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## DENSITY OF SOME SODA-POTASH-SILICA GLASSES AS A FUNCTION OF THE COMPOSITION

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### ABSTRACT

Certain relations between the specific volume and composition of glasses of the soda-silica series have previously been reported. Additional glasses were made in the same series and also in the potash-silica and soda-potash-silica series. The glasses were analyzed and their densities measured. Analysis of the new data suggests that the specific volume-composition relations change definitely at compositions approximating those of eutectics, instead of at those of simple molecular ratios, as previously reported.

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### I. INTRODUCTION

The densities of 37 soda-silica glasses, containing from 13 to 50 percent of soda, were reported in a previous publication [1],<sup>1</sup> and certain relations between specific volume and composition were suggested. Since those data were published, many new glasses in the same range of composition have been made. These, together with all of the former glasses still available for further experimentation, constitute a new series of 50 soda-silica glasses whose densities and compositions are given in the present report. This report also includes data on 66 potash-silica glasses varying in composition from 17 to 40 percent of potash, and on 78 soda-potash-silica glasses whose compositions were distributed throughout the field between the composition-ranges of the two binary series. All the glasses were given the same heat treatment, which treatment differed materially from that described in the earlier report.

Study of the new data, which are considerably more numerous as well as more reproducible than those previously reported, indicated the desirability of some revision of the conclusions based on the previous data. In addition to presenting the new data, therefore, the present paper replaces that part of the earlier one which dealt with the soda-silica glasses.

<sup>1</sup> Figures in brackets indicate the literature references at the end of this paper.

## II. PREPARATION OF THE GLASSES

The reader is referred to previous publications [1, 2] for a description of the methods of preparation and analysis, and determination of density of the glasses.

"Weathering" of some of the soda-silica glasses remaining from the original series was found to have occurred when these specimens were examined for inclusion in the new series. This weathering appeared to be merely a surface effect, however, and in each case tried the original density of the glass was restored (which implied that the "interior composition" had not been changed) by removal of the surface to a depth of about 1 mm by grinding with carborundum and water. The use of water proved satisfactory provided the glasses were dried immediately after grinding. For most of the glasses the procedure was to grind off the surface twice or more immediately following heat treatment, measuring the density after each grinding, until no further change in density was noted. The densities of the few glasses which had been "reconditioned" by grinding before heat treatment were measured directly after heat treatment. These densities remained unchanged by subsequent grinding.

## III. HEAT TREATMENT

In the previously reported work, each glass was held for an hour at approximately its highest "annealing temperature." Subsequent experience indicates, however, that insufficient knowledge and control of the rates of cooling from these temperatures had introduced errors which would hinder a satisfactory evaluation of the data. In an effort, therefore, to develop a method for giving reproducible and "comparable"<sup>2</sup> heat treatments, five different heat treatments were applied successively to each of 11 glasses (indicated by the symbol  $\odot$  in figure 4) representative of the entire field of compositions studied. The heat treatments (represented schematically) and the densities after each treatment (tabulated directly beneath the respective schematic curves) are given in figure 1 in the order in which the heat treatments were applied.

### 1. DEVELOPMENT OF FIGURE 1

The "critical temperatures" and "softening temperatures" were determined for each glass as described in a former paper [11]. The term "critical range", as used herein, may be defined as a temperature interval (for each glass) above and below which large variations in cooling rate did not appreciably affect the final, or room temperature, density. An indication of the practical limits of this range is given below in the discussion of results of heat-treating experiments.

In treatments 2, 3, and 5 the glasses were necessarily treated individually since the critical temperature, softening point, and critical range are different for each glass. The difficulties involved in the individual treatments are, however, such that the results obtained are merely indicative. First, the various critical temperatures, etc., cannot be determined with assurance; and second, the extreme nature of the quenching process is such that no single experiment could be repeated in every detail. For treatment 1, however, in which it was

<sup>2</sup> The word "comparable", as used here, is intended to convey the idea of a common basis for comparison of the densities of glasses of different compositions so far as the densities may be affected by differences in heat treatment.

desired to maintain the cooling rate constant, and the same for all glasses, not only through the respective critical ranges but also for a considerable temperature interval on either side, these two difficulties were largely obviated by cooling all the glasses simultaneously in a furnace whose cooling rate was essentially constant through a temperature interval ( $550^{\circ}$  to  $300^{\circ}$  C), which included by a fairly wide margin the critical range of every glass to be treated. For treatment 4 the glasses were similarly treated except that a different furnace (one having a much greater cooling rate) was used. All glasses made for this investigation were given treatment 1, whereas the 11 glasses for which data are given in figure 1 and, in addition, 14 soda-silica, 12 potash-silica, and 7 soda-potash-silica glasses were selected for treat-

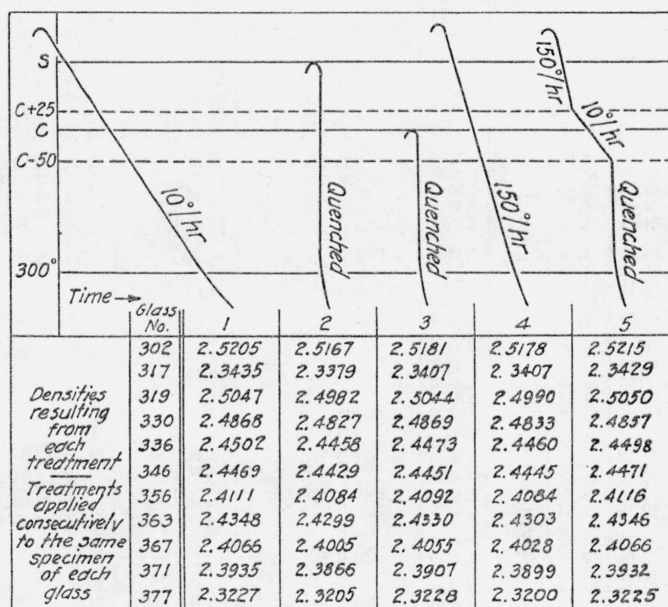


FIGURE 1.—Schematic cooling curves for each of the 11 glasses, with densities resulting from each treatment shown at the base of the corresponding curve.

S, softening temperature—different for each glass; C, critical temperature—different for each glass. The curves are not drawn to scale.

ment 4. Complete density data for these two treatments are included in table 1.

It is noteworthy that the density of a specimen after treatment does not depend upon the size of the specimen, providing size, as such, is not a factor in determining the rate of cooling (as, for instance, when the specimen is quenched in air). For example, it was found that a 46-g block of borosilicate crown glass (density approximately 2.51) and some small chips of the same material, each chip weighing about 0.2 g, had within experimental limits the same density after having been cooled simultaneously at about  $70^{\circ}$  C per hour from above the critical range. The density of the chips was measured as a composite sample. When the same specimens were quenched in air from the temperature just indicated, the large block had a considerably greater density ( $\Delta=0.0050$ ) than the small chips, the "free" cooling rate of the former necessarily being much less than those of the chips.

TABLE 1.—Density and composition for 194 soda-potash-silica glasses

Glass number <sup>1</sup>	Composition			Density		Glass number <sup>1</sup>	Composition			Density	
	SiO <sub>2</sub>	Na <sub>2</sub> O	K <sub>2</sub> O	Treat-ment 1	Treat-ment 4		SiO <sub>2</sub>	Na <sub>2</sub> O	K <sub>2</sub> O	Treat-ment 1	Treat-ment 4
	%	%	%				%	%			
1 (3)-----	51.62	48.38	-----	2.5676	-----	211-----	68.10	31.90	2.4131	2.4079	
2 (4)-----	52.60	47.40	-----	2.5622	-----	212-----	68.60	31.40	2.4108	-----	
3 (5)-----	53.62	46.38	-----	2.5557	-----	213-----	68.86	31.14	2.4086	-----	
4 (8)-----	57.45	42.55	-----	2.5361	2.5325	214-----	69.17	30.83	2.4074	-----	
5 (9)-----	58.98	41.02	-----	2.5286	2.5254	215-----	69.32	30.68	2.4064	-----	
6-----	59.04	40.96	-----	2.5290	-----	216-----	69.50	30.50	2.4046	-----	
7 (10)-----	59.69	40.31	-----	2.5262	-----	217-----	69.77	30.23	2.4047	-----	
8 (11)-----	59.71	40.29	-----	2.5257	-----	218-----	70.11	29.89	2.4015	-----	
9 (12)-----	59.97	40.03	-----	2.5236	-----	219-----	70.35	29.65	2.3994	-----	
10-----	61.17	38.83	-----	2.5188	-----	220-----	70.58	29.42	2.3984	-----	
11-----	61.77	38.23	-----	2.5147	-----	221-----	70.64	29.36	2.3972	-----	
12-----	62.58	37.42	-----	2.5111	-----	222-----	70.93	29.07	2.3966	-----	
13 (13)-----	62.77	37.23	-----	2.5092	2.5053	223-----	71.35	28.65	2.3927	-----	
14-----	62.78	37.22	-----	2.5094	-----	224-----	71.45	28.55	2.3940	-----	
15 (14)-----	62.86	37.14	-----	2.5084	-----	225-----	71.65	28.35	2.3929	-----	
16 (15)-----	63.06	36.94	-----	2.5079	-----	226-----	71.67	28.33	2.3923	2.3884	
17-----	63.13	36.87	-----	2.5080	-----	227-----	71.68	28.32	2.3928	-----	
18-----	63.60	36.40	-----	2.5056	-----	228-----	72.26	27.74	2.3888	-----	
19-----	64.00	36.00	-----	2.5028	-----	229-----	72.30	27.70	2.3884	-----	
20-----	64.71	35.29	-----	2.4979	-----	230-----	72.33	27.67	2.3891	-----	
21 (17)-----	65.32	34.68	-----	2.4961	2.4925	231-----	72.45	27.55	2.3886	-----	
22-----	65.72	34.28	-----	2.4912	-----	232-----	72.49	27.51	2.3874	-----	
23-----	66.16	33.84	-----	2.4904	-----	233-----	72.60	27.40	2.3868	-----	
24 (18)-----	66.52	33.48	-----	2.4869	-----	234-----	72.74	27.26	2.3862	-----	
25-----	66.58	33.42	-----	2.4859	-----	235-----	73.00	27.00	2.3850	-----	
26-----	67.07	32.93	-----	2.4830	-----	236-----	73.14	26.86	2.3840	-----	
27-----	67.22	32.78	-----	2.4821	-----	237-----	73.15	26.85	2.3828	-----	
28-----	67.35	32.65	-----	2.4816	-----	238-----	73.25	26.75	2.3830	-----	
29-----	68.10	31.90	-----	2.4770	-----	239-----	73.45	26.55	2.3815	-----	
30-----	68.81	31.19	-----	2.4722	2.4692	240-----	73.63	26.37	2.3807	-----	
31-----	69.10	30.90	-----	2.4706	-----	241-----	73.67	26.33	2.3812	2.3767	
32 (20)-----	69.65	30.35	-----	2.4670	2.4645	242-----	74.15	25.85	2.3781	-----	
33 (22)-----	70.44	29.56	-----	2.4623	-----	243-----	74.37	25.63	2.3756	-----	
34-----	71.10	28.90	-----	2.4571	2.4540	244-----	74.89	25.11	2.3728	2.3693	
35 (23)-----	72.15	27.85	-----	2.4519	-----	245-----	74.99	25.01	2.3734	-----	
36 (24)-----	72.33	27.67	-----	2.4505	-----	246-----	75.00	25.00	2.3728	2.3692	
37-----	73.00	27.00	-----	2.4450	2.4420	247-----	75.32	24.68	2.3721	-----	
38-----	73.92	26.08	-----	2.4387	-----	248-----	75.49	24.51	2.3701	-----	
39 (25)-----	74.16	25.84	-----	2.4378	-----	249-----	75.76	24.24	2.3691	-----	
40 (26)-----	74.69	25.31	-----	2.4356	2.4332	250-----	76.34	23.66	2.3646	-----	
41 (27)-----	75.29	24.71	-----	2.4289	2.4261	251-----	76.46	23.54	2.3629	-----	
42 (28)-----	76.60	23.40	-----	2.4152	2.4126	252-----	76.57	23.43	2.3636	-----	
43 (29)-----	76.65	23.35	-----	2.4152	2.4133	253-----	76.65	23.35	2.3625	-----	
44 (30)-----	76.70	23.30	-----	2.4141	-----	254-----	76.82	23.18	2.3595	-----	
45 (31)-----	77.85	22.15	-----	2.4031	2.4013	255-----	77.27	22.73	2.3579	-----	
46 (32)-----	78.61	21.39	-----	2.3945	-----	256-----	77.74	22.26	2.3529	2.3499	
47-----	81.54	18.46	-----	2.3677	-----	257-----	77.93	22.07	2.3523	-----	
48-----	82.72	17.28	-----	2.3553	-----	258-----	78.74	21.26	2.3470	-----	
49 (36)-----	85.15	14.85	-----	2.3329	2.3310	259-----	79.08	20.92	2.3450	2.3414	
50-----	86.12	13.88	-----	2.3230	-----	260-----	79.22	20.78	2.3430	-----	
(38)-----	100	-----	-----	2.203	-----	261-----	80.02	19.98	2.3366	2.3339	
201-----	60.59	-----	39.41	2.4522	-----	262-----	80.58	19.42	2.3336	-----	
202-----	60.81	-----	39.19	2.4481	-----	263-----	80.99	19.01	2.3303	2.3277	
203-----	60.88	-----	39.12	2.4489	2.4433	264-----	81.00	19.00	2.3304	-----	
204-----	61.90	-----	38.10	2.4449	-----	265-----	82.87	17.13	2.3178	-----	
205-----	62.93	-----	37.07	2.4383	2.4326	266-----	82.99	17.01	2.3172	-----	
206-----	63.93	-----	36.07	2.4347	-----	301-----	60.14	34.75	5.11	2.5198	
207-----	65.08	-----	34.92	2.4279	-----	302-----	60.43	37.06	2.51	2.5205	
208-----	65.92	-----	34.08	2.4242	2.4196	303-----	60.53	25.09	14.38	2.5117	
209-----	67.02	-----	32.98	2.4185	-----	304-----	61.62	29.81	8.57	2.5097	
210-----	67.24	-----	32.76	2.4166	-----						

<sup>1</sup> Figures in parentheses are the numbers of the same glasses in the earlier density paper.

TABLE 1.—Density and composition for 194 soda-potash-silica glasses—Continued

Glass number	Composition			Density		Glass number	Composition			Density	
	SiO <sub>2</sub>	Na <sub>2</sub> O	K <sub>2</sub> O	Treat-ment 1	Treat-ment 4		SiO <sub>2</sub>	Na <sub>2</sub> O	K <sub>2</sub> O	Treat-ment 1	Treat-ment 4
305	62.89	35.85	1.26	2.5084	-----	342	70.61	15.18	14.21	2.4409	-----
306	62.99	27.00	10.01	2.5004	-----	343	70.60	12.90	16.50	2.4348	-----
307	63.48	32.78	3.74	2.5044	-----	344	70.76	20.03	9.21	2.4464	-----
308	64.89	33.56	1.55	2.4968	-----	345	70.87	10.04	19.09	2.4301	-----
309	65.42	28.38	6.20	2.4911	-----	346	72.30	25.20	2.50	2.4469	2.4445
310	66.00	30.84	3.12	2.4886	-----	347	71.75	7.04	21.21	2.4191	-----
311	68.32	27.70	3.98	2.4740	2.4706	348	72.77	10.45	16.78	2.4176	-----
312	68.33	29.85	1.82	2.4741	-----	349	72.92	15.27	11.81	2.4268	-----
313	71.20	27.50	1.29	2.4564	-----	350	73.28	12.69	14.03	2.4181	-----
314	77.19	21.08	1.73	2.4087	-----	351	73.66	7.51	18.83	2.4050	-----
315	80.61	10.13	9.26	2.3630	-----	352	73.60	22.55	3.85	2.4360	-----
316	80.58	14.99	4.43	2.3706	-----	353	73.90	16.63	9.47	2.4218	-----
317	83.43	12.00	4.58	2.3435	2.3407	354	74.76	23.99	1.25	2.4312	-----
318	84.52	8.66	6.82	2.3275	2.3253	355	75.12	12.57	12.31	2.4048	-----
319	59.31	16.97	23.72	2.5047	2.4990	356	75.38	14.80	9.82	2.4111	2.4084
320	60.51	20.24	19.25	2.5048	-----	357	75.52	7.75	16.73	2.3943	2.3912
321	60.73	15.12	24.15	2.4926	-----	358	75.87	10.21	13.92	2.3968	-----
322	61.12	10.27	28.61	2.4825	-----	359	76.30	16.79	6.91	2.4063	-----
323	61.41	12.00	26.59	2.4837	-----	360	78.94	9.27	11.79	2.3764	-----
324	62.33	22.83	14.84	2.4982	-----	361	61.54	5.04	33.42	2.4653	-----
325	62.62	7.61	29.77	2.4685	-----	362	64.14	2.09	33.77	2.4422	-----
326	62.90	15.85	21.25	2.4826	-----	363	66.44	3.30	30.26	2.4348	2.4303
327	63.70	12.69	23.61	2.4726	-----	364	68.73	4.40	26.87	2.4258	-----
328	64.78	17.21	18.01	2.4757	-----	365	69.11	1.54	29.35	2.4140	-----
329	65.08	15.39	19.53	2.4700	-----	366	70.75	5.25	24.00	2.4191	-----
330	65.23	24.86	9.91	2.4868	2.4833	367	71.00	2.55	26.45	2.4066	2.4028
331	65.25	8.55	26.20	2.4542	-----	368	71.28	0.96	27.76	2.3999	-----
332	65.37	20.10	14.53	2.4775	-----	369	71.61	5.35	23.04	2.4145	-----
333	65.82	10.08	24.10	2.4572	-----	370	73.50	2.19	24.31	2.3898	-----
334	66.50	13.00	20.50	2.4591	-----	371	74.35	4.44	21.21	2.3935	2.3899
335	66.97	21.80	11.23	2.4700	-----	372	75.48	3.37	21.15	2.3797	2.3764
336	67.43	12.09	20.48	2.4502	2.4460	373	75.62	5.12	19.26	2.3851	-----
337	67.92	10.01	22.07	2.4436	-----	374	76.05	1.03	22.92	2.3696	-----
338	68.02	15.09	16.89	2.4544	-----	375	79.30	4.55	16.15	2.3581	-----
339	68.12	6.93	24.95	2.4384	2.4337	376	81.05	5.24	13.71	2.3467	2.3443
340	69.98	24.60	5.42	2.4603	2.4573	377	83.17	2.16	14.67	2.3227	2.3200
341	69.94	8.08	21.98	2.4300	-----	378	84.69	5.27	10.04	2.3197	-----

The size of the specimen does, however, affect the accuracy to which the density may be measured. In general, the sizes of the specimens were such that the experimental error in density varied from about  $\pm 0.0002$  to  $\pm 0.0004$ . The 11 specimens used in the several heat-treating experiments, however, were so selected as to be of nearly the same size, in order that the results of the quenching experiments might be comparable.

## 2. RESULTS OF HEAT-TREATING EXPERIMENTS

It will be noted that for any given glass the two densities resulting from treatments 1 and 5, respectively, are essentially the same, although the cooling rates were alike only through a temperature interval between approximately 25° C above to 50° C below the critical temperature. It may be said, therefore, that this interval includes the critical range as defined above. Although no attempt was made to determine the limits for this range more closely, it is interesting to note that a considerable increase in the density of a given glass over that found by quenching from the softening temperature (treatment 2) was

obtained merely by heating the glass rapidly to the critical temperature and immediately quenching it in air (treatment 3). In 3 of the 11 cases, the densities increased enough to equal those resulting from treatment 1.

From a comparison of the results of treatments 4 and 1, respectively, it was found that the difference between the corresponding specific volumes ( $V_4$  and  $V_1$ ) is a linear function of the percentages of soda ( $B$ ) and potash ( $E$ ) (no serious error was introduced in assuming the factor for silica ( $A$ ) to be zero). Specifically, for each glass

$$V_4 - V_1 = (1.6B + 2.5E) \times 10^{-5}$$

The individual deviations from this expression were found not to exceed the errors incident to the measurement of specific volume.<sup>3</sup> Thus, it follows (provided the cooling rate is constant, and the same for all the glasses, through their respective critical ranges) that the actual rate used does not determine the character of the surface showing the relationship between specific volume and composition, nor establish the existence or location of "critical points" such as those noted in the following sections. For if

$$V_1 = f_1(A, B, E),$$

then

$$V_4 = f_1(A, B, E) + (1.6B + 2.5E) \times 10^{-5},$$

and at any given composition the rate of change of slope is the same for both surfaces, as the second derivatives are identical. Since the two cooling rates used are wholly arbitrary, it is reasonable to infer that a similar relation would hold had some other rates been chosen.

For the purpose of correlating specific volume with composition, the specific volumes used were those resulting from treatment 1.<sup>4</sup>

#### IV. EVALUATION OF THE DATA

In figure 2, specific volume,  $V$  ( $=1/D$ ), is plotted as a function of silica content for the potash-silica ( $A$ ) and soda-silica glasses ( $B$ ), as well as for a short series of lithia-silica glasses ( $C$ ) discussed in the next section. For a closer study of these data, in order to determine the type of function which would best represent them, the points were replotted as in figure 3, the ordinates ( $\Delta V$ ) representing, for each binary series, the departures of the observed  $V$ -values from the respective straight lines  $a$  and  $b$ , indicated in figure 2.

<sup>3</sup> Specific volume is, of course, determined from the same measurements as density—weight and volume. In figure 1, and also in table 1, density, being the more familiar quantity, is recorded rather than specific volume.

<sup>4</sup> The densities of the soda-silica glasses reported by Morey and Merwin [3], which were "annealed to maximal density", correspond very closely to those obtained from treatment 1, except in the very low soda range of composition. In this range the densities of our glasses were lower. After table 1 was prepared, the 14 soda-silica glasses used in treatment 4 (see table 1) were re-treated by the same procedure, except that a cooling rate of about 2° per hour was used. The resulting densities were as follows:

(4) Devitrified.	(21) 2.4992.	(34) 2.4587.	(41) 2.4307.	(45) 2.4059.
(5) 2.5312.	(30) 2.4752.	(37) 2.4468.	(42) 2.4168.	(49) 2.3336.
(13) 2.5120.	(32) 2.4697.	(40) 2.4381.	(43) 2.4179.	

As was expected, these densities are greater than those resulting from treatment 1. The new densities, although remaining lower than those of Morey and Merwin for glasses containing less than about 23 percent of soda, were somewhat higher than theirs (in the order of about 0.003) for glasses containing more than about 27 percent of soda.

Certainly none of the simpler types of curves would be adequate for representing these data since, from figure 3, the best fit appears to be provided for each series by a line undergoing a "deflection" (that is, a rapid, although perhaps "continuous," alteration in direction) through each of two narrow ranges of composition, while remaining relatively straight outside these ranges.<sup>5</sup> For the purpose of simplicity in expressing this fact mathematically, it was decided that three straight lines could be fitted to the points of each binary series, the intersections

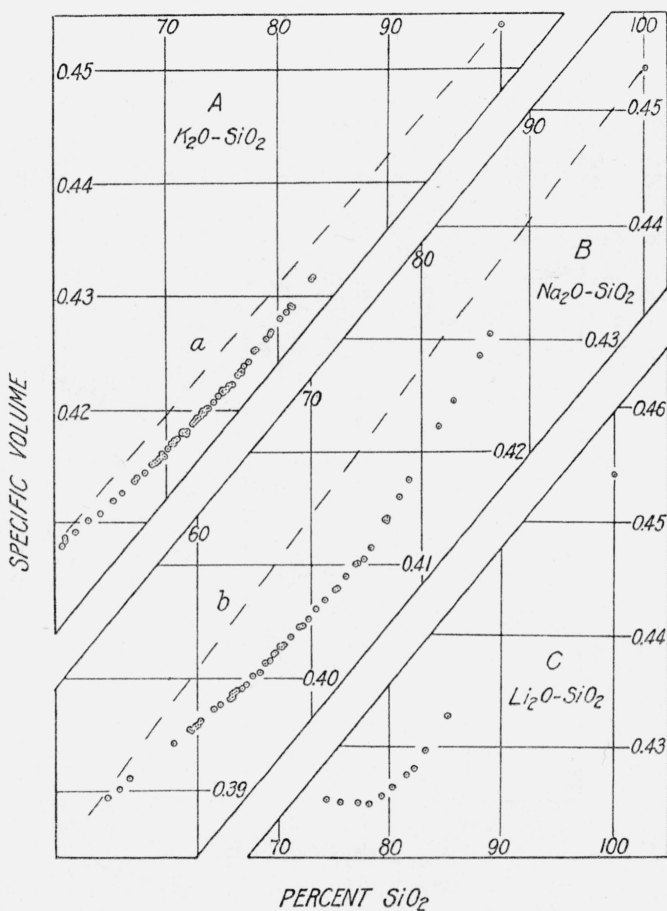


FIGURE 2.—Specific volume of A the potash-silica, B the soda-silica, and C the lithia-silica glasses plotted against the percentage of silica.

of these lines representing the deflections in the "true" function—in whatever way that function should "properly" be expressed.

The dashed lines in figure 3 for each series represent the maximum expected deviation on either side of the "best-fitting" straight lines (the constants of which were evaluated as part of the correlation of the ternary relations described below), taking into account only the esti-

<sup>5</sup> In the previously mentioned report by Morey and Merwin the "smooth curve," which they have drawn through their density-composition (and also their refractive index-composition) points, exhibits similar characteristics—a fact made sharply evident by plotting the first derivative of their curve.

mated maximum error in making the chemical analyses (expressible as  $\pm 0.2$  percent  $\text{SiO}_2$ ), and in the measurement of specific volume ( $\pm 0.0001$ ). The fact that only three or four points in each series lie outside these limits is further indication that the method of heat treatment is reproducible.

To determine the relation between specific volume and composition of the soda-potash-silica glasses, their compositions were plotted on a triordinate diagram (fig. 4). Then several lines of constant  $V$

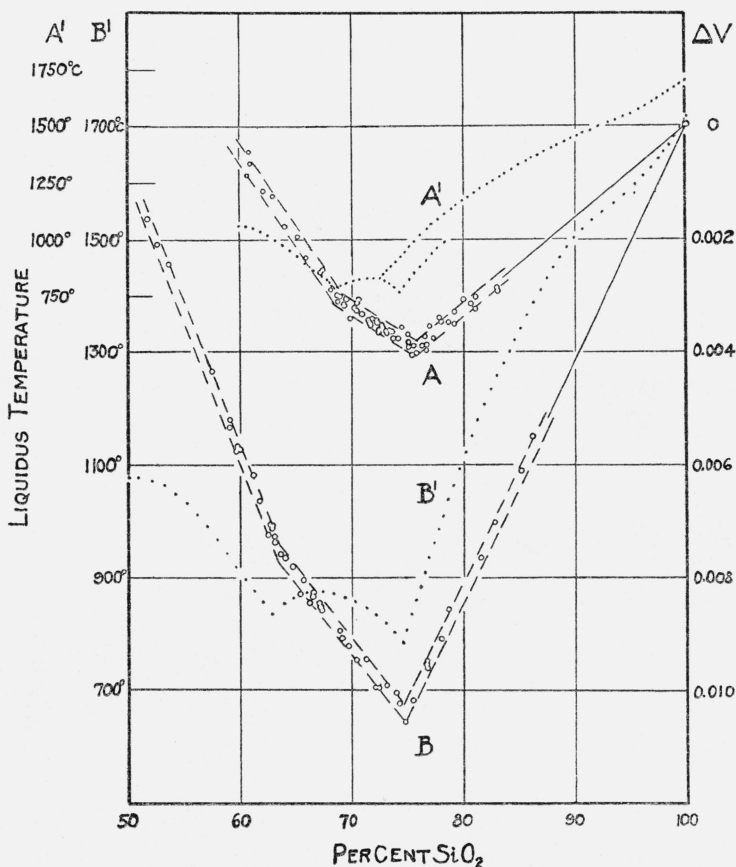


FIGURE 3.—Departures ( $\Delta V$ ) of the observed specific volumes of *A* the potash-silica, and *B* the soda-silica glasses from the corresponding straight lines *a* and *b* in figure 2.

*A'* and *B'* are the corresponding liquidus curves, taken from references [3] and [4].

(specific volume) were obtained by first interpolating graphically each selected constant  $V$  value between several appropriate pairs of neighboring points, and then sketching a smooth line through each of the sets of interpolated points representing the same  $V$  value. The trend and separation of the constant  $V$  lines thus drawn revealed that the over-all surface, representing the variation in specific volume with composition, divided itself into several relatively flat regions of different slopes. Accordingly, each region was treated arbitrarily as a



plane of the form  $V = aA + bB + eE$ , in which  $A$ ,  $B$ , and  $E$  are the percentages of silica, soda, and potash, respectively, and  $a$ ,  $b$ , and  $e$  are constants which are different for each region. The constants were evaluated for each region by the method of least squares, using the foregoing graphic representation as a guide for the appropriate grouping of the points. The data for the two binary series were, of course, also included in these calculations, but the points are not recorded on the triordinate diagrams. The constants thus derived are shown in table 2 and are given to  $5 \times 10^{-7}$  in order to avoid numerical errors in the "computed specific volumes" as great as the error in measuring

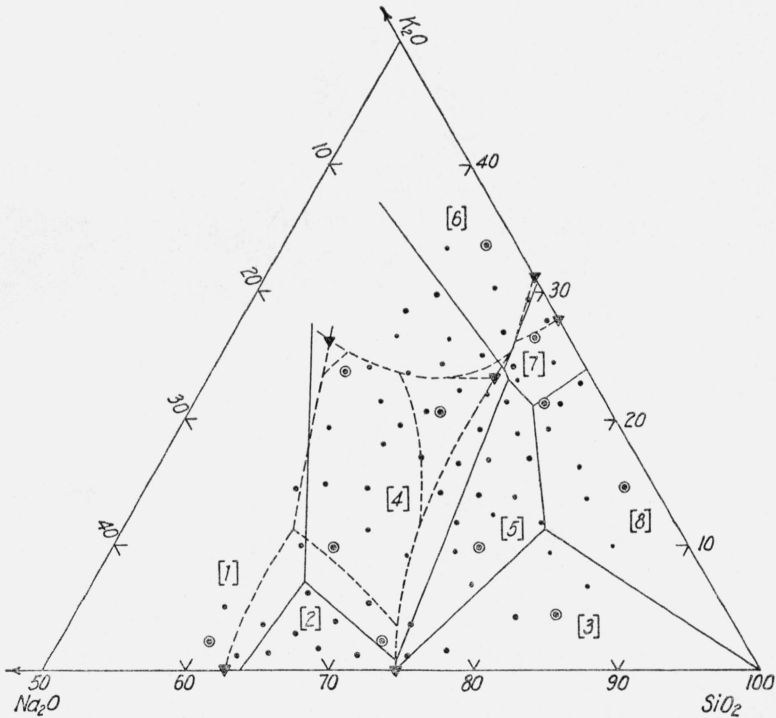


FIGURE 4.—Compositions of the soda-potash-silica glasses and the boundaries (solid lines) of the various planes (numbered as in table 2) representing the surface on which the specific volumes lie.

Dotted lines are the boundary curves of the phase-equilibrium diagram. Points in circles identify the glasses used in the heat-treating experiments outlined in figure 1.

specific volume. The statistical error in the constants themselves is, of course, much greater than  $5 \times 10^{-7}$ .

Simultaneous solutions, by pairs, of the equations of adjacent planes yielded the lines of intersection of the planes (shown by heavy lines, fig. 4).

TABLE 2.—Numerical value of constants for the different regions

Constant×100	Region→1	2	3	4	5	6	7	8
<i>a</i> .....	0.428 00	0.437 10	0.454 00	0.441 05	0.448 35	0.441 65	0.447 85	0.454 00
<i>b</i> .....	.348 60	.332 65	.282 95	.320 55	.299 60	.290 85	.273 20	.265 15
<i>e</i> .....	.358 90	.339 15	.308 00	.350 75	.334 00	.356 25	.342 50	.322 85

It is not the authors' contention, however, that physical significance is necessarily to be attached to the system of planes as derived, nor that the individual deviations represent merely experimental or random errors. On the contrary, if, by an examination of the magnitude and distribution of plus and minus departures from the calculated surface (recorded in fig. 5), the reader is enabled to form some conception of what the "true" surface would be, making due allowance for experimental error, then the solution here presented has fulfilled its chief purpose. If the latter viewpoint is taken, then the fact that a different number of parameters or some other grouping of the

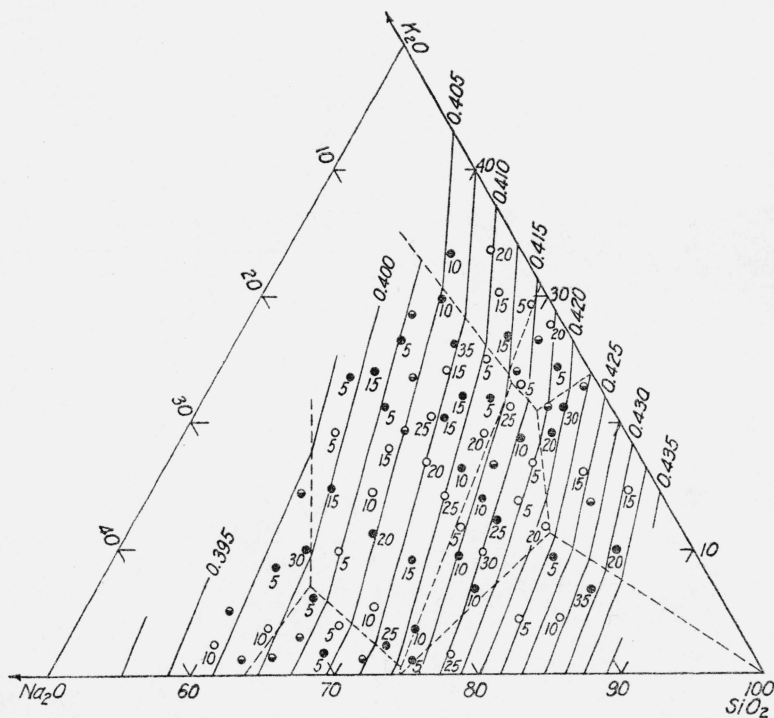


FIGURE 5.—Relation between specific volume and composition of some soda-potash-silica glasses.

Observed specific-volume values lie on (●), above (●), or below (○), the computed surface, by the amounts indicated, times  $10^{-3}$ . Density deviations equal approximately six times these amounts.

points would lead to a different statistical solution becomes of minor importance, since the discrepancies between two such solutions would be compensated for by a correspondingly different pattern of departures. That is, the "true" surface conceivably would be indicated in either case. To the authors, the calculated surface here presented is a very good approximation to the "true" surface in view of the comparative absence of systematic deviation.

## V. DISCUSSION

If the specific volume-composition relations are compared to the liquidus curves of the appropriate equilibrium diagrams presented by Kracek [4, 5, 6, 7], one finds a surprising, if not significant, correspond-

ence. In the case of the soda-silica data (*B*, fig. 3), the two deflections occur at or near the eutectic compositions (See *B'*, fig. 3, at 63 and 75 percent  $\text{SiO}_2$ ). In the potash-silica data (*A*, fig. 3), the deflection at 69 percent  $\text{SiO}_2$  adequately reflects the eutectic at this composition (see *A'*, fig. 3). The other deflection occurs at about 76 percent  $\text{SiO}_2$ , and appears to correspond, not to the "true" eutectic (which is at about 72 percent  $\text{SiO}_2$ ), but to the minimum point along the metastable cristobalite liquidus curve, which point is found under conditions more nearly comparable to the glassy state than those under which the "true" eutectic composition is found.

Concerning the soda-potash-silica data, the deflections (represented as lines of demarcation, fig. 4) correspond roughly to the boundary curves of the equilibrium diagram (dotted lines, fig. 4) connecting the eutectic compositions (small triangles, fig. 4) or other invariant points.

The plot of the specific volume-composition data thus far obtained on the lithia-silica glasses (*C*, fig. 2) is included here only for comparative purposes, since this phase of the work is far from complete. The two deflections—at about 78 and 82 percent  $\text{SiO}_2$ —are, however, the most pronounced observed for any series of glasses studied at this Bureau. The deflection at 82 percent  $\text{SiO}_2$  corresponds to the eutectic at that composition, but the equilibrium diagram does not indicate a "critical composition" at 78 percent  $\text{SiO}_2$ .

A series of 10 boric oxide-silica glasses was reported by Cousens and Turner [8]. They plotted density as a function of composition and concluded that these data should be represented by a "smooth curve" showing no deflections—an apparent contradiction to the interpretation just given concerning our own data. It appears, however [9], that the eutectic, if any, in the boric oxide-silica system would occur very near to 100 percent  $\text{B}_2\text{O}_3$ , so that the existence of a corresponding deflection in the density-composition line could not easily be demonstrated, even if one were expected. Of interest, furthermore, is the fact that, when the specific volumes instead of the densities of their glasses are plotted against the compositions, the points are found to lie upon a single *straight* line connecting the specific volume of vitreous silica (0.4540) with that of vitreous  $\text{B}_2\text{O}_3$ , using for the latter the value 0.5440 as computed from the density reported by the same laboratory in a subsequent paper [10].<sup>6</sup>

## VI. SUMMARY

The foregoing presentation may be briefly summarized as follows:

1. Data are presented on the density and composition of 194 glasses in the ternary system soda-potash-silica.

2. A reproducible and, it is felt, consistent method of heat treatment is described which, when applied simultaneously to all the glasses, results in the cooling of each glass at the same rate through its own "critical range" as defined.

3. A study of the data presented does not indicate that there is a simple relationship between specific volume and composition, but that the relation is best expressed by a system of planes in a three-dimensional triordinate diagram whose intersections correspond fairly well to the boundary curves and eutectic compositions indicated by

<sup>6</sup> In that paper, the value reported in the earlier paper [8] was said to have been erroneous.

the equilibrium diagram of Kracek. The correspondence is very close for the soda-silica and potash-silica data, but it is only approximate for glasses containing all three constituents.

4. Data on the specific volume of 10 lithia-silica glasses are presented graphically.

## VII. REFERENCES

- [1] F. W. Glaze, J. C. Young, and A. N. Finn, *The density of some soda-lime-silica glasses as a function of the composition*, BS J. Research **9**, 799 (1932) RP507.
- [2] C. A. Faick and A. N. Finn, *The index of refraction of some soda-lime-silica glasses as a function of the composition*, BS J. Research **6**, 993 (1931) RP320; J. Am. Cer. Soc. **14**, 518 (1931).
- [3] G. W. Morey and H. E. Merwin, *Relation between the composition and the density and optical properties of glass. I. The soda-lime-silica glasses*, J. Optical Soc. Am. **22**, 632 (1932).
- [4] F. C. Kracek, *The system sodium oxide-silica*, J. Phys. Chem. **34**, 1583 (1930); J. Am. Cer. Soc. **16**, 491 (1933).
- [5] F. C. Kracek, N. L. Bowen, and G. W. Morey, *The system potassium metasilicate-silica*, J. Phys. Chem. **33**, 1857 (1929); J. Am. Cer. Soc. **16**, 490 (1933).
- [6] F. C. Kracek, *The ternary system  $K_2SiO_3$ - $Na_2SiO_3$ - $SiO_2$* , J. Phys. Chem. **36**, 2529 (1932); J. Am. Ceram. Soc. **16**, 509 (1933).
- [7] F. C. Kracek, *The binary system lithium oxide-silica*, J. Phys. Chem. **34**, 2641 (1930); J. Am. Ceram. Soc. **16**, 491 (1933).
- [8] Arnold Cousen and W. E. S. Turner, *A study of the glasses boric oxide-silica*, J. Soc. Glass Tech. **12**, 169 (1928).
- [9] E. P. Flint and Lansing S. Wells, *The system lime-boric oxide-silica*, J. Research NBS **17**, 727 (1936) RP941.
- [10] E. J. Gooding, and W. E. S. Turner, *A Study of the series of glasses containing sodium oxide, boric oxide and silica*, J. Soc. Glass Tech. **18**, 32 (1934).
- [11] B. S. Schmid, A. N. Finn, and J. C. Young, *Thermal expansions of some soda-lime-silica glasses as functions of the composition*, BS J. Research **12**, 422 (1934) RP667.

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