of the 15 samples under each

[13] Computations of trilinear coordinates, r, g, b, and of apparent reflectance, A, for these 75 illuminant-sample combinations were carried out by Mabel E. Brown.

of the 5 illuminants. The illuminant point is indicated by a square. The samples under sunlight illumination are shown by circles, with large circles for the five samples having the most saturated colors of the five Munsell hues, red, yellow, green, blue, and purple. For the strongly chromatic illuminants, the samples are shown by crosses, with large crosses for the same five samples shown by large circles for sunlight illumination. For the red illuminant, only the green (G) and the purple (P) samples are shown; points for the other samples fall on the straight line connecting these two points, and, for the sake of clarity, crosses to indicate them have been omitted. It will be noted that points representing the samples for other illuminants than red do not cluster as closely about the illuminant point. The small cluster of points for the red illuminant is related to the greater selectivity of the red filter (see table 4).

TABLE 5.—*Trilinear coordinates (r, g) of the light reflected from each sample and the apparent reflectances (A) of the samples for each of the five illuminants used*

Sample	Abbot-Priest sunlight			Red illuminant			Yellow illuminant			Green illuminant			Blue illuminant		
	r	*g*	*A*	*r*	*g*	*A*	*r*	*g*	*A*	*r*	*g*	*A*	*r*	*g*	*A*
MgO	0.458	0.458	1.000	0.958	0.042	1.000	0.705	0.295	1.000	0.372	0.613	1.000	0.237	0.351	1.000
R 3/7	.606	.342	0.087	.961	.039	0.276	.792	.208	0.151	.390	.597	0.049	.243	.338	0.048
R 5/10	.628	.327	.221	.959	.041	.706	.796	.204	.404	.391	.595	.110	.241	.344	.110
R 7/6	.527	.408	.459	.958	.042	.781	.744	.255	.618	.380	.606	.357	.240	.347	.346
YR 4/5	.574	.394	.163	.959	.041	.297	.735	.265	.246	.401	.589	.110	.248	.365	.069
Y 5/7	.525	.448	.241	.957	.043	.272	.700	.299	.288	.393	.598	.221	.254	.373	.097
Y 7/8	.532	.449	.490	.958	.042	.577	.703	.296	.588	.393	.598	.455	.265	.398	.159
G 3/4	.437	.490	.094	.961	.039	.072	.686	.313	.078	.367	.619	.107	.236	.369	.093
G 5/7	.407	.539	.285	.953	.047	.087	637	.362	.171	.362	.624	.377	.237	.413	.267
G 7/7	.439	.497	.522	.956	.044	.329	.677	.322	.428	.367	.619	.596	.237	.384	.493
B 3/5	.374	.485	.114	.958	.042	.064	.685	.314	.072	.351	.629	.133	.232	.353	.217
B 5/6	.386	.491	.277	.959	.041	.161	.684	.315	.185	.354	.627	.324	.232	.363	.472
B 7/4	.427	.478	.531	.957	.043	.381	.690	.309	.445	.365	.619	.581	.236	.356	.642
P 3/6	.440	.423	.108	.962	.038	.153	.730	.270	.119	.370	.612	.097	.237	.325	.153
P 5/6	.456	.434	.253	.961	.039	.359	.731	.269	.273	.370	.614	.235	.236	.337	.310
PB 7/4	.428	.467	.458	.960	.040	.428	.704	.294	.405	.364	.619	.485	.236	.350	.600

Table 5 also gives the apparent reflectance of each sample computed for each of the five illuminants. The three numbers (A, r, g) serve to specify the beam reflected from the sample toward the observer relative to that which would be reflected from a surface of magnesium oxide, which was the reflectance standard used in the spectrophotometric work [68].

V. EXPERIMENTAL METHOD

The observer was shown the samples (irregularly shaped papers of about 1 in.^{2}) spread out in disarray on one of two large cardboards (either a light one, $A_0=0.80$, which appeared white in daylight, or a dark one, $A_0=0.10$, which appeared dark gray). He was asked to arrange them in order of lightness, placing the lightest at the top and the darkest at the bottom. After this task had been done, he was asked to estimate the lightness on a scale of 10 equal visual steps such that white would have lightness, $L=10$, and black would have lightness, $L=0$. These estimates were recorded. Then he was asked to arrange the samples in order of saturation, placing the one having the

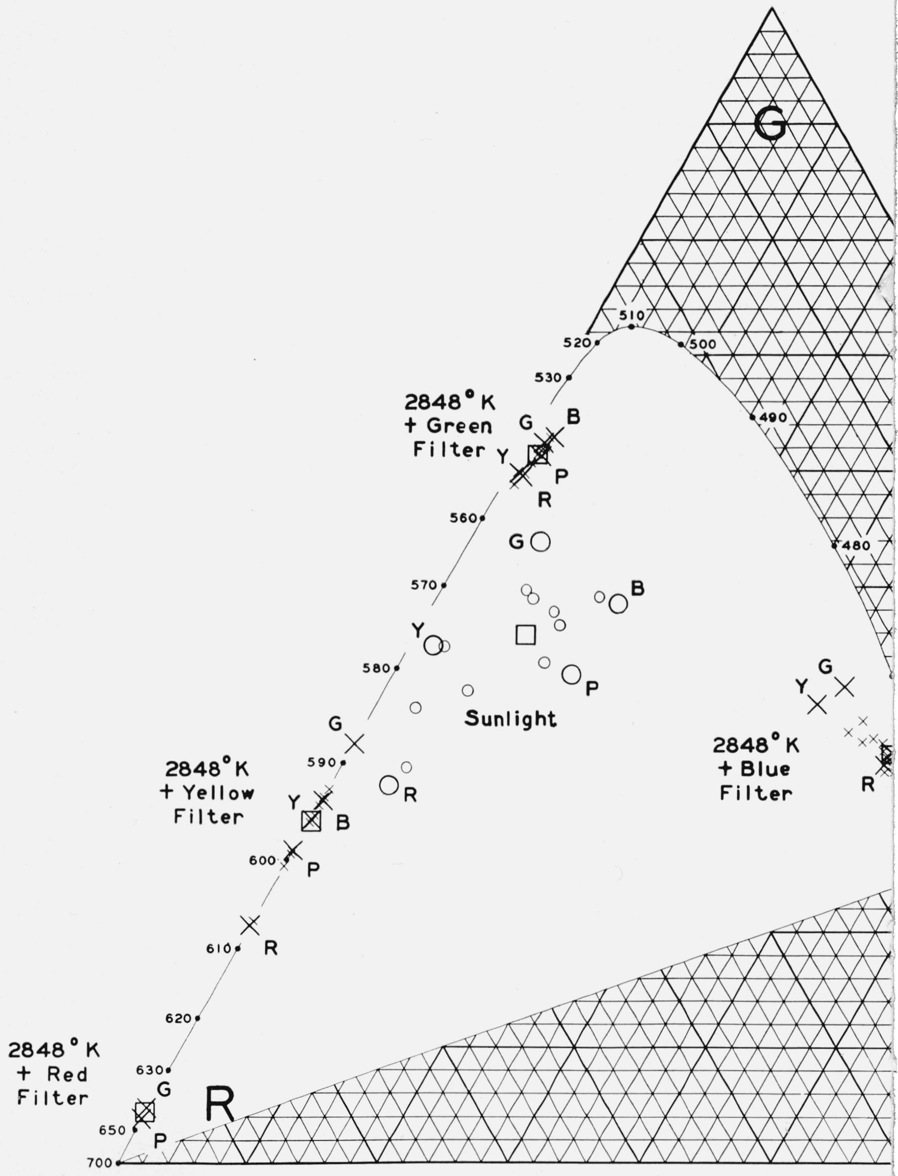

FIGURE 1.—*The 75 illuminant-sample combinations represented on the unif*

The samples with Abbot-Priest sunlight as the illuminant are shown by circles; those for the four strongly selecti illuminants are indicated by squares.

most saturated (strongest) color first and the one most closely resembling an achromatic color last. After this task had been done, he was asked to estimate the saturation on a scale of 10 equal visual steps, such that the strongest daylight color produced by any of the 15 samples would be assigned the saturation, $S=10$, and such that an achromatic color would be given a saturation, $S=0$. Then he was asked to name the hue of the sample on the eight-point hue scale; see definition of H, section II–3. These saturation and hue estimates were also recorded. If, as was frequently the case, the observer was not satisfied with one of the 8 or 11 answers suggested by these 8- and 11-point scales, his estimate was recorded on a more finely divided scale. This procedure was carried out for each observer for all five illuminants, and for each of the two backgrounds. In his manipulation of the samples, the observer was not permitted to place the samples next to one another except in forming estimates of lightness; for hue and saturation estimates the samples were kept about one-half inch apart. Furthermore, the observer was requested to avoid fixating any one sample very long at a time; he was encouraged to look rapidly from one sample to another in order to avoid as much as possible the influence of any one sample on estimates of the color of another.

Although estimates of this sort have sometimes been criticized as meaningless (see, for example, Smith [84] and Guild [22]), they are frequently made and constitute a recognized general psychophysical method called by Boring [8, p. 389] "sense distance, method of fractionation." Richardson and Maxwell [59, 78, 79] and Newhall [70] have applied it to color.

VI. COMPUTED VALUES OF HUE, SATURATION, AND LIGHTNESS

Values of hue, saturation, and lightness have been computed from eq 1, 2, 3, 4, 5a, and 5c for the 15 samples used in the preliminary experiments and for 3 hypothetical nonselective samples identified by the appropriate Munsell notations, N 2/, N 5/, and N 9/.[14] These computations refer to all five illuminants studied and to observing situations described under cases 3 and 4. Values of hue and saturation have also been computed for cases 1 and 2.

TABLE 6.—*Constants used in computing the trilinear coordinates* (r_n, g_n, b_n) *of the achromatic points for the five illuminants studied (eq 3)*

Illuminant	r_f $(=r_0)$	g_f $(=g_0)$	b_f $(=b_0)$	D_f	T_{2848}	I	$\bar{A}$	A_0	A_f (eq 5a)
						fc			
Abbot-Priest sunlight	0.458	0.458	0.084	0.02	--------	50	0.287	0.10 .80	0.194 .544
Red	.958	.042	.000	.68	0.052	36	.330	.10 .80	.215 .565
Yellow	.705	.295	.000	.33	.362	253	.298	.10 .80	.199 .549
Green	.372	.613	.015	.18	.038	27	.282	.10 .80	.191 .541
Blue	.237	.351	.412	.40	.0061	4	.272	.10 .80	.186 .536

[14] These hypothetical samples because of their nonselectivity have trilinear coordinates (r,g,b) the same as those given in table 5 for MgO. They have been assigned values of apparent reflectance: 0.036, 0.193, and 0.738, respectively.

In table 6 are given values of the constants used in computing, by means of eq 3, the trilinear coordinates (r_n,g_n,b_n) of the achromatic points for viewing by case 3 for the five illuminants. Since the samples and background taken together are nearly nonselective in spectral reflectance, trilinear coordinates (r_f,g_f,b_f) for the field points are taken equal to those for the illuminant points (r_0,g_0,b_0) which, in turn, are taken equal to the trilinear coordinates (r,g,b) for MgO in table 5 by the assumption that MgO is spectrally nonselective [75]. The illuminances (I) supplied by the chromatic illuminants are computed as 700 T_{2848}, where T_{2848} is the transmission of the chromatic filter for light of color temperature 2,848° K. Values of T_{2848} were computed in the usual way [23, 39] from values of spectral transmission given in table 4.

Application of the constants in table 6 to eq 3 gives the following specification of the trilinear coordinates (r_n,g_n,b_n) of the achromatic point as a function of lightness, L':

Illuminant	Equation 3
Abbot-Priest sunlight	$r_n=0.458-0.00022\ L'+\begin{matrix}0.00009\\(.00003)^{15}\end{matrix}(L')^2$; $g_n=0.459-0.00035\ L'$
Red	$r_n=0.958-0.0407\ L'$; $g_n=0.062+0.0175\ L'$
Yellow	$r_n=0.705-0.0114\ L'$; $g_n=0.305+0.000165\ L'$
Green	$r_n=0.372-0.00022\ L'+\begin{matrix}0.000124\\(.000044)^{15}\end{matrix}(L')^2$; $g_n=0.618-0.0056\ L'$
Blue	$r_n=0.237+0.0049\ L'+\begin{matrix}0.0063\\(.0022)^{15}\end{matrix}(L')^2$; $g_n=0.363-0.00204\ L'$

[15] This constant refers to the dark background ($A_0=0.10$); the one given just above to the light background ($A_0=0.80$).

Figure 2 shows the locus (dotted line) of achromatic points plotted to a large scale on the Maxwell (UCS) triangle for the red illuminant. Only the extreme lower left-hand corner of the triangle (fig. 1) appears on this plot. Since eq 3 reduces for the red illuminant to a first-degree equation (see above), this locus is a straight line. The limits of the locus for samples viewed by momentary fixation (case 3) on a dark background ($A_0=0.10$) are shown by shaded circles, one for a dark (Bk) sample ($A=0.03$), and one for a light (W) sample ($A=0.80$); similar limits are indicated for a light background ($A_0=0.80$) by open circles. It will be noted that the achromatic point for the dark sample ($A=0.03$), whether viewed on the light or on the dark background, falls outside the region of the triangle representing mixtures of real stimuli. That is, according to eq 3, when an observer has been in a room filled with red light for 5 minutes or more, his visual mechanism is in such a state of chromatic adaptation that no sample exists of apparent reflectance as low as 0.03, such that it will look gray or black against a background of apparent reflectance, $A_0=0.10$, or greater. Figure 2 also shows the illuminant point (square) and the limits (crosses) of the locus of points representing the 15 samples studied. Two of the vectors whose direction is correlated with hue according to eq 2 are also shown; the one associated with a dark sample viewed on a light background indicates a blue hue, the one associated with a light sample on a light ground indicates a yellow-red hue; see graphical representation of eq 2 identifying hue with direction on the triangle, shown in the upper left-hand portion of the figure.

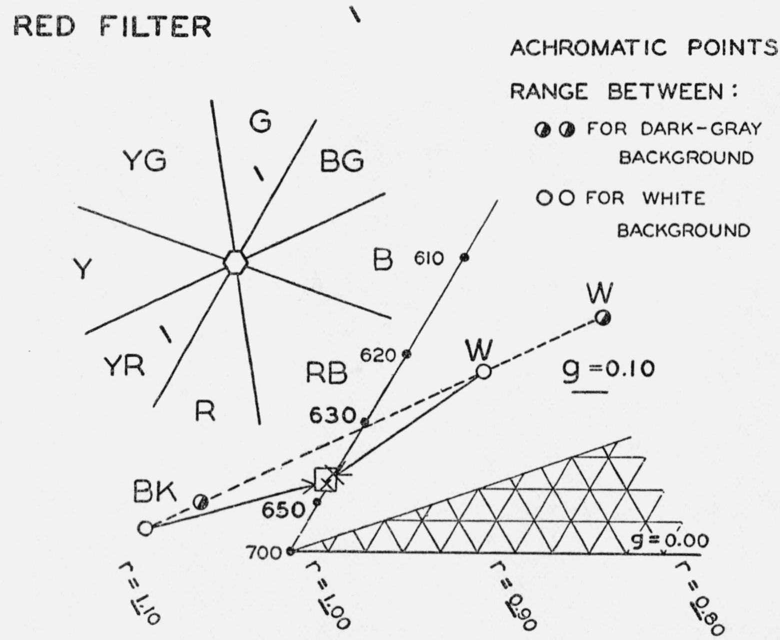

Figure 2.—*Large-scale plot of a portion of figure 1, which shows, for the red illuminant (square), the locus of the achromatic points (dotted line) and two of the vectors defining hue and saturation by eq 1, 2, and 3.*

The definition of hue, according to angle by eq 1, is indicated in the upper left-hand part of this figure. The vector indicating a blue hue refers to a dark nonselective sample on a light ground; and that indicating yellow-red hue, to a light nonselective sample on a light ground. Both vectors refer to momentary fixation of the samples by an observer who has been exposed to the field with red illumination for 5 minutes or more (case 3, section III, 2, (e)).

Figure 3 shows loci of achromatic points for all five illuminants studied. Like the locus for the red illuminant, that for the yellow illuminant is a straight line; the other loci are parabolas, those for Abbot-Priest sunlight and the green illuminant being relatively short because of the relatively small chromatic adaptation (measured by D_f) associated with them according to eq 3; note particularly the short locus for Abbot-Priest sunlight, which falls entirely within the square indicating this illuminant point. Note also how the direction of these loci is controlled by the position of the point, indicated by a double hexagon, whose trilinear coordinates (r=0.360, g=0.300) appear explicitly in eq 3. The graphical representation of eq 2 defining hue according to direction on this diagram is centered about the daylight point (0.44, 0.47, 0.09) indicated by a single hexagon.

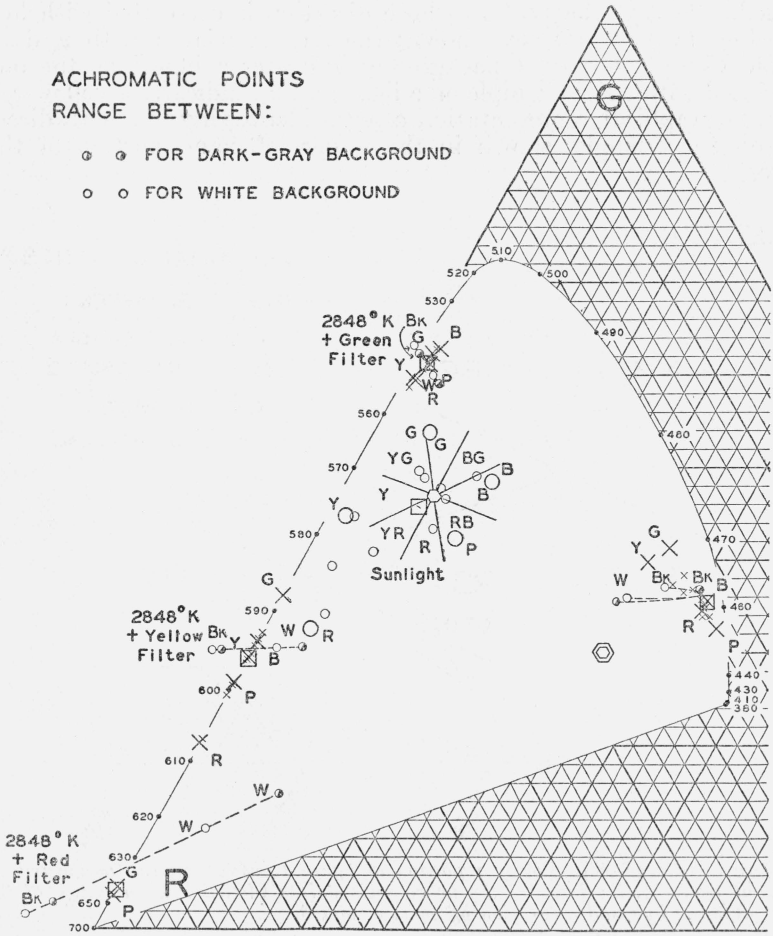

FIGURE 3.—*Loci of achromatic points (dotted lines) on the uniform-chromaticity-scale Maxwell triangle for the five illuminants studied.*

Note the varying lengths of the loci (determined chiefly by D_f), and the curvature of the loci of achromatic points for the blue illuminant (determined by b_f). Note also how the direction of each nearly straight locus is determined by the position of the point (r=0.36, g=0.30), indicated by a double hexagon, whose trilinear coordinates appear explicitly in eq 3. The definition of hue, according to angle by eq 1, is indicated graphically by lines intersecting at the daylight point (indicated by a small single hexagon).

Table 7 gives the computed hues and saturations for the hypothetical nonselective samples for both backgrounds for the five illuminants, and for the four observing situations (cases 1 to 4); it also gives lightness computed for cases 3 and 4. It should be noted that the hue to be expected for the aperture colors derived from nonselective samples (case 1) with a chromatic illuminant is approximately the hue of the illuminant, and the saturation is high. This result is independent of the apparent reflectance of the sample. Similarly, the surface color with the illuminant color discounted (case 2) is also expected to be independent in hue and saturation of the apparent reflectance of the sample; that is, as may be seen from table 7, the colors are all expected to be achromatic (zero saturation, no hue). Cases 3 and 4 yield hues, lightnesses, and saturations for nonselective samples which depend upon the relation of the apparent reflectance, A, of the sample to that of the field, A_f. From a comparison of these computations for case 3 and case 4, the formulation may be seen to indicate that fixation of the sample for a few seconds contrary to instructions may produce a considerable change in hue and saturation of samples viewed in strongly chromatic illumination, but produces a negligible effect for samples viewed by Abbot-Priest sunlight.

TABLE 7.—*Computed hues, lightnesses, and saturations for three hypothetical nonselective samples for both light and dark backgrounds, for the five illuminants, and for four observing situations (cases 1 to 4)*

Illuminant	A_0	Sample	Computed for different viewing situations: Momentary fixation of sample (Case 3)	Fixation of sample for several seconds (Case 4)	Illuminant color discounted (Case 2)	Aperture color for daylight-adapted eye (Case 1)
Abbot-Priest sunlight	0.80	N 2/0	RB 0.2/0.2	RB 0.3/0.2	N /0.0	YR /1.1
		N 5/0	RB 3.5/0.1	RB 5.3/0.1	N /0.0	YR /1.1
		N 9/0	N 8.8/0.0	Y 9.4/0.1	N /0.0	YR /1.1
	.10	N 2/0	RB 0.3/0.1	RB 0.6/0.1	N /0.0	YR /1.1
		N 5/0	N 5.2/0.0	N 6.9/0.0	N /0.0	YR /1.1
		N 9/0	YG 9.4/0.1	YG 9.7/0.1	N /0.0	YR /1.1
Red	.80	N 2/0	B 0.2/6.3	B 0.3/5.2	N /0.0	YR /33.9
		N 5/0	RB 3.5/1.8	R 5.3/1.7	N /0.0	YR /33.9
		N 9/0	YR 8.7/6.8	YR 9.4/5.8	N /0.0	YR /33.9
	.10	N 2/0	B 0.3/4.3	B 0.6/2.3	N /0.0	YR /33.9
		N 5/0	YR 5.0/4.6	YR 6.9/5.1	N /0.0	YR /33.9
		N 9/0	YR 9.3/12.0	YR 9.7/7.8	N /0.0	YR /33.9
Yellow	.80	N 2/0	B 0.2/2.3	B 0.3/2.0	N /0.0	YR /16.5
		N 5/0	RB 3.5/0.9	R 5.3/0.6	N /0.0	YR /16.5
		N 9/0	Y 8.8/1.9	Y 9.4/1.6	N /0.0	YR /16.5
	.10	N 2/0	RB 0.3/1.7	RB 0.6/1.1	N /0.0	YR /16.5
		N 5/0	YR 5.1/1.3	Y 6.9/1.4	N /0.0	YR /16.5
		N 9/0	Y 9.4/3.6	Y 9.7/2.2	N /0.0	YR /16.5
Green	.80	N 2/0	RB 0.2/1.3	RB 0.3/1.1	N /0.0	G /8.8
		N 5/0	RB 3.5/0.5	RB 5.3/0.2	N /0.0	G /8.8
		N 9/0	YG 8.8/0.7	YG 9.4/0.6	N /0.0	G /8.8
	.10	N 2/0	RB 0.3/0.9	RB 0.6/0.6	N /0.0	G /8.8
		N 5/0	YG 5.2/0.4	YG 6.9/0.4	N /0.0	G /8.8
		N 9/0	YG 9.4/1.6	YG 9.7/0.9	N /0.0	G /8.8
Blue	.80	N 2/0	RB 0.2/2.5	RB 0.3/1.9	N /0.0	RB /20.0
		N 5/0	RB 3.5/0.9	RB 5.3/0.9	N /0.0	RB /20.0
		N 9/0	B 8.8/4.7	B 9.4/3.4	N /0.0	RB /20.0
	.10	N 2/0	RB 0.3/1.0	RB 0.6/0.9	N /0.0	RB /20.0
		N 5/0	B 5.2/1.7	B 6.9/1.7	N /0.0	RB /20.0
		N 9/0	B 9.4/5.5	B 9.7/2.8	N /0.0	RB /20.0

The computed color descriptions for these hypothetical nonselective samples agree, in general, both for case 3 and for case 4 with experimental results by Helson [24]. Detailed comparison has yet to be carried out, but evaluations of adaptation reflectance (see table 2) suggest that improved agreement with Helson's results would be obtained by using a definition of apparent reflectance of the field, A_f, slightly different from eq 5a, and a slightly different definition of L'.

VII. COMPARISON OF COMPUTATION WITH EXPERIMENT

Table 8 summarizes the experimental results obtained in south-daylight illumination; table 9, those obtained under the four strongly chromatic illuminants studied. These tables give the average hue, lightness, and saturation of the 15 samples estimated by the 6 observers who completed the series, namely, G. B. Reimann, G. W. Haupt, B. H. Carroll, K. S. Gibson, R. S. Hunter, and D. B. Judd. The individual variation among these observers was considerably larger than the uncertainty of estimates by the two observers who made repeat runs; this individual variation is indicated in tables 8 and 9 by the ranges of the estimates of hue, lightness, and saturation. Similar individual variation is reported by Bocksch [7, p. 366]. In parallel columns are also given the results of computations of hue, lightness, and saturation for the four observing situations (cases 1 to 4).

TABLE 8.—*Comparison of observed and computed color descriptions*

[Hue, lightness, and saturation (*H L/S*) of the 15 selective samples studied. Illuminant: Abbot-Priest sunlight taken as representative of south daylight]

Sample	Average observed values, *H L/S*	Individual observer range $\Delta H\ \Delta L/\Delta S$	Computed for different viewing situations: Momentary fixation of sample	Fixation of sample for several seconds	Illuminant color discounted	Aperture color for daylight-adapted eye
			(Case 3)	(Case 4)	(Case 2)	(Case 1)
LIGHT BACKGROUND (A_0=0.80)						
R 3/7	R 2.6/5.1	R 1–3/2–7	**YR** 1.6/9.6	**YR** 2.6/9.6	**YR** /9.5	**YR** /10.6
R 5/10	R 5.2/10	R 4–6/10	**YR** 4.0/10.9	**YR** 5.8/10.9	**YR** /10.9	**YR** /12.0
R 7/6	R 6.9/5.8	RB-YR 5–8/3–8	YR 6.8/4.4	YR 8.2/4.4	YR /4.4	YR /5.5
YR 4/5	YR 3.8/4.8	YR 3–5/1–7	YR 3.0/7.1	YR 4.7/7.1	YR /7.1	YR /8.3
Y 5/7	gY 4.6/4.4	Y-YG 4–5/2–6	Y 4.3/4.4	Y 6.1/4.4	Y /4.4	Y /5.4
Y 7/8	Y 6.7/9.4	Y 6–8/9–10	Y 7.1/4.9	Y 8.3/4.9	Y /5.0	Y /5.9
G 3/4	yG 2.0/2.9	YG-G 1–3/1–6	G 1.7/1.9	G 2.8/1.9	G /2.0	YG /1.3
G 5/7	G 5.3/9.8	G 4–6/9–10	G 4.9/5.0	G 6.7/5.0	G /5.0	G /4.2
G 7/7	yG 7.3/5.2	YG-G 7–8/4–8	G 7.4/2.4	G 8.5/2.4	G /2.4	YG /1.9
B 3/5	B 3.0/6.2	B 2–4/5–7	B 2.1/5.3	B 3.4/5.3	B /5.2	B /4.2
B 5/6	B 4.9/7.9	B 4–6/7–9	**BG** 4.8/4.4	**BG** 6.6/4.4	**BG** /4.4	B /3.3
B 7/4	gB 7.3/3.4	BG-B 7–8/1–8	BG 7.4/1.9	BG 8.6/1.9	BG /1.9	BG /0.8
P 3/6	RB 2.9/6.8	RB 2–4/6–8	RB 2.0/3.4	RB 3.3/3.4	RB /3.3	RB /3.3
P 5/6	RB 5.1/5.3	RB 5–6/4–8	RB 4.5/1.8	RB 6.3/1.8	RB /1.8	**R** /2.2
PB 7/4	B 6.8/3.6	rB-B 6–7/1–8	B 6.8/1.9	B 8.1/1.9	B /1.9	**RB** /1.0

TABLE 8.—*Comparison of observed and computed color descriptions*—Continued

[Hue, lightness, and saturation (*H L/S*) of the 15 selective samples studied. Illuminant: Abbot-Priest sunlight taken as representative of south daylight]

Sample	Average observed values, *H L/S*	Individual observer range $\Delta H\ \Delta L/\Delta S$	Computed for different viewing situations			
			Momentary fixation of sample	Fixation of sample for several seconds	Illuminant color discounted	Aperture color for daylight-adapted eye
			(Case 3)	(Case 4)	(Case 2)	(Case 1)
DARK BACKGROUND (A_0=0.10)						
R 3/7	R 3.1/5.7	R 1-5/3-8	**YR** 2.7/9.6	**YR** 4.2/9.6	**YR /9.5**	**YR /10.6**
R 5/10	R 6.0/10	RB-R 5-7/10	**YR** 5.7/**11.0**	**YR** 7.3/**10.9**	**YR /10.9**	**YR /12.0**
R 7/6	R 7.4/6.8	RB-YR 7-9/3-9	YR 8.1/4.4	YR 8.9/4.4	YR /4.4	YR /5.5
YR 4/5	YR 4.1/5.9	YR 2-5/2-9	YR 4.7/7.1	YR **6.4**/7.1	YR /7.1	YR /8.3
Y 5/7	Y 4.6/4.5	Y 2-6/2-6	Y 6.0/4.5	Y **7.5**/4.4	Y /4.4	Y /5.4
Y 7/8	Y 7.4/9.9	Y 6-9/9-11	Y 8.3/**4.9**	Y 9.0/**5.0**	Y /**5.0**	Y /**5.9**
G 3/4	G 2.4/2.2	G 1-4/2-4	G 2.8/1.9	G 4.5/1.9	G /2.0	**YG /1.3**
G 5/7	G 5.7/9.0	G 4-8/8-10	G 6.6/**5.0**	G 7.9/**5.0**	G /**5.0**	G /4.2
G 7/7	G 7.4/4.7	YG-BG 6-9/3-8	G 8.5/**2.4**	G 9.2/**2.4**	G /**2.4**	YG /**1.9**
B 3/5	B 3.2/7.2	B 2-4/7-8	B 3.3/**5.3**	B **5.1/5.3**	B /**5.2**	B /**4.2**
B 5/6	B 5.4/6.8	B 4-6/5-8	**BG** 6.4/**4.3**	**BG 7.9/4.3**	**BG /4.4**	B /**3.3**
B 7/4	gB 7.6/1.8	BG-B 5-10/1-4	BG 8.5/1.9	BG 9.2/1.9	BG /1.9	BG /0.8
P 3/6	RB 3.1/5.7	RB 2-4/4-9	RB 3.2/**3.4**	RB **4.9/3.4**	RB /**3.3**	RB /**3.3**
P 5/6	RB 5.2/3.8	RB 4-6/1-7	RB 6.1/1.7	RB 7.6/1.7	RB /1.8	**R** /2.2
PB 7/4	rB 7.2/1.7	RB-B 5-8/1-3	B **8.6**/1.9	B **8.9**/1.8	B /1.9	RB /1.0

It may be seen from table 8 that individual saturation estimates for most of the samples viewed in south daylight are notably less certain than those for hue and lightness. For example, PB 7/4 observed on the light background was called blue by five of the six observers, and reddish blue by the sixth; hence the hue range is indicated as rB–B; the lightness estimates ranged from 6 to 7, with an average of 6.8; but the saturation estimates range all the way from 1 to 8. For each of 9 of the 15 samples viewed on the light background, all 6 observers gave the same hue name; this is indicated in table 8 by expressing the hue range for each of these 9 samples with a single hue symbol (R, YR, and so on).

Whenever the computed hue, lightness, or saturation falls within the corresponding individual-observer range, we have taken it as an indication that the formulation is substantially correct; and similarly, whenever the computed saturation is 0.5 or less, and one observer reports zero saturation we have taken the computed hue to be correct even though it does not agree with reports by the other observers. Computed hues, lightnesses, and saturations which do not agree with our experimental results are given in bold-faced type for easy identification.

1. SOUTH DAYLIGHT

It will be noted from table 8 that eq 2 is fairly successful in duplicating the hue estimates, there being only 3 samples of the 15 for which the computed result by any of the cases (1 to 4) falls outside the observed range; and indeed these 3 discrepancies are by but one division of the 8-point hue scale. Two of them (R 3/7, R 5/10) could have been eliminated by abandoning the simplifying principle of making red by eq 2 the exact opposite of green.

Computation of lightness by case 3 is in even closer agreement, no discrepancy for the light background, and only one for the dark, being shown. Case 4 (eq 5c) yields consistently higher values of lightness, and for nearly half of the samples yields values too high to be acceptable. It would appear, therefore, that our observers followed instructions to judge lightness without fixating the sample in question more than a few seconds at a time; that is, eq 5a rather than eq 5c describes the observing habits of our observers in south daylight during their judgment of lightness.

In spite of the large individual variation in the estimates of saturation, eq 3 fails for about half of the samples to yield acceptable values of saturation. This failure indicates that the UCS triangle does not yield as uniform a spacing of surface colors viewed in a field subtending visual angles large relative to the macular pigment as it does for aperture colors of small angular extent (assumption (c), section III, 1). For improved agreement, the red end of the spectrum locus must be moved closer to the daylight point (0.44, 0.47, 0.09), and the yellow part must be moved further away. Experiments on relative size of small color differences between large surfaces have already yielded the same conclusion [42, 43].

It may also be noted from table 8 that the computed hues and saturations are much alike for the observing situations (cases 1 to 4) evaluated; that is, it makes little difference according to the formulation whether an observer (a) fixates the sample momentarily, or (b) for several seconds, or (c) discounts the faint yellow-red color of south daylight compared to average daylight, or (d) is even adapted to average daylight and judges the samples merely as color areas or patches of chromatic light. A far different conclusion applies, however, to strongly chromatic illuminants, as may be seen from table 9.

2. STRONGLY CHROMATIC ILLUMINANTS

Table 9 shows that the formulation is as successful for strongly chromatic illuminants as for south daylight. As before, case 3 is seen to yield nearly perfect agreement with estimated lightnesses, while case 4 yields higher lightnesses which, for about 40 percent of the sample-illuminant-background combinations, are too high to be acceptable. Figure 4 compares the average estimates of lightness with the lightnesses computed by eq 4 and 5a (case 3) for the 15 samples both on the light and the dark background for all 5 illuminants. It is seen that the Adams and Cobb formula renders correctly the effect on lightness of changing from light to dark background, provided the apparent reflectance of the field, A_f, be computed by eq 5a, an arithmetical mean. (The quadratic mean recommended by Adams and Cobb [2] was found not to give agreement with our experimental results; neither does eq 5c give agreement.) It would seem, therefore, that our observers in their estimates of lightness followed instructions not to fixate the sample being judged too long; and,

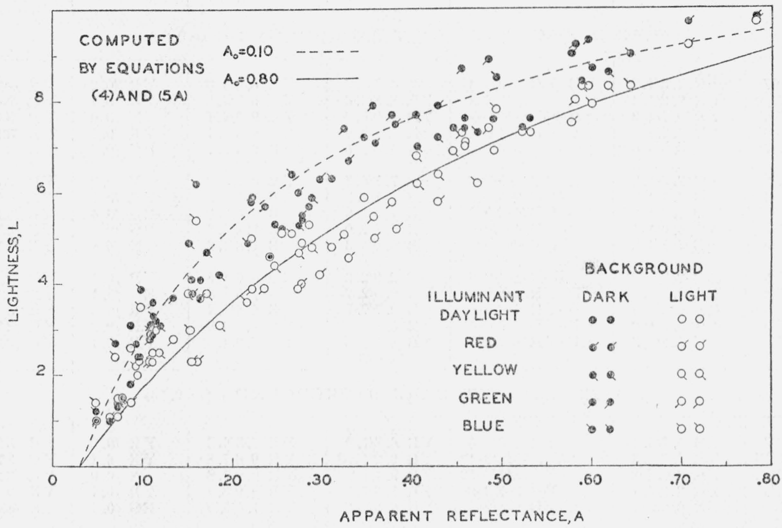

FIGURE 4.—*Comparison of estimated lightness, L, with that computed from eq 4 and 5a.*

The agreement is generally good with the exception of dark samples on a light background for nonred illuminants. See section VIII for the discussion of this discrepancy.

indeed, since light and dark samples, such as could exemplify approximately the end points (black and white) of the lightness scale, were always present in the field, there was an incentive to shift the fixation point often from the sample being judged to those representative of the end points. For estimates of hue and saturation, however, no fixed points were exemplified in the field of view, and table 9 shows that case 4 yields agreement with our observers which is not significantly different from that yielded by case 3. It is possible, therefore, that some of our observers fixated the sample whose color was being judged long enough to make case 4 apply. Case 2 gives fairly good predictions for samples on the light background, but for the red and yellow illuminants is definitely inapplicable to those on the dark. Case 1 yields hues in agreement with many of those actually reported, but, as might be expected, the saturations are much higher than those reported.

TABLE 9.—*Comparison of observed and computed color descriptions*

[Hue, lightness, and saturation (*H L/S*) of the 15 selective samples studied under the 4 chromatic illuminants; see table 4]

Sample	Average observed values, *H L/S*	Individual observer range, ΔH ΔL/ΔS	Computed for different viewing situations: Momentary fixation of sample (Case 3)	Fixation of sample for several seconds (Case 4)	Illuminant color discounted (Case 2)	Aperture color for daylight-adapted eye (Case 1)
RED ILLUMINANT, LIGHT BACKGROUND (A_0=0.80)						
R 3/7	bR 4.0/1.2	RB–YR 3–5/0–4	R 4.7/1.7	YR 6.6/3.0	YR /0.2	YR /34.1
R 5/10	rYR 9.2/1.4	R–rY 8–10/0–4	YR 8.5/6.6	YR 9.3/5.9	YR /0.1	YR /33.9
R 7/6	YR 9.7/1.3	yR–rY 8–10/0–4	YR 9.0/7.0	YR 9.5/6.1	N /0.0	YR /33.9
YR 4/5	bR 4.2/1.1	RB–YR 3–5/0–4	R 5.0/1.9	YR 6.8/3.1	YR /0.1	YR /33.9
Y 5/7	bR 3.9/1.0	RB–YR 3–5/0–4	R 4.6/1.5	YR 6.5/2.7	BG /0.1	YR /33.8
Y 7/8	yR 7.5/1.0	R–YR 6–9/0–2	YR 7.7/5.3	YR 8.8/5.3	N /0.0	YR /33.9
G 3/4	B 1.1/1.9	BG–BR 0–2/0–6	B 1.1/4.9	B 2.1/3.3	YR /0.2	YR /34.1
G 5/7	B 1.4/1.9	BG–RB 1–2/0–6	B 1.5/4.8	B 2.5/3.0	BG /0.4	YR /33.6
G 7/7	yR 4.6/1.1	R–YR 4–6/0–4	R 5.4/2.0	YR 7.1/3.3	BG /0.1	YR /33.7
B 3/5	gB 1.1/1.8	BG–B 0–2/0.6	B 0.9/5.3	B 1.6/3.6	N /0.0	YR /33.9
B 5/6	RB 2.3/1.4	B–R 1–4/0–6	B 2.9/2.6	RB 4.6/1.5	YR /0.1	YR /33.9
B 7/4	yR 5.2/1.0	R–YR 4–6/0–4	YR 6.0/2.7	YR 7.6/3.8	BG /0.1	YR /33.8
P 3/6	RB 2.3/1.5	B–YR 1–4/0–6	B 2.7/2.7	RB 4.4/1.4	YR /0.3	YR /34.2
P 5/6	bR 5.0/1.0	RB–YR 4–6/0–4	YR 5.8/2.8	YR 7.4/3.9	YR /0.2	YR /34.1
PB 7/4	R 5.8/1.3	RB–YR 5–7/0–4	YR 6.5/3.6	YR 7.9/4.3	YR /0.1	YR /34.0
RED ILLUMINANT, DARK BACKGROUND (A_0=0.10)						
R 3/7	YR 5.5/3.4	YR 5–6/2–5	YR 6.3/7.0	YR 7.8/6.3	YR /0.2	YR /34.1
R 5/10	YR 9.7/6.3	YR 7–11/3–12	YR 9.2/11.8	YR 9.6/7.8	YR /0.1	YR /33.9
R 7/6	YR 9.8/6.3	YR 7–11/3–12	YR 9.4/12.2	YR 9.8/7.8	N /0.0	YR /33.9
YR 4/5	YR 6.3/4.1	YR 5–7/3–6	YR 6.6/7.1	YR 8.0/6.1	YR /0.1	YR /33.9
Y 5/7	YR 6.0/3.4	YR 5–8/2–5	YR 6.2/6.5	YR 7.8/6.0	BG /0.1	YR /33.8
Y 7/8	YR 9.0/6.0	YR 7–10/3–11	YR 8.6/10.7	YR 9.3/7.5	N /0.0	YR /33.9
G 3/4	RB 1.3/0.4	RB–R 1–2/0–2	RB 1.9/1.9	R 3.5/2.0	YR /0.2	YR /34.1
G 5/7	R 1.8/0.6	RB–R 1–3/0–4	RB 2.4/1.5	YR 4.2/2.0	BG /0.4	YR /33.6
G 7/7	YR 6.7/3.9	YR 6–8/3–6	YR 6.9/7.6	YR 8.3/6.2	BG /0.1	YR /33.7
B 3/5	RB 1.0/0.6	B–RB 0–2/0–4	B 1.6/2.3	R 3.0/1.5	N /0.0	YR /33.9
B 5/6	YR 3.7/1.7	R–YG 3–4/1–4	YR 4.4/3.5	YR 6.3/4.4	YR /0.1	YR /33.9
B 7/4	YR 7.5/4.6	YR 6–8/3–8	YR 7.4/8.4	YR 8.6/6.7	BG /0.1	YR /33.8
P 3/6	rYR 3.8/2.0	R–YR 3–4/1–4	YR 4.2/3.5	YR 6.1/4.4	YR /0.3	YR /34.2
P 5/6	YR 6.9/4.2	YR 5–8/2–8	YR 7.2/8.4	YR 8.5/6.8	YR /0.2	YR /34.1
PB 7/4	YR 7.9/5.0	YR 6–9/3–10	YR 7.8/9.4	YR 8.8/7.2	YR /0.1	YR /34.0
YELLOW ILLUMINANT, LIGHT BACKGROUND (A_0=0.80)						
R 3/7	rBR 3.8/10.0	RB–R 3–6/4–16	R 2.8/6.3	R 4.4/6.5	YR /6.2	YR /22.4
R 5/10	yR 6.8/9.7	R–YR 5–9/5–16	YR 6.3/7.3	YR 7.8/7.5	YR /6.4	YR /22.7
R 7/6	rYR 8.3/6.1	R–Y 8–10/3–10	YR 8.0/4.2	YR 9.0/4.1	R /2.8	YR /19.1
YR 4/5	bR 4.4/3.3	RB–Y 3–5/1–4	R 4.3/2.6	R 6.2/2.8	YR /2.1	YR /18.5
Y 5/7	YG 4.8/0.2	Y–G 4–6/0–1	RB 4.9/0.4	YR 6.7/0.5	BG /0.3	YR /16.3
Y 7/8	gY 8.3/0.4	Y–YG 8–10/0–1	Y 7.8/1.4	Y 8.8/1.4	BG /0.1	YR /16.4
G 3/4	bRB 1.5/4.2	B–RB 1–2/0–8	B 1.3/2.6	B 2.1/1.9	BG /1.3	YR /15.2
G 5/7	gBG 3.8/6.1	G–B 3–6/3–8	BG 3.1/4.8	BG 4.8/4.4	BG /4.8	YR /12.1
G 7/7	yG 6.4/4.3	YG–G 5–7/2–8	G 6.5/1.1	G 7.9/1.1	BG /1.9	YR /14.7
B 3/5	bRB 1.5/4.7	B–RB 1–2/0–10	B 1.2/2.6	B 1.9/2.1	BG /1.4	YR /15.2
B 5/6	gB 3.1/1.4	BG–RB 2–4/0–3	B 3.3/1.7	BG 5.1/1.0	BG /1.4	YR /15.1
B 7/4	yG 6.7/2.7	YG–G 5–7/1–4	YG 6.6/0.4	YG 8.0/0.5	BG /1.0	YR /15.5
P 3/6	rRB 2.5/5.5	RB–YR 2–3/2–10	RB 2.2/1.8	R 3.5/2.1	YR /1.8	YR /18.2
P 5/6	bR 4.7/3.3	RB–Y 4–6/1–4	R 4.7/2.3	YR 6.5/2.6	YR /1.8	YR /18.2
PB 7/4	YG 6.2/0.2	Y–G 5–7/0–1	YR 6.3/0.9	YR 7.8/1.1	RB /0.1	YR /16.5

TABLE 9.—*Comparison of observed and computed color descriptions*—Continued

[Hue, lightness, and saturation (*H L/S*) of the 15 selective samples studied under the 4 chromatic illuminants; see table 4]

Sample	Average observed values, *H L/S*	Individual observer range, $\Delta H\ \Delta L/\Delta S$	Computed for different viewing situations: Momentary fixation of sample (Case 3)	Fixation of sample for several seconds (Case 4)	Illuminant color discounted (Case 2)	Aperture color for daylight-adapted eye (Case 1)
YELLOW ILLUMINANT, DARK BACKGROUND (A_0=0.10)						
R 3/7	bR 4.9/8.4	RB–R 4–6/7–12	YR 4.2/7.0	YR 6.1/7.2	YR /6.2	YR /22.4
R 5/10	rYR 7.7/10.4	R–YR 6–10/8–16	YR 7.6/8.5	YR 8.7/8.0	YR /6.4	YR /22.7
R 7/6	yYR 8.6/8.9	R–Y 7–10/6–16	YR 8.9/5.6	YR 9.4/4.6	R /2.8	YR /19.1
YR 4/5	YR 5.3/5.2	YR 3–6/4–7	YR 5.9/3.6	YR 7.6/3.5	YR /2.1	YR /18.5
Y 5/7	gY 5.8/3.5	Y–YG 5–6/1–6	Y 6.5/1.8	Y 8.0/3.6	BG /0.3	YR /16.3
Y 7/8	gY 8.4/6.6	Y–yYG 7–10/2–14	Y 8.8/3.1	Y 9.4/2.0	BG /0.1	YR /16.4
G 3/4	yG 1.5/0.6	YG–G 1–2/0–2	B 2.2/1.4	BG 3.6/0.7	BG /1.3	YR /15.2
G 5/7	G 4.7/8.6	G 4–6/7–12	G 4.7/3.8	G 6.5/3.7	BG /4.8	YR /12.1
G 7/7	gYG 7.2/6.6	YG–G 6–8/4–10	YG 7.8/1.9	YG 8.8/1.4	BG /1.9	YR /14.7
B 3/5	yG 1.4/0.4	YG–G 1–2/0–2	B 2.0/1.4	BG 3.4/0.9	BG /1.4	YR /15.2
B 5/6	gYG 4.2/4.3	YG–G 3–5/3–6	YG 5.0/0.7	YG 6.8/0.7	BG /1.4	YR /15.1
B 7/4	YG 7.2/5.3	Y–G 6–8/2–8	Y 7.9/2.2	Y 8.9/1.4	BG /1.0	YR /15.5
P 3/6	rYR 3.1/3.4	R–YR 2–5/2–6	R 3.4/2.4	YR 5.2/2.6	YR /1.8	YR /18.2
P 5/6	YR 5.3/4.9	YR 4–6/3–6	YR 6.3/3.6	YR 7.8/3.3	YR /1.8	YR /18.2
PB 7/4	Y 7.0/4.7	Y–gY 6–8/2–8	Y 7.6/2.6	Y 8.7/1.9	RB /0.1	YR /16.5
GREEN ILLUMINANT, LIGHT BACKGROUND (A_0=0.80)						
R 3/7	RB 1.0/3.3	RB 1–2/0.8	R 0.5/2.1	R 1.0/1.8	YR /1.2	G /7.8
R 5/10	R 2.3/2.4	RB–yG 2–3/0–6	R 2.0/1.9	R 3.2/1.7	YR /1.3	G /7.7
R 7/6	rY 5.5/1.4	R–G 4–7/0–4	YR 5.8/0.5	YR 7.4/0.5	YR /0.5	G /8.4
YR 4/5	YR 2.5/2.8	R–YG 2–3/0–6	R 2.0/2.4	R 3.2/2.2	YR /1.9	YG /7.4
Y 5/7	gY 4.1/2.8	YR–YG 3–5/1–4	YR 4.0/1.5	YR 5.8/1.3	YR /1.3	YG /7.9
Y 7/8	gY 7.1/5.1	Y–YG 6–8/3–8	YR 6.7/1.3	YR 8.1/1.3	YR /1.3	YG /7.9
G 3/4	B 2.3/1.8	G–RB 2–3/0–6	RB 2.0/0.7	RB 3.2/0.4	G /0.4	G /9.1
G 5/7	G 5.8/2.4	G 4–8/0–4	G 6.0/0.8	G 7.6/0.9	G /0.7	G /9.4
G 7/7	G 8.3/0.9	G 7–9/0–2	G 7.9/0.8	G 8.9/0.9	G /0.4	G /9.1
B 3/5	B 2.8/3.8	B 2–4/2–6	B 2.4/1.3	BG 3.9/1.3	BG /1.3	G /9.8
B 5/6	B 5.1/3.9	B 3–6/1–7	BG 5.4/1.1	BG 7.1/1.2	BG /1.2	G /9.6
B 7/4	B 8.0/0.5	BG–B 6–9/0–2	G 7.8/0.8	G 8.8/0.9	BG /0.5	G /9.1
P 3/6	bRB 2.3/2.1	B–RB 2–3/0–6	RB 1.7/1.1	RB 2.8/0.8	B–RB /0.2	G /8.7
P 5/6	B 4.1/0.8	BG–RB 2–5/0–4	RB 4.2/0.4	B 6.1/0.1	BG /0.1	G /8.8
PB 7/4	B 7.2/1.0	B 6–8/0–2	G 7.0/0.7	G 8.3/0.7	BG /0.5	G /9.1
GREEN ILLUMINANT, DARK BACKGROUND (A_0=0.10)						
R 3/7	bR 1.0/2.2	RB–R 0–2/0–6	R 1.0/1.7	R 1.9/1.4	YR /1.2	G /7.8
R 5/10	yYR 2.9/2.4	YR–Y 2–6/1–4	YR 3.2/1.4	YR 4.9/1.3	YR /1.3	G /7.7
R 7/6	gY 7.7/3.6	Y–YG 6–10/1–10	YG 7.2/0.9	Y 8.4/0.8	YR /0.5	G /8.4
YR 4/5	yYR 3.6/3.8	YR–Y 3–6/2–6	YR 3.2/2.0	YR 4.9/1.9	YR /1.9	YG /7.4
Y 5/7	gY 6.1/4.8	Y–YG 4–8/2–10	Y 5.6/1.4	Y 7.3/1.3	YR /1.3	YG /7.9
Y 7/8	Y 8.5/6.7	Y 8–12/3–16	Y 8.0/1.6	Y 8.9/1.4	YR /1.3	YG /7.9
G 3/4	G 2.8/1.2	G 2–4/0–4	BG 3.2/0.3	G 4.8/0.4	G /0.4	G /9.1
G 5/7	yG 7.7/4.4	YG–G 7–10/2–10	G 7.4/1.6	G 8.6/1.3	G /0.7	G /9.4
G 7/7	gYG 9.3/4.6	YG–G 9–12/1–12	YG 8.8/1.7	YG 9.4/1.2	G /0.4	G /9.1
B 3/5	gB 3.7/3.2	BG–B 3–4/2–6	BG 3.8/1.3	BG 5.6/1.4	BG /1.3	G /9.8
B 5/6	gB 7.2/4.0	G–B 6–10/2–10	G 7.0/1.7	G 8.2/1.5	BG /1.2	G /9.6
B 7/4	yG 9.2/4.2	YG–G 8–12/1–12	YG 8.7/1.6	YG 9.4/1.2	BG /0.5	G /9.1
P 3/6	RB 2.4/0.5	RB 1–4/0–4	RB 2.8/0.4	B–RB 4.6/0.2	B–RB /0.2	G /8.7
P 5/6	bG 5.9/2.0	G–BG 4–8/0–6	YG 5.7/0.6	YG 7.5/0.6	BG /0.1	G /8.8
PB 7/4	G 8.7/6.1	YG–BG 8–12/2–12	YG 8.2/1.5	G 9.0/1.0	BG /0.5	G /9.1

TABLE 9.—*Comparison of observed and computed color descriptions*—Continued

[Hue, lightness, and saturation (*H L/S*) of the 15 selective samples studied under the 4 chromatic illuminants; see table 4]

Sample	Average observed values, *H L/S*	Individual observer range, ΔH $\Delta L/\Delta S$	Computed for different viewing situations			
			Momentary fixation of sample	Fixation of sample for several seconds	Illuminant color discounted	Aperture color for daylight-adapted eye
			(Case 3)	(Case 4)	(Case 2)	(Case 1)
			BLUE ILLUMINANT, LIGHT BACKGROUND (A_0=0.80)			
R 3/7	rRB 1.4/1.8	RB-R 1-2/0-6	RB 0.5/2.7	RB 1.0/2.1	R /0.8	RB /20.3
R 5/10	bR 3.0/3.6	RB-R 3/2-6	RB 2.0/1.6	RB 3.3/1.3	R /0.4	RB /20.1
R 7/6	RB 5.9/3.1	RB-rRB 5-7/1-5	RB 5.7/1.3	RB 7.3/1.8	R /0.3	RB /20.0
YR 4/5	R 2.4/2.5	RB-YR 2-3/1-4	RB 1.1/0.2	Y 1.8/0.6	YG /1.5	RB /18.5
Y 5/7	rYR 3.5/2.8	RB-Y 3-4/1-4	YG 1.7/1.0	YG 2.9/1.6	YG /2.4	RB /17.6
Y 7/8	yYR 5.4/4.6	R-Y 4-6/1-6	YG 2.9/3.8	YG 4.6/3.9	YG /4.6	B /15.6
G 3/4	bRB 2.2/0.5	B-RB 2-3/0-2	B 1.6/0.5	YG 2.8/0.3	YG /1.2	B /19.0
G 5/7	yG 5.1/3.8	YG-BG 4-6/2-5	YG 4.7/3.5	YG 6.5/3.4	YG /4.4	B /16.7
G 7/7	yG 7.6/1.5	Y-G 6-9/0-4	BG 7.1/1.9	BG 8.4/2.1	YG /2.3	B /18.2
B 3/5	B 3.8/2.2	B 3-4/0-4	RB 3.9/0.9	RB 5.7/1.2	B /0.3	RB /20.2
B 5/6	gB 6.2/4.4	BG-B 5-7/2-6	B 6.9/2.1	B 8.2/2.3	G /0.7	RB /19.6
B 7/4	B 8.3/2.2	G-B 7-9/1-4	B 8.2/3.5	B 9.1/3.1	YG /0.3	RB /19.7
P 3/6	rB 3.0/3.4	B-RB 2-4/0-4	RB 2.8/2.8	RB 4.4/2.6	RB /1.8	RB /21.4
P 5/6	rB 4.8/2.8	R-rRB 4-5/0-5	RB 5.3/2.0	RB 6.9/2.5	RB /1.0	RB /20.8
PB 7/4	B 7.9/2.3	B 6-9/1-4	B 7.9/3.5	B 8.9/3.1	RB /0.1	RB /20.1
			BLUE ILLUMINANT, DARK BACKGROUND (A_0=0.10)			
R 3/7	bR 1.2/2.0	RB-R 0-2/0-4	RB 1.0/1.7	RB 1.8/1.6	R /0.8	RB /20.3
R 5/10	rRB 3.3/4.2	RB-R 2-4/1-6	RB 3.2/1.3	RB 4.9/1.4	R /0.4	RB /20.1
R 7/6	rRB 7.2/2.5	RB-R 6-8/0-4	B 7.2/3.1	B 8.4/2.3	R /0.3	RB /20.0
YR 4/5	YR 2.7/3.3	rYR-YR 2-4/2-5	Y 2.0/1.0	Y 3.3/0.8	YG /1.5	RB /18.5
Y 5/7	yYR 3.9/4.2	YR-Y 3-5/3-5	YG 2.9/1.7	YG 4.6/1.5	YG /2.4	RB /17.6
Y 7/8	rY 6.2/6.2	YR-Y 5-7/4-8	YG 4.5/3.4	YG 6.2/3.3	YG /4.6	B /15.6
G 3/4	yG 2.7/1.7	YG-G 2-4/1-3	YG 2.8/0.4	G 4.4/0.5	YG /1.2	B /19.0
G 5/7	yG 6.4/5.1	YG-G 6-7/4-8	G 6.3/3.4	G 7.8/3.4	YG /4.4	B /16.7
G 7/7	gBG 8.3/2.1	G-B 8-9/1-8	BG 8.3/3.7	BG 9.1/2.2	YG /2.3	B /18.2
B 3/5	B 5.1/5.1	gB-B 4-6/4-8	B 5.6/2.2	B 7.2/2.0	B /0.3	RB /20.2
B 5/6	B 7.3/5.5	B 7-8/4-10	B 8.2/4.1	B 9.0/2.4	G /0.7	RB /19.6
B 7/4	B 9.0/3.4	B 8-10/1-10	B 9.0/5.1	B 9.5/2.6	YG /0.3	RB /19.7
P 3/6	bRB 4.1/4.2	B-RB 3-5/2-8	RB 4.3/2.9	RB 6.1/3.1	RB /1.8	RB /21.4
P 5/6	rB 6.3/4.4	B-RB 6-7/3-8	B 6.8/3.7	RB 8.1/2.9	RB /1.0	RB /20.8
PB 7/4	B 8.7/3.9	B 8-10/2-10	B 8.9/5.2	B 9.4/2.8	RB /0.1	RB /20.1

Table 10 summarizes the success of the formulation for the four observing situations. Each computed result, correct in the sense that it is within the range of the individual estimates, is given in tables 8 and 9 in plain (not bold-faced) type. The total numbers correct in this sense out of the 15 samples are given in table 10 for each illuminant-background combination. If the total should be listed 15 15/15, perfect agreement between formulation and preliminary experiment would be indicated for hue, lightness, and saturation; but if the total should be 0 0/0, no correspondence at all would be indicated. Totals for all illuminants are given for each background, and grand totals for each observing situation. The grand totals are also expressed in percentages of the total number (150) of sample-illuminant-background combinations. It will be seen that the formulation for case 3 yields 97 percent of these combinations in agreement with the experiment in regard to lightness, 84 percent in regard to hue, and 73 percent in regard to saturation. The superiority of case 3 over cases 2 and 4 is gratifying, because it indicates that the attempt to take into account the actual observing situation has been largely successful. On the other hand, there is need for further work before it can be said that this success is complete.

TABLE 10.—*Number of sample-illuminant-background combinations correct*

Illuminant	Apparent reflectance of background A_0	Number correct			
		Case 3	Case 4	Case 2	Case 1
South daylight	0.80	12 15/7	12 10/7	12 /7	11 /5
	.10	12 14/7	12 8/7	12 /7	11 /6
Red	.80	15 15/12	15 7/12	15 /15	11 /0
	.10	15 15/11	14 7/13	9 /3	12 /0
Yellow	.80	14 15/13	14 9/14	10 /15	5 /0
	.10	12 15/12	12 6/10	5 /3	5 /0
Green	.80	10 15/13	9 12/13	10 /13	7 /2
	.10	14 15/12	14 12/10	10 /8	8 /8
Blue	.80	11 11/14	10 12/14	7 /10	8 /0
	.10	11 15/9	11 9/9	8 /8	6 /0
Total	.80	62 71/59	60 50/60	54 /60	42 /7
	.10	64 74/51	63 42/49	44 /29	42 /14
Grand total correct		126 145/110	123 92/109	98 /89	84 /21
Percentage correct		84 97/73	82 61/73	65 /59	56 /14

Out of the 56 combinations for which case 3 failed to yield acceptable color descriptions, all but 4 combinations can apparently be adjusted by a more careful choice of constants in the definition of L' and in eq 3, by a redefinition of hue (eq 2) without the simplifying restriction that red shall be taken as the exact opposite of green, and by the use of a coordinate system giving improved chromaticity spacing. The preliminary data do not afford a very reliable basis for such revaluation of the constants; and it is planned next to examine carefully the more complete data of Helson and Jeffers [26], which have already served as guides in several aspects of the formulation. The four combinations which are still a puzzle are two on the light background (B 7/4, PB 7/4) under the green illuminant, and two under the blue illuminant, both involving Y 7/8, one for the light, the other for the dark background. It is interesting to note that the observers reported more nearly the daylight colors of these samples than apparently can be justified from the computations, but we do not believe that the observers remembered the daylight colors. Helson and Jeffers [25, 26] find color descriptions in accord with our computations for samples B 7/4 and PB 7/4 under the green illuminant; this finding suggests an unknown source of error in our preliminary experiments.

But they find the same colors as our observers did for sample Y 7/8 under the blue illuminant. This finding suggests that the formulation may have to be revised.

The large individual variations in estimates of saturation and the smaller variations in hue can be explained; first, by the ambiguity of the observing situation which permits allocation by the observer of any chromatic component of the illumination hue in indeterminate proportion between the illumination and the surface color;[16] and second by variations in eye movements such as distinguish case 3 from case 4. The need for these explanations in regard to saturation would be more acute, however, if our observers had agreed better in their saturation estimates in south daylight (table 8). A large part of the individual variation is to be ascribed to uncertainty inherent in our experimental method.

3. SUBSTITUTION OF A DARK FOR A LIGHT BACKGROUND

The comparison between computed and experimentally obtained hues and saturations has so far dealt only with the ranges in the experimental values. One might readily suppose that the agreement between computation and experiment for both light and dark backgrounds arises from an experimental uncertainty sufficiently large to cover any differences between them. The success of the formulation is more detailed and complete, however, than has yet been demonstrated, and to show this success we have compared the direction of hue differences computed for each sample-illuminant combination with that observed, taking the average observed values in tables 8 and 9. Similar comparisons have been made for the direction of lightness and saturation differences and have been summarized in table 11. It will be noted that for about 65 percent of the sample-illuminant combinations the computed change agrees with that indicated by the average of the actual estimates in the sense that for these combinations either the direction of the difference agrees, when both observed and computed differences are significant, or both computed and observed differences are too small to be significant. About 33 percent of the combinations show disagreement in the limited sense either that no significant difference was observed even though predicted by the computation, or the reverse. There are 28 combinations (37 percent) which showed limited disagreement of this sort in lightness; for these combinations no significant difference was observed from substitution of a dark for a light background, but eq 4 indicates for each combination an increase in lightness greater than 0.5. This may also be seen from figure 4. It is probably significant that nearly all (13 out of 15) of the samples viewed by south daylight fall in this category. Helson [25–26] has discussed the conditions under which the expected increase in lightness is found by observation.

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[16] Katz [48, p. 219] ascribes the large individual differences principally to differences in attitude. Gelb [16, p. 677] likewise mentions attitude, which he found to be "fluctuating and unstable," corresponding to different organizations of the visual field. This view is close to ours in section III, 2, (c).

TABLE 11.—*Effect on the color of a sample caused by substitution of a dark for a light background*

HUE

Computed ↓ / Observed →	Number of sample-illuminant combinations: Observed to change in the RYGBR sense by more than one-sixteenth of the hue circuit	Observed to change by less than one-sixteenth of the hue circuit	Observed to change in the RBGYR sense by more than one-sixteenth of the hue circuit
Computed to change in the RBGYR sense	0	5	7
Same hue on the 8-point hue scale	6	32	3
Computed to change in the RYGBR sense	13	8	1

LIGHTNESS

Computed ↓ / Observed →	Number of sample-illuminant combinations: Observed to decrease by more than 0.5	Observed to change by 0.5 or less	Observed to increase by more than 0.5
Computed to increase by more than 0.5	0	28	44
Computed to change by 0.5 or less	0	3	0
Computed to decrease by more than 0.5	0	0	0

SATURATION

Computed ↓ / Observed →	Number of sample-illuminant combinations: Observed to decrease by more than 0.5	Observed to change by 0.5 or less	Observed to increase by more than 0.5
Computed to increase by more than 0.5	3	1	34
Computed to change by 0.5 or less	8	5	14
Computed to decrease by more than 0.5	8	1	1

The combinations showing direct contradiction between computation and experiment are few; one in hue, none in lightness, and four in saturation. The contradiction in hue is found for sample PB 7/4 under the yellow illuminant (see table 9); the computed hue change is from YR to Y; the observed, from YG to Y. There is some doubt as to the significance of this apparent contradiction because of the low saturation (0 to 1) of the reported color. The four contradictions in saturation are found for samples R 3/7, G 5/7 and P 3/6 under the yellow illuminant, and R 7/6 under the blue (see table 9). These discrepancies are by about 2 steps on the saturation scale of 10; considering the large individual variation in estimates of saturation, a few such contradictions are to be expected even in an average of estimates by six observers. At least one observer corroborated each computed result. It may be concluded that the formulation is successful in specifying the observed effect on color of the sample caused by substitution of a dark for a light background.

VIII. INFLUENCE OF LIGHT-SCATTERING ELEMENTS BETWEEN SAMPLE AND RETINA

If the optical media between the sample plane and the retinal image of that plane contain light-scattering elements, it is to be expected that the aperture colors corresponding to all surfaces will be modified in the sense corresponding to admixture of light reflected from the field as a whole. The amount of the admixture would correspond to the number of light-scattering elements; these elements might be in the space between the observer and the samples (air molecules, dust, or fog), on the surface of the filters through which the observers looked (imperfect polishing of the surface, dust, or grease), or in the optical media of the eye itself (incipient cataract, floating particles in the aqueous humour, inclusions, or haze in the vitreous humor). Although such elements were present in all these places, no specific account of them has been taken; first, because it has been assumed that the amount of light scattered by them is too small to produce a significant effect; and second, because such light would chiefly serve as a clue to the illumination [10] rather than influence the estimates of hue, lightness, and saturation of the surface color.

Certain aspects of the formulation and comparison with experiment indicate, however, that the influence of light-scattering elements may not be negligible. The situation in which such influence would make itself most evident is the observation of dark samples on a light background. Light from the background would be scattered onto the retinal image of the dark sample, and even though amounting to a very small fraction of the retinal illuminance corresponding to the background, it might constitute a considerable fraction of that corresponding to the dark sample itself. In such a case the chromaticity of the color would be altered toward that of the scattered light, and the observed lightness would be higher than could be accounted for by eq 4. Reference to figure 4 shows, except for the red illuminant, that all samples of apparent reflectance less than 0.18 were observed to have lightnesses higher than those computed from eq 4, the discrepancies being particularly large for the blue illuminant. It may therefore be significant that in table 9 the only sample-illuminant-background combinations yielding computed values too low to fall within the individual variation are four samples of low apparent reflectance on the light background under the blue illuminant. These discrepancies suggest the presence of light-scattering elements in the eye media so small that they are selective in their scattering power, scattering more short-wave than long-wave light somewhat in accord with the Rayleigh law that such scattering is inversely proportional to the fourth power of the wavelength. Light-scattering elements of this sort were postulated by Holladay [32] to explain certain of his measurements on the amount of stray light in the eye. Further studies of this quantity have been carried out by Bartley and Fry [5], by le Grand [20] and by Schouten [81]. Helmholtz [30, p. 274, 278] suggested that fluorescence of the eye media, as well as scattering of light, might be a contributing cause.

It should be noted in this connection that a part of the effect of the $(L')^2$ term in the formula for r_n (eq 3) is to contribute to a red-blue component for dark samples on a light ground. This component is proportional to the apparent reflectance, A_f, and blue trilinear coordinate, b_f, of the field; so it is already closely of the form required to take account of Rayleigh scattering by the eye media. Since this term combined with 0.030 D_f in the expression for g_n agrees qualitatively with Karwoski's findings [46] of the prevalence of red-blue in the after-images produced by intense stimuli, we have so far viewed it as an approximate representation of some photochemical or neurological fact of the retina or central nervous system. Perhaps, however, it is to be considered as partly based upon Rayleigh scattering and fluorescence by the eye media. A better approximation to Rayleigh scattering could be obtained by the substitution of the term $1+10b_f$ for b_f; and it might also be worth while to modify eq 4 by the appropriate introduction of such a term. Appraisal of the value of such modifications will be easier after a detailed check of the formulation against the extensive experimental results of Helson and Jeffers [26] has been carried out.

IX. BEZOLD-BRÜCKE PHENOMENON AND THE HUE CHANGE BY ADMIXTURE OF ACHROMATIC LIGHT

It was observed by Von Bezold [6] and by Brücke [9] that an increase in luminance not only increases the brightness of the aperture color but also introduces a change in hue. Red, yellow-red, and yellow-green shift toward yellow; red-blue and blue-green toward blue. By reverse application of assumption (d) (section III) it will be noted that the present formulation should apply to this hue change, often called the "Bezold-Brücke" phenomenon. Inspection of eq 3 shows that the formulation agrees in a broad way with the Bezold-Brücke phenomenon, provided the formulation be rewritten to apply to aperture colors by substitution of luminance for apparent luminous reflectance, A. There have been some quantitative studies of this phenomenon, notably by Janicki and Lau [36] and by Purdy [76],[17] against which it is possible to check the present formulation, and, perhaps, obtain better values for the constants; but computations by eq 1 and 3 for comparison with these quantitative results have not yet been made.

The hue change by admixture of achromatic light has been studied by Müller [77] both for dark and for light backgrounds, and by Abney [1] for dark background. Abney found red, yellow-red, and yellow-green to shift toward yellow, and red-blue to shift toward red. The hue change for a light background is analogous to that for dark with colors of red-blue hue, but reverses for red, yellow-red and yellow-green, which shift away from yellow. It may be shown from eq 3 that the present formulation is in qualitative agreement with these known facts, but quantitative comparison has yet to be made.

These hue changes by change of luminance and by admixture of achromatic light make the concept of dominant wavelength an unre-

[17] Consult Purdy [76] for 14 other references to the Bezold-Brücke phenomenon, experimental and theoretical.

liable indication of hue. The present formulation may therefore lead to a substitute for dominant wavelength by means of which colorimetric results (such as obtained by the spectrophotometer) may be interpreted in accord with the actual hue of the surface color. Aids to computation of hue by eq 1, which would make it readily applicable in practice, have yet to be worked out.

X. RELATION TO VISUAL THEORY

The present formulation giving approximately the hue, saturation, and lightness of a surface color in any of a wide variety of illuminants for both light and dark backgrounds is, of course, in no sense a theory of vision. It does not separate photochemical activity from neural activity in the retina, nor retinal activity from cortical, nor even photochemical effects from fluorescence and scattering of light by the optical media of the eye. The primaries of the coordinate system used are not intended to represent colors produced by structurally or functionally separated parts of the visual mechanism; they merely compose one of an infinite number of sets giving approximately uniform chromaticity spacing. It is true that the $(L')^2$ term is confined in eq 3 to the expression for r_n; similarly the term $0.030D_f$ in the expression for g_n finds no counterpart in the expression for r_n. This simplicity does not however, indicate a fundamental significance to be attached to the primaries of the coordinate system so much as it indicates the preliminary character of eq 3, which has been derived empirically. Probably, further work will show the necessity of a small $(L')^2$ term in the equation for g_n, and of a term composed of D_f multiplied by a small constant in that for r_n. The sole reason for choice of the coordinate system used is its degree of approach to uniform chromaticity spacing, and it would be logical to transfer the formulation to some other coordinate system if one could be devised to yield a more uniform chromaticity-spacing.

The validation of the present empirical formulation does, however, clear the ground for the construction of a theory. The relation of the chromatic components (hue and saturation) of the surface color to the corresponding aperture color and the different character of the correspondence between the achromatic components (lightness and brightness) foreshadowed by the work of Bocksch [7, p. 395] has been formulated and verified. The variables defining the viewing situation (A_f, r_f, g_f, b_f) have been formulated and their effect upon hue, saturation, and lightness of the surface color have been quantitatively evaluated within a coordinate system defined in terms of the standard ICI system [23, 39]. Various components of these effects have been well known, but the total amounts of them have not been evaluated until now.

From the variables found to be pertinent in defining the viewing situation it may be concluded that the more familiar theories of vision (Helmholtz, Hering, Ladd-Franklin) require elaboration before they are applicable to this problem. These theories refer with some adequacy to aperture color with a dark surrounding field but not to the more complicated situation of objects in an illuminated space. The suggestion by Ives [34] for an extension of the Helmholtz theory [30, p. 235, 276] to this case is seen (section III, 2, (e)) to be an approximation to the truth for the limited groups of samples for which

$A \doteq A_f$, for illuminants that are not too selective, and probably also for durations less than 10 seconds. It may be seen from figures 2 and 3, however, that some strongly selective illuminants give rise to achromatic points which are outside the region corresponding to real stimuli. We may conclude, therefore, that to follow out Ives' suggestion would require a coordinate system in which the spectrum locus falls well inside the Maxwell triangle, such as those suggested by Helmholtz [28] and Sinden [83] on other grounds.

The theory of G. E. Müller [64] goes far toward allocation of the various color effects to activities of one or another portion of the visual mechanism, and it is furthermore elaborated to take account of light and dark surrounding fields. The information supplied by our formulation might be used to quantify the Müller theory. An initial difficulty arising from lack of any distinction in the Müller theory between brightness and lightness has been discussed by Bocksch [7, p. 436ff], who suggested a way of elaborating the Müller theory in this respect. To extend the Müller theory, however, to the perception of object and illumination color (Katz [48], Gelb [16]) in such a way as to embrace the present formulation, including the appearance of surface-color hues nearly complementary to the illuminant hue by after-image projection would be a long, involved task; but an attempt might be worth while because any resultant quantitative color theory would stimulate further visual research by being amenable to quantitative verification.

XI. SUMMARY

The stimulus for an aperture color can be expressed as a function of three variables. Surface color, however, includes the idea of a surface viewed in the presence of other surfaces in an illuminated space; it requires at least six variables. The observer may perceive the visual field yielded by such surfaces as a pattern of juxtaposed color areas, or aperture colors; or he may perceive this field organized into objects in an illuminated space. The surfaces of the objects possess surface colors, and the space possesses an illumination color; surface color and illumination color are both related to the pattern of aperture colors, illumination color being derived from clues obtained from the whole visual field and its organization. Simple visual fields, such as those studied here, can be produced by more than one combination of objects with illumination, and accordingly, it is usual to find that observers will report more than one organization of them. This ambiguity generates large individual variations. Other variations arise because the eye movements of the observer are uncontrolled; that is, the surface color may depend somewhat upon whether he has just previously been looking at the darkest or the lightest object in the field.

From a study of the literature, formulas for hue, lightness, and saturation of surface colors have been set up for all illuminants (daylight and chromatic); for all backgrounds (black, gray, white, and chromatic); and for four viewing situations. These formulas for the most common viewing situation are based upon the principle known in Helmholtz' time [30, p. 267] that the hue and saturation of the surface color seen at any given instant are the result of projecting upon the field provided by the surface an after-image of the average field

seen for the previous 15 or 20 seconds. It has been found impractical to represent changes in retinal sensitivity which produce these after-images by changes in coordinate system so as to make achromatic colors correspond to equal stimulation of the red, green, and blue processes, as in the Young-Helmholtz theory. Instead, the same coordinate system has been used for all states of chromatic adaptation, a central part of the formulation being the definition of the trilinear coordinates of the point in this system representing for each state of adaptation the stimulus for an achromatic color. This achromatic point is found to wander about in the neighborhood of the point representing the average of the visual field. There is, therefore, scarcely any real stimulus which cannot under some viewing situation give rise to an achromatic color, and, indeed, many of the achromatic points are to be found outside of the area representing real stimuli (that is, outside of the area of the Maxwell triangle bounded by the spectrum locus and the straight line joining its extremes).

The formulas have been checked and their constants approximately evaluated by means of preliminary observations by 6 observers viewing 15 samples of known spectral apparent reflectance under 5 illuminants (daylight and four strongly chromatic illuminants) of known spectral-energy distribution with two spectrally nonselective backgrounds (in daylight, dark gray, and white) of known apparent reflectance. The formulas have been improved by a preliminary study of more careful and extensive observations of the same samples under the same illuminants by Helson [24, 25, 26]. The hues, lightnesses, and saturations computed by the improved formulas have been compared with the present results and have been shown to be in agreement with estimated lightness in 97 percent of the 150 sample-illuminant-background combinations studied, to be in agreement with estimated hue in 84 percent, and to be in agreement with estimated saturation in 73 percent of the combinations studied. The connection of the formulas to theories of vision has been pointed out, and there has been given a brief discussion of their application to visual phenomena known by various names (the "dimming effect," light-scattering by the optical media of the eye, Bezold-Brücke phenomenon, and the hue change by admixture of achromatic light).

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