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APPLICATION OF VERTICAL-INCIDENCE IONOSPHERE MEASUREMENTS TO OBLIQUE-INCIDENCE RADIO TRANSMISSION

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ABSTRACT

An extension of the transmission curves described in a previous paper is given. The effects of the earth's curvature and magnetic field are considered, as well as absorption or reflection from lower ionosphere layers. A chart is included for rapid calculation of transmission curves.

CONTENTS

I. INTRODUCTION

In a recent number of this Journal[1] the author outlined a graphic method of obtaining, from vertical-incidence ionosphere measurements, the limiting frequencies and virtual heights of reflection of radio waves incident obliquely upon the ionosphere. In that paper was developed a type of "transmission curve" which, when superimposed on a curve of frequency against virtual height, measured at vertical incidence, gave directly the virtual height of reflection for a given frequency at a given distance. A logarithmic sec ϕ_0 transmission curve was also described by which, for a given distance, one curve could be used to determine the heights, to a fair approximation, at any frequency.

It is the purpose of the present paper to evaluate in more detail the effect of the earth's curvature upon the ray theory of ionosphere transmission, to indicate the modifications consequently necessitated in the application of the transmission curves, and to consider briefly how the earth's field may affect limiting frequencies and virtual heights.

II. EFFECT OF THE EARTH'S CURVATURE

The influence of the earth's magnetic field will be neglected in this section. In the next section the effect of the magnetic field on radio

[1] Newbern Smith. *Extension of normal-incidence ionosphere measurements to oblique-incidence radio transmission.* J. Research NBS **19**, 89 (1937) RP1013.

transmission will be considered for the case of the plane earth, and it will be assumed that the results hold approximately for the curved earth, also.

The elementary theory of propagation of electromagnetic waves in an ionized medium whose surfaces of equal ionic density are planes

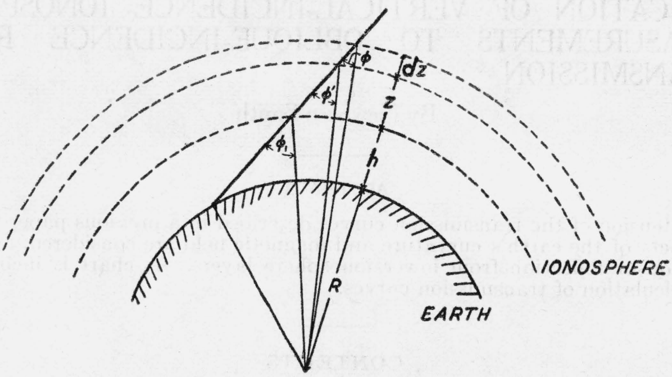

FIGURE 1.—*Variation, with height, of angle a straight line makes with the normal to the earth's surface.*

shows that, by Snell's law, the waves will penetrate the medium until the refractive index μ' is reduced to the value $\sin \phi_1$, where ϕ_1 is the angle of incidence of the waves upon the ionosphere, i. e., the angle the wave normal makes with the vertical on entering the ionosphere. To obtain the corresponding relation for a curved ionosphere it is necessary to consider the variation in the angle ϕ which a straight line makes with the vertical, or normal to the earth's surface, at various altitudes (z).

The geometry of figure 1 leads to the relation

$$\sin \phi' = \sin \phi \left(1 + \frac{dz}{R+h+z}\right). \tag{1}$$

Assuming Snell's law to be valid for a ray traversing an infinitely thin layer of the ionosphere, of thickness dz, we can use this relation to obtain the differential equation:

$$\frac{d(\mu' \sin \phi)}{\mu' \sin \phi} = -\frac{dz}{R+h+z}$$

Integrating this from the lower boundary of the ionosphere, where $\mu'=1$, $\phi=\phi_1$, and $z=0$ (see fig. 2) up to the altitude z, we get

$$\mu' \sin \phi = \frac{\sin \phi_1}{1+\frac{z}{R+h}} \tag{2}$$

as the form of Snell's law appropriate to the curved earth.

This means that the wave will penetrate the curved ionosphere until the refractive index μ' is reduced to the value μ_0', given by

$$\mu_0' = \frac{\sin\phi_1}{1+\dfrac{z_0}{R+h}}, \tag{3}$$

where ϕ_1 is the angle of incidence of the wave on the lower boundary of the ionosphere and z_0 is the maximum height of penetration above this lower boundary.

From the geometry of figure 1, we have also

$$\sin\phi_1 = \sin\phi'\left(1+\frac{z}{R+h}\right). \tag{1a}$$

If we put $\phi'=\phi_0$ and $z=z_v$, the values which these quantities have at the top of the equivalent triangular path in figure 2, this becomes

$$\sin\phi_1 = \sin\phi_0\left(1+\frac{z_v}{R+h}\right), \tag{4}$$

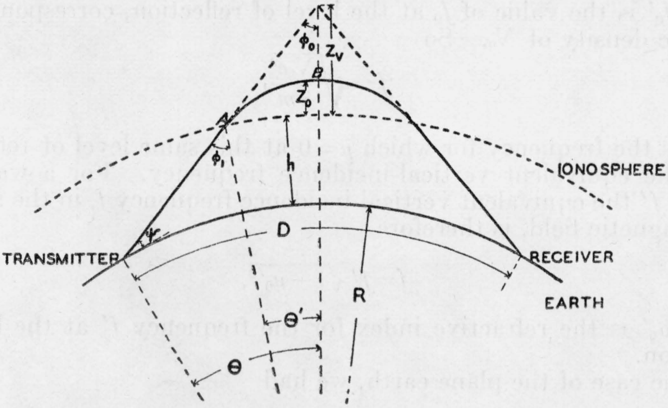

FIGURE 2.—*Transmission through curved ionosphere.*

z_v=height of equivalent triangular path; z_0=true height of reflection, at level where $\mu'=\sin\phi_1\left(1-\frac{z_0}{R+h}\right)$, R=radius of earth; and D=distance of transmission.

and so the refractive index at the level of reflection must be

$$\mu_0' = \sin\phi_0 \frac{\left(1+\dfrac{z_v}{R+h}\right)}{\left(1+\dfrac{z_0}{R+h}\right)} \tag{5}$$

instead of $\mu_0'=\sin\phi_0$, as is the case for the plane earth.

If, as is usually the case, the penetration $z_0 << R+h$, we may write this condition

$$\mu_0' = \sin\phi_0\left(1+\frac{z_v-z_0}{R+h}\right) \tag{5a}$$

The effect of the earth's curvature is thus to cause the radio wave to be reflected from a region of smaller ionization density than would be necessary were the earth flat.

In considering the problem of radio transmission over a distance, it will often be convenient to speak of an "equivalent vertical-incidence frequency" for the given wave frequency and transmission path. This is the frequency which is reflected at vertical incidence at the same height and undergoes approximately the same absorption as does the actual wave at oblique incidence. It will be denoted by f, and the actual wave frequency will be denoted by f'.

Since, for a frequency f',

$$\mu=\sqrt{1-\frac{Ne^2}{\pi m f'^2}}=\sqrt{1-\frac{f_0^2}{f'^2}},$$

where $f_0=\sqrt{\frac{Ne^2}{\pi m}}$, [2,3] it follows that reflection of this frequency will take place at a level where

$$\mu_0'=\sqrt{1-\frac{f_0'^2}{f'^2}},$$

where f_0' is the value of f_0 at the level of reflection, corresponding to an ionic density of N_0. So

$$f=\sqrt{\frac{N_0e^2}{\pi m}},$$

and f is the frequency for which $\mu=0$ at this same level of reflection, i. e., the equivalent vertical-incidence frequency. For a wave frequency f' the equivalent vertical-incidence frequency f, in the absence of a magnetic field, is therefore

$$f=f'\sqrt{1-\mu_0'^2}, \tag{6}$$

where μ_0' is the refractive index for the frequency f' at the level of reflection.

In the case of the plane earth, we had

$$\mu_0'=\sin\phi_1=\sin\phi_0,$$

which gives

$$f=\frac{f'}{\sec\phi_0},$$

the well-known secant law. In the case of the curved earth, however, we must use the values of μ_0' given by eq 3 or eq 5, and

$$\sqrt{1-\mu_0'^2}=\sqrt{1-\frac{\sin^2\phi_1}{\left(1+\frac{z_0}{R+h}\right)^2}}=\sqrt{1-\sin^2\phi_0\left(\frac{1+\frac{z_v}{R+h}}{1+\frac{z_0}{R+h}}\right)^2} \tag{7a}$$

Under practical conditions z_0 is almost always less than 400 km, and values of z_v greater than 600 km contribute nothing to trans-

[2] W. H. Eccles, Proc. Roy. Soc. (London) [A] **87,** 79 (1912).
[3] J. Larmor, Jahrb. drahtlosen Telegraphie **25,** 141 (1925).

mission over appreciable distances, so that we can always consider z_0 and $z_v \ll R+h$. Using this approximation, eq 7a becomes

$$\sqrt{1-\mu_0'^2}=\cos\phi_1\sqrt{1+\frac{2z_0}{R+h}\tan^2\phi_1}=\cos\phi_0\sqrt{1-\frac{2(z_v-z_0)}{R+h}\tan^2\phi_0}\quad(7b)$$

Figure 3 gives values of $\frac{1}{\sqrt{1-\mu_0'^2}}$ plotted against $\tan\phi_1$ for various

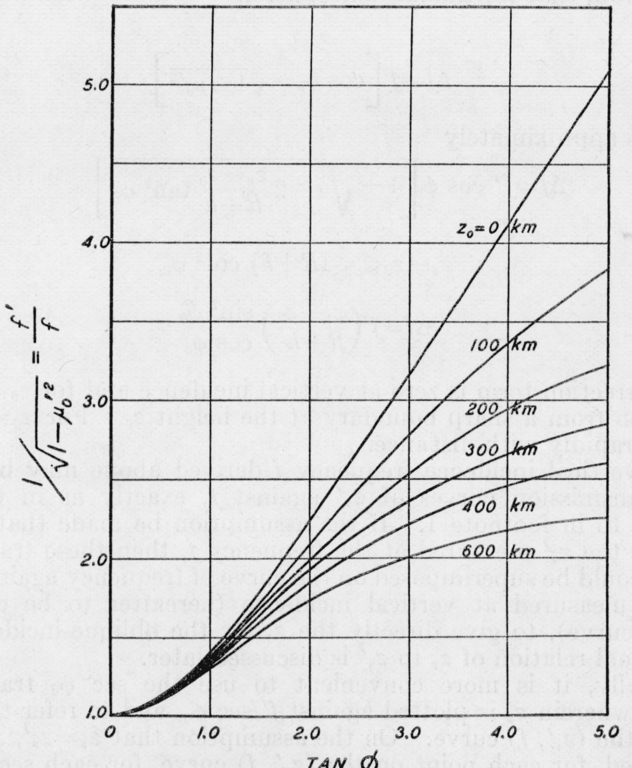

FIGURE 3.—*Variation of* $\frac{1}{\sqrt{1-\mu_0'^2}}$ *with* z_0 *and* $\tan\phi_0$.

values of z_0, assuming $h=100$ km (at the bottom of the E layer). These curves may be used to determine the relation of f to f' for various values of z_0 and ϕ_1. By substituting ϕ_0 for ϕ_1 and z_0-z_v for z_0, similar curves may also be plotted to determine the relation of f to f' for various values of z_0, z_v, and ϕ_0.

If, further, $z_0 \ll (R+h)\cot^2\phi_1$ or $(z_v-z_0) \ll (R+h)\cot^2\phi_0$ we may write eq 7b approximately,

$$\sqrt{1-\mu_0'^2}=\cos\phi_1\left[1+\frac{z_0}{R+h}\tan^2\phi_1\right]=\cos\phi_0\left[1-\frac{z_v-z_0}{R+h}\tan^2\phi_0\right]\quad(7c)$$

a form which it is convenient to use in some discussions. This approximation leads to results good to 1 percent or better for E-layer transmission, where z_0 is less than 50 km, and for single-reflection F-layer transmission over distances less than 1,500 km, or multireflection F-layer transmission where each reflection covers less than 1,500 km. For transmission over distances greater than 1,500 km for each reflection it is necessary to use the more exact expression.

The correction term, or frequency Δf which it is necessary to subtract from $f'/\sec \phi_0$ in order to obtain f, is the amount by which the equivalent vertical-incidence frequency for the curved ionosphere differs from that for the flat ionosphere.

This is

$$\Delta f = f'\left[\cos \phi_0 - \sqrt{1-\mu_0'^2}\right],$$

which is approximately

$$\Delta f = f' \cos \phi_0\left[1-\sqrt{1-2\frac{z_v-z_0}{R+h}\tan^2 \phi_0}\right]$$

and, if

$$z_v - z_0 << (R+h) \cot^2 \phi_0,$$

$$\Delta f \doteq f'\left(\frac{z_v-z_0}{R+h}\right)\frac{\sin^2 \phi_0}{\cos \phi_0}$$

This correction term is zero at vertical incidence and for $z_0 = z_v'$, i. e., reflection from a sharp boundary at the height z_0. For $z_0 \neq z_v'$ it increases rapidly with distance.

The vertical incidence frequency f derived above may be used to plot transmission curves of z_v' against f, exactly as in the paper referred to in footnote 1. If the assumption be made that z_v is the same as the z_v' measured at the frequency f, then these transmission curves could be superimposed on the curve of frequency against virtual height, measured at vertical incidence (hereafter to be called the (z_v', f) curve), to give directly the z_v for the oblique-incidence case. The actual relation of z_v to z_v' is discussed later.

Actually, it is more convenient to use the "sec ϕ_0" transmission curves, wherein z_v is plotted against $f'/\sec \phi_0$, and to refer the correction to the (z_v', f) curve. On the assumption that $z_v = z_v'$, Δf can be calculated, for each point on the (z_v', f) curve, for each sec ϕ_0 transmission curve. This transmission curve can then be superimposed on the (z_v', f) curve displaced toward the higher frequencies by the amount Δf, and the oblique-incidence values of z_v read off.

The calculation of Δf for each distance from z_v' and z_0, without any assumption as to z_v, and for a ϕ_0 calculated for an equivalent triangular path of height z_v', is discussed below. This value of Δf can be used in the same manner as was described in the preceding paragraph, and the sec ϕ_0 curves applied, ϕ_0 being calculated for an equivalent triangular path of height z_v' instead of z_v.

If the $(z_v' f)$ curve and the sec ϕ_0 curves are plotted logarithmically, as in the paper referred to,[4] we may calculate a factor $1+\Delta f/f$, as follows:

$$1+\frac{\Delta f}{f} = \frac{\cos \phi_0}{\sqrt{1-\mu_0'^2}}$$

[4] *Extension of normal-incidence ionosphere measurements to oblique-incidence radio transmission.* J. Research NBS **19**, 89 (1937) RP1013.

The frequencies on the (z_v', f) curve can then be easily multiplied by this factor by adding the factor logarithmically to the curve, and the log sec ϕ_0 curves can be applied to the resulting corrected (z_v', f) curve. It should be noted that in this expression the ϕ_0 refers only to an equivalent triangular path of height z_v', and not z_v. The z_v enters only into the expression for μ_0'.

A typical (z_v', f) curve corrected in this manner for a given distance (2,000 km) is shown in figure 4. The virtual heights are lower on the corrected curve and the curve extends out to frequencies higher than the critical frequency for the ordinary ray. The correction to the curve is different for different distances, increasing with the distance. This means that the virtual heights of reflection are lower, for greater distances, than if the curvature of the earth were not considered. The limiting frequency of transmission, or maximum usable frequency, over considerable distances is correspondingly increased, the difference becoming as much as 20 percent at the greater distances, depending of course on the form of the (z_v', f) curve.

When the (z_v', f) curve has been corrected in the above manner it may be plotted logarithmically, as described in the previous paper mentioned above, and the logarithmic sec ϕ_0 curves may be applied to give directly the maximum usable frequencies and virtual heights over transmission paths of given lengths.

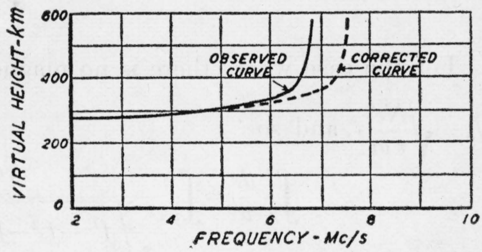

FIGURE 4.—*(z_v', f) curve corrected for a given distance (2,000 km) for the effect of the earth's curvature.*

Since the first draft of this paper was written there has come to the author's attention an excellent unpublished paper "Skip-Distance Analysis," by T. L. Eckersley and G. Millington, in the form of a contribution to the November 1937 London meeting of the Special Radio Wave Propagation Committee held in preparation for the Cairo Radio Conference. In this they undertake an analysis of radio transmission over a curved earth and obtain curves for determining the retardation of sky wave over ground wave at a distance, in terms of vertical-incidence measurements. They limit the analysis, however, to the case where z_0 is very small compared with h. The analysis can thus apply only to E-layer transmission, since for F-layer transmission, z_0 must be measured from the lower boundary of the ionosphere in order to include the effect of retardation in the E layer.

Their work, however, suggests a method of approximate analysis for larger values of z_0 and a means for easily estimating the error involved in the assumption made above that the z_v for a given frequency f' was equal to the vertical-incidence z_v' for the equivalent vertical-incidence frequency f.

The development given by Eckersley and Millington begins with the following relation, for the ray path shown in figure 2:

$$\theta' = \int_A^B d\theta = \int_A^B \frac{ds \sin \phi}{R+h+z},$$

where ds is an element of the ray path making an angle with the vertical. Since, from eq 2

$$\sin\phi=\frac{1}{\mu'}\frac{\sin\phi_1}{1+\frac{z}{R+h}},$$

this becomes

$$\theta'=\frac{\sin\phi_1}{R+h}\int_A^B\frac{ds}{\mu'\left(1+\frac{z}{R+h}\right)^2}. \tag{8}$$

They also used the relation

$$\int_A^B\frac{ds}{\mu'}=\int_0^{z_0}\frac{dz}{\mu'\cos\phi}=\int_0^{z_0}\frac{dz}{\sqrt{\mu'^2-\frac{\sin^2\phi_1}{\left(1+\frac{z}{R+h}\right)^2}}}. \tag{9}$$

For the case where there is no magnetic field $\mu'=\sqrt{1-\frac{f_0^2}{f'^2}}$, where $f_0=\sqrt{\frac{Ne^2}{\pi m}}$, and so

$$\int_A^B\frac{ds}{\mu'}=\int_0^{z_0}\frac{f'dz}{\sqrt{f'^2-f_0^2-f'^2\frac{\sin^2\phi_1}{\left(1+\frac{z}{R+h}\right)^2}}}. \tag{10}$$

Putting into this expression the wave frequency f' in terms of the equivalent vertical-incidence frequency given above in eq 6.

$$\int_A^B\frac{ds}{\mu'}=\frac{1}{\sqrt{1-\mu_0'^2}}\int_0^{z_0}\frac{fdz}{\sqrt{\frac{f^2}{1-\mu_0'^2}\left(1-\frac{\sin^2\phi_1}{\left(1+\frac{z}{R+h}\right)^2}\right)-f_0^2}}. \tag{11}$$

At this point Eckersley and Millington assumed that they were dealing with a thin layer, and that in consequence (1) θ' was only slightly less than $\frac{\sin\phi_1}{R+h}\int_A^B\frac{ds}{\mu'}$, from eq 8, (2) $\int_A^B\frac{ds}{\mu'}$ was only slightly greater than $\frac{1}{\sqrt{1-\mu_0'^2}}\int_0^{z_0}\frac{fdz}{\sqrt{f^2-f_0^2}}$, from eq 11. This simplification is justified for E-layer transmission, but is unfortunately not justified for F_2-layer transmission, since the lower boundary of the ionosphere must be taken at the beginning of the E-layer in each case. We shall not, therefore, confine our discussion to this limited case. It must also be noted that the simplification introduced by assuming $z_0 << (R+h)\cot^2\phi_1$ also is not valid except at comparatively short distances, where $\phi_1=45°$ or less. For practical purposes, it is, however, justifiable to assume $z_0 << R+h$. Using this relation then, we obtain

$$\theta'=\frac{\sin\phi_1}{R+h}\int_A^B \frac{ds}{\mu'}\left(1-\frac{2z}{R+h}\right). \tag{12}$$

Eckersley and Millington combined the approximate forms of eq 8 and 11 and introduced the value of $\sqrt{1-\mu_0'^2}=\cos\phi_1$, to obtain $\theta'=\frac{\tan\phi_1}{R+h}\int_0^{z_0}\frac{f\,dz}{\sqrt{f^2-f_0^2}}$. We shall, however, combine eq 11 and 12 to obtain the more exact relation

$$\theta'=\frac{\sin\phi_1}{(R+h)\sqrt{1-\mu_0'^2}}\int_0^{z_0}\frac{f\,dz}{\sqrt{f^2(1-A)-f_0^2}}\left(1-\frac{2z}{R+h}\right), \tag{12a}$$

where

$$1-A=\frac{1-\dfrac{\sin^2\phi_1}{\left(1+\dfrac{z}{R+h}\right)^2}}{1-\mu_0'^2}=\frac{1+\dfrac{2z}{R+h}\tan^2\phi_1}{1+\dfrac{2z_0}{R+h}\tan^2\phi_1} \tag{12b}$$

If we let

$$1+B=\frac{1}{\sqrt{1-A\left(\dfrac{f^2}{f^2-f_0^2}\right)}}, \tag{12c}$$

$$\theta'=\frac{\sin\phi_1}{(R+h)\sqrt{1-\mu_0'^2}}\int_0^{z_0}\frac{dz}{\sqrt{1-\dfrac{f_0^2}{f^2}}}(1+B)\left(1-\frac{2z}{R+h}\right) \tag{13}$$

For values of z_0 considerably less than $(R+h)\cot^2\phi_1$ B is usually a small number, and for $z\to z_0$ (near the level of reflection) it is also usually small. It may attain, however, relatively large values.

In general, the evaluation of B involves a knowledge of the distribution of ionic density, and a precise evaluation of the quantity θ' involves graphic integration for each case considered. We shall for the present consider two principal cases, $z_0\to 0$ and $z_0=z_v'$, the virtual height at vertical incidence for the frequency $f=f'\sqrt{1-\mu_0'^2}$, and discuss some cases of linear distributions of ionic density, where $z/z_0=f_0^2/f^2$.

(*a*) $z_0\ll(R+h)\cot^2\phi_1$. This case applies to *E*-layer transmission at any distance or to *F*-layer transmission at short distances (ϕ_1 small).

Combining eq 6 and 7 we get

$$f=f'\cos\phi_1\left[1+\frac{z_0}{R+h}\tan^2\phi_1\right], \tag{14b}$$

and from eq 12b

$$1-A=1+2\left(\frac{z-z_0}{R+h}\right)\tan^2\phi_1.$$

From eq 12c

$$1+B=1+\left(\frac{z_0}{R+h}\right)\frac{1-\dfrac{z}{z_0}}{1-\dfrac{f_0^2}{f^2}}\tan^2\phi_1,$$

if we assume that $(1-z/z_0)/(1-f_0^2/f^2)$ is nowhere so large as to make B comparable with unity. This is equivalent to assuming that f_0^2/f^2 does not approach 1 much more rapidly than does z/z_0, an assumption that is valid except quite close to a critical frequency. Transmissions involving equivalent vertical-incidence frequencies close to the critical frequency are of interest, however, only when the distance of transmission is short, and in this case ϕ_1 is small, so that B is a small number, anyhow.

Thus

$$\theta'=\frac{\tan\phi_1}{R+h}\left[1-\frac{z_0}{R+h}\tan^2\phi_1\right]\int_0^{z_0}\frac{dz}{\sqrt{1-\dfrac{f_0^2}{f^2}}}$$

$$\left\{1+\frac{z_0}{R+h}\tan^2\phi_1\frac{\left(1-\dfrac{z}{z_0}\right)}{\left(1-\dfrac{f_0^2}{f^2}\right)}-\frac{2z}{R+h}\right\}.$$

Now $\displaystyle\int_0^{z_0}\frac{dz}{\sqrt{1-\dfrac{f_0^2}{f^2}}}=z_v'$, the virtual height measured at vertical incidence for the equivalent vertical-incidence frequency f. Since the terms involving $z_0/R+h$ are small, we may write

$$\theta'=\frac{z_v'\tan\phi_1}{R+h}\left[1-\frac{z_0}{R+h}\tan^2\phi_1(1-C)-C'\right],\tag{14c}$$

where

$$C=\frac{1}{z_v'}\int_0^{z_0}\frac{dz}{\sqrt{1-\dfrac{f_0^2}{f^2}}}\left(\frac{1-\dfrac{z}{z_0}}{1-\dfrac{f_0^2}{f^2}}\right),\text{ and}$$

$$C'=\frac{2}{z_v'(R+h)}\int_0^{z_0}\frac{zdz}{\sqrt{1-\dfrac{f_0^2}{f^2}}}$$

For a linear distribution of ionic density

$$1-\frac{z}{z_0}=1-\frac{f_0^2}{f^2},\ z_v'=2z_0,\text{ and } C=1,\text{ and}$$

$$\theta'=\frac{\tan\phi_1}{R+h}\int_0^{z_0}\frac{dz}{\sqrt{1-\dfrac{z}{z_0}}}\left[1-\frac{2z}{R+h}\right],$$

$$=\frac{z_v'\tan\phi_1}{R+h}\left[1-\frac{2}{3}\left(\frac{z_v'}{R+h}\right)\right].\tag{15}$$

For z_0 vanishingly small $C' \rightarrow 0$ and eq 14 reduces simply to

$$\theta' = \frac{z_v' \tan \phi_1}{R+h} \tag{15a}$$

(*b*) $z_0 \rightarrow z_v'$. This case applies to reflection from a fairly sharp boundary at the level z_0, the refractive index being nearly unity up to nearly this level. An example of this would be reflection from the sporadic E region. The angular distance of the part of the ray path where μ departs appreciably from unity is small and so the ionosphere can be considered as essentially flat. This is the condition assumed by the author in the paper already referred to (RP1013). Here $z_v = z_v'$ and

$$\tan \phi_0 = \frac{\sin \theta'}{\frac{z_v'}{R+h} + 1 - \cos \theta'}. \tag{16}$$

In this case

$$\sqrt{1-\mu_0'^2} = \cos \phi_0 \left(1 - \frac{z_v' - z_0}{R+h} \tan^2 \phi_0\right) \tag{16a}$$

For $z_v' - z_0$ vanishingly small compared with $(R+h) \cot^2 \phi_0$, this reduces to the case for the plane ionosphere, where

$$\sqrt{1-\mu_0'^2} = \cos \phi_0.$$

(*c*) $z_0 = \frac{1}{2} z_v'$, i. e., a linear distribution of ionic density. Here $1 - f_0^2/f^2 = 1 - z/z_0$ and eq 12a becomes

$$\theta' = \frac{\sin \phi_1}{(R+h)\sqrt{1-\mu_0'^2}} \int_0^{z_0} \frac{dz\left(1 - \frac{2z}{R+h}\right)}{\sqrt{1 - A - \frac{z}{z_0}}}$$

Putting in the value of $1-A$ and integrating

$$\theta' = z_v' \frac{\sin \phi_1 \sqrt{1 + \frac{2z_0}{R+h} \tan^2 \phi_1}}{(R+h)\sqrt{1-\mu_0'^2}} \left(1 - \frac{2}{3} \frac{z_v'}{R+h}\right). \tag{17}$$

Now since we may consider $z_0 \ll R+h$ we can write, as in eq 7b

$$\sqrt{1-\mu_0'^2} = \cos \phi_1 \sqrt{1 + \frac{2z_0}{R+h} \tan^2 \phi_1},$$

and thus eq 17 becomes

$$\theta' = \frac{z_v' \tan \phi_1}{R+h} \left[1 - \frac{2}{3} \frac{z_v'}{R+h}\right], \tag{15}$$

just as in the case where $z_0 \ll (R+h) \cot^2 \phi_1$. We must, however, write for the equivalent vertical-incidence frequency in this case

$$f = f' \cos \phi_1 \sqrt{1 + \frac{2z_0}{R+h} \tan^2 \phi_1} \tag{17a}$$

instead of the value

$$f=f' \cos \phi_1 \left[1+\frac{z_0}{R+h} \tan^2 \phi_1\right], \tag{14b}$$

which we could use when $z_0 \ll (R+h) \cot^2 \phi_1$.

The three examples just treated give an idea of what happens on transmission through the ionosphere, and of the angular distance θ' the wave travels in the ionosphere as a function of angle of incidence ϕ_1 (or vertex angle of equivalent triangular path ϕ_0), of true height of reflection z_0 and of virtual height measured at vertical incidence z_v'. We must now consider the part of the path from the earth to the lower boundary of the ionosphere. If we consider the incident ray to traverse an angular distance $(\theta-\theta')$ in going from the earth to the lower boundary of the ionosphere, the geometry of figure 2 tells us that

$$\tan \phi_1=\frac{\sin (\theta-\theta')}{\frac{h}{R}+1-\cos (\theta-\theta')} \tag{18}$$

For a given ϕ_1, then, we may solve this equation for $(\theta-\theta')$, and, by adding this angle to the angle θ' already computed, we obtain the entire angular distance θ traversed by the wave from the ground to the point of reflection in the ionosphere. For $h=100$ km, as we are assuming, it is sufficiently accurate to replace $\sin (\theta-\theta')$ by $(\theta-\theta')$ and $1-\cos (\theta-\theta')$ by $\frac{1}{2}(\theta-\theta')^2$ in this equation. We can thus solve for $(\theta-\theta')$, obtaining

$$\theta-\theta'=\cot \phi_1-\sqrt{\cot^2 \phi_1-\frac{2h}{R}} \tag{18a}$$

We can now write the expression for the total distance of transmission D in terms of ϕ_1 (or ϕ_0) z_0 and z_v':

$$D=2R\left(\cot \phi_1-\sqrt{\cot^2 \phi_1-\frac{2h}{R}}\right)+\theta', \tag{19}$$

where θ' is computed as above, from eq 12a.

The time T required for the sky wave to travel from the transmitter to the receiver is, if c=velocity of the wave in vacuum,

$$T=\frac{2}{c}\left(R\frac{\sin (\theta-\theta')}{\sin \phi_1}+\int_A^B \frac{ds}{\mu}\right).$$

If we put in the value of $\int_A^B ds/\mu$ obtained on the basis of the above analysis, and replace $\sin(\theta-\theta')$ by its value in terms of ϕ_1, we may express T in terms of the quantities ϕ_1 (or ϕ_0), z_0 and z_v', as was done with D.

The relation between the height z_v of the equivalent triangular path and the virtual height z_v' measured at vertical incidence for the equivalent vertical-incidence frequency $f=f'\sqrt{1-\mu_0'^2}$ can now be written. From the geometry of figure 2,

$$\tan \phi_0=\tan (\phi_1-\theta')=\frac{\sin \theta'}{\frac{z_v}{R+h}+1-\cos \theta'},$$

so that

$$z_v=(R+h)[\sin \theta' \cot (\phi_1-\theta')-1+\cos \theta'] \tag{20}$$

The relation between z_v, z_v', z_0, ϕ_1, ϕ_0, θ', D, f', and f may be summarized for the cases discussed here.

(*a*) $z_0<<\frac{1}{2}(R+h) \cot^2 \phi_1$

$$f=f' \cos \phi_1\left[1+\frac{z_0}{R+h} \tan^2 \phi_1\right],$$

$$\theta'=\frac{\tan \phi_1}{R+h}\left[1-\frac{z_0}{R+h}\tan^2 \phi_1\right]\int_0^{z_0}\frac{dz}{\sqrt{1-\frac{f_0^2}{f^2}}}$$

$$\left\{1+\frac{z_0}{R+h}\tan^2 \phi_1\left(\frac{1-\frac{z}{z_0}}{1-\frac{f_0^2}{f^2}}\right)-\frac{2z}{R+h}\right\},$$

$$T=\frac{2}{c}\left[\frac{R(\theta-\theta')}{\sin \phi_1}+\frac{1}{\cos \phi_1}\int_0^{z_0}\frac{dz}{\sqrt{1-\frac{f_0^2}{f^2}}}\left\{1-\frac{z_0}{R+h}\tan^2 \phi_1\left(1-\frac{1-\frac{z}{z_0}}{1-\frac{f_0^2}{f^2}}\right)\right\}\right],$$

$$\theta-\theta'=\cot \phi_1-\sqrt{\cot^2 \phi_1-\frac{2h}{R}},$$

$$D=2R\theta,$$

$$z_v=(R+h)[\sin \theta' \cot (\phi_1-\theta')-1+\cos \theta'],$$

$$z_v'=\int_0^{z_0}\frac{dz}{\sqrt{1-\frac{f_0^2}{f^2}}},$$

(*a'*) $z_0=0$,

$$f=f' \cos \phi_1,$$

$$\theta'=\frac{z_v' \tan \phi_1}{R+h},$$

$$T=\frac{2}{c}\left[\frac{R(\theta-\theta')}{\sin \phi_1}+\frac{z_v'}{\cos \phi_1}\right],$$

$$\theta-\theta'=\cot \phi_1-\sqrt{\cot^2 \phi_1-\frac{2h}{R}},$$

$D=2R\theta$, and

$$z_v=(R+h)[\sin \theta' \cot (\phi_1-\theta')-1+\cos \theta'],$$

(*b*) linear gradient of ionic density $\left(z_0=\frac{1}{2}z_v'\right)$

$$f=f' \cos \phi_1 \sqrt{1+\frac{z_v'}{R+h} \tan^2 \phi_1},$$

$$\theta'=\frac{z_v' \tan \phi_1}{R+h}\left[1-\frac{2}{3}\frac{z_v'}{R+h}\right],$$

$$T=\frac{2}{c}\left[\frac{R(\theta-\theta')}{\sin \phi_1}+\frac{z_v'}{R+h}\right],$$

$$\theta-\theta'=\cot \phi_1-\sqrt{\cot^2 \phi_1-\frac{2h}{R}},$$

$$D=2R\theta,$$

$$z_v=(R+h)[\sin \theta' \cot (\phi_1-\theta')-1+\cos \theta'],$$

(*c*) $z_0=z_v'$,

$$f=f' \cos \phi_0,$$

$$\tan \phi_0=\frac{\sin \theta'}{\frac{z_v'}{R+h}+1-\cos \theta'}=\frac{\sin \theta}{\frac{z_v'+h}{R}+1-\cos \theta},$$

$$T=\frac{2}{c}\frac{(R+h) \sin \theta}{\sin \phi_0},$$

$$D=2R\theta, \text{ and}$$

$$z_v=z_v'$$

Case (a') corresponds to the special case treated by Eckersley and Millington, except that, for simplicity of analytical computation, they took $R+h=R$ in the expression for θ', which introduces an error of about $0.016h$ percent (h expressed in km)—not a serious error for $h=100$ km, as we have assumed. This error becomes appreciable, however, if it is attempted to extend this treatment, as they have done, to a height of 400 km or so.

Martyn's equivalence theorem,[5] developed for a plane ionosphere, tells us that

$$z_v=z_v',$$

$$T=\frac{2}{c}\frac{z_v'}{\cos \phi_0}=\frac{2}{c}\frac{z_v'}{\cos \phi_1},$$

$$D'=2z_v' \tan \phi_0=2z_v' \tan \phi_1, \text{ and}$$

$$f=f' \cos \phi_0=f' \cos \phi_1,$$

where

ϕ_1=angle of incidence of waves upon the ionosphere,
ϕ_0=half the vertex angle of the equivalent triangular path,
z_v=height of equivalent triangular path,
T'=time the wave spends in the ionosphere, and
D'=horizontal distance the wave travels in the ionosphere.

We see that the relation $z_v=z_v'$ is valid in case (*c*) but not in any other case. The relation $T=\frac{2}{c}\frac{z_v'}{\cos \phi_1}$ is valid in cases (a') and (*b*), but is not valid in general. The relation $D'=2z_v' \tan \phi_1$ holds approximately only in case (a'). And finally, $f=f' \cos \phi_0$ in case (*c*), $f=f'$

[5] D. F. Martyn, Proc. Phys. Soc. (London) 47, 332 (1935).

$\cos \phi_1$ in case (a'), and f is a complicated function in every other case. It is therefore concluded that the equivalence theorem, in the form given, cannot be applied to the curved-earth problem.

Referring again to the summaries of cases (a'), (b), and (c), we may for a given value of z_v' take a set of values of z_0, for each of which can be calculated the variation of D with ϕ_1, in each of the three cases. We may also, for these values of z_v' and z_0, calculate the variation of f'/f with ϕ_1. By eliminating ϕ_1 graphically, we can determine the variation of f'/f with D for a given z_v' and z_0, or, what is the same thing, the variation of f'/f with z_v' for a given D and z_0. We can plot a family of transmission curves, with f as abscissas and z_v' as ordinates, for each value of D and several values of z_0, corresponding to each of these cases.

Such curves may be superimposed on the (z_v', f) curve obtained at vertical incidence, and the (z_0, f) curve deduced therefrom, to give directly the wave frequency f' corresponding to reflection at a given level z_0, which is characterized by a given z_v'. The value of z_0 appropriate to reflection at the given height determines which curve of the family for a given D is to be used, and the point of reflection is determined as the intersection of this transmission curve with the (z_v', f) curve.

The value of z_v depends only on D and ϕ_1, and so lines of equal z_v may be plotted on the curve sheet for each D, so that z_v, as well as f', may be read off directly.

Because of the approximations made in case (b) and the assumption of a linear variation of ionic density with height, this case is of only special significance. It will be assumed, until further investigation determines more precisely the variation of conditions with z_0, that the curves vary smoothly between those calculated for $z_0=0$ and those calculated for $z_0=z_v'$.

A family of curves for each distance is rather cumbersome for rapid use. It is, as was said above, more convenient to use the log sec ϕ_0 transmission curves, and apply a correction to the (z_v', f) curve by multiplying each vertical-incidence frequency by the factor $1+\Delta f/f$, where

$$1+\frac{\Delta f}{f}=\frac{\cos \phi_0}{\sqrt{1-\mu_0'^2}}.$$

This factor is obtained, for a given D, z_v' and z_0, by determining corresponding values of D and $\sqrt{1-\mu_0'^2}$ for arbitrary values of ϕ_0 or ϕ_1. It is unity for $z_v'=z_0$ and is quite easily obtained for $z_0=0$. For intermediate values of z_0 it will be assumed that the factor $1+\Delta f/f$ varies in a manner similar to that determined from the relation eq 7b with $z_v=z_v'$, i. e.,

$$\frac{\cos \phi_0}{\sqrt{1-\mu_0'^2}}=\sqrt{1-\frac{2(z_v'-z_0)}{R+h}\tan^2 \phi_0},$$

but drawn through the values for $z_0=0$ and $z_0=z_v'$ determined in the more precise analysis, rather than those indicated on the assumption that $z_v=z_v'$. Cos ϕ_0 is here calculated for an equivalent triangular path of height z_v'.

Figure 5 gives the approximate factors by which f must be multiplied to give $f'/\sec\phi_0$ for values of z_v' from 200 to 500 km and for distances up to 4,000 km.

III. EFFECT OF THE EARTH'S MAGNETIC FIELD

The presence of the earth's magnetic field introduces some complications in the use of these transmission curves. These complications are often of minor importance compared with some of the unknown

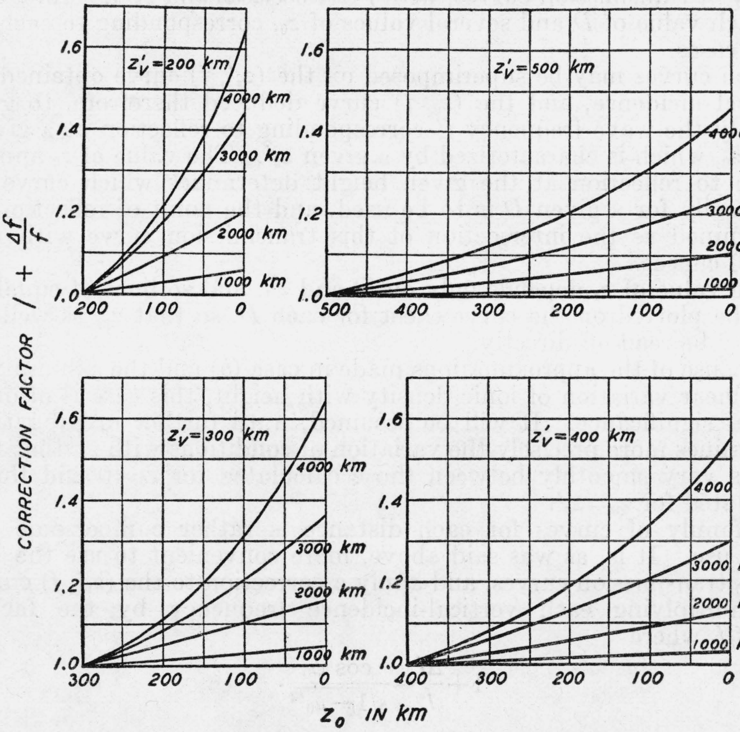

FIGURE 5.—*Approximate factors by which the equivalent vertical-incidence frequency f must be multiplied to give the frequency $f'/\sec\phi_0$ which is to be used in conjunction with the log sec ϕ_0 curves.*

factors (e. g., the geographic uniformity of the ionosphere over the transmission path), especially over long distances, but the effect of the earth's field must at times be taken into account. The anisotropy of the ionosphere due to this field causes the effect of the field on radio transmission to vary with the length, direction, and geographic location of the transmission path.

One effect of the field is to cause the received signal to be split in general into two main components, the one with the lower maximum usable frequency known as the "o" component and the other as the "x" component. The refractive index for a frequency f', in the presence of a magnetic field H, whose components along and transverse to the direction of phase propagation are, respectively, H_L and H_T is given[6] by

$$\mu'^2 = 1 - \frac{f_0^2}{f'^2 - \frac{f'^2 f_T^2}{2(f'^2 - f_0^2)} \pm \sqrt{\left[\frac{f'^2 f_T^2}{2(f'^2 - f_0^2)}\right]^2 + f'^2 f_L^2}},$$

where
$$f_0 = \sqrt{\frac{Ne^2}{\pi m}},$$

$$f_T = \frac{e}{2\pi mc} H_T,$$

$$f_L = \frac{e}{2\pi mc} H_L, \text{ and}$$

N = ionization density.
Upper sign refers to o-component; lower to x-component.

The frequency of the wave whose x-component is returned from a given ionization density, at a given height, is different from the frequency of the wave whose o-component is returned from the same level. This frequency separation is in general a function both of frequency and distance and may often become negligible at great distances.

For practical calculation, it may be assumed that only the field and direction of wave propagation in the region near the level of reflection will appreciably affect the propagation of the wave. This assumption is probably better for the o- than for the x-component, and is, it must be emphasized, only a good approximation.

With this limitation, therefore, the value of μ'^2 may be calculated for a given transmission frequency and transmission path. If we put $\mu' = \sin\phi_0$ and deduce z_v from $\sin\phi_0$ as was done in the previous paper, we may plot transmission curves, of virtual height against equivalent vertical-incidence frequency, for the o- and x-components. When these curves are applied to the corrected (z_v', f) curves, they may be expected to give reasonably good results. An example of this type of transmission curve is shown in figure 6. The one curve gives transmission conditions for the x-component, and the other for the o-component. The frequency used is well over the gyrofrequency $f_H = eH/2\pi mc$ so that the x-component is returned from a lower level than is the o-component and has a higher limiting frequency.

It is not now justifiable to plot the transmission curves logarithmically, since the form of the curves will vary with the transmission frequency f'. For practical purposes, however, a logarithmic curve may be used within a limited range of wave frequencies about the frequency f' for which the curve is plotted; a practical limit might be, say, within ± 15 percent of this frequency.

The logarithmic sec ϕ_0 transmission curves may be used in estimating the maximum usable frequency for each component over a given path by adding or subtracting the separation between the limiting frequencies for the two components, evaluated at that frequency and distance. This separation is in general a function only of the trans-

[6] E. V. Appleton. J. Inst. Elec. Engrs. (London), 71, 642 (1932).

mission frequency and the quantity sec ϕ_0, and may be estimated within the limits of experimental error in most cases.

Figure 7 gives the frequency to be added or subtracted from the maximum usable frequency given by the logarithmic sec ϕ_0 transmission curves for the cases $H_L=0$, $H_T\doteq 0$, and $H_T=H_L$. The $H_T=0$ case is uninteresting save for single-hop transmission over the magnetic equator, close to the magnetic meridian. For transmission in the continental United States H_L is much less than H_T and, indeed, is negligible over east-west paths, so that such transmission we may consider as essentially transverse transmission. In this case,

$$\mu'^2=1-\frac{f_0^2}{f'^2-\delta\frac{f'^2f_H^2}{f'^2-f_0^2}},$$

where $\delta=0$ for the *o*-component and 1 for the *x*-component, and the logarithmic sec ϕ_0 transmission curves give the correct maximum

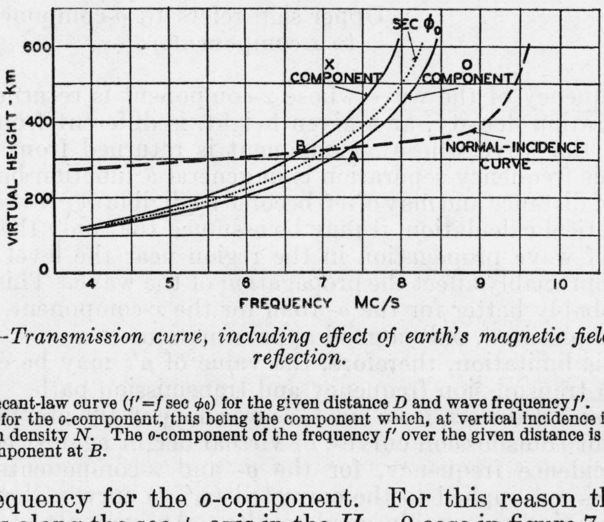

FIGURE 6.—*Transmission curve, including effect of earth's magnetic field at level of reflection.*

Dotted line=secant-law curve ($f'=f$ sec ϕ_0) for the given distance D and wave frequency f'. Dashed line=(z_r', f) curve for the *o*-component, this being the component which, at vertical incidence is a measure of the ionization density N. The *o*-component of the frequency f' over the given distance is reflected at A, and the *x*-component at B.

usable frequency for the *o*-component. For this reason the *o*-component lies along the sec ϕ_0 axis in the $H_L=0$ case in figure 7.

For a maximum usable frequency for the *o*-component below the gyrofrequency $f_H=\sqrt{f_T^2+f_L^2}$ the *x*-component always has a maximum usable frequency above f_H. This is only important for *E*-layer transmission and only in cases where the *x*-component is not too highly absorbed at frequencies near f_H. This case must not be confused with the case of a transmission frequency f' less than f_H, in which case the *x*-component is reflected from a level above the level where the *o*-component is reflected.

Another effect of the anisotropy of the ionosphere due to the earth's field is to cause a difference in the directions of phase and energy propagation in the medium. This results in the wave's being reflected, not at the level where the direction of phase propagation is horizontal, but where the direction of energy flow (group direction) is horizontal. This effect has not been considered in these curves, and is probably not important to the degree of accuracy to which these calculations are carried.

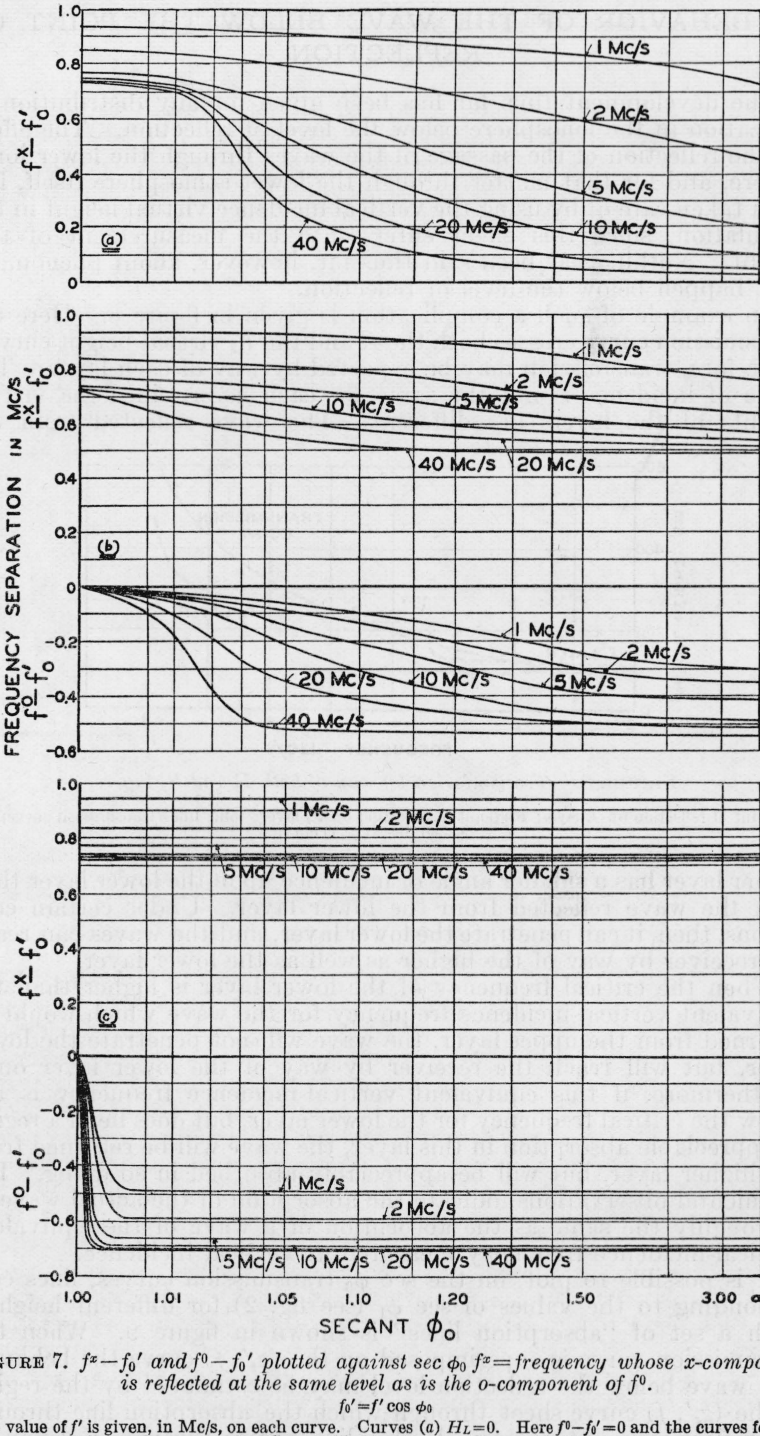

FIGURE 7. *f^x-f_0' and f^0-f_0' plotted against sec ϕ_0 f^x=frequency whose x-component is reflected at the same level as is the o-component of f^0.*

$f_0'=f'\cos\phi_0$

The value of f' is given, in Mc/s, on each curve. Curves (a) $H_L=0$. Here $f^0-f_0'=0$ and the curves for the o-component lie on the sec ϕ_0 axis. Curves (b) $H_L=H_T\neq 0$. Curves (c) $H_T\doteq 0$. Here f^x-f_0' and f^0-f_0' are independent of sec ϕ_0, save near sec $\phi_0=1$.

IV. BEHAVIOR OF THE WAVE BELOW THE POINT OF REFLECTION

The development thus far has been given for any distribution of ionization in the ionosphere below the level of reflection. The effect on the reflection of the passage of the waves through the lower ionosphere, and for that matter through the lower atmosphere itself, has been taken care of by using the vertical incidence virtual height in the calculation, since this effect enters into the measurement of this height. Nothing has been said thus far, however, about phenomena that happen below the level of reflection.

An example of such a complication is given in figure 8. Here the transmission curve crosses both the E and the F_2 virtual-height curves. Therefore, transmission may be expected by way of each layer. The angle of incidence is not the same for each layer, since the virtual heights of the layers are different. The wave reflected from the

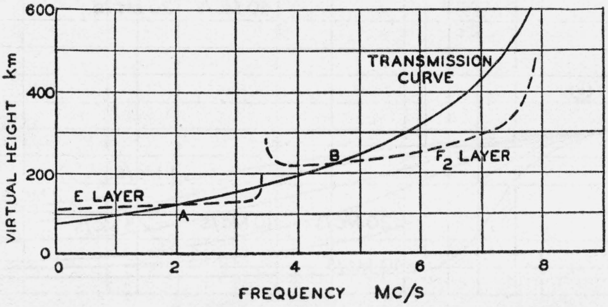

FIGURE 8.—*Transmission by way of both E and F_2 layers.*

A=point of reflection on E layer; B=point of reflection on F_2 layer; solid line=transmission curve; and dashed line=(z_v', f) curve.

higher layer has a smaller angle of incidence upon the lower layer than does the wave reflected from the lower layer. Under certain conditions, then, it can penetrate the lower layer, and the waves can reach the receiver by way of the higher as well as the lower layer.

When the critical frequency of the lower layer is higher than the equivalent vertical-incidence frequency for the wave which would be returned from the upper layer, the wave will not penetrate the lower layer, but will reach the receiver by way of the lower layer only. Furthermore, if this equivalent vertical-incidence frequency is not below the critical frequency for the lower layer, but does lie in a region of appreciable absorption in this layer, the wave will be returned from the higher layer, but will be appreciably absorbed in so doing. Experimental observations indicate the absorption of the actual wave to be roughly the same as the absorption of a wave of the equivalent vertical-incidence frequency measured at vertical incidence.

It is possible to plot, on the sec ϕ_0 transmission curves, lines corresponding to the values of sec ϕ_1 (see fig. 2) for different heights. Such a set of "absorption lines" is shown in figure 9. When the transmission curve is superimposed on the (z_v', f) curve the behavior of a wave below the reflection level may be estimated by the region of the (z_v', f) curve sheet through which the absorption line through the reflection point passes. If this line passes through a region of absorption or cuts a lower layer the wave will be absorbed or will not penetrate through to the higher layer.

These absorption lines are the lines sec ϕ_1=const. for a plane earth and for short distances on a curved earth. They curve toward larger

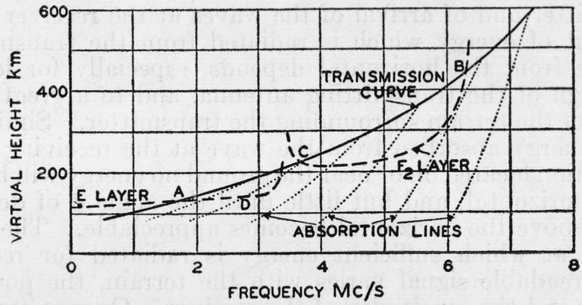

FIGURE 9.—*Absorption lines on logarithmic transmission curve.*

Dashed line=(z_v', f) curve; dotted lines=absorption lines. E reflection takes place at point A and F_2 reflection at point B. F_2 reflection at C is shielded by the E layer at D.

values of sec ϕ_1 for lower heights in the case of greater distances over a curved earth, and approach the transmission curve itself for great distances. An example of the use of the transmission curves and

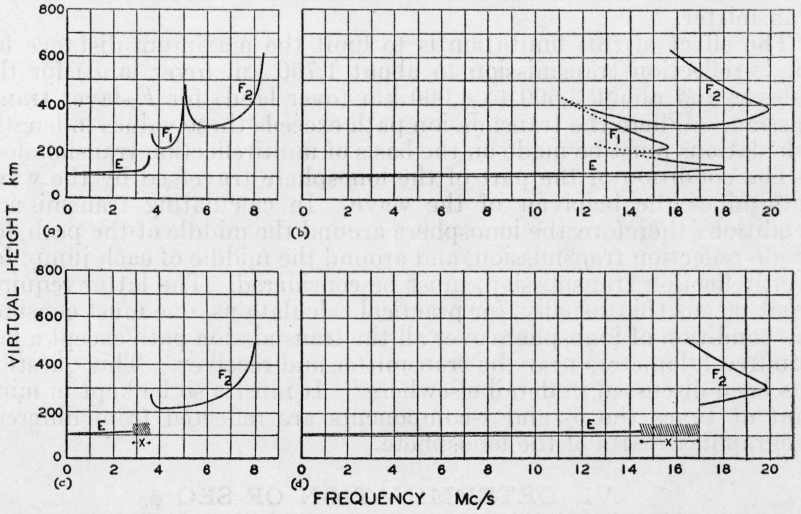

FIGURE 10.—*Virtual heights at vertical and oblique incidence.*

(*a*) (z_v', f) curve showing E, F$_1$, and F_2 layers. (*b*) oblique-incidence heights corresponding to (*a*). Dotted lines show parts of curves shielded by E layer. Note transmission of some frequencies by several layers. This shows how the F_1-layer reflections are relatively unimportant for transmission. (*c*) (z_v', f) curve showing absorption (×) above critical frequency for the E layer. (*d*) oblique-incidence heights corresponding to (*c*). Absorption here (×) completely blocks out some frequencies. All curves are for the *o*-component.

absorption lines is shown in figure 10, which gives vertical-incidence curves and the corresponding frequency-height curves derived therefrom for the transmitted wave over a distance. From these latter curves can be deduced, of course, the equivalent paths or group retardation of the waves over the transmission path.

V. ANGLE OF DEPARTURE AND ARRIVAL OF THE WAVES

Disregarding any possible asymmetry of the wave trajectory due to the earth's magnetic field, the angles of departure of the waves from the transmitter and of arrival of the waves at the receiver are equal. The amount of energy which is radiated from the transmitter at a given angle from the horizontal depends, especially for low angles, on the design of the transmitting antenna, and to a great extent on the nature of the terrain surrounding the transmitter. Similar factors affect the energy absorbed from the wave at the receiving station.

For a station located on or near the ground no energy will be radiated below the horizontal, and but little until the angle ψ of departure of the waves above the horizontal becomes appreciable. The minimum value of ψ at which sufficient energy is radiated (or received) to produce a readable signal varies with the terrain, the power of the transmitter and the sensitivity of the receiver. Over sea water ψ can be very nearly zero; over land the minimum value may be several degrees. A fair average approximation may be that ψ must exceed about 3.5°.

A simple geometrical calculation gives ψ in terms of sec ϕ_0 for various distances, and these may be noted on the transmission curves. The point of reflection must then correspond to an angle ψ greater than the minimum value assumed for the terrain at the receiver and transmitter.

The effect of this limitation is to limit the maximum distance for single-reflection transmission to about 1,750 km (over land) for the E-layer and about 3,500 to 4,000 km (over land) for F_2-layer transmission. Where the transmission path exceeds these values in length, calculations must be made on the basis of multireflection transmission.

The condition of the part of the ionosphere traversed by the wave determines the behavior of the wave. In calculating transmission conditions, therefore, the ionosphere around the middle of the path, for single-reflection transmission, and around the middle of each jump, for multireflection transmission, must be considered. The latter requirement means that usually, for practical calculations, one must consider the condition of ionosphere over all the transmission path except a few hundred kilometers near the transmitter and receiver. This question has been discussed in detail elsewhere.[7] It must also be kept in mind that at times the *o*- and *x*-components are reflected from different geographical parts of the ionosphere.

VI. DETERMINATION OF SEC ϕ_0

Figure 11 gives an alignment chart for the rapid determination of the factor sec ϕ_0 to be used in calculating the logarithmic transmission curves. To use the chart, place a straightedge so that it passes through the desired virtual height and the desired distance laid off on the distance scale at the lower left-hand edge of the chart (increasing distances lie to the left). The ordinate of the intersection of the straightedge with the vertical line corresponding to the same

[7] T. R. Gilliland, S. S. Kirby, N. Smith, and S. E. Reymer. *Characteristics of the ionosphere and their applications to radio transmission.* J. Research NBS 18, 645 (1937) RP1001; Proc. Inst. Radio Engrs. 25, 823 (1937).
The weekly radio broadcasts of the National Bureau of Standards on the ionosphere and radio-transmission conditions. Letter Circular LC499.

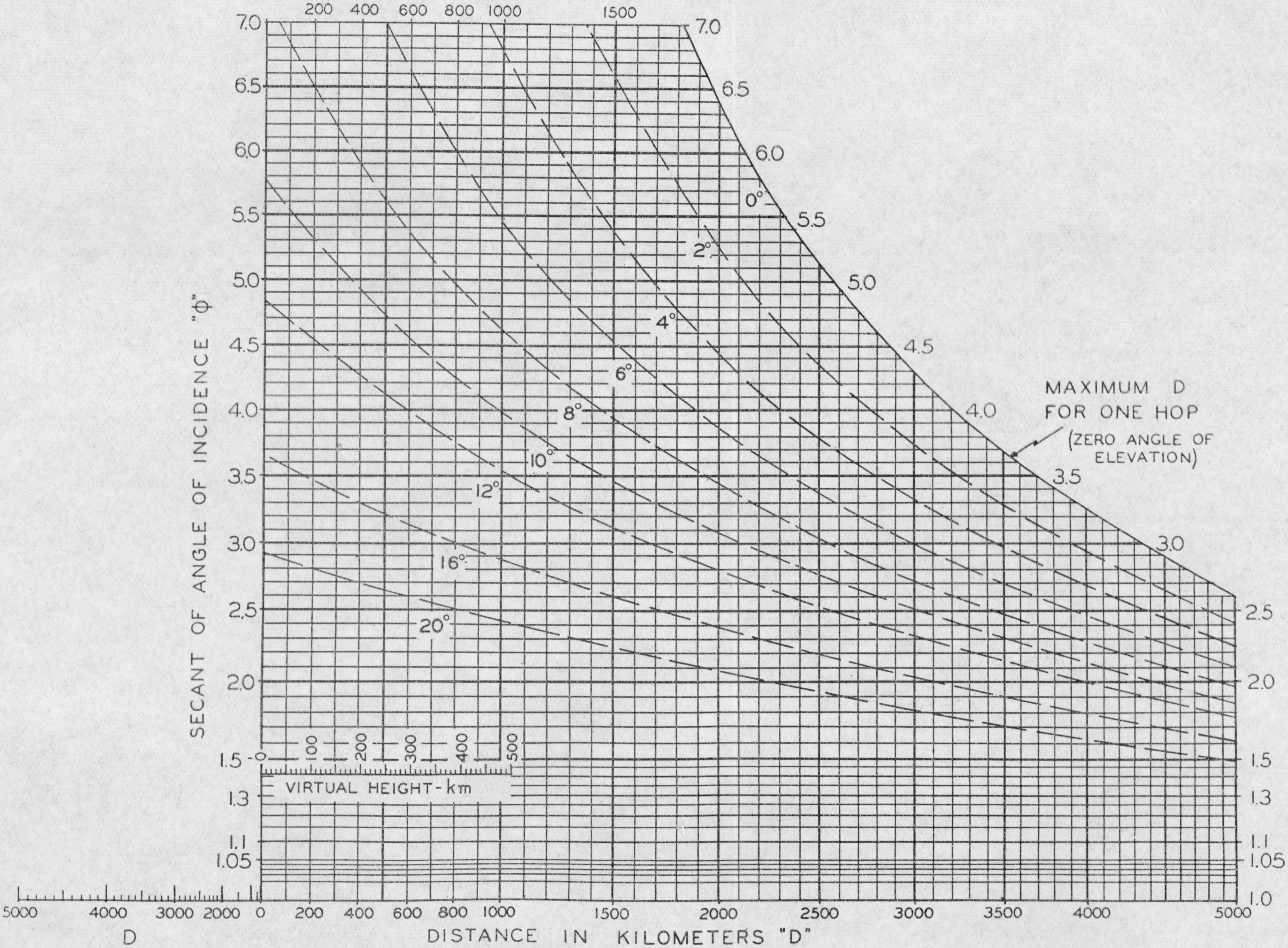

Figure 11.

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desired distance laid off on the main distance scale (increasing distances lie to the right) gives the value of sec ϕ_0. The relation of the point of intersection to the curved dashed lines of equal ψ gives the value of the angle of departure of the waves from the horizontal. A point of intersection falling above the $\psi=0°$ line indicates an impossible case, where the ray would have to depart at an angle below the horizon.

For example, a distance of 2,400 km and a virtual height of 300 km corresponds to a sec ϕ_0 of 3.07, and an angle of departure of 8.2°.

VII. TRANSMISSION FACTORS

When average transmission conditions over a period of time or when a variety of transmission paths are to be considered, or when an estimate of the maximum usable frequencies is to be made without a precise knowledge of the ionosphere over the transmission path, it is convenient to have available a means by which the maximum usable frequencies may be quickly estimated from an approximate value of the vertical-incidence critical frequency.

The National Bureau of Standards is now beginning a compilation of factors by which the critical frequency for the *o*-component, measured at vertical incidence, may be multiplied in order to obtain the maximum usable frequencies. These factors are based on average observations over a period of time, and may be applied either to average critical frequencies to give average transmission conditions, or to a given observation of a critical frequency to obtain approximate transmission conditions at a given time.

VIII. CONCLUSIONS

The type of transmission curves described above have been in use at the National Bureau of Standards for the past 2 years in studying the correlation of high-frequency radio-transmission conditions with regular ionosphere observations. The results of continuous measurements of high-frequency broadcasting stations and observations on other high-frequency signals, as well as the results of some specific experiments have been compared with vertical-incidence data. Practically all the available data agree with what would be expected on the basis of the theory outlined above, and the exceptions may in most cases be accounted for.

WASHINGTON, November 22, 1937.