INTERNATIONAL TEMPERATURE SCALE AND SOME RELATED PHYSICAL CONSTANTS

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ABSTRACT

The situation in regard to the radiation constants is reviewed primarily for the purpose of selecting the value of c_2 to use in Planck's equation for securing, above 1063° C, a temperature scale which most nearly conforms with the thermodynamic scale. The values in the literature are discussed, and some data are presented for deriving c_2 from gas thermometry data. The constants arrived at are the following:

$\sigma = (5.70 \pm 0.02) \times 10^{-5}$ erg sec $^{-1}$ cm^{-2} deg $^{-4}$
$c_1 = (3.732 \pm 0.006) \times 10^{-5}$ erg cm^2 sec^{-1}
$c_2 = 1.436 \pm 0.001$ cm deg
$\lambda_m T = (2892 \pm 2) \times 10^{-4}$ cm deg
M = least mechanical equivalent of light = 0.00151 ± 0.00001 watt ("new" lumen)$^{-1}$

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I. INTERNATIONAL TEMPERATURE SCALE

In 1927 the Seventh General Conference of Weights and Measures, representing 31 nations, unanimously adopted the International Temperature Scale [1].[1] If the ratio of any two temperatures be defined by the equation $T_2/T_1 = 1 - E$, where E is the efficiency of a completely reversible heat engine operating between a source at temperature T_1 and a sink at temperature T_2, and if further the temperature interval between the steam point and the ice point be defined as 100°, the resultant scale is the thermodynamic scale advocated by Lord Kelvin. Temperatures on this scale, known as the Kelvin Scale, are designated as "°K" and denoted by the symbol T. The lower limit of temperature is 0°K, and the normal freezing point of water, called the ice point and designated by the symbol T_0, is found by experiment to be approximately 273° K.

[1] Numbers in brackets indicate the literature references at the end of this paper.

The Kelvin Scale and other scales on which numerical values of temperature are a linear function of T are the only thermodynamic scales which have been used to any considerable extent. If we write $t = T - T_0$, t is the temperature on a scale which is called the Thermodynamic Centigrade Scale. Any temperature *interval* has the same numerical value when expressed on this scale as when expressed on the Kelvin Scale. The International Temperature Scale "conforms with the thermodynamic [centigrade] scale as closely as is possible with present knowledge" and "is to be regarded as susceptible of revision and amendment as improved and more accurate methods of measurement are evolved."

The possibility of revising the scale to secure some improvement appeared soon after the scale was adopted. Below 660° C the scale is defined by the standard resistance thermometer and from 660° C to the gold point, defined as 1063.0° C, the scale is defined by the standard thermocouple. The standard thermocouple and the standard resistance thermometer are made to agree at the freezing point of antimony, 630.5° C, but agreement at 660° C was left to chance. In 1929 Roeser [2] found that the agreement was within 0.01° C at the melting point of the sample of aluminum, 659.23° C, at which he compared thermocouples and resistance thermometers. Roeser used two thermocouples and obtained agreement to 0.01° C with one and 0.00° C with the other. It subsequently developed [3], however, that the silver used in this work had a freezing point 0.48° C lower than that of pure silver, corresponding to a difference of 5.5 microvolts in the electromotive force yielded by Roeser's thermocouples at the silver point. Making this correction, it is found that these thermocouples indicate the freezing point of the aluminum used to be 659.05° C, whereas the standard resistance thermometer indicated the temperature to be 659.23° C. Inasmuch as the standard thermocouple defines the scale only from 660° to 1063° C, the temperature in question is 659.23° C (Int. 1927), because it is in the range of the scale defined by the standard thermometer. Subsequently, the freezing point of a lot of aluminum to be used for standard samples was measured [2] with a standard resistance thermometer as 660.01° C and with a standard thermocouple as 695.87° C. The temperature in question is above the range defined in terms of the resistance thermometer and below the range defined in terms of the thermocouple and therefore cannot be expressed on the International Temperature Scale of 1927. This situation can be remedied by limiting the range of the scale defined by the resistance thermometer to temperatures below the freezing point of antimony and extending the range of the thermocouple down to that point.

Above the melting point of gold, the scale might well be based on the Planck formula instead of the Wien formula, which was used in defining the 1927 scale [1] up to about 5000° C, where $\lambda (t + 273)$ is less than 0.3 cm deg. The use of the Planck formula would make no significant change in the scale below 5000° C and would result in a scale which conforms closely to the thermodynamic scale at all temperatures above the gold point.

A study of the question of whether other revisions might secure still closer conformity has led, among other things, to a re-evaluation of the

radiation constant which is used in defining the scale at high temperatures. A true perspective of the data on the radiation constants cannot be obtained without consideration of a number of other physical constants more or less directly related to the radiation constants. The numerical values of some of these related constants have been established so accurately that, in combining them with other constants which are much less accurately established, they may be treated as exact. Nevertheless, it seemed worth while to discuss all the constants involved in some detail, either to indicate how little the value selected contributes to the uncertainty of the derived radiation constants or to make the incidental results of this study available to readers who may find them of interest.

II. THE RADIATION CONSTANTS

The constants by means of which the energy radiated by a black body may be characterized are c_1, c_2, σ, $\lambda_m T$, and ν_m/T defined by the formulas

$$J_\lambda = c_1 \lambda^{-5}\left(e^{\frac{c_2}{\lambda T}} - 1 \right)^{-1} \tag{1}$$

$$J_\nu = c_1 c^{-4} \nu^3 \left(e^{\frac{c_2 \nu}{T}} - 1 \right)^{-1} \tag{2}$$

$$\int_0^\infty J_\lambda d\lambda = \int_0^\infty J_\nu d\nu = \sigma T^4 \tag{3}$$

$$\lambda_m T = \frac{c_2}{A} = 0.201405 c_2, \tag{4}$$

where

$$A = 5(1 - e^{-A}) \tag{5}$$

and

$$\frac{\nu_m}{T} = \frac{Bc}{c_2} = 2.82144\frac{c}{c_2}, \tag{6}$$

where

$$B = 3(1 - e^{-B}) \tag{7}$$

In the above formulas
λ is the wavelength
ν is the frequency
c ($=\lambda\nu$) is the velocity of light
T is temperature in degrees K
e is the Naperian base, 2.7182818 . . .
J_λ is the radiant energy (of wavelength λ) per unit wavelength interval emitted per unit time by unit area of a black body throughout the solid angle 2π, i. e., "hemispherical radiation."
J_ν is the radiant energy (of frequency ν) per unit frequency interval emitted per unit time by unit area of a black body throughout the solid angle 2π.
λ_m and ν_m are the values of λ and ν at which J_λ and J_ν, respectively, are a maximum for any given value of T.
It should be pointed out that, for a given value of T, λ_m and ν_m do not correspond to the same region of the spectrum. In other words,

their product is not c but $c(B/A)$, or $0.56825\ c$, so that ν_m is displaced toward the red end of the spectrum from the value of ν corresponding to λ_m. This may be seen by writing $d\nu = -(c/\lambda^2)d\lambda$.

All magnitudes will be expressed in cgs units, i. e., λ in cm, J_λ in erg $\sec^{-1}\mathrm{cm}^{-3}$, c_1 in erg $\sec^{-1}\mathrm{cm}^2$, c_2 in cm deg, σ in erg $\sec^{-1}\mathrm{cm}^{-2}\mathrm{deg}^{-4}$, etc. In giving numerical values, factors of 10^n and the cumbersome designation of units such as those above will often be omitted after the first writing when it is felt that no ambiguity will result.

We have the following relations

$$c_1 = 2\pi c^2 h = \frac{15}{\pi^4}\sigma c_2^4, \tag{8}$$

$$c_2 = \frac{ch}{k} = \left(\frac{h}{e}\right)\frac{cF}{R}, \text{ and} \tag{9}$$

$$\sigma = \frac{2\pi^5 c^2 h}{15 c_2^4} = 40.8026\frac{c^2 h}{c_2^4}, \tag{10}$$

where $c =$ the velocity of light in cm $\sec^{-1}$
$h =$ Planck's constant of action in erg sec
$k =$ the Boltzmann constant in erg $\deg^{-1}$
$e =$ electronic charge in electrostatic units (esu).
$F =$ the faraday constant in (esu) mole^{-1}
$R =$ the gas constant in erg $\deg^{-1}\mathrm{mole}^{-1}$.

There are a number of reviews available on the best values of the physical constants derivable from experimental data, but the radiation constants have not received much attention in recent years. The experimental values of σ and c_2 have usually been used to calculate values of h, but not much weight has been given to the results. Better values of the radiation constants can be derived from atomic constants than from the published experimental values, and it seems worth while to make a critical study of these related constants for the primary purpose of deducing the most probable value of the constants which appear in the formulas for black-body radiation.

For practical work, a knowledge of the numerical value of any radiation constant, excepting c_2, to better than 1 or 2 percent is seldom required. Measurements involving J_λ are usually so made as to depend only on *ratios* in which c_1 cancels out. Even when this is not the case, an accurate value of c_1 is seldom of interest, because we have at present no technique for accurate measurements of J_λ. The situation in regard to σ is substantially the same. The only experiments in which anything besides an approximate value of σ is of interest are those made for the purpose of determining σ.

Our interest in the constant c_2 lies in the fact that it is one of the constants used in defining the International Temperature Scale above the melting point of gold. This definition is "by means of the formula

$$\log_e \frac{J_2}{J_1} = \frac{c_2}{\lambda}\left(\frac{1}{1336} - \frac{1}{(t+273)}\right) \tag{11}$$

The constant c_2 is taken as 1.432 cm degrees. The equation is valid if $\lambda(t+273)$ is less than 0.3 cm degrees." In this equation J_1 and J_2 are the values of J_λ at the melting point of gold, defined as $1063°$, and at the temperature t, respectively. A change in c_2 of 1 percent results

in a change in t of more than $10°$ at the melting point of platinum and of more than $60°$ at the melting point of tungsten.

III. AUXILIARY CONSTANTS

In giving the numerical value of a quantity it is usually advisable to add a numerical statement of the uncertainty associated with the value given. The $\pm$ terms in this paper represent the limits within which it is estimated that the true value is included with a probability of about 0.9. They represent merely the author's opinion and may bear little relation to the mean deviation.

1. VELOCITY OF LIGHT

The velocity of light is so accurately known that, for the calculations made in this paper, it may be treated as exact. Karolus and Mittlestaedt [4] obtained the value 2.99778×10^{10}, while (Michelson), Pease, and Pearson [5] obtained the value 2.99774×10^{10}. The velocity of light is taken as

$$c = (2.99776 \pm 0.00020) \times 10^{10} \text{ cm sec}^{-1}.$$

2. KELVIN TEMPERATURE OF THE ICE POINT AND THE VALUE OF R

The temperature of the ice point on the Kelvin Scale, T_0, has been derived in a number of laboratories. The most recent and reliable of the values are listed in table 1.

TABLE 1.—*Recent values of* T_0

Authors	Laboratory	Date	T_0
Heuse and Otto [6]	PTR	1930	273.16
Keesom and Tuyn [7]	Leiden	1936	273.14₄
Roebuck [8]	University of Wisconsin	1936	273.16
Kinoshita and Oishi [9]	Tokyo University	1937	273.16₇
Beattie [10]	MIT	1937	273.16–273.17
		Mean	273.16

The value 273.16 to 273.17 listed for Beattie is provisional as Beattie's final calculations have not been completed. On the basis of the determinations listed in table 1,

$$T_0 = 273.16 \pm 0.02°\text{K}.$$

The value of RT_0 needs little comment as it is known more accurately than T_0. The value for RT_0 given by Birge [11] reduces to 2.2711×10^{10} erg mole^{-1}. Combining this with $T_0 = 273.16$, the value of R is found to be

$$R = (8.3142 \pm 0.0010) \times 10^7 \text{ erg deg}^{-1} \text{ mole}^{-1}.$$

3. INTERNATIONAL ELECTRICAL UNITS

Many of the experimental data to be considered are expressed in International Electrical Units. In order to reduce the published

results to electrostatic units (esu) for substitution in eq 9 the ratios of the international to the absolute units must be used. The most recent published determinations of the ratios of the international to the absolute (practical) units are summarized on page 113 of the Procès Verbaux des Seances, Comité International des Poids et Mesures of 1937. Inasmuch as this publication is not generally available the summary is reproduced below:

ETL (Japan)	1 international ohm	$= 1.00046_5$ abs. ohms.
NBS (U. S.)	1 international ohm	$= 1.00045_4$ abs. ohms.
NPL (Great Britain)	1 international ohm	$= 1.00050_4$ abs. ohms.
PTR (Germany)	1 international ohm	$= 1.00048_3$ abs. ohms.
ETL	1 international ampere	$= 0.99993_8$ abs. ampere.
NBS	1 international ampere	$= 0.99989_5$ abs. ampere.
NPL	1 international ampere	$= 0.99984_8$ abs. ampere.
ETL	1 international volt	$= 1.000403$ abs. volts.
NBS	1 international volt	$= 1.000349$ abs. volts.
NPL	1 international volt	$= 1.000352$ abs. volts.

The most recent NBS value [12] for the ampere, however, is 0.99986, which is in remarkably good agreement with the NPL value. From these data the following values have been derived:

$$\frac{1 \text{ International Ohm}}{1 \text{ Absolute Ohm}} = p = 1.00048.$$

$$\frac{1 \text{ International Ampere}}{1 \text{ Absolute Ampere}} = q = 0.99986.$$

$$\frac{1 \text{ International Volt}}{1 \text{ Absolute Volt}} = pq = 1.00034.$$

4. THE FARADAY

In 1929 Birge [11] recommended 96,487 abs. coul. for the faraday and is still using this value, since his latest value [13] of 96,513.1 abs. coul. per mole on *the physical scale* (atomic weight of oxygen = 1.00027 ×16) is simply 1.00027 times his 1929 value. This value is based entirely on the determinations of F with the silver voltameter. It is the proper value to use in equations connecting atomic constants (as Birge himself points out) only if the measured amount of silver deposited in a silver voltameter by the passage of a given charge of electricity truly represents the mass associated with that amount of charge. There is reason to believe that this is not the case. While it is true that the difference is of no consequence in deriving values of the radiation constants, it cannot be ignored if one wishes to express the faraday to 0.1 coul.

In the first place, on account of inclusions and similar causes, the mass *deposited* by one coulomb is not the mass of silver ions with which one coulomb of charge is associated. Correction is usually made for inclusions, but there may be other effects which are not known. In the second place, the amount of silver *deposited* is not necessarily the amount of silver *weighed*. It may be worth while to quote a few sentences from a discussion of this question by Rosa and Vinal [14]. "The precaution taken by some observers to soak the silver deposits over night in distilled water to remove the last traces

of electrolyte was shown to be harmful because it was discovered that silver in contact with platinum is appreciably soluble in distilled water. This was shown by repeated tests. The silver and the platinum differ slightly in potential so that a current passes from the silver to the platinum through the water." If acid is present in the electrolyte there will, of course, be a re-solution of the deposit. Although there is no difficulty in avoiding loss of deposit due to acidity of the electrolyte, the question of re-solution in a neutral electrolyte still remains, inasmuch as in the usual form of silver voltameter, the silver is deposited on platinum.

All of these effects, as long as they are constant, have no bearing on the use of the silver voltameter as a device for reproducing the international ampere defined as the current which will deposit 0.001118 g of silver per second in a silver voltameter. The technique of the silver voltameter has been so well worked out that it is a precision device capable of being used to check the voltage of the Weston normal cell. Rosa and Vinal [14] list six determinations of this voltage in five different countries with an average deviation of 1 part in 10^5 from the mean and a *maximum* deviation of less than 2 parts in 10^5.

The international ampere, so defined, is thus transferred to standard cells and material resistance standards (wire coils) which are then used to reproduce the international ampere in the various national laboratories. There are small differences in these material standards but they amount to only a few parts in 10^5 and can be neglected. We have then, from the silver experiments, that

$$F = \frac{\text{at. wt. of Ag}}{0.001118} = \frac{107.880}{0.001118} = 96,494 \text{ int. coul. mole}^{-1}.$$

While the statement "in a silver voltameter a weighed deposit of 1 mole will result from the passage of 96494 int. coul." is true to about 1 part in 10^5, the uncertainty in the above figure as a value of F is determined by the magnitude of the effects described earlier. We will not question the fact, which must be true if values of the specific electronic charge, e/m, are to be deduced from F, that each univalent ion carries the charge e.

It is evident, then, that the definition of the international ampere in terms of the silver voltameter does not confer any special virtue on silver in the determination of the faraday. In fact, if we used a different element which was free from all effects of the kind described, it would yield a more reliable value of F. Unfortunately, there are very few elements that are suitable. There must be no question of the valence. If we have, for example, both univalent and divalent ions, the result will be entirely useless. Iodine is one of the few that can be used. It is free from the effect of inclusions, but the mass determined by titration may differ from the mass deposited due to oxidation by the arsenious acid solution used in the titration. The value of the faraday obtained with the iodine voltameter is 96,514 int. coul. per g equiv. [15]. There seems no logic, whatever, in ignoring the iodine value. The effects in the two cases are not known, but it seems fairly certain that the faraday lies somewhere between the two values whose mean is 96,504 int. coul. There is no point in weighting the silver value higher than the other simply because more work has been done with the silver voltameter. This difference of 20 parts

per 10^5 is inherent in the two methods and would probably not be greatly changed by more determinations.

To get F in units which can be used by atomic physicists we must know the relation between the absolute and international ampere. For this we now have a reliable conversion factor q. The various determinations yield the ratio of the size of the absolute ampere to the international ampere as represented in each case by the material standards, standard cells and standard resistance coils, at the national laboratory making the determination. The differences in these material standards can be neglected and well within the accuracy with which we can at present evaluate the faraday, the conversion factors apply to the international ampere as defined in terms of the silver voltameter, that is, to the value of $F=96,504$.

In 1906–07 an extended investigation was carried out at the National Physical Laboratory [16] in which the silver deposited by one absolute coulomb in the "New Form" of silver voltameter was $0.0011181{,}5_1$. The absolute ampere was realized with the Ayrton-Jones current balance. This is not a determination of F, but a determination of

$$q=\frac{0.0011180}{0.0011181{,}5_1}=0.99986.$$

This value, while not comparable in accuracy with recent determinations, agrees surprisingly well with the best of these.

The value of the faraday is then 96,504 int. coul. mole^{-1}, or 96,490 abs. coul. mole^{-1}. This value does not differ much from the one Birge is now using but is based on different data. His use of only the silver data was to a large extent balanced by his use of an obsolete value of q.

Expressing the result in electrostatic units

$$F=(2.8926 \pm 0.0002)\times 10^{14}\ \text{(esu)}\ \text{mole}^{-1}$$

5. VALUES OF e AND h

For the purposes of this paper an accurate value of the ratio h/e is of far more concern than accurate values of either h or e. There is some divergence in the fourth significant figure of the value advocated for e. It appears that the value derived by Von Friesen [17] is based upon impartial consideration of the data available. This value is

$$e=(4.800 \pm 0.005)\times 10^{-10}\ \text{(esu)}$$

The value of $e=4.800$ and the value of h/e listed on page 392 lead to a value of

$$h=(6.61 \pm 0.01)\times 10^{-27}\ \text{erg sec.}$$

This value agrees to three significant figures with all the recent values, see table 5.

IV. VALUES OF c_2 FROM $\lambda_m T$.

One of the most direct methods of evaluating c_2 is by the use of eq 4. The published results are listed in table 2.

TABLE 2.—*Values of c_2 from measurements of $\lambda_m T$*

Authors	Year	Published values	Selected values
Lummer and Pringsheim [18]	1899 1901	1.43 and 1.46 1.458	1.45
Paschen and Wanner [19]	1899 1900 1901	1.435 1.441 1.447	1.44
Holborn and Valentiner [20]	1907 1912	1.42 1.435	1.435
Warburg, Leithauser, Hupka, and Müller [21]	1910 1911 1912 1913 1915	1.457 1.42 to 1.46 1.436 and 1.438 1.4374 1.425 and 1.43 to 1.44	1.43
Coblentz [22]	1914 1916 1922	1.4465 1.4369 1.4318	1.432
Mendenhall [23]	1914 1917	1.44 1.4394	1.439
Michel [24]	1922	1.427	1.427
Unweighted mean			1.4361
Mean deviation			0.0059

All the values listed in the third column except those of Coblentz and of Holborn and Valentiner represent separate determinations. Holborn and Valentiner's value of 1.435 is merely a correction by Valentiner of the value originally reported as 1.42. Coblentz's values of 1916 and 1922 represent only corrections of the value originally published in 1914.

It is evident that a mean of all the published values would have little or no significance. The best value derivable from these data would be a weighted mean of the selected values, but it is very difficult to assign weights to such observations. These selected values are the final values arrived at by the various groups of experimenters. In cases where reported values of c_2 were based on the equation $c_2 = 5\lambda_m T$ obtained by using Wien's formula instead of eq 4, the values have been recalculated.

V. VALUES OF c_2 FROM σ

The second method for obtaining c_2 is from measurements of σ as defined by eq 3 and the relation given in eq 8. Substitution of the numerical values of c and h from section III in the latter yields the relation

$$c_2 = 0.12477\sigma^{-1/4} \tag{12}$$

In table 3 are listed the available data on σ with values of c_2 calculated from eq 12.

TABLE 3.—*Experimental values of σ and corresponding values of* c_2

Authors	Year	Reported values of $\sigma 10^5$	Selected values of $\sigma 10^5$	$c_2 = 0.12477\ \sigma^{-1/4}$
Lehnebach [25]	1874	4.58		
Kundt and Warburg [26]	1878	4.62		
Graetz [27]	1880	4.5		
Christiansen [28]	1883	5.04		
Scheiner [29]	1909	4.8		
Kurlbaum [30]	1898	5.33	5.33	1.460
Féry and Drecq [31]	1909 1911 1912–13	6.30 6.51 5.57 and 6.20.	5.89	1.424
Todd [32]	1909	5.48	5.48	1.450
Bauer and Moulin [33]	1909 1910	6.0 5.3	5.3	1.462
Valentiner [34]	1910 1912	5.36 5.58	5.58	1.443
Shakespear [35]	1912	5.67	5.67	1.438
Puccianti [36]	1912	5.96	5.96	1.420
Westphal [37]	1912–13	5.57	5.57	1.444
Gerlach [38]	1912 1916 1920	5.803 5.85 5.80	5.80	1.430
Keene [39]	1913	5.89	5.89	1.424
Coblentz [40]	1914 1916	5.6 5.72	5.72	1.435
Foote [41]	1918	5.7	5.7	1.436
Kahanowicz [42]	1919	5.61	5.61	1.442
Wachsmuth [43]	1921	5.723	5.723	1.435
Müller and Császár [44]	1923	5.72	5.72	1.435
Hoffmann [45]	1923	5.764	5.764	1.432
Kussmann [46]	1924	5.795	5.795	1.430
Hoare [47]	1928	5.735	5.735	1.434
Müller [48]	1928	5.771	5.771	1.432
Mendenhall [49]	1929	5.79	5.79	1.430
Mean			5.69	1.4368
Mean deviation			0.13	0.0081

There is considerable uncertainty in these values arising from the corrections of about 2 percent which are usually made for the lack of blackness of the receiver. The values reported by Hoare and by Mendenhall are the only ones which do not involve any uncertainty from such corrections and are considered the most reliable. Many of the values originally reported have been corrected in later publications, but the original values are used except in cases where the corrections were made by the men who originally reported the results. Thus Valentiner corrected his value of 5.36 to 5.58 and Kurlbaum's value of 5.33 to 5.45. Valentiner's corrected and Kurlbaum's original values are selected for use in obtaining the mean. The first five values reported have been included in the table merely because of their historical interest.

A weighted mean of the values in table 3 would probably yield a better value than the one derived, but the assigning of such weights is not attempted. It is evident that in general the earlier values of σ are smaller and the derived value of c_2 larger than the later ones. Thus the first 10 values of c_2 listed in table 3 have a mean of 1.439,

and the last 10 have a mean of 1.434. On the assumption that the later values are more reliable, they might be given more weight, but this assumption is open to some question. It is felt that there are enough values to minimize the effect of including a few which are believed to be somewhat less reliable than the others and the unweighted mean seems the wiser choice.

VI. VALUES OF c_2 BASED ON OPTICAL-PYROMETER MEASUREMENTS

Another method which was used by Mendenhall [23] and by Day and Sosman [50] has not been given much attention because of the scarcity of the required experimental data. Equation 11, defining the International Temperature Scale of 1927, may be written

$$c_2 = \lambda \ln \frac{J_t}{J_{Au}} \frac{(t+273)(t_{Au}+273)}{t - t_{Au}} = \lambda \ln \frac{J_t}{J_{Au}} f(t) \tag{13}$$

The product $\lambda \ln J_t/J_{An}$ may be measured with an optical pyrometer and c_2 calculated for any case where the corresponding temperatures are known on the thermodynamic scale.

Day and Sosman and Holborn and Day [51] have measured the temperature of a large number of fixed points on the constant-volume nitrogen gas scale with a pressure at $0°$ C. of approximately 300 mm of mercury. The corrections to convert [52] these temperatures to the Centigrade Thermodynamic Scale are only a fraction of a degree even at the palladium point. Some of these temperatures, so corrected, are given in table 4. Day and Sosman did not make measurements at the copper-silver eutectic point, but their scale was so well described in terms of other fixed points, that temperatures in this region can be determined on their scale with considerable precision. Mendenhall and Forsythe [53] have determined the ratio of the temperatures of the gold and palladium points on the Kelvin Scale by means of eq 3. They found that the energy radiated by a black body at the palladium point was 3.4626 times that radiated by a black body at the gold point. From this

$$\frac{t+273}{1336} = \sqrt[4]{3.4626}, \text{ or } t = 1549.5° \text{ C.}$$

The value of $\lambda \ln J_t/J_{Au}$ for palladium listed in table 4 is that of Fairchild, Hoover, and Peters [54]. These authors compared their palladium with samples used by Day and Sosman and found the melting point to be the same within $0.1°$ C. The value for the nickel point is that of Wensel and Roeser [55] and the remaining values represent unpublished work by these same experimenters.

In the case of copper, the size of the interval, about $20°$ C, precludes the determination of c_2 to better than about 1 percent. Although Day and Sosman ascribe an uncertainty of about $\frac{3}{4}°$ to each of the gold and silver points, the nature of their measurements was such that the systematic or constant errors in their work would have the same sign and be of about the same magnitude at these two points. The difference between them is probably good to $\pm 0.2°$ C, in the same

way that the interval between gold and copper is certainly not in error by more than 0.2°. At the copper-silver eutectic point the uncertainty in c_2 is caused largely by the inability to make highly accurate optical-pyrometer measurements at the comparatively low brightness level corresponding to that temperature.

TABLE 4.—*Values of c_2 based on optical pyrometry*

Fixed points	Temperature	Authors	Method	$f(t)$	$\left(\lambda\ln\frac{J_t}{J_{Au}}\right)\times10^7$	c_2
	°C					
Pd____	1549.8	Day and Sosman_____	Gas thermometer_____	4997	2878	1.438
Ni_____	1453.0	____do____	____do____	5905	2430	1.435
Cu_____	1082.8	____do____	____do____	896×10²	159.5	1.429
Au_____	1062.6	____do____	____do____			
Ag_____	960.2	____do____	____do____	−16085	−891.5	1.434
Cu-Ag__	778.2	(Indirect)____	____do____	−4937	−1912	1.438
Cu_____	1084.3	Holborn and Day_____	____do____	903×10²	159.5	1.441
Au_____	1064.2	____do____	____do____			
Ag_____	961.7	____do____	____do____	−16108	−891.5	1.436
Pd_____	1549.5	Mendenhall and Forsythe.	Radiation pyrometer__	5003	2878	1.440
Au_____	1063.0	_____	(Used as base)_____			
Unweighted mean_____						1.4364
Mean deviation_____						0.0029

VII. VALUES OF c_2 FROM ATOMIC CONSTANTS

In view of the material presented in the preceding sections, the adoption in 1927 of $c_2=1.432$ cm deg in the definition of the International Temperature Scale seems to call for some explanation.

The values of c_2 derivable from experimental values of $\lambda_m T$ and from the gas-thermometer determination of the palladium point were essentially the same in 1927 as those given in this paper, namely, 1.436 and 1.438, respectively. The value of c_2 derivable from σ, however, was about $\frac{1}{4}$ percent lower than that derived today from the same value of σ, because the accepted value of h was then about 6.55, or 1 percent, lower than that accepted today. A value of $h=6.55$ and $\sigma=5.7$ corresponds to a value of $c_2=1.432_8$. Moreover, the values of h/e advocated in 1927 yielded values of 1.432 to 1.433. As the confidence placed in the available values of h and h/e at that time was such that they were generally expressed to four or five significant figures, the discrepancies in c_2 were not considered serious. The data on $\lambda_m T$ are not such as, in themselves, to inspire much confidence, and the value of c_2 deduced from the temperature of the palladium point was considered to indicate that Day and Sosman were overly optimistic in claiming an accuracy of 2° for their determination of this temperature. The history of the values of h and of h/e, as outlined in table 5 and figure 1, explains the choice of the value of c_2 (shown by the star at 1927 in fig. 1) used in defining the temperature scale.
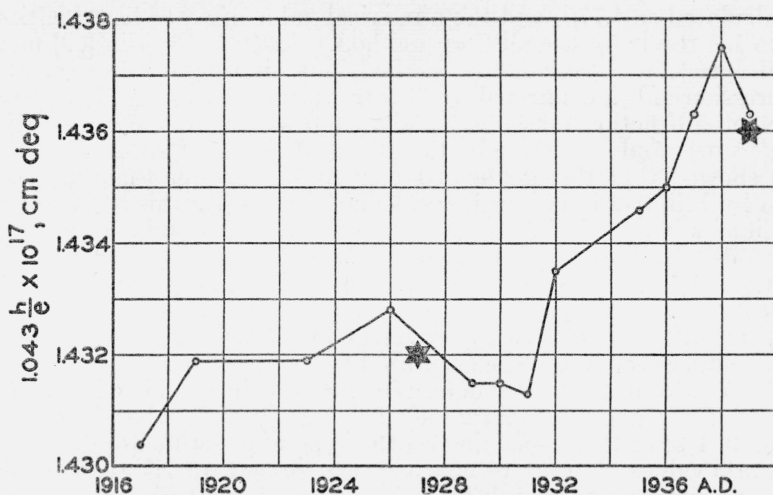
FIGURE 1.—*Values of c_2 calculated from h/e.*

TABLE 5.—*Values of h and of h/e which have been recommended by various reviewers*

Year	Recommended by	$h \times 10^{27}$	$(h/e) \times 10^{17}$	$1.0430 \ (h/e) \times 10^{17} = c_2$
1917	Millikan [56]	6. 547	1. 3714	1. 4304
1919	Birge [57]	6. 550	1. 3729	1. 4319
1923	Int. Crit. Tables [58]	6. 554	1. 3729	1. 4319
1926	Birge [59]	6. 560	1. 3737	1. 4328
1927	ITS [1]	--------------	1. 373	1. 432
1929	Birge [11]	6. 547	1. 3725	1. 4315
1930	Millikan [60]	6. 547	1. 3725	1. 4315
1931	Bond [61]	6. 5586	1. 3723	1. 4313
1932	Birge [62]	6. 5443	1. 3744	1. 4335
1935	Beardon [63]	6. 558	1. 3755	1. 4346
1936	Birge [64]	6.58 and 6.61	1. 37588	1. 4350
1937	Von Friesen [65]	6. 61	1. 3771	1. 4363
1938	Millikan [66]	6. 61	1. 3782	1. 4375
1939	Dunnington [67]	6. 6133	1. 3771	1. 4363

Since the values of c, F, and R are all very accurately known, the quantity $cF/R = 1.0430 \times 10^{17}$ may be taken as exact and eq 9 written as

$$c_2 = 1.0430 \ h/e \times 10^{17}. \tag{14}$$

The confidence which may be placed in this value of cF/R is indicated by the fact that the constants in the oldest reference available in which all three are given, the 1897 edition of the Smithsonian Tables, yield 1.0429, those in the first (1910) volume of "Tables Annuelles de Constantes" yield 1.0428, while those recommended by Birge [11] in 1929 yield 1.0431. Using the constants in the ICT, the result is 1.0435. The value of c_2 calculated from formula 14 therefore, will be as accurate as the value used for h/e.

The reason for the sharp upturn in the accepted values of h and h/e about 1932 is to be found in a change in the views held in regard to the electronic charge, e. Up to 1928 no determination had been made since Millikan [68] obtained the value 4.774×10^{-10} (esu) (later [60]

recalculated as 4.770) and the numerical value of e was based entirely upon his result by the oil drop method. In 1928 Bäcklin [69] measured e as 4.793 by an X-ray method, but his work made very little impression until confirmed by other workers both by X-ray and electron-diffraction methods. This led to a reexamination of Millikan's work, and a redetermination of the viscosity of air by Kellström [70] showed that this is about 1 part in 150 greater than the value used by Millikan in his original calculations. With this modification, Millikan's value is in agreement with values obtained by other methods as well as with a more recent [71] and accurate determination of 4.800 by the oil drop method. In the last 10 years there have appeared seven determinations [65] of e, ranging from 4.793 to 4.806, by three different methods, the average of which is 4.800×10^{-10} (esu). This value is believed to be accurate to ± 0.1 percent.

The constant h is never determined except in combination with e, the quantity actually determined being $Q = h/e^{n+1}$, where n has the value 0, 1/3, or 2/3, depending on the type of experiment. The most accurate values of Q are obtained in experiments of X-ray diffraction from calcite and from ruled gratings [3] yielding values of $h/e^{4/3}$ and $h/e^{5/3}$, respectively. We have then that

$$\frac{d(h/e)}{h/e} = n\frac{de}{e} + \frac{dQ}{Q}$$

so that, aside from the error in Q, the percentage error in h/e is always less than that in e. The various determinations of h/e^{n+1} have been carefully examined and the values of h/e shown in table 6 have been derived on the following basis:

<div align="center">

Calcite grating space$= 3.0356 \times 10^{-8}$ [72]

$c = 2.99776 \times 10^{10}$

$e = 4.800 \times 10^{-10}$

1 International Volt$= 1.00034$ Absolute Volts

</div>

TABLE 6.—*Experimental values of h/e*

Authors	Year	Method	$(h/e) \times 10^{17}$
Lawrence [73]	1926	Ionization potentials	1.3753
Millikan [74]	1916	Photoelectric effect	1.3777
Lukirsky and Prilëzaev [75]	1928	do	1.3711
Olpin [76]	1930	do	1.3748
Wagner [77]	1920	X-rays	1.3727
Duane, Palmer, and Yeh [78]	1921	do	1.3784
Feder [79]	1930	do	1.3757
Kirkpatrick and Ross [80]	1934	do	1.3752
Schaitberger [81]	1935	do	1.3768
From Rydberg constant [82]		$e/m = 5.2730 \times 10^{17}$	1.3787
Gnan [83]	1934	Electron diffraction	1.3765
Von Friesen [84]	1935	do	1.3769
Eddington [85]	1933	$ch/2\pi e^2 = 137$	1.3783

[3] The latter method yields

$$\frac{h}{e^{4/3}m^{1/3}} = \frac{h}{e^{5/3}}\left(\frac{e}{m}\right)^{1/3},$$

but the value of e/m is an independently determined constant, the measurement of which is not affected by e, and thus becomes part of the measurable quantity Q. In fact the measured value e/m, together with e, constitutes the determination of h/e.

There are many determinations from ionization potentials but, in most of these, accuracy beyond two significant figures was not even sought. Lawrence's result is the only one worth considering here. Lawrence's reported value of 1.3737 from the ionization potential of mercury vapor and the equation

$$Ve=\frac{hc}{\lambda}, \text{ or } \frac{h}{e}=\frac{V\lambda}{c} \tag{15}$$

was based on the international volt and $c=3\times10^{10}$. On the basis used here we get

$$h/e=1.00034\times\frac{3}{2.99776}\times1.3737=1.3753$$

Strictly speaking, this is in no sense a "correction" of Lawrence's result, but an expression of h/e on a different basis. Lawrence's actual result is a value of 10.40 international volts for the limit of the mercury absorption spectrum $\lambda=1.188\times10^{-5}$. From this we have that

$$\frac{h}{e}=\frac{1.00034\times10.40\times10^8}{2.99776\times10^{10}}\times\frac{1.188\times10^{-5}}{2.99776\times10^{10}}=1.3753\times10^{-17}$$

This result is entirely independent of the value of e.

From measurements of the photoelectric effect, Millikan obtained a value of $h=6.57\times10^{-27}$ based on $c=3\times10^{10}$ and $e=4.774\times10^{-10}$. On our basis, therefore, his work yields

$$h/e=\frac{6.57}{4.774}\times\frac{3}{2.99776}\times1.00034=1.3777.$$

Lukirsky and Prilĕzaev obtained $h=6.543$ based on $e=4.774$. This result requires adjustment for e and for reduction to the absolute volt, but not for c. Olpin obtained $h=6.541$, using sodium treated with sulfur vapor, and states in his prefatory abstract "An almost identical value is obtained for untreated sodium." However, in the body of the paper his result for untreated sodium is given as $h=6.6$, with the observation that it is not as reliable as the value 6.541. Von Friesen [65] in his table IV quotes Olpin's result as $h=6.561$, which is the mean obtained by giving the value 6.6 one-half as much weight as the value 6.541. This seems to be as good a way as any to treat Olpin's result, and from it we get

$$h/e=\frac{6.561}{4.774}\times1.00034=1.3748.$$

Determinations of h/e by this method likewise do not require a knowledge of e.

The X-ray method also determines h/e from eq 15, that is,

$$\frac{h}{e}=\frac{v\lambda}{c}=\frac{V\lambda}{c^2}\times10^8=\frac{2dV^1(1.00034)\sin\theta\times10^8}{c^2}, \tag{16}$$

where v is in (esu) of potential difference, V is in absolute volts, V^1 is in international volts, and d is the grating space of calcite. The result does not depend on e unless the value used for d is made to

depend on e. In the past, d has usually been calculated from the physical properties of calcite, using an equation in which d is proportional to $\sqrt[3]{e}$. When e was put equal to 4.770 or 4.774 a value of d (about 3.028×10^{-8}) was obtained which required X-ray wavelengths found with crystal spectrometers to be multiplied by 1.00248 to make them agree with X-ray wavelengths measured with ruled gratings [72]. In other words, measurements by the two methods will agree if d for calcite is taken as $1.0025 \times 3.028 \times 10^{-8} = 3.0356 \times 10^{-8}$. If, on the other hand, we calculate d on the basis of $e = 4.800$, we get $d = 3.028$ $(4.800/4.770)^{1/3} = 3.0344$. We get about the same values of h/e, therefore, from the crystal X-ray work by using the multiplying factor $(4.800/e)^{1/3}$ or by using Beardon's [72] value of $d = 3.0356$, which is obtained by comparing ruled-grating measurements with crystal measurements of X-ray wavelengths. The latter method is more direct and will be used. The following values of $V^1 \sin \theta$ are available for calcite.

Wagner_____2031.5
Duane, Palmer, and Yeh_____2039.9
Feder_____2036.0
Kirkpatrick and Ross_____2035.3

Putting $d = 3.0356$ and the above values of $V^1 \sin \theta$ into eq 16, we get the corresponding values of h/e listed in table 6. Schaitberger, on the other hand, used a rock-salt crystal, taking $d = 2.814$. He expressed his result as $h/e\alpha = 1.3743$ on the basis of the absolute volt, $e = 4.774$, and $c = 2.99774$, where α is the factor which converts the conventional d to true d. The simplest way to get h/e from this is to put

$$\alpha = \sqrt[3]{\frac{4.800}{4.774}} = 1.0018,$$

whence $h/e = 1.3768$. Kirkpatrick and Ross have characterized Duane's result as too high and Wagner's as too low. They apply corrections to make the former 2034.6 but do not correct the latter. It seems advisable to take each result as reported, as it makes very little difference in the result sought here.

To get h/e from the Rydberg number, R_∞, we use the equation

$$\frac{h}{e} = \sqrt[3]{\frac{2\pi^2 e^2}{R_\infty c e/m}} \qquad (17)$$

The value of R_∞ is 109737.4 [11], and, on the basis of the selected values of e and c, h/e is simply a question of the numerical value of e/m. However, Birge [82] has recently published a review of 10 determinations of e/m and concludes that the best derivable value is 1.7591×10^7 $(\text{emu})\text{g}^{-1}$, or 5.273×10^{17} $(\text{esu})\text{g}^{-1}$, on a basis substantially equivalent to the one used in this paper. Dunnington [67] arrives at the value 1.7590×10^7 $(\text{emu})\text{g}^{-1}$. Inasmuch as the highest and the lowest of the 10 values listed by Birge yield values of h/e differing by less 0.1 percent, it is evident that the value of h/e from equation 17 is largely dependent on and not much more accurate than the value used for e. For this reason it should not be given as much weight as would be the case if a more accurate value of e were available.

Gnan reported h/e as 1.374, but points out that his result depends on the value of e. He then states that his explicit result is $h/e^{4/3} = 1.758 \times 10^{-14}$, which is independent of his crystal constant. From this, $h/e = \sqrt[3]{.4800} \times 1.758 \times 10^{-17} = 1.3765 \times 10^{-17}$. Unfortunately, Von Friesen's Uppsala dissertation is not available to the writer, but Von Friesen [57] himself gives his result for h as 6612/6610 of that of Gnan. From this we obtain $h/e = 1.3769$ for Von Friesen's work.

Eddington's theoretical value of 1/137 for the fine structure constant is listed as a matter of interest. The mean obtained from the data listed in table 7, however, does not include this value, although, as a matter of fact, the mean is not appreciably changed if this value is included.

TABLE 7.—*Summary of values of h/e, in erg sec $(esu)^{-1}$ and the corresponding values of c_2 in cm deg*

Method	Determinations	$h/e \times 10^{17}$	c_2	Weight
Ionization potentials	1	1.3753	1.4344	1
Photoelectric effect	3	1.3745	1.4336	3
X-rays	5	1.3758	1.4350	5
Rydberg number	?	1.3787	1.4380	5
Electron diffraction	2	1.3767	1.4359	2
Fine structure		(1.3783)	(1.4376)	0
Weighted mean			1.4358 cm deg	
Mean deviation			0.0014	

It might be pointed out that according to Bond [86], all experimental determinations of e/m are in reality determinations of $(136/137)$ (e/m). On Bond's theory the value of c_2 from the Rydberg number is reduced by about $\frac{1}{4}$ percent, or, in this case, from 1.4380 to 1.4344.

Perhaps it should also be mentioned that Anderson [87] on the basis of a pseudo-frictional effect between light quanta has deduced the relation

$$\sigma = \left(\frac{\pi^2}{90}\right)^{1/3} \frac{2^4 \pi^{11} c^2 h}{3^7 5^2 c_2^4} = 1.01\sigma \text{ (from eq. 10)}$$

This theory was apparently devised to explain discrepancies between experimental values of $\lambda_m T$ and σ which have disappeared with the abandonment of the old value of e.

VIII. SUMMARY

All of the experimental data available have been used to derive values of c_2. This was merely a matter of choice, and the same data can be used in each case to calculate values of the other radiation constants. For example, the relation between $\lambda_m T$ and c_2 is exact. Similarly, the values of σ and c_1 listed on page 392 have been calculated from c_2. The values of c_2 from various sources are summarized in table 8, and the final mean value of c_2 there recorded will yield values of the other radiation constants which embody all the experimental data which have been considered.

TABLE 8.—*Summary of values of c_2 from all sources*

Source	c_2 in cm deg	Weight
$\lambda_m T$	1.4361 ±0.0050	1
σ	1.4368 ± .0040	1
Optical pyrometry	1.4364 ± .0020	2
Atomic constants	1.4358 ± .0015	3
Weighted mean	1.4362 ±0.001	
Mean deviation	0.0003	

It would seem that the probability is small that the true value of c_2 lies outside the limits 1.436 ±0.001 cm deg. The values of a set of mutually consistent constants derived from all available data are listed below.

$$c_1 = (3.732 \pm 0.006) \times 10^{-5} \text{ erg cm}^2 \text{ sec}^{-1}$$
$$c_2 = 1.436 \pm 0.001 \text{ cm deg}$$
$$\lambda_m T = (2892 \pm 2) \times 10^{-4} \text{ cm deg}$$
$$\nu_m / T = (5890 \pm 4) \times 10^7 \text{ deg}^{-1} \text{ sec}^{-1}$$
$$\sigma = (5.70 \pm 0.02) \times 10^{-5} \text{ erg sec}^{-1} \text{ cm}^{-2} \text{ deg}^{-4}$$
$$c = (2.9978 \pm 0.0002) \times 10^{10} \text{ cm sec}^{-1}$$
$$F = (2.8926 \pm 0.0003) \times 10^{14} \text{ (esu) mole}^{-1}$$
$$T_0 = 273.16 \pm 0.02 \text{ deg}$$
$$R = (8.3142 \pm 0.0010) \times 10^7 \text{ erg deg}^{-1} \text{ mole}^{-1}$$
$$k = (1.378 \pm 0.002) \times 10^{-16} \text{ erg deg}^{-1}$$
$$e = (4.800 \pm 0.004) \times 10^{-10} \text{ (esu)}$$
$$p = 1.00048$$
$$q = 0.99986$$
$$pq = 1.00034$$
$$h = (6.61 \pm 0.01) \times 10^{-27} \text{ erg sec}$$
$$h/e = (1.377 \pm 0.001) \times 10^{-17} \text{ erg sec (esu)}^{-1}$$
$$e/m = (5.273 \pm 0.002) \times 10^{17} \text{ (esu) } g^{-1}$$
$$M = \text{Least mechanical equivalent of light} = (151 \pm 1) \times 10^{-5} \text{ watt}$$
("new" lumen)$^{-1}$.

Calcite-grating space $= (3.0356 \pm 0.001) \times 10^{-8}$ cm.
Fine-structure constant (reciprocal $= 137.0 \pm 0.2$).

The writer has attempted to handle the data in such a manner as to secure results which reflected his own personal opinions as little as possible. An attempt has been made to avoid criticism by refusing to assign weights to published results or to apply corrections not pointed out by those who reported the original results. The practice of weighting values by different observers in the inverse ratio of the uncertainties which the observers attach to their own results seems a very questionable one.

In conclusion, it may be of interest to say something about the least mechanical equivalent of light, M. This is not a constant in the ordinary sense because it is not fixed by the units of length, mass, time, and luminous intensity but involves, in addition, the physiological and psychological characteristics which are arbitrarily assigned to some hypothetical individual designated as standard, that is, the so-called "ICI standard observer." In the past a determination of M has been merely a matter of determining the luminous intensity of a black body at some measured temperature and then calculating

M on the basis of c_2, σ or c_1, and the selected standard observer. In terms of the present "International" candle, 1 cm^2 of a black body at the freezing point of pure platinum emits 58.9 π, or 185.0 $\pm$0.2 international lumens [88]. The freezing temperature of platinum is 1773.5 on the international scale ($c_2=1.432$). On January 1, 1940, a new system of photometric units is scheduled to go into effect. The numerical value of M, in terms of the "new" units, will increase by approximately 2 percent, because on that date the luminous emission from 1 cm^2 of a black body at the freezing point of pure platinum becomes 60 π "new" lumens by definition. M, thereupon, becomes fixed by c_2, σ, and the standard observer (luminosity factors). On the basis of $c_1=3.732\times10^{-5}$ and $c_2=1.436$, we have that the freezing point of platinum is 2043.8° K and that $c_2/2043.8=7.0261\times10^{-4}$, so that

$$M=A\int_0^\infty L_\lambda\lambda^{-5}\Big(e^{\frac{7.0261}{\lambda\times10^4}}-1\Big)^{-1}d\lambda,$$

where the values of L_λ are the internationally accepted [80] luminosity factors describing the ICI standard observer and $A=c_1/60\pi=1.980\times10^{-7}$ erg sec^{-1} cm^2 ("new" lumen)$^{-1}$. From this relation M is found to be 1.508×10^4 ergs per "new" lumen second, or 0.001508 watt per "new" lumen. On the basis of $c_2=1.432$, $M=0.00157$ watt per "new" lumen, which is identical with the former value of $M=0.00160$ watt per "international" lumen.

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