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RESISTIVITY AND POWER INPUT IN THE CESIUM DISCHARGE AT HIGH CURRENT DENSITY

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ABSTRACT

Measurements have been made of the potential gradient and the ion current to the tube wall in a tube 5 mm in diameter for current densities ranging from 5 to 20 amp/cm^2 and vapor pressures from 0.0045 mm to 0.33 mm. Potential gradients have also been measured in a 1-mm tube for current densities ranging from 30 to 150 amp/cm^2 and vapor pressures from 0.33 to 2 mm. Published results give electron concentrations and temperatures for this range of conditions. The resistivity agrees approximately with the theoretical value for a completely ionized gas. The power dissipation comes predominantly from wall recombination at low pressures. Above 0.17 mm the radiation is appreciable, and above about 1 cm the radiation would account for most of the power input. Theoretical equations are given for the case where pressure and degree of ionization are so high that the vapor is opaque.

CONTENTS

I. INTRODUCTION

In another paper [1] data have been given on the electron concentration and electron temperature in cesium discharges with current densities sufficient to give nearly complete single ionization over a wide range of pressures. In connection with these experiments, measurements of potential gradient were made for pressures up to 2 mm of mercury, while values of the positive-ion current density to the tube walls were obtained at pressures up to 0.3 mm. These measurements permit for the first time an experimental investigation of the resistivity and power input in an ionized gas.

A number of physicists have been interested in the theoretical problem of the mean free path and mobility of electrons in an ionized gas. A complication in the problem is that an isolated ion has an infinite effective cross section for electron scattering. However, if there are N_+ ions per cm^3, then an electron is in the field of a single ion over a path less than $1/N_+^{1/3}$ cm, and the scattering cross section has a finite value.

[1] F. L. Mohler, J. Research NBS **21**, 697 (1938) RP1150.

Gvosdover [2] gives the equation for the cross section in square centimeters of a singly charged ion as

$$\sigma^{+}=\frac{\pi}{4}\frac{e^4}{k^2T_e^2}\ln\frac{k^2T_e^2}{e^4N_+^{2/3}}=\frac{5.02\times10^{-6}}{T_e^2}\log 0.72\times10^6\frac{T_e^2}{N_+^{2/3}}, \qquad (1)$$

where T_e is the electron temperature, N_+ the ion concentration (equal to electron concentration under discharge conditions), and the other symbols follow conventional usage. Most other derivations give equations of a similar form but with various values for the constants.[3] Gabor, however, has used a very different statistical method of solving the problem and derived a quite different equation.

$$\sigma^{+}=\frac{0.8\times10^{-2}}{T_eN_+^{1/3}} \qquad (2)$$

Boeckner and the author [4] have shown that scattering of electrons by ions becomes important in a low-pressure discharge with a current density of the order of an ampere per square centimeter. The mean free path at constant pressure decreases with increasing current, and measurements of this decrease give a method of computing the ion-scattering area. The effect of a mixture of ions and atoms is conveniently expressed in terms of the reciprocal of the mean free path, which is the total scattering area per cubic centimeter.

$$1/\lambda=S=N_1\sigma_1+N_+\sigma_+, \qquad (3)$$

where N_1 and σ_1 pertain to the neutral atoms. The resistivity (gradient/current density) in ohms centimeters is

$$X/I=\frac{2.94\times10^9T_e^{1/2}S}{N_+} \qquad (4)$$

Because the scattering area of ions is 20 to 40 times that of neutral atoms, it is practicable to obtain conditions where the scattering by neutral atoms is negligible. Equations 4, 3, and 1 then give

$$X/I=2.94\times10^9T_e^{1/2}\sigma^{+} \qquad (5)$$

$$X/I=\frac{1.48\times10^4}{T_e^{3/2}}\log 0.72\times10^6\frac{T_e^2}{N_+^{2/3}} \qquad (6)$$

In this equation the log term changes so slowly that it is satisfactory to use the form

$$X/I=K/T_e^{3/2}, \qquad (7)$$

where K is nearly constant for a range of conditions where $T_e^2/N_+^{2/3}$ remains of the same magnitude. Equation 6 is quite general, for it is assumed to give the limiting value of the resistivity with high currents in any gas or discharge tube, assuming only that the ions are singly ionized.

The energy balance in the positive column can be expressed symbolically

$$\pi a^2XI=R+H+W, \qquad (8)$$

[2] Physik. Z. Sowjetunion **12**, 164 (1937).
[3] Langevin, Ann. chim. phys. **5**, 271 (1905); Thomas, Proc Roy. Soc. (London) [A] **121**, 470 (1928); Landau, Physik. Z. Sowjetunion **10**, 154 (1936); Gabor, Z. Physik **84**, 474 (1933).
[4] BS J. Research **10**, 357 (1933) RP535.

where the term on the left is the power input per centimeter, R is the power radiated by the vapor, H is the power carried to the walls by heat conduction, and W is the power dissipated by recombination of ions and electrons on the walls. The last term can be expressed as follows: [5]

$$W=2\pi aI_{+}A(V_{i}+8T_{e}/11,600)=2\pi aI_{+}AV_{R}, \qquad (9)$$

where $2\pi aI_{+}$ is the ion current to unit length of the tube wall, V_i is the ionization potential, and $8T_e/11{,}600$ is the kinetic energy of the ion and electron. V_R is the total recombination energy in electron volts and A is an accommodation coefficient to take account of the fact that the neutral atom after recombination will have some kinetic energy. A value nearly equal to one is however to be expected. V_R is of the order of 10-electron volts in these experiments, and it is evident that with 50-percent ionization, H will be only a few percent of W and can safely be neglected. R is probably small compared with W for extreme currents. At low pressures the ratio R/W decreases with increasing current to a value of about 0.5 for a current of 4 amp in a tube 1.8 cm in diameter,[6] but there is no safe basis for predicting the ratio for the conditions here used.

II. EXPERIMENTAL RESULTS

Experimental details have been covered in another paper.[7] At pressures above 0.3 mm of mercury, the electron concentration and temperature in a capillary discharge were measured by measuring the intensity distribution of the continuous recombination spectrum. In one series of experiments the potential difference was measured between two probes near the two ends of a capillary 1 cm long and 1.1 mm in diameter. This potential difference is assumed equal to the gradient in the capillary. The experiments covered a pressure range from 0.3 to 2 mm with current densities from 30 to 150 amp/cm^2. The percentage of ionization ranges from 25 to 98.

Another series of experiments base measurements of T_e and N_e on the characteristics of a probe surface flush with the wall of a 5-mm tube. N_e was derived from the "ion current equation",[8]

$$I_{+}=0.48\times10^{-16}N_{e}T_{e}^{1/2}. \qquad (10)$$

The potential gradient was measured between two probes 7 cm apart in the 5-mm tube. These experiments cover a pressure range from 0.0045 mm to 0.3 mm with current densities from 5 to 20 amp/cm^2. The percentage of ionization is not known but is probably of the same magnitude as was found for higher pressures.

Table 1 gives the complete experimental data. Experimental uncertainties are probably of the order of 10 percent for T_e, N_e, and X. It was impossible to attain the precision usually obtained in low current discharges under these extreme conditions. Evolution of foreign gas and thermionic emission from probes from overheating may account for the uncertainty.

[5] Tonks and Langmuir, Phys. Rev. **34,** 876 (1929).
[6] F. L. Mohler, BS J. Research **9,** 25 (1932) RP455.
[7] F. L. Mohler, J. Research NBS **21,** 697 (1938) RP1150.
[8] See footnote 5.

TABLE 1.—*Resistivity and power input*

Vapor pressure	I	X	$\pi a^2 XI$	X/I	T_e	N_e	$aXI/2I_+$	Mean free path
				TUBE DIAMETER, 5 mm				
mm	*amp/cm²*	*v/cm*	*w/cm*	*ohms-cm*	°*K*		*v/ion*	*cm*
0.0045	5	0.60	0.60	0.122	6,600	1.0×10^{13}	10	1.97×10^{-1}
	7.5	.53	.80	.073	8,000	1.22	9.6	2.9
	10	.44	.88	.045	9,500	1.15	10.3	5.5
	12.5	.38	.95	.031	10,800	1.25	9.5	7.8
0.009	5	.8	.80	.163	5,500	1.44	9.9	0.94
	7.5	.87	1.31	.119	6,600	2.54	8.3	.80
	10	.87	1.74	.089	7,800	3.7	6.9	.78
	12.5	.67	1.68	.055	9,100	3.0	7.6	1.70
	15	.54	1.63	.037	10,300	2.3	9.2	3.5
0.0175	5	.86	.86	.175	4,200	1.9	9.1	0.57
	7.5	.99	1.48	.135	5,100	3.2	8.6	.45
	10	1.02	2.04	.104	6,200	4.5	7.7	.49
	12.5	.99	2.48	.081	7,100	5.0	7.8	.62
	15	.98	2.94	.067	8,000	5.0	8.65	.77
	17.5	.90	3.15	.053	8,900	4.7	9.23	1.12
	20	.78	3.12	.040	10,000	3.9	10.4	1.9
0.039	5	1.01	1.01	.206	3,700	2.5	8.5	0.35
	10	1.34	2.68	.136	5,200	6.1	8.0	.25
	12.5	1.42	3.55	.116	5,900	7.7	7.9	.25
	15	1.43	4.29	.098	6,600	9.3	7.5	.26
	17.5	1.59	5.6	.092	7,300	10.8	7.9	.25
	20	1.68	6.72	.085	8,000	12.0	8.2	.25
0.080	5	1.1	1.1	.224	3,400	2.7	9.2	.28
	10	1.53	3.06	.156	4,700	6.9	9.5	.185
	15	1.85	5.55	.127	5,300	12.7	7.8	.132
	17.5	1.89	6.6	.110	5,800	15.1	7.5	.134
	20	1.85	7.4	.094	6,400	17	7.2	.147
0.17	10	1.65	3.30	.168	3,800	0.81×10^{14}	8.7	13.4×10^{-3}
	12.5	1.86	4.65	.151	4,300	1.14	8.15	11.1
	15	2.0	6.0	.136	4,600	1.53	7.6	9.6
	17.5	2.16	7.6	.126	5,000	1.97	7.2	8.4
	20	2.28	9.1	.116	5,300	2.32	7.0	7.9
0.33	10	1.86	3.72	.19	3,400	0.64	13.2	14.1
	12.5	2.0	5.0	.163	3,600	.88	12.2	12.3
	15	2.22	6.66	.151	3,800	1.14	12.2	10.6
	17.5	2.36	8.3	.138	4,100	1.48	11.4	9.2
	20	2.43	9.7	.124	4,200	2.46	8.0	6.2
				TUBE DIAMETER, 1 mm				
0.33	30	7.5	2.25	0.24	3,360	2.8×10^{14}	---------	2.5×10^{-3}
	50	8.8	4.40	.167	3,830	3.6	---------	3.0
	75	9.5	7.14	.12	4,400	3.6	---------	4.5
	100	9.5	9.5	.09	5,050	3.1	---------	7.5
	150	9.0	13.5	.057	6,200	2.7	---------	15.1
0.60	30	8.5	2.54	.27	3,360	4.2	---------	1.51
	50	10.0	5.0	.19	3,830	5.9	---------	1.63
	75	11.6	8.7	.147	4,400	6.6	---------	2.02
	100	11.9	11.9	.113	5,050	6.5	---------	2.8
	150	11.5	17.2	.074	6,200	4.9	---------	6.4
1.1	30	10	3	.32	3,360	6.2	---------	0.85
	50	11.4	5.7	.22	3,830	10.0	---------	.83
	75	13.4	10.1	.17	4,400	13	---------	.88
	100	14.4	14.4	.14	5,050	14	---------	1.07
	150	15.4	23.2	.10	6,200	12	---------	1.93
2.0	30	11.7	3.5	.37	3,360	8.8	---------	0.52
	50	13.1	6.6	.25	3,830	13	---------	.55
	75	15.1	11.3	.19	4,400	18	---------	.59
	100	16.4	16.4	.156	5,050	21	---------	.63

III. RESISTIVITY

It follows from eq. 6 that the resistivity is conveniently expressed as a function of electron temperature. In figure 1 the data in the pressure range 0.0045 to 0.08 mm have been plotted and a single curve drawn through the points. Values tend to increase with decreasing ion concentration, but experimental uncertainties do not justify drawing separate curves through points obtained at different pressures. The theoretical eq. 6 has been plotted as a broken line, using a single mean value of 4.4 for the log term (values range from 5.3 to 3.7).

Figure 2 gives data in the pressure range 0.17 to 2.0 mm. Crosses give values at 0.17 mm in the 5-mm tube; circles for 0.33 mm in the 5-mm tube; dots give data for the 1-mm tube at pressures indicated at the right of each curve. A single theoretical curve has been drawn as a broken line, using a mean value of 3.26 for the log term. Values of the log term decrease from 3.8 to 3.0 as the pressure increases, but the experimental values in the 1-mm tube increase with increasing ion concentration.

Values of the scattering area σ^+ derived from values of X/I by eq 5 range from 0.14×10^{-12} at 10,000° to 1.52×10^{-12} at 3,000° for data of figure 1. Published values[9] based on measurements of the change in total scattering area S of eq 4 with current give somewhat smaller values. Thus at $T_e=4650$, $P=0.0012$, Boeckner and Mohler obtained $\sigma_+=0.60\times10^{-12}$, while the curve of figure 1 gives 0.81×10^{-12}. A complication in measuring the change in S with current is that with increasing current there is an increase in vapor temperature, so that a decrease in $N_1\sigma_1$ of eq 4 partially masks the change in $N_+\sigma_+$. The difference between the two results may come from this effect.

The absolute value of the resistivity from figure 1 is on the average 20 percent less than the value given by Gvosdover's equation (eq 6). Other theoretical equations give higher values and are less satisfactory.

The observed change in resistivity with temperature is somewhat greater than $T_e^{-3/2}$ though the mean curve of figure 1 conceals this trend. The experiments are definitely inconsistent with the Gabor equation (eq 2) both in magnitude and in dependence on T_e and N_+.

In comparing experiment and theory it is to be noted that eq 6 is an ideal equation for a completely ionized gas without boundaries. It does not take into consideration the distribution of current across the tube and the effective current density is not accurately the mean current density, I. This may well account for most of the discrepancy between experiment and theory. The contribution of scattering by neutral atoms to the resistivity is probably small. The scattering area of a neutral atom is about $.03\times10^{-12}$ and with 30 percent ionization the neutral atoms contribute less than 10 percent of the total scattering area. Gvosdover[10] has recently evaluated the contribution of the tube walls to the resistivity. This is small except for long mean free paths and is negligible in these experiments.

Table 1 includes values of the mean free path computed by eq 3 and 4. Except at the lowest pressures the mean free path is much less than the tube diameter. Values decrease with increasing N_e but

[9] Boeckner and Mohler, BS J. Research **10**, 357 (1933) RP535.
[10] Physik. Z. Sowjetunion **13**, 133 (1938).

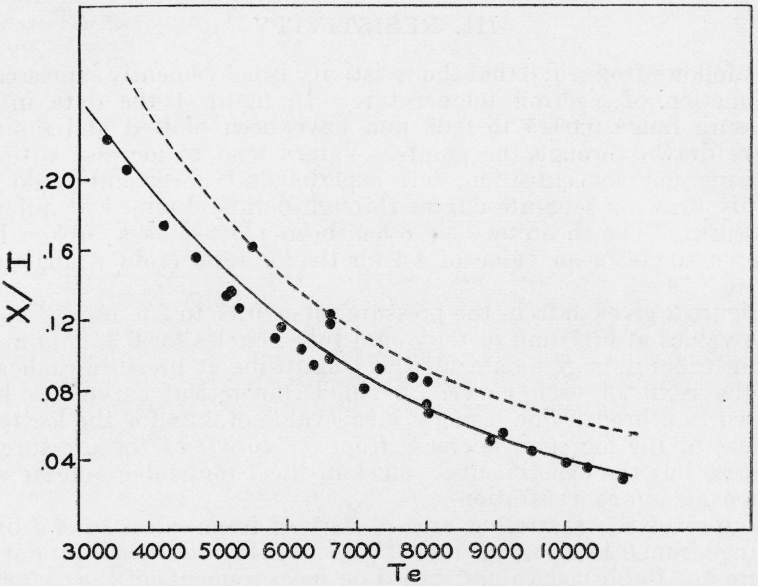

Figure 1.—*Resistivity in ohms-centimeters as a function of electron temperature at pressures from 0.0045 to 0.080*

Broken-line curve is eq 6 with a mean value 4.4 for the log term

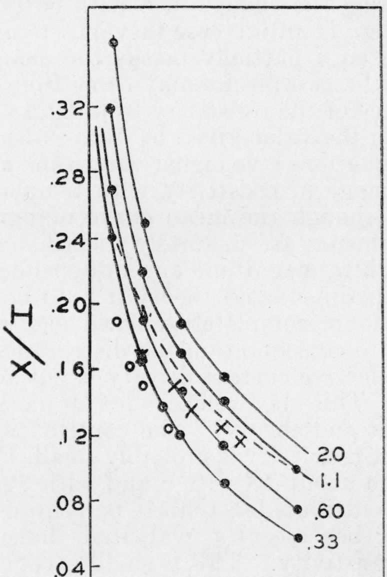

Figure 2.—*Resistivity in ohms-centimeters as a function of electron temperature*

Crosses for 5-mm tube at 0.17-mm pressure, circles 5-mm tube at 0.33-mm pressure, dots 1-mm tube at pressures indicated. Broken-line curve is eq. 6 with 3.26 for the log term begin to increase with increasing current as N_e approaches its maximum value. Theory predicts an increase proportional to T_e^3 for nearly complete ionization.

IV. POWER INPUT AND LOSS

In section I it is shown that recombination on the walls probably accounts for most of the power loss though the power radiated cannot be definitely evaluated. The power input per unit wall current ($aXI/2I_+$ of table 1) is then a quantity that is not much greater than the available recombination energy, AV_R, defined by eq 9. The tabulated values are always of the same magnitude as V_R of eq 9 and at high temperatures definitely less than V_R. A plot of power input per unit wall current versus temperature is given in figure 3. Experimental uncertainties preclude a detailed analysis but some definite conclusions can be drawn. A value of A somewhat less than 1 must

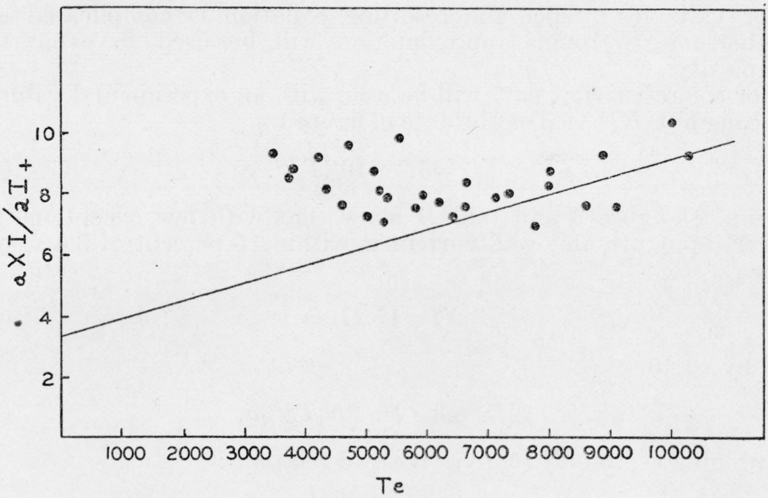

FIGURE 3.—*Power input per ion in electron volts as a function of temperature.*

Pressures from 0.009 to 0.17, inclusive. The straight line is a plot of AV_R of eq 9 with A=0.85.

be assumed to fit the high temperature values and the straight line is a plot of AV_R for A=0.85. Results are consistent with the assumption that radiation is appreciable only at temperatures less than 6,000° corresponding to lower currents and higher pressures and the vertical distance between the line and the points is the energy radiated per unit ion current. These experiments alone do not exclude the possibility of a lower value of A and more radiation. However, published work[11] on power input and loss at low current densities is inconsistent with a value of A much less than 0.85.

[11] F. L. Mohler, BS J. Research 9, 25 (1932) RP455.

V. CONCLUSIONS

For the range of conditions covered in table 1, the resistivity, X/I, is nearly equal to the theoretical value for a completely ionized gas, as given by eq 6. It depends primarily on the electron temperature and neither the kind of gas nor the tube radius appears explicitly in the equation.

For a more limited range of conditions the power input is nearly equal to the wall recombination as given by eq 9. That is,

$$XI=\frac{2}{a}I_+A(V_i+8T_e/11{,}600). \tag{12}$$

In contrast to the resistivity, the quantity, XI depends specifically on the tube radius, the kind of gas through V_i, and perhaps on the material of the tube through A. One can eliminate X from eq 6 and 12 and obtain a relation between the electron temperature and the other variables. Since the resulting equation is complicated and cumbersome, a rough approximation will be used to avoid this complexity.

For the resistivity, eq 7 will be used with an experimental value of the constant, K, based on the data of figure 1.

$$X/I=5.2\times10^4/T_e^{3/2}. \tag{13}$$

Data of figure 3 and table 1 show that with few exceptions the power input per unit wall current is within 15 percent of 8.6 v and, hence,

$$XI=17.2I_+/a \tag{14}$$

and by eq 10

$$XI=8.3\times10^{-16}N_eT_e^{1/2}/a. \tag{15}$$

Combining eq 13 and 15 gives the two relations:

$$X^2=4.3\times10^{-11}N_e/T_ea, \tag{16}$$

$$I^2=1.6\times10^{-20}N_eT_e^2/a. \tag{17}$$

It is to be emphasized that these simple relations are rough approximations valid for a limited range. The equations serve merely to give a rough picture of the relation of T_e to other variables under conditions of relatively low pressure and extreme currents. Thus it is of interest that T_e increases with increasing tube radius and constant current density.

Power loss by radiation is evidently appreciable at pressures above 0.3 mm and will be predominant at pressures above a few centimeters in a 5-mm tube for the radiation increases as the square of the pressure for sufficiently high currents. One can obtain relations analogous to eq 16 and 17 for high pressures by a purely theoretical approach. Because there is nearly temperature equilibrium at high pressure, for a given pressure and temperature one can derive N_e by Saha's equation. Equation 6 then gives the resistivity, provided T is sufficiently high to give over 30-percent ionization and the power input is assumed equal to the total radiation. An approximate theoretical evaluation of the total radiation of ionized cesium would, in general, be complicated. However, for the case that pressure, temperature, and tube diameter are so great that the ionized vapor is nearly opqaue, the radiation approaches that given by the Stefan-Boltzman law, namely, $5.71\times10^{-12}T_e^4$ w/cm². The power equation is then simply

$$XI=\frac{11.4}{a}\times10^{-12}T_e^4. \tag{18}$$

This, combined with the resistivity equation, 6, gives

$$X^2=\frac{17\times10^{-8}T_e^{5/2}}{a}\log\frac{0.72\times10^6T_e^2}{N_+^{2/3}}. \tag{19}$$

$$I^2=7.7\times10^{-16}T_e^{11/2}/a\log\frac{0.72\times10^6Te^2}{N_+^{2/3}}. \tag{20}$$

Neglecting the variation in the log term, one has equations of the simple but unusual form,

$$T_e=k_1a^{2/11}I^{4/11}. \tag{21}$$

$$X=k_2a^{-3/11}I^{5/11}. \tag{22}$$

These relations are independent of the pressure except for the variation of the log term. An approximate evaluation of the total radiation has been made for cesium vapor at a pressure of 760 cm of mercury and a temperature of 6,000° K in a tube 0.5 cm in radius. There is approximately 50-percent ionization for these conditions and $N_e=6\times10^{17}$. The total radiation is 80 percent of that of a black body and $XI=2.4\times10^4$ w. $X/I=0.05$ ohms-cm. Hence, X equals 34 v and $I=700$ amp/cm². It may be entirely impractical to realize such conditions experimentally, but it remains of some interest to see the relations which must exist under conditions of extreme current density and pressure. Three independent variables in three quite general equations, the Saha equation, the total radiation equation, and the resistivity equation, completely describe the discharge conditions. This is in marked contrast to the complexities encountered under all other discharge conditions. The generality of the Gvosdover resistivity equation for extreme ion concentrations may perhaps be open to question.

The results of the numerical example give what may appear as a low value of X and a correspondingly high value of I, but there is no safe basis of comparison. Very high pressure mercury arcs have been operated at a surface brightness comparable to the above, but the ionization is far from complete, which makes X very much greater and I correspondingly smaller.[12]

WASHINGTON, October 3, 1938.

[12] Elenbaas, Z. Tech. Physik **17**, 61 (1936).

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