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## A PORTABLE APPARATUS FOR MEASURING VIBRATION IN FRESH CONCRETE

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### ABSTRACT

A portable vibrograph was designed and constructed for measuring vibration in fresh concrete during the process of placing by vibration. An electromagnetic type of pickup unit, submerged with definite orientation within the concrete, picks up the vibrations and conveys an electric response through either of three electric circuits, and then through an amplifier to a cathode-ray oscillograph. Particle displacements, velocities, or accelerations can be measured and displacement, velocity, or acceleration wave forms obtained, depending upon the circuit chosen. Frequencies which form simple ratios with 60 cycles can be measured accurately, and other frequencies can be estimated.

The integrating or differentiating circuit-amplifier-oscillograph system was found to cause a variation of less than 3 percent in the recording of a constant input for frequencies ranging from approximately 20 to 300 c/s.

The pickup unit was calibrated by use of microscope observations of amplitudes of vibration of a steel plate, vibrating in a horizontal direction at approximately 60 c/s, to which the pickup unit was attached. The amplitudes included in this calibration ranged from approximately 0.045 to 0.00035 in. Lower amplitudes were evaluated by extrapolation.

Calibrations of the acceleration and velocity scales were made by replacing the pickup unit with an audio-frequency oscillator supplying a sine wave of known frequency and voltage, corresponding to calibrated amplitudes. Accelerations ranging from approximately  $\frac{1}{16}$  to 40 times gravity, and velocities ranging from 0.01 to 2.5 f/s were included in the calibration.

Typical curves are presented showing the variation of amplitude of vibration in concrete with distance from the vibrator. Photographs are given showing corresponding particle displacement, velocity, and acceleration wave forms produced by vibration of concrete with two different types of vibrators.

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### I. INTRODUCTION

Within the past decade, numerous studies have been made of the effects of the vibration of fresh concrete. Meager data, however, are available on the actual displacements, velocities, and accelerations,<sup>1</sup>

<sup>1</sup> Throughout this paper the terms amplitude, displacement, velocity, and acceleration refer to particle motion unless otherwise specifically stated.

within the concrete. It was thought that an apparatus capable of measuring these vibration characteristics at any point within the concrete could be of value in numerous studies. Such studies might include the effectiveness of various types and sizes of vibrators, the effect of size and rigidity of forms, and the effect of different mixes and  $C/W$  ratios on the propagation of vibrations within the concrete. Accordingly, an apparatus was designed and built at the National Bureau of Standards for measuring the foregoing vibration characteristics, as well as the frequencies and the propagation velocities of the vibrations.

This paper describes the apparatus in detail and also gives a brief description of various types of measurements which have been made.

## II. DESIGN REQUIREMENTS

The following requirements were considered essential for an apparatus to be used for measuring vibration in fresh concrete over a wide range of both field and laboratory conditions.

1. The unit for picking up the vibrations should be small, light, portable, mechanically rugged, and adapted to being immersed to any desired depth in the concrete, with a definite orientation.
2. The indicating device should give a continuous indication which can be read rapidly.
3. The reading should be proportional to the actual displacement, velocity, or acceleration, as the case may be, at a given point and in a given direction in the concrete.
4. With proper adjustment, the pickup unit should be capable of registering vibrational motion in either a horizontal or the vertical direction.
5. The apparatus should respond accurately over the range of amplitudes<sup>2</sup> and frequencies encountered in the commercial vibration of concrete.
6. A means of obtaining wave forms should be provided.

## III. THEORY AND CONSTRUCTION OF ELECTRICAL AND MECHANICAL SYSTEMS

### 1. GENERAL CONSIDERATIONS

An electrical method of measuring the vibration in the concrete appeared to be most promising in satisfying the requirements. There are a number of well-known methods through which vibrational motion may be detected by electrical means. The motion may be caused to affect an electrical circuit through change of resistance, capacitance, or inductance, or through electromagnetic or piezoelectric effects. While certain of these methods<sup>3 4 5</sup> have been utilized to measure one vibration characteristic (either displacement, velocity, or acceleration) it is desirable to measure all three characteristics with one detecting circuit.

It was found that all three of these vibration characteristics could be measured by using a pickup of the electromagnetic type, which has a voltage output proportional to the velocity, and by combining

<sup>2</sup> Amplitude refers to the maximum displacement from the position of rest.

<sup>3</sup> U. S. Bureau of Mines Tech. Paper 518 and 556.

<sup>4</sup> C. D. Greentree, Gen. Elec. Rev. 40, No. 9 (Sept. 1937).

<sup>5</sup> M. Mary, Ann. Ponts et Chaussées 106, 338 (March 1936).

this pickup with electrical circuits for integrating and for differentiating the voltage from the pickup. In addition, the electromagnetic type of pickup unit is adapted to the desired simple, compact, and rugged construction, and requires a minimum of adjustment and recalibration.

A similar method of utilizing an electromagnetic detecting system in conjunction with electrical circuits to obtain displacements, velocities, and accelerations has been used by Meyer and Böhm.<sup>6</sup> The principles of their circuits are considerably different, however, from those described in the present paper, a different frequency range was covered in their work, and less accurate results were considered satisfactory.

Beale and Stansfield<sup>7</sup> have described an electromagnetic pickup and a resistance-capacitance type of integrating circuit used in pressure studies of diesel-engine operation. Draper and Bentley<sup>8</sup> have described a similar pickup unit and integrating circuit used in studying aircraft vibration during flight.

The present apparatus uses a pickup and an integrating circuit similar to those used by the above-mentioned investigators. In addition, it contains a differentiating circuit, so that by a simple switching arrangement either the displacement, velocity, or acceleration as a function of time may be shown on the oscillograph screen.

## 2. PICKUP UNIT

The mechanical design of the pickup unit follows the general principles of seismic vibration measuring devices. The unit consists essentially of an electromagnet (the seismic element) coupled through a spring of low stiffness to the vibrating member, the mass of the electromagnet and the spring constant being adjusted so that the electromagnet remains relatively motionless throughout the frequency range to be covered. The pickup coil is rigidly attached to the vibrating member (the case). Hence, the relative motion between the electromagnet and the pickup coil causes an induced electromotive force in the coil which is proportional to the instantaneous velocity of the vibrating member.

The first instrument made was constructed so that with the proper adjustment, measurements could be made in either a horizontal or the vertical direction. During experimentation in the measurement of vibration in concrete, it became evident that for the most widely used types of vibrators, including the internal and form types which vibrate in a horizontal plane, it was sufficient for the instrument to respond only to horizontal vibrational motion. Hence a new instrument was constructed which, by making it suitable for horizontal measurement only, was made more compact and rugged.

This second instrument is shown diagrammatically in figure 1. The external dimensions are approximately  $2\frac{1}{4}$  by  $2\frac{1}{4}$  by  $2\frac{1}{2}$  in. The weight of the pickup unit and supporting rod together is about  $2\frac{1}{2}$  lb. The bulk specific gravity of the pickup unit is approximately 3.2.

As shown in figure 1, the suspension of the stationary magnet in this instrument consists of two phosphor-bronze strips, about 0.005 in.

<sup>6</sup> Elek. Nachr. Tech. 12, 404 (1935).

<sup>7</sup> Engineer 160, 667 (1935).

<sup>8</sup> J. Aero. Sci. 3, 116 (1936); 4, s. 281 (1937).

in thickness. The width and thickness of the middle section of each strip was decreased to obtain a natural period of the magnet of about  $\frac{1}{4}$  second. A phosphor-bronze leaf spring was adjusted to exert a light pressure upon the surface of the magnet in order to produce sufficient damping to minimize the effect of transient vibration.

A semiflexible joint between the pickup-unit case and the supporting rod (section A-A', fig. 1) allows free movement of the unit in the direction of measurement, but simultaneously provides high strength

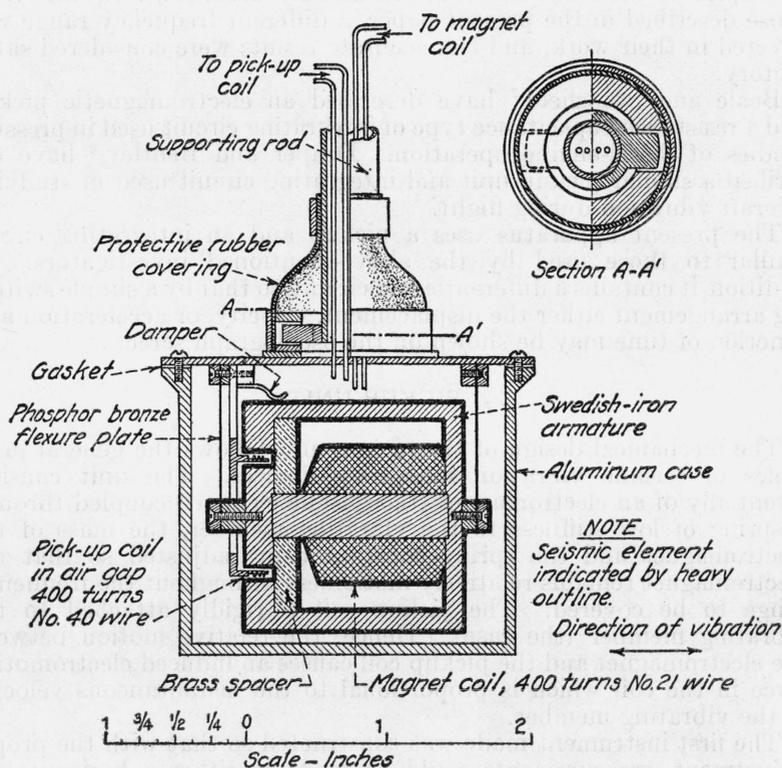


FIGURE 1.—Schematic drawing of pickup unit.

to allow the unit to be pushed, twisted, or pulled in any manner to permit rapid placement and alignment in the desired position in the concrete. The instrument is watertight, but its internal parts are readily accessible for any adjustment or repair.

### 3. INTEGRATING CIRCUIT

The integrating circuit<sup>9</sup> is shown in figure 2. The pickup coil,  $P$ , moving in a uniform magnetic field, has at any instant an induced electromotive force which is proportional to the pickup coil velocity, that is,  $e=K(dx/dt)$ , where  $x$  is the displacement of the coil from its

<sup>9</sup> For discussions of the theory involved, see Beale and Stansfield, *Engineer* 160, 667 (1935); also Draper and Bently, *J. Aero. Sci.* 3, 116 (1936); 4, 281 (1937).

position of rest and  $K$  is a constant. For a single sinusoidal vibration of frequency  $\omega/2\pi$  the current in the circuit will be

$$i = \frac{e}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \tag{1}$$

where  $R$  is the resistance, in ohms, in series with the condenser of capacitance  $C$  farads, and  $L$  is the inductance in henries of the pickup

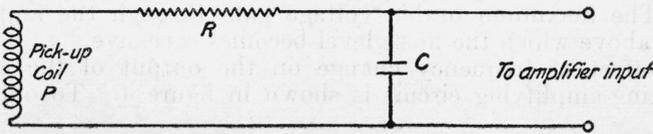


FIGURE 2.—Diagram of integrating circuit.

coil. If  $R$  and  $C$  are increased and  $L$  decreased sufficiently to make the term  $(\omega L - 1/\omega C)^2$  small compared with  $R^2$ , then approximately,  $i = e/R = K(dx/dt)/R$ .

In a complex wave the circuit introduces a phase distortion, due to the difference in phase lead of the various harmonic components.

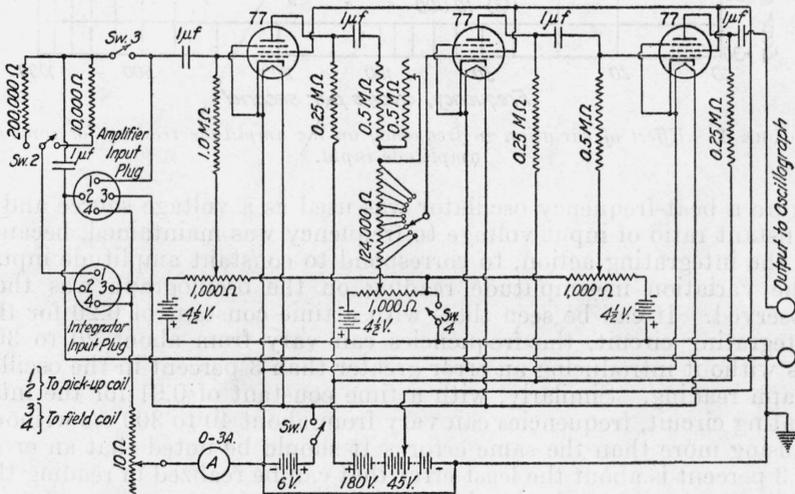


FIGURE 3.—Diagram of integrating and amplifying circuits.

Temporarily neglecting this phase distortion, the voltage across the condenser terminals at any instant,  $t$ , is then

$$E_c = \frac{1}{C} \int_0^t i dt = \frac{K}{CR} \int_0^t \frac{dx}{dt} dt = \frac{K}{CR} x_t, \tag{2}$$

that is,  $E_c$  is proportional to the displacement at the time,  $t$ . If  $E_c$  then is applied through an amplifier to the deflecting plates of an oscillograph having a linear time axis, the trace on the oscillograph

screen will give an accurate picture of the displacement of the pickup as a function of time.

The integrating and amplifying circuits are shown in figure 3. The value of  $C$  used in the integrating circuit is  $1\ \mu\text{f}$ ; two values of  $R$  are included, 10,000 and 200,000 ohms, respectively. Using the resistance of 10,000 ohms, making the time constant ( $CR$ ) equal to 0.01, the sensitivity is approximately 20 times that when the resistance  $R$  equals 200,000 ohms ( $CR=0.20$ ). The amplifier has three resistance-capacitance coupled stages using pentode tubes having high gain. The maximum usable voltage gain through the amplifier is 35,000, above which the noise level becomes excessive.

The effect of frequency change on the output of the complete integrating-amplifying circuit is shown in figure 4. To obtain this

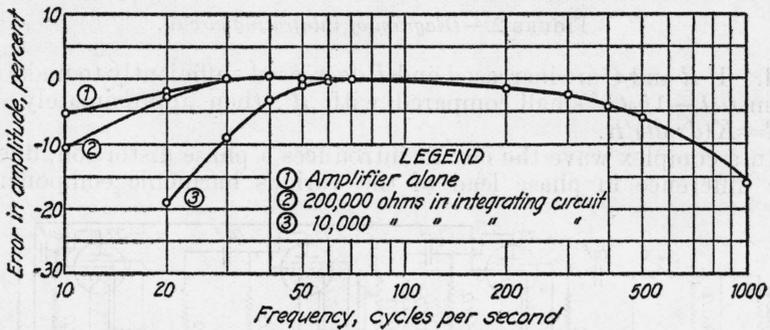


FIGURE 4.—Effect of variation in frequency on the amplitude reading for constant amplitude input.

figure a beat-frequency oscillator was used as a voltage source and a constant ratio of input voltage to frequency was maintained, because of the integrating action, to correspond to constant amplitude input. The variation in amplitude reading on the oscillograph was then observed. It can be seen that, with a time constant of 0.20 for the integrating circuit, the frequencies can vary from about 20 to 300 c/s without introducing an error greater than 3 percent in the oscillograph reading. Similarly, with a time constant of 0.01 for the integrating circuit, frequencies can vary from about 40 to 300 c/s without causing more than the same error. It should be noted that an error of 3 percent is about the least error that can be realized in reading the oscillograph even under ideal conditions.

Above 300 c/s the error increases, reaching 16 percent at 1,000 cycles. In commercial vibration of concrete, practically all vibrators operate in the frequency range of 50 to 150 c/s, and it is probable that most significant harmonics occur below 1,000 c/s.

The phase lead for a sinusoidal current of frequency  $\omega/2\pi$  is  $\theta = \tan^{-1} 1/\omega RC$ , and represents the maximum phase error for any harmonic. Substituting the circuit constants in this expression, it is found that the phase distortion for the circuit with the time constant of 0.20 is negligible. In this case the angle  $\theta$  is about 2.3 degrees at 20 cycles and decreases at higher frequencies. The circuit with the time constant of 0.01 (designed to be used only at frequencies above 100 c/s) gives a value of 9 degrees for the angle  $\theta$  at 100 cycles. Even this maximum distortion, representing a shift of  $1/40$  wave length, is

normally insignificant, particularly as the higher harmonics are generally of comparatively low magnitude in the original wave and contribute even less proportionally to the shape of the integrated wave.

#### 4. DIFFERENTIATING CIRCUIT

Several different electrical circuits may be used to perform what corresponds to a mathematical differentiating process and thus to convert an alternating voltage representing a velocity-time wave into its corresponding acceleration-time wave. The voltage measured across an inductor of low impedance, in series with a resistor of high resistance, is approximately the time derivative of the current in the circuit. Similarly, the voltage across a resistor of a low resistance in series with a capacitor of high impedance is approximately the time derivative of the voltage across the circuit. However, the circuit adopted as most satisfactory for the present purpose is shown in figure 5, wherein  $e_1$  is an alternating voltage obtained from the pickup unit, and  $e_2$  is the voltage output after passing through the differentiating circuit. For convenience, it may be assumed that  $e_1 = E \sin \omega t$ , where  $E$  is a constant,  $\omega = 2\pi f$ ,  $f$  is the frequency in cycles per second, and  $t$  is the time in seconds. For a complex wave in a particular case this sinusoidal wave may be considered as representing a single term of a Fourier's series giving the actual wave.

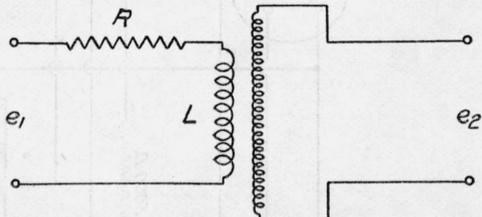


FIGURE 5.—Diagram of differentiating circuit.

The circuit shown in figure 5 is essentially an air-core transformer, with a high resistance in series with the primary. With the alternating voltage  $e_1 = E \sin \omega t$  placed across the input, the current  $i$  in the primary circuit, with a small current in the secondary, is given by the equation

$$E \sin \omega t = iR + L \frac{di}{dt} \tag{3}$$

where  $R$  is the total resistance, and  $L$  is the inductance, in henries, of the primary circuit.

With  $R$  large with respect to  $L\omega$ , the voltage across the secondary becomes

$$e_2 = \frac{M}{R} E \omega \cos \omega t, \tag{4}$$

$$\text{or } e_2 = \frac{M}{R} \frac{de_1}{dt} = \frac{MK}{R} \frac{d^2x}{dt^2} \text{ (from eq 2)} \tag{5}$$

where  $d^2x/dt^2$  is the acceleration at the time,  $t$ , and  $M$  is the mutual inductance, in henries, of the primary and secondary coils.

The phase distortion due to the difference in phase lag  $\theta = \tan^{-1} L\omega/R$  is negligible with the given constants for all significant harmonics.

The present circuit has an advantage over the other differentiating circuits mentioned, in that the relation between the resistance and

the reactance of the coil in the input circuit is of small consequence, a condition which becomes of greater importance at low frequencies. Also, while a large voltage loss occurs in any of the three circuits, a portion of the loss may be regained in the present one by increasing the amount of winding on the secondary coil. The amount which the number of secondary turns may be increased is limited by the size of the coil and the amount of shielding required.

A diagram of the differentiating circuit with the constants and the switching arrangement used, is shown in figure 6. For the

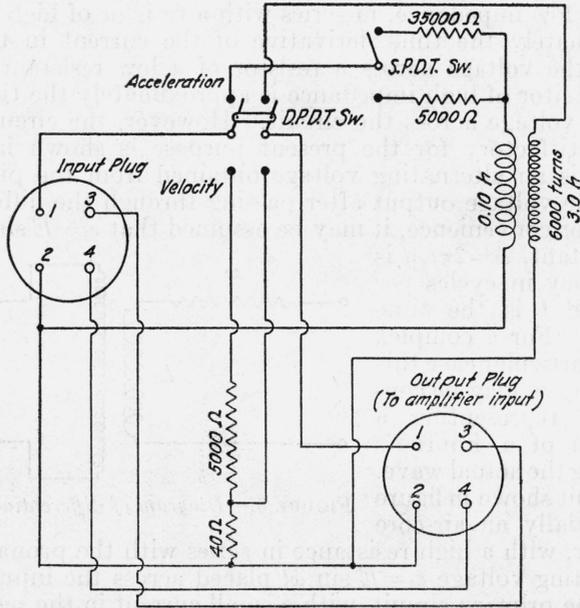


FIGURE 6.—Diagram of the acceleration and velocity measuring circuits.

constants chosen, as shown in the diagram, and with the input voltage corresponding to constant acceleration, the relation between acceleration reading on the oscillograph and the frequency follows the curve shown for the amplifier alone in figure 4.

Included in figure 6 is a voltage attenuating circuit with constants adjusted so that the desired range of velocity readings may be obtained to correspond to the displacement and acceleration scales.

## 5. INDICATING SYSTEM

A cathode-ray oscillograph with 5-in. screen serves as an indicating device, the screen being ruled in 0.10-in. divisions to permit rapid readings of the beam deflection. The oscillograph also contains a single-stage amplifier, which gives a voltage gain of about 25, making the total available voltage gain in the apparatus approximately 900,000.

For precise results, such high amplification might be undesirable. In the present case, variations of 5 percent are not considered excessive, and, with a simple rapid method of checking the calibration at any time having been devised, described in a later section, the magnitude of amplification used is not objectionable.

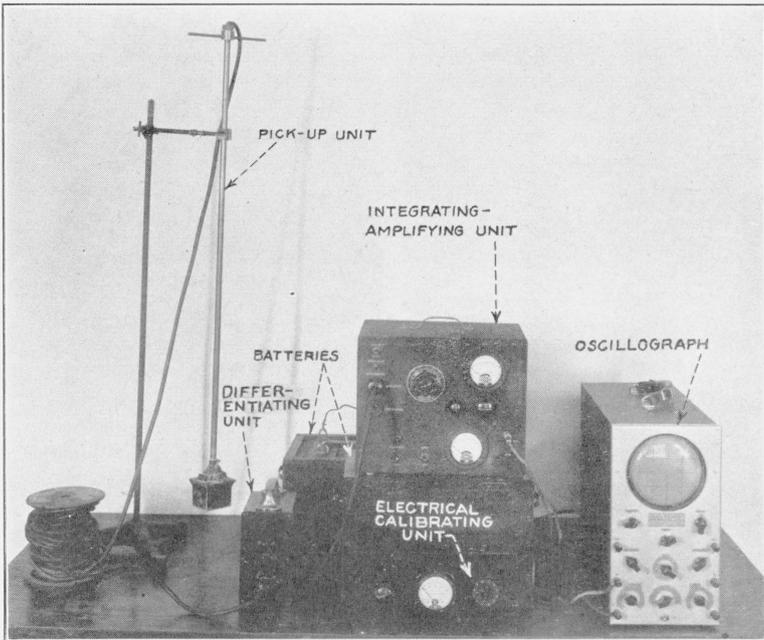


FIGURE 7.—Assembled vibrograph.

The oscillograph may be utilized to measure frequency of vibration, although it is often simpler to measure the frequency of the vibrator directly with some form of frequency meter. If the frequency to be measured, however, forms a simple ratio with 60 cycles, it can be determined directly through the patterns it forms on the oscillograph screen when the sweep circuit has been synchronized with the 60-cycle power supply. The accurate determination of any other frequency by means of the oscillograph required the use of a source of alternating voltage of known variable frequency.

Figure 7 shows the complete apparatus, assembled as it is used in making measurements.

#### IV. CALIBRATION

Calibration of the apparatus for displacement measurement was accomplished through the use of a steel plate (approximately 24 by 5 by  $\frac{1}{16}$  in.) held in a vertical plane by being clamped at both ends to heavy concrete blocks. The pickup unit was bolted to the center of the plate, and the plate caused to vibrate horizontally by a rotating, motor-driven eccentric mass also attached to the center of the plate. The amplitude of vibration of the pickup unit was observed through a microscope, having a calibrated reticule in the eyepiece, sighted upon a fine crosshair on the top of the pickup unit. It was found that with the ends of the steel plate held in this manner, and the rotating eccentric driven at frequencies from 50 to 70 c/s, the vibration obtained was smooth and linear in character, and in general could be held to a very constant value. The amplitudes obtained for the calibration in this manner ranged from 0.045 in. to 0.00035 in.

During the calibration process, the various resistors in the tapped gain-control of the amplifier were adjusted in value to give the desired beam deflections on the oscillograph for the chosen amplitude ranges. A 3-in. total deflection of the oscillograph beam was obtained for amplitudes of approximately 0.045, 0.015, 0.0045, and 0.0015 in., respectively, with 200,000 ohms in the integrating circuit, and for amplitudes of approximately 0.0020 and 0.0010 in., respectively, with 10,000 ohms in the integrating circuit. For each range a linear relation was obtained between the optically measured amplitude and the oscillograph beam deflection.

Although in this scale calibration the frequency was maintained between 50 and 70 c/s, other tests were made at constant amplitude in which the frequency was varied from approximately 30 to 100 c/s (the frequency limits of the available vibration source) to check the linearity of the pickup unit output. No deviation from linearity was found in that range.

In order to utilize all of the available amplification, and to increase the range of the apparatus for the measurement of amplitudes too small to be conveniently observed by optical methods, an extrapolation was made. Observations were taken of the input voltages, at 60 c/s, necessary to give a definite oscillograph-beam deflection as the controls were set, first for each of the calibrated amplitude ranges, and then for each of the uncalibrated ranges. Assuming that the straight line relation found for the ranges covered by direct calibration still holds, the calibration can be extended for the evaluation of lower amplitudes. In this manner four additional ranges were obtained,

the lowest of which gives a 3-in. total oscillograph deflection for an amplitude of approximately 0.00002 in.

The calibration of the acceleration and velocity measuring circuits was accomplished through the use of a beat-frequency audio oscillator with an output wave containing a harmonic content of approximately 0.5 percent. With a constant voltage output from the oscillator, at a known frequency, the signal was applied to the calibrated integrating circuit and then to the uncalibrated acceleration and velocity measuring circuits, respectively, and corresponding readings were obtained under the three conditions. The amplitude corresponding to the given input signal was known from the first reading, and, with the signal being a sine wave of known frequency, the corresponding maximum velocity and acceleration could be accurately computed.

The acceleration scale was calibrated for six arbitrary ranges, covering accelerations from 2 to 1,250 f/s/s, or approximately  $\frac{1}{6}$  to 40 times gravity. The velocity scale was calibrated for five ranges, covering velocities from 0.01 to 2.5 f/s.

In order to detect any changes in calibration which might appear, due to variations in the integrating, amplifying, or oscillograph circuits, a portable calibration checking device was constructed. This consists of an attenuator, supplied with 60-cycle alternating current, with which a known voltage may be supplied to the input of the integrating circuit. Immediately after calibration in each range, oscillograph beam deflection readings were taken with this attenuator in the circuit. A volume control, incorporated in the amplifier, permits the amplification to be adjusted at any time to obtain these standard readings. With this device, the calibration may be rapidly checked for any range before that range is used.

In the calibration of the integrating circuit, observations were made of the oscillograph reading obtained with the pickup unit being vibrated with large amplitude at right angles to the direction in which the unit is designed to respond. It was found that even such large transverse vibration apparently had no effect on the reading. In concrete, the pickup unit should thus respond only to vibrational motion in the direction of its axis.

## V. VIBRATION MEASUREMENTS

A short laboratory study was made to test the adaptability of the vibrograph to the purposes for which it was designed. Two commercial internal vibrators,<sup>10</sup> of rotating eccentric weight type, the outside diameters of the vibrating shafts being 1 and 2 $\frac{1}{4}$  in., respectively, were used for vibrating the concrete. The small vibrator is directly driven by a  $\frac{1}{2}$ -hp electric motor, and operates at frequencies from 9,000 to 14,000 rpm. This vibrator is of a type designed to compact laboratory specimens, and was not intended for field work. The large vibrator is hydraulically operated, being driven by a 3-hp electric motor and can be operated at any frequency below 6,500 rpm. The amplitudes of vibration of the two vibrators when suspended in air were approximately 0.012 and 0.050 in., respectively. There was no appreciable diminution in vibrator amplitude when either vibrator was immersed in concrete, according to measurements made just above the immersed section of the vibrator.

<sup>10</sup> The vibrators were generously lent to the Bureau for these studies by the Viber Co., Ltd., and by the Electric Tamper and Equipment Co.

For most of the study a clay concrete was used, proportioned 1:2.2:3.8 parts by weight of clay, sand, and graded aggregate, respectively. The form used was 4 by 4 by 2 ft. high, inside dimensions, having rigid 4-in. concrete walls and base. During vibration of the concrete, almost no vibration was perceptible in the walls of the form.

With the limited data obtained, no attempt was made to evaluate the effects upon amplitude distribution in the concrete of such variables as size and shape of vibrator, size, shape, and rigidity of form, the frequency of vibration and variations in mix. However, several typical curves are presented in which some of these factors are varied.

Figure 8 shows the variation of amplitude of horizontal vibration at different depths in the concrete at various distances from the

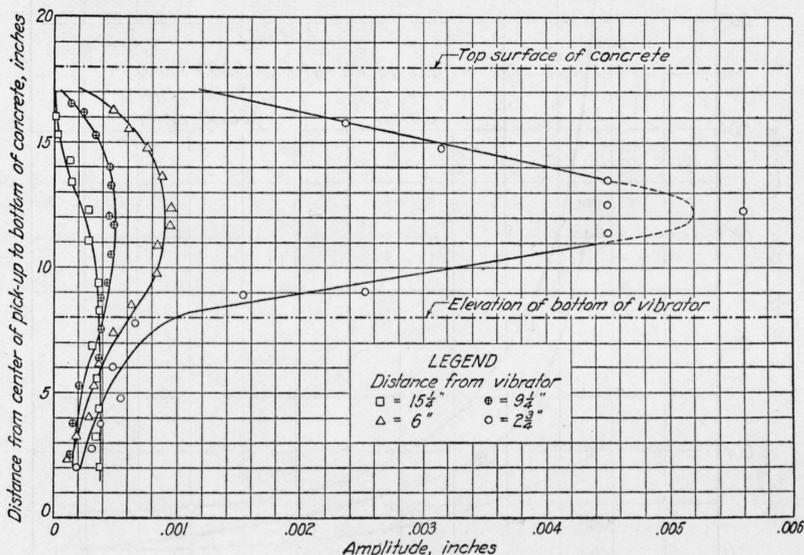


FIGURE 8.—Variation of displacement amplitudes of vibration with depth in concrete at various distances from the vibrator.

vibrator. The latter was held in the same position throughout (near the midpoint of the concrete) while the pickup unit was moved along the diagonal of the form. This figure indicates the necessity for carefully controlling the depth of immersion of the pickup unit during measurements, especially near the vibrator.

Figure 9 shows the variation of horizontal amplitude with distance from the vibrator for two different speeds of the large vibrator and one speed of the small vibrator. In obtaining these data, the vibrator was moved towards the pickup unit, the latter being maintained in one position at approximately the midpoint of the immersed section of the vibrator.

The accuracy of the points shown in figure 9 depends upon the degree in which the pickup unit will follow the actual motion of the concrete which surrounds it. This in turn depends upon (1) the ratio of the wave length of the vibratory wave in the concrete to the size of the pickup unit, (2) the change in amplitude in the concrete over the

length of the pickup unit, (3) the density of the pickup unit, and (4) the ease with which the concrete will flow around the unit, or the "internal friction" of the concrete. It is probable that in this respect the pickup unit acts as a piece of aggregate of the same approximate dimensions.

It will be shown later that with the frequencies ordinarily used in vibrating concrete the wave length is large with respect to the length of the pickup unit. The average specific gravity of the entire pickup unit is about 3.2, which is somewhat higher than the average value

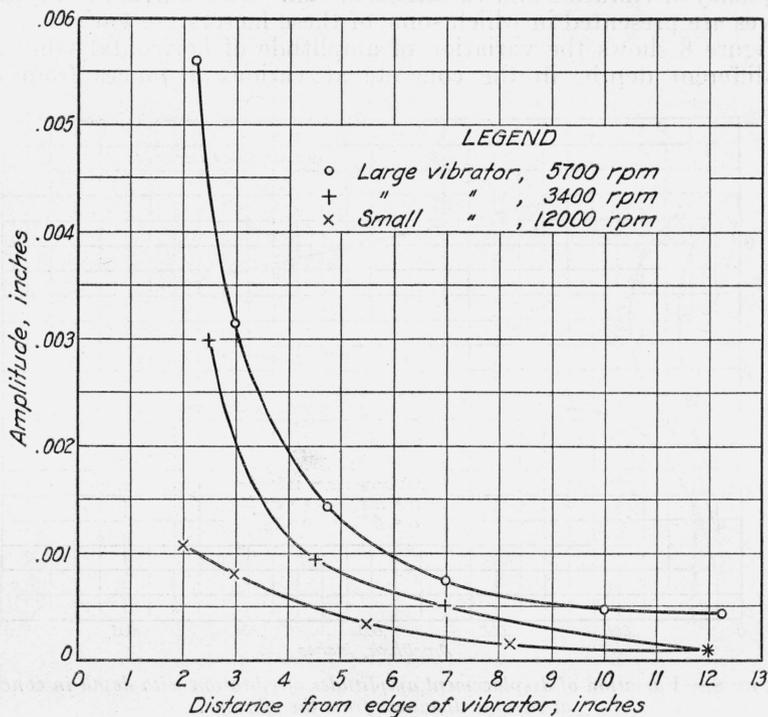


FIGURE 9.—Variation of displacement amplitude with distance from vibrator for clay concrete contained in rigid concrete form.

for concrete. However, as the seismic element remains nearly stationary, the mass being accelerated is less than the total mass of the pickup unit, and unduly large inertia forces are not expected to be present. The "internal friction" of concrete is considerable, even with relatively large amounts of water present, and in the dry concretes generally best adapted to placement by vibration is probably particularly great in magnitude. The amplitude-distance curves given in figure 9 for the large vibrator show that for distances from the source of vibration greater than 4 or 5 in. the amplitude gradient is not excessively large. Considering all of these factors it may be assumed that beyond 4 or 5 in. the pickup unit registers some average value of the amplitudes distributed over its surface. For distances less than about 4 in. the absolute value of the amplitude reading is to be questioned. However, from consideration of the regularity of the points

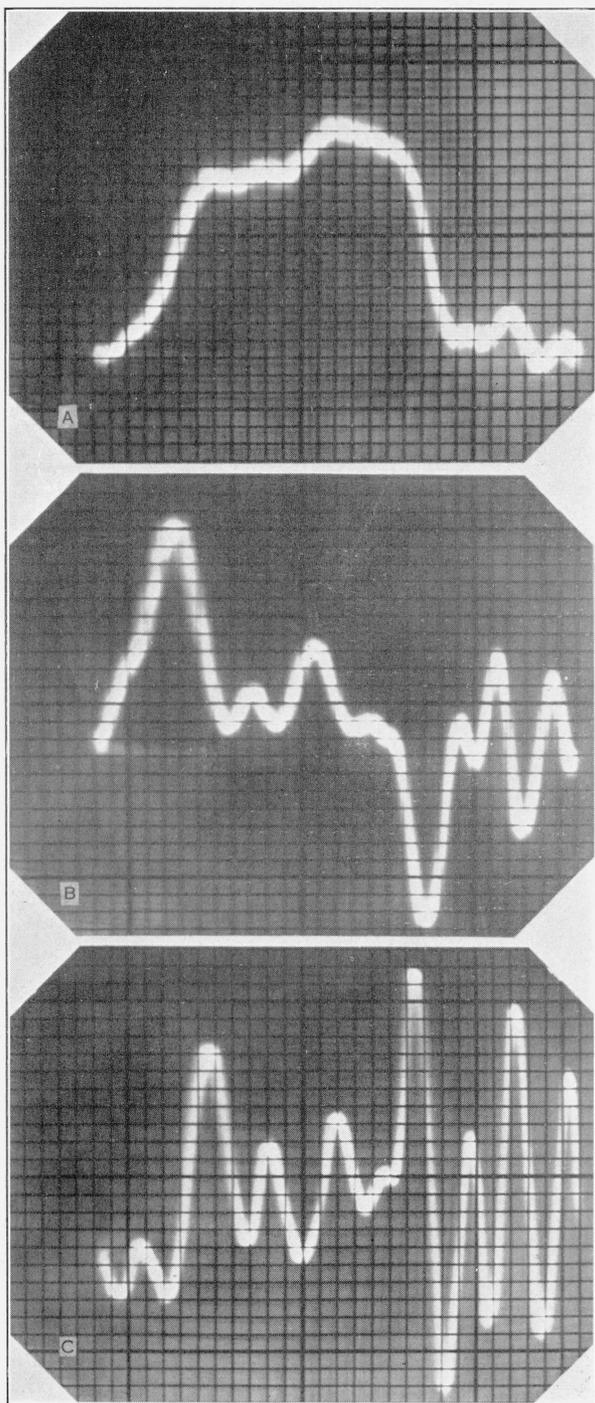


FIGURE 11.—Corresponding displacement, velocity, and acceleration wave forms, A, B, and C, respectively, with correct phase relations maintained, of vibration produced in concrete with a compressed-air vibrator.

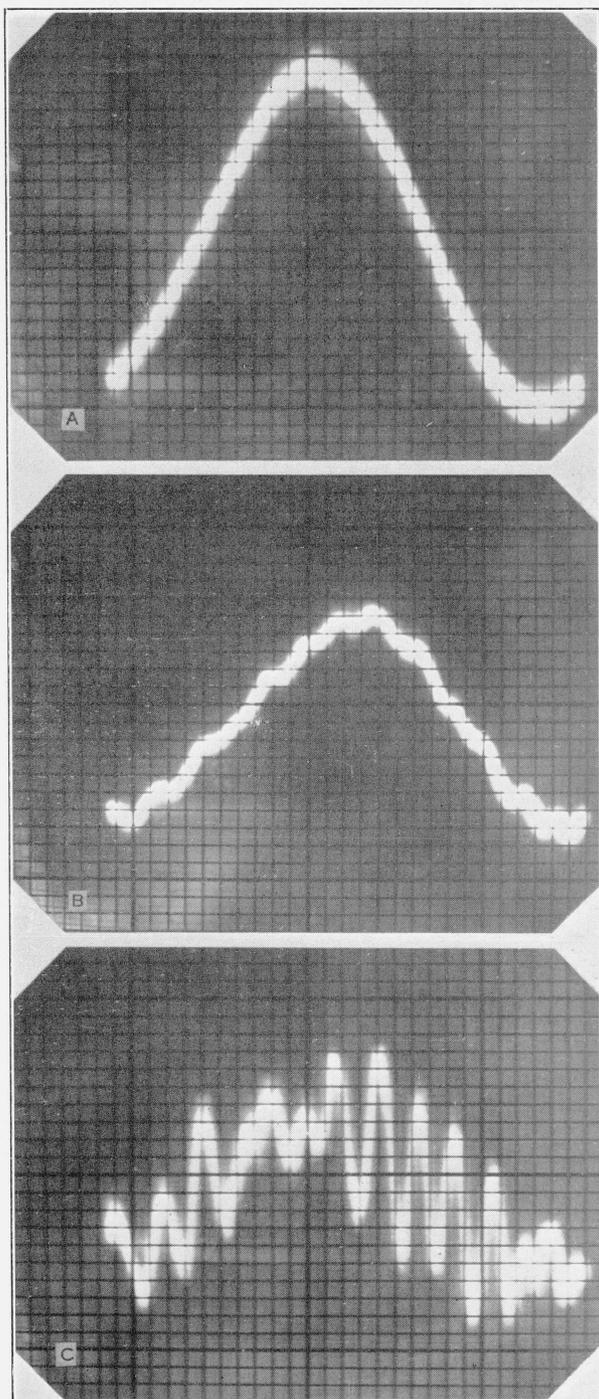


FIGURE 12.—Displacement, velocity, and acceleration wave forms, A, B, and C, respectively, of vibration in concrete produced by a revolving eccentric-weight type vibrator.

in figure 9 and their relation to the amplitude of the freely suspended vibrator, it is probable that even in this region the readings give a fair approximation of the true amplitudes in the concrete.

Figure 10 shows the variation of horizontal amplitude in portland cement concrete contained in a heavy wooden form, 2½ by 4½ ft. by 2 ft. high inside dimensions. In this case, although the vibrator was moved along the middle line of the box, the form itself vibrated very perceptibly, and the concrete was affected much more by the vibrator than was the clay concrete contained in the rigid concrete form (see fig. 9). (The clay concrete in the rigid form appeared, in general, to be compacted only 4 to 7 in. from the vibrator; the cement concrete in the wooden form was nearly all compacted in a short time.)

For the purpose of obtaining typical wave forms of several types of concrete vibrators, and to demonstrate qualitatively the actions of the integrating and differentiating circuits, records were made of the oscillograph traces obtained with the pickup unit immersed in clay

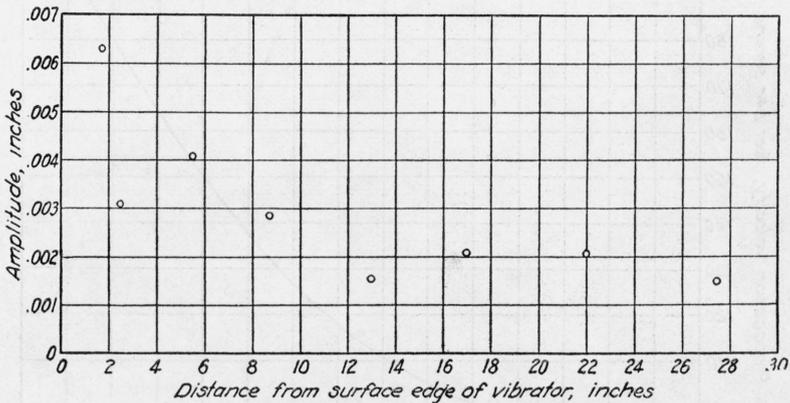


FIGURE 10.—Variation of displacement amplitude with distance in portland cement concrete contained in heavy wooden form.

concrete to which vibration was being applied. Figure 11 shows wave forms obtained with a reciprocating compressed-air vibrator applied to the form, *A*, *B*, and *C* being respectively the displacement, velocity, and acceleration waves. The correct phase relation was maintained between the photographs by synchronizing the oscillograph sweep circuit with the vibrator frequency by means of a second pickup unit. The fundamental wave of *A* has a frequency of approximately 110 c/s. A close inspection of the three waves will reveal that, going from *A* to *B* to *C* in that order, for the same abscissa the ordinate of each succeeding wave is proportional to the slope of the preceding one. Thus the maximum peak of velocity wave *B* corresponds precisely to the region of maximum slope of the displacement wave *A*.

As a further check on the validity of the relation indicated in the sequence of the three curves, the areas under the acceleration and velocity curves were measured with a planimeter and the corresponding velocity and displacement wave forms plotted (with arbitrary adjustment of scale). The plotted wave forms agreed closely with the pictured forms, although lack of linearity of the time base (oscillograph

sweep circuit) prevented precise superposition of the plotted wave on the picture.

The predominant part played in the velocity and acceleration waves by the high-frequency components, which are relatively unimportant in the displacement wave, is to be particularly noted, and appears to be characteristic of the vibration produced by piston-type vibrators.

Figure 12 gives the corresponding wave forms obtained with the large internal vibrator described in an earlier section, operating at a frequency of approximately 3,600 rpm. As before, *A*, *B*, and *C* are respectively the displacement, velocity, and acceleration waves, but in this case the correct phase relation was not maintained between the three traces. It may be noted that, although the displacement wave is superficially sinusoidal, high-frequency components, again,

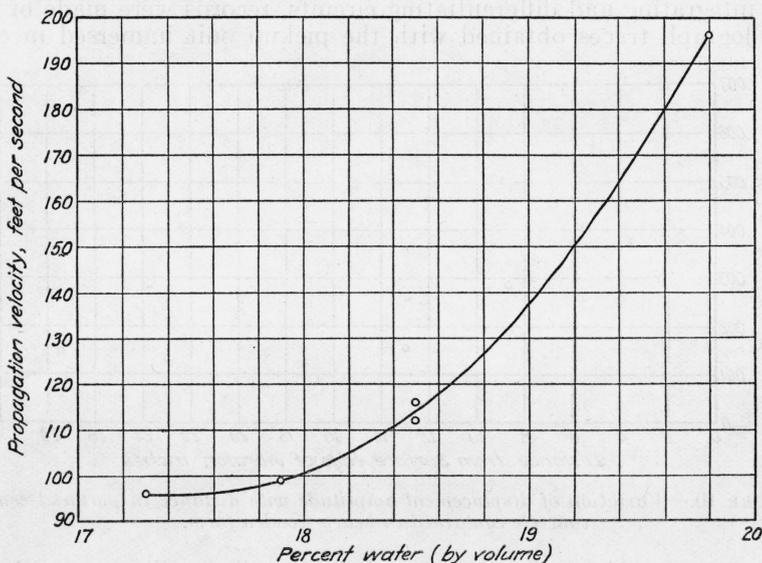


FIGURE 13.—Variation of the velocity of propagation of vibration through concrete, proportioned 1:2.2:3.8 by weight, with change of water content of the concrete.

are very evident in the acceleration wave form. The maximum acceleration associated with the harmonic waves alone, shown in figure 12 (*A*), is of the same order of magnitude as the maximum acceleration associated with the fundamental wave alone. It seems probable, therefore, that attempts to correlate efficacy of vibration with acceleration may yield misleading results when the values for acceleration are obtained by computations from amplitude and frequency data under the assumption that no harmonics are present. Unless the absence of appreciable harmonics is demonstrated (i. e., a truly sinusoidal displacement wave obtained), the possible effects of the harmonics should be considered.

Another type of measurement that can be made with this instrument is illustrated by figure 13, which shows the variation of velocity of propagation of vibration with change in the water content of portland cement concrete proportioned 1:2.2:3.8 by weight. When two pickup

units, one at the source of vibration and one several feet away in the concrete are used, and the oscillograph sweep is synchronized with the closest pickup unit, then as the second pickup unit is moved toward or from the vibrator, a phase shift occurs in the oscillograph pattern which is equal to the phase shift within the concrete over the distance moved. From this the wave length in the concrete can be determined. Knowing the frequency  $f$  and the wave length  $\lambda$ , the velocity can be calculated according to the relation  $v=f\lambda$ .

The technique used in making these particular measurements was not refined and accordingly the values given are first approximations only.<sup>11</sup>

The trend indicated in figure 13 was found to hold for concretes having much greater water content than shown, and measurements of the velocity in water gave results within 15 percent of the accepted value of the velocity of sound in water.

For the velocities indicated in figure 13 the wave lengths varied between 8 and 16 in., the frequency being 9,000 rpm. At a frequency of 3,600 rpm, the wave length for the same concrete would be from 20 to 40 in. Thus, as mentioned in an earlier section, the length of the pickup unit is small compared with the wave length, for the driving frequencies encountered in most of the commercial vibrators now in use.

## VI. CONCLUSIONS

An electromagnetic pickup unit, and amplifier, and special electrical integrating and differentiating circuits used in conjunction with a cathode-ray oscillograph for measuring instantaneous particle velocities, displacements, and accelerations, for measuring the frequencies and for showing the wave forms of mechanical vibrations in fresh concrete are described. It has been shown that the apparatus will give a measure of these characteristics with sufficient accuracy for conditions found in present-day concrete-vibration practice, and, in addition, is adapted to a variety of more precise laboratory tests.

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<sup>11</sup> Wave velocities as low as 260 f/s in peat and 360 f/s in fine sand have been reported. R. Koehler and A. Ramspeck, Veröffentl. Inst. Deut. Forschungsges. Bodenmechanik. 4, 9 (1936).

