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IMPROVED CONTINUOUSLY VARIABLE SELF AND MUTUAL INDUCTOR

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ABSTRACT

The features to be desired in a continuously variable self and mutual inductor, and the extent to which a particular existing type of inductor meets these requirements are stated. The investigation here reported had as object the improvement of the design of this inductor.

Two new general theorems concerning a pair of mutually inductive coils are derived. They apply to coils of any form, capable of change of relative position. If the self inductance of the two coils in series has an upper value L_u for a certain relative position and a lower value L_l for a second relative position, and if either coil may be rewound with any desired number of turns without change of space factor, the first theorem states that the ratio L_u/L_l will have its maximum value when the self inductances of the two coils are equal.

The second theorem relates to a quantity which is arbitrarily called the "characteristic time-constant" of the pair of coils for the given relative positions and is equal to $(L_u-L_l)/(R_1+R_2)$, where R_1 and R_2 are the resistances of the coils. This time-constant will have its maximum value when $R_1=R_2$.

When $L_1=L_2$ and $R_1=R_2$ the requirements for maximum L_u/L_l and for maximum characteristic time-constant are satisfied. An added advantage of this set of conditions is that when the two coils are connected in parallel the value of the self inductance, for any given relative position of the coils, will be exactly one-fourth of the corresponding value for the series connection.

The procedure which was used in applying the new theorems to the design of an improved inductor and the performance of the finished instrument are given in detail.

CONTENTS

I. INTRODUCTION

1. DESIRABLE FEATURES IN A CONTINUOUSLY VARIABLE INDUCTOR

A satisfactory continuously variable self inductor should have (a) a large value of the ratio of the useful upper limiting value of self inductance L_u to the useful lower limiting value L_l; (b) a large value of the useful upper limiting value of time-constant L_u/R for the given mass of active material and the space occupied; (c) astaticism with respect to a uniform magnetic field produced by an external source; (d) a close approach to a linear scale law over nearly the whole scale; (e) a minimum dependence of the inductance on the frequency, over a considerable range of frequency. Because the continuous variation of the inductance is obtained by the relative motion of parts of the apparatus, with resulting wear, we may well add that the inductor should have (f) inherent compensation for the effect of wear of the bearing surfaces. Some of these requirements being antagonistic, the design of an inductor involves the inevitable compromises. For example, to satisfy requirements (a) and (b) it is necessary that the coils (or groups of coils) of which the relative motion effects the variation of inductance should be closely coupled at their extreme relative positions. This close coupling, however, introduces relatively large values of capacitance between turns, between layers, and between coils. The effective self inductance will therefore depend upon the frequency to a greater extent than would be the case in an inductor in which the individual turns and layers are more widely spaced and the individual coils are further apart, and which therefore will be inferior as regards requirements (a) and (b).

2. FEATURES OF AN EXISTING TYPE OF CONTINUOUSLY VARIABLE INDUCTOR

The variable self and mutual inductor devised by one of the authors [1] and F. C. Weaver is shown in cross section in figure 1. Figure 2 shows a plan view, to scale, of the two groups of link-shaped coils. This

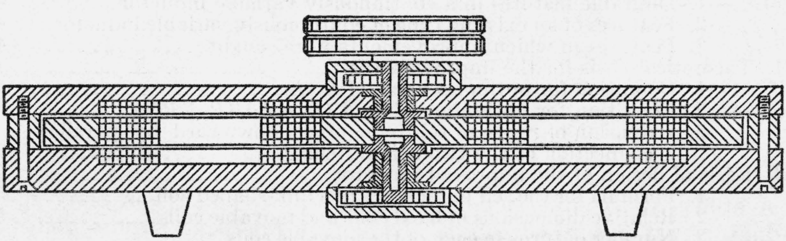

FIGURE 1.—*Cross-sectional view of variable self and mutual inductor.*

This illustration refers to the earlier form of variable inductor which it was the object of the present investigation to improve.

variable inductor meets requirements (a) and (b) by using interleaved groups of coils developed from Maxwell's circular coil of square cross section and maximum time-constant. It meets requirement (c) by having two like groups of fixed coils having opposite magnetic polarities and two like movable coils, also of opposite polarities. It meets

[1] H. B. Brooks and F. C. Weaver. BS Sci. Pap. 13, 569 (1916–17) S290.

requirement (d) by having link-shaped coils of which the empirical proportions and relative positions are the result of extensive experiments. The interleaving which is necessary to meet requirements (a) and (b) also satisfies requirement (f) because a lowering of the movable-coil group as a result of wear of the step bearing increases the mutual inductance between the movable-coil group and the lower fixed coils by nearly the same amount that it decreases the mutual inductance between the movable-coil group and the upper fixed coils. This feature of construction also makes the inductance substantially inde-

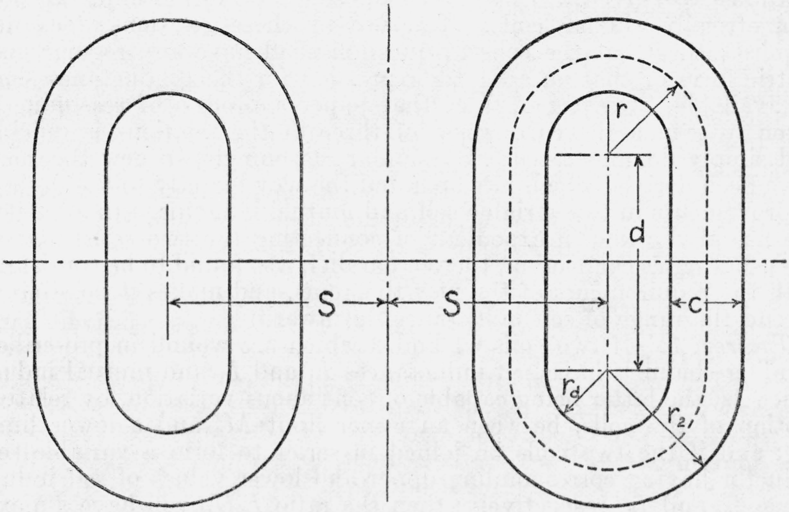

FIGURE 2.—*Plan view of the two groups of link-shaped coils used in the earlier form of variable inductor.*

pendent of warping of the movable plate, and is doubtless one of the principal reasons for the extensive use of inductors of this type during the 20 years since their introduction. Inductors of this type are preferably used on frequencies not exceeding 3,000 to 5,000 c/s, depending on the range of self inductance for which they are wound. Those for the higher ranges, having more turns per layer and more layers of wire, will show a greater change of inductance for a given change of frequency.

3. FEATURES IN WHICH IMPROVEMENTS WERE SOUGHT

A study of the question of possible improvements in the type of inductor just described led to the conclusion that requirements (c) to (f), inclusive, were probably satisfied as well as is possible without complication in construction. Urgent reasons, however, existed for an effort to meet requirements (a) and (b) to the greatest practicable extent.[2] As compared with adjustable rheostats, for example, these inductors are expensive, and, consequently, any improvement which will increase the ratio L_u/L_l will reduce the number of inductors necessary for a given piece of work. Since the useful property of an

[2] Requirement (a) could be more fully met by a greater degree of interleaving of the fixed coils and the movable coils, but this would complicate and weaken the construction and increase the cost.

inductor is its inductance, and its undesirable (and often its limiting) property is its resistance, it was felt that any improvement in the time-constant would also be valuable.

II. THEORETICAL BASIS FOR THE IMPROVEMENTS

1. CONDITION FOR OPTIMUM VALUE OF L_u/L_l

In a previous paper by one of the authors a number of general relations [3] are given which concern the self and mutual inductance of geometrically similar coils. The use of these relations does not require either that the coils in question shall have any regular geometric form or that an analytic expression for their inductance shall be available. On page 297 of that paper a process of reasoning is given by which the correctness of three of the relations is quickly and simply demonstrated. By similar reasoning, two new theorems have been derived which have pointed the way directly to the desired improvements in the variable self and mutual inductor. In addition, the use of the known expedient of connecting the two coils, first in the usual series connection, then in parallel, was found to be consistent with the requirements of the new theorems, and makes it possible to extend the range of self inductance downward.

Theorem 1.—If two coils [4] 1 and 2 which are wound in prescribed winding channels have self inductances L_1 and L_2 and mutual inductance M, the latter being capable of continuous variation, by relative motion of the coils, between an upper limit M_u and a lower limit M_l; and if the two coils be joined in series to form a variable self inductor having corresponding upper and lower values of self inductance L_u and L_l, respectively; then the ratio L_u/L_l will have a maximum value when the channels are wound to make $L_1=L_2$.

In this theorem and in the one to follow it is assumed that the space factor of each coil is constant, regardless of the number of turns of wire in it, and hence that the time-constant of each coil is independent of the number of turns and depends solely on its geometric form, its absolute dimensions, the space factor, and the resistivity of the conducting material of the winding. The first theorem may be proved in the following manner:

The coupling coefficient k is defined by the relation

$$k=M/(L_1L_2)^{\frac{1}{2}} \tag{1}$$

from which

$$M=k(L_1L_2)^{\frac{1}{2}} \tag{2}$$

For the two given limiting relative positions of the coils, denoted by the subscripts u and l, the corresponding coefficients are k_u and k_l. They depend on the geometry of the individual coils and their relative positions and are not altered by a change in the number of turns in either coil or in both coils if the change in the number of turns leaves unaltered the form of the coil and the distribution of the turns over its cross section.

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[3] H. B. Brooks. BS J. Research 7, 296 (1931) RP342.

[4] Throughout this paper, for brevity, the word "coil" will be used to denote either a coil or a group of electrically and mechanically connected coils which function as a single coil.

If L_2 be denoted by p^2L_1, where p^2 is a coefficient as yet unknown, the ratio L_u/L_l, which may be denoted for brevity by y, may be written

$$y=\frac{L_1+p^2L_1+2k_upL_1}{L_1+p^2L_1+2k_lpL_1} \tag{3}$$

$$=\frac{1+p^2+2k_up}{1+p^2+2k_lp} \tag{4}$$

Differentiation gives the relation

$$\frac{dy}{dp}=\frac{2(k_l-k_u)(p^2-1)}{(1+p^2+2k_lp)^2} \tag{5}$$

Because k_u is not equal to k_l, this derivative vanishes only when $p=1$, that is, when

$$L_1=L_2 \tag{6}$$

Examination of the second derivative shows that the condition $p=1$ corresponds to a maximum value of y. It is evident, furthermore, that the theorem holds for any pair of values of L between the limiting values L_u and L_l which were chosen in the statement of the theorem because of their practical importance.

The substitution of $p=1$ in eq 4 gives the simple relation

$$y_{\max}=\frac{1+k_u}{1+k_l} \tag{7}$$

If the change in the mutual inductance is accomplished either by rotating one coil 180° about a suitable axis of symmetry or by interchanging the connections between the coils so as to reverse their relative magnetic polarities, k_l becomes $-k_u$ and eq 4 becomes

$$y'=\frac{1+p^2+2k_up}{1+p^2-2k_up} \tag{8}$$

which for the condition $L_1=L_2$, that is, $p=1$, assumes the simple form

$$y'_{\max}=\frac{1+k_u}{1-k_u} \tag{9}$$

It is important to note that the closer the coupling between the coils the greater will be the relative loss in the ratio L_u/L_l if the requirement that L_1 should be equal to L_2 is not fulfilled.

The requirement $L_1=L_2$ for $y_{\max}$ imposes no restriction on the relative sizes of the two coils. It applies to any two winding channels, and requires only that the numbers of turns shall be such as to make the two self inductances equal. Consideration of eq 4 shows that a pair of coils, one large and having a relatively small number of turns, the other small and having a relatively large number of turns, would not constitute a good basis for the design of a variable inductor even if their inductances were equal because of the relatively small possible values of coupling coefficient. Theoretically, any pair of winding spaces, however dissimilar in their values of inductance for equal numbers of turns, could be wound to give any desired large value of self inductance of the pair of coils in series. Such an inductor might be useful in cases where a relatively low time-constant is unobjectionable, but in general the design of any inductor should be such as to give the largest value of time-constant consistent with other requirements of the case. A study of this point led to the second general theorem.

2. CONDITION FOR OPTIMUM VALUE OF $(L_u-L_l)/(R_1+R_2)$

In a fixed standard of self inductance it is usually desirable that the resistance R should be as low as possible in comparison with the inductance L, in other words, that the time-constant $\tau=L/R$ should be as large as possible, in order to minimize the weight and cost of the coil and the space occupied by it. For a variable inductor, several different values of time-constant may be defined, including the following:

$$\tau_u=L_u/(R_1+R_2) \tag{10}$$

$$\tau_l=L_l/(R_1+R_2) \tag{11}$$

and of course an infinite number of intermediate values for intermediate relative positions of the two coils. None of these, however, is suited for use as a criterion of the excellence of the design of a given system of coils and their relative positions. A characteristic which one strives to make as great as possible, however, is the ratio of the total change of self inductance L_u-L_l to the resistance R_1+R_2 of the two coils in series. We may arbitrarily call this ratio the "characteristic time-constant" τ' of the set of coils; that is,

$$\tau'=(L_u-L_l)/(R_1+R_2) \tag{12}$$

The change L_u-L_l defines the range of available values of self inductance, and is numerically equal to $2(M_u+M_l)$, where the M's denote the values of mutual inductance for the limiting positions of the coils. An investigation to discover the conditions under which τ' will have a maximum value led to the second general theorem, as follows:

Theorem 2.—If two coils 1 and 2 which are wound in prescribed winding channels have self inductances L_1 and L_2, resistances R_1 and R_2, and mutual inductance M, the latter being capable of continuous variation, by relative motion of the coils, between an upper limit M_u and a lower limit M_l; and if the two coils be joined in series to form a variable self inductor having corresponding upper and lower values of self inductance L_u and L_l, respectively; then the characteristic time-constant $\tau'=(L_u-L_l)/(R_1+R_2)$ will have its maximum value when the channels are wound to make $R_1=R_2$.

It is assumed in this case, as in that of the first theorem, that the space factor of each of the coils is independent of the number of turns of wire in it, and hence that the time-constants τ_1 and τ_2 of the separate coils are also independent of the number of turns of wire in them. The theorem may be proved in the following manner:

The time-constants of the individual coils are

$$\begin{aligned}\tau_1&=L_1/R_1\\ \tau_2&=L_2/R_2\end{aligned} \tag{13}$$

from which

$$R_1 = L_1/\tau_1$$

$$R_2 = L_2/\tau_2 \tag{14}$$

and the characteristic time-constant of the system of coils, as defined, is

$$\tau' = (L_u - L_l)/\left(\frac{L_1}{\tau_1} + \frac{L_2}{\tau_2}\right) \tag{15}$$

If we denote L_2 by p^2L_1, where p^2 is a coefficient as yet unknown, and the coupling coefficients for the two limiting positions by k_u and k_l, eq 15 becomes

$$\tau' = \frac{L_1 + p^2L_1 + 2k_upL_1 - L_1 - p^2L_1 - 2k_lpL_1}{\frac{L_1}{\tau_1} + \frac{p^2L_1}{\tau_2}} \tag{16}$$

$$= \frac{2p(k_u - k_l)}{\frac{1}{\tau_1} + \frac{p^2}{\tau_2}} \tag{17}$$

The choice of definite winding channels and definite limiting positions of the coils, with the assumption that the space factor of each coil is independent of the size of wire in it, determines the values of τ_1 and τ_2 as well as of k_u and k_l, leaving p as the only independent variable in this equation. It remains to discover the value of p for which τ' is a maximum. Differentiation of eq 17 gives

$$\frac{d\tau'}{dp} = 2(k_u - k_l)\frac{\frac{1}{\tau_1} - \frac{p^2}{\tau^2}}{\left(\frac{1}{\tau_1} + \frac{p^2}{\tau_2}\right)^2} \tag{18}$$

This vanishes only when

$$p^2 = \frac{\tau_2}{\tau_1} \tag{19}$$

that is, when

$$\frac{L_2}{L_1} = \frac{\tau_2}{\tau_1} = \frac{L_2}{R_2} \cdot \frac{R_1}{L_1} \tag{20}$$

that is, when

$$R_2 = R_1 \tag{21}$$

It should be noted that the separate attainment of each of the optimum conditions, L_u/L_l a maximum, $(L_u - L_l)/(R_1 + R_2)$ a maximum, imposes no restrictions on the dimensions of either winding channel with respect to those of the other channel. This ceases to be true, however, when one requires that the system of coils shall satisfy *both* conditions; in which case

$$L_2 = L_1 \tag{22}$$

$$R_2 = R_1$$

and consequently

$$\tau_2 = \tau_1 = L_1/R_1 \tag{23}$$

To satisfy this condition, when the two coils are geometrically similar and their windings (of the same conducting material) have the same space factor, the two coils must have identical dimensions. If the coils are not geometrically similar, no simple general relation exists between their dimensions but it is obvious that in general the coils cannot differ greatly in size. It is also obvious that two coils differing greatly in size would have a relatively low value of coupling coefficient and therefore would be unsuitable also on this account for use as a variable self and mutual inductor.

3. EXTENSION OF RANGE OF SELF INDUCTANCE DOWNWARD BY PARALLEL CONNECTION OF THE TWO COIL SYSTEMS

When both of the preceding conditions for optimum values are satisfied, a third very desirable result immediately follows, provided the ratio of L_u to L_l is 4 or more. The usual connection of coils 1 and 2 in series gives a range of self inductance having upper and lower limits L_u and L_l. The connection in parallel of two coils for which $L_1=L_2$ and $R_1=R_2$ in such a way that the instantaneous directions of the currents in the coils are the same as in the series connection, gives a range of self inductance having an upper limit $L_u/4$ and a lower limit $L_l/4$. For every relative position of the coils the inductance for the parallel connection will be exactly one-fourth of the corresponding value for the series connection. The range of self inductance is thus effectively from an upper limit L_u to a lower limit $L_l/4$. The time-constant of the inductor, for any relative position of the coils, is the same for either method of connection.

If L_u/L_l is less than 4, as may occur in variable inductors for frequencies high enough to intensify the effects of capacitance between coils and hence to require a greater separation of the coils, the full advantage of changing the connection from series to parallel cannot be realized because there will be a gap between the lower limit L_l of the series arrangement and the upper limit $L_u/4$ of the parallel arrangement.

Inductors having the two coil systems equal in resistance and in inductance are regularly made by at least one American manufacturer. A similar method of extending the range of a variable inductor was used by Drysdale.[5] He made the movable system of two coils having equal self inductances and equal resistances; and similarly for the fixed system. This arrangement provides not only the two ranges of self inductance but also two ranges of mutual inductance, values for the lower range in each case being one-fourth of the corresponding values for the upper range.

III. BASIS OF DESIGN OF THE IMPROVED INDUCTOR

1. FORMULA FOR THE SELF INDUCTANCE OF A LINK-SHAPED COIL

In designing link-shaped coils to meet the conditions laid down by the two theorems developed in section II it is necessary to use approximate formulas because no exact formula for the inductance of such

[5] Electrician 84, 108 (1920); H. Tinsley & Co. list 45 (April 1916).

coils is available. It is known, however, that to an accuracy of 1 or 2 percent the self inductance of a link-shaped coil of a given number of turns bears the same ratio to the self inductance of a circular coil having the same cross-sectional area, the same number of turns, and the same length of mean turn as the area inclosed by the mean turn of the link-shaped coil bears to the area inclosed by the mean turn of the circular coil.[6] For the coils used here this ratio is 0.83, so that we may take the self inductance of the link-shaped coil as equal to 0.83 times the self inductance of the equivalent circular coil, which latter is given conveniently and with sufficient accuracy by Maxwell's approximate formula.[7] We have, then, as an empirical formula which is sufficiently accurate for design purposes,

$$L_{\text{link}}=0.83\left[4\pi aN^2\left(-2+\log_e\frac{8a}{G}\right)\right] \tag{24}$$

where

L_{link} = self inductance of link-shaped coil in abhenries (millimicrohenries).

a = radius of the mean turn of the equivalent circular coil.

= length of the mean turn of the link-shaped coil, divided by 2π.

N = number of turns.

G = geometric mean distance of the cross section of the winding channel.

It has been shown by Rosa and Grover[8] that

$$G=0.2235\ (b+c)\ \text{approximately} \tag{25}$$

where (see fig. 3)

b = axial depth of the coil.

c = radial width of the coil.[9]

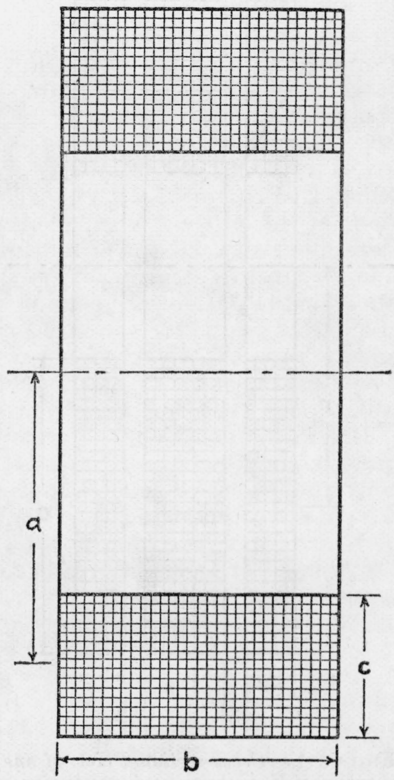

FIGURE 3.—*Cross-sectional view of circular coil of square cross section, to illustrate equation 25.*

[6] BS Sci. Pap. **13**, 578 (1916–17) S290.

[7] Maxwell Elec. and Mag., 3d ed., **2**, p. 345.

[8] BS Sci. Pap. 8, 168 (1916) S169.

[9] We have found, by actual computations by the two methods, that an equally good approximation to the inductance of a link-shaped coil can be obtained either by the use of eq 24 or by assuming that the inductance of the link-shaped coil is equal to the inductance of a rectangular coil of the same cross section and number of turns, the mean turn of the rectangular coil circumscribing the mean turn of the link-shaped coil. For the computation of the inductance of a rectangular coil we have used a formula given by Jeans, Math. Theory of Elec. and Mag., 5th ed., p. 446. While not as convenient for routine calculations as eq 24 given above, this rectangular formula is very useful in investigating the effects of various ratios of length to breadth, etc., on the inductance of a link-shaped coil.

2. RELATIVE DIMENSIONS OF FIXED COILS AND MOVABLE COILS

Figure 4 represents a cross-sectional view of two of the fixed coils and one of the movable coils of a variable inductor which consists of two coils which may be rotated between two equal pairs of fixed coils. It is obvious that if the self inductance of the two systems of coils is to have the optimum value for any given configuration, certain of their geometrical dimensions must be identical; that is, the fixed coils and the movable coils must be identical as to dimensions c, r, and d (see fig. 2) and hence must have the same value of the equivalent radius a. As eq 24 indicates, the inductance of a coil is a function of the four variables, N, a, b, and c. In designing coils for a variable inductor we are at liberty to adjust these variables for one of the coil systems, say the movable system, almost at will. When the movable coils have been designed, however, the dimensions a and c for the fixed coils must be equal to the corresponding dimensions of the movable coils. We are therefore left with two variables, the axial depth b_f, of the fixed coil, and the number of turns, N_f, with which to satisfy the two conditions for optimum performance, namely, equality of inductance and equality of resistance.

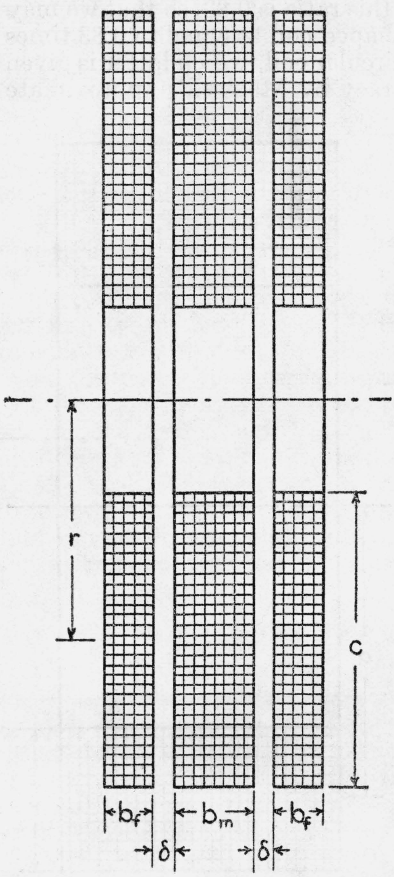

FIGURE 4.—*Cross-sectional view of one group of link-shaped coils of the proportions used in the improved inductor.*

The cross section is made by a plane whose trace is the horizontal center line in figure 2.

In designing an actual inductor we are not at liberty, however, to choose the dimensions even of the movable system entirely at will because the linearity of the scale law of an inductor constructed with link-shaped coils is very sensitive to the relative proportions of the coils.[10] It was felt desirable, therefore, that the relative proportions of the new coils should not depart too widely from those that had already been proved satisfactory. It was decided also that the movable coils of the new inductor should have substantially the same radius of mean turn and the same axial depth as the corresponding coils in inductors of the design then in commercial production.[11] Dimensions d, r, and b_m, see figures 2 and 4, were therefore left sub-

[10] BS Sci. Pap. **13**, 572–575 (1916–17) S290.

[11] It should be noted that the absolute dimensions of the coils in the inductors as manufactured on the basis of the paper by Brooks and Weaver (BS Sci. Pap. 290) are not identical with those given in that article, in which, for example, r=36.8 mm. In the commercial model r=30.2 mm, and the other dimensions of the coils were such as to have relative proportions given in the paper.

stantially unchanged. It was proposed to increase the radial width, c, of the system by a slight amount, to be experimentally determined, to give a greater coupling coefficient between the fixed system and the movable system and consequently a greater range between L_u and L_l.

3. NUMBER OF TURNS IN EACH OF THE MOVABLE COILS

With the equivalent radius a and the dimension b_m (fig. 4) fixed as has just been explained, and with provisional values chosen for c and for the number of turns N_m in one of the movable coils, we are in position to calculate L_m with the aid of eq 24. Tentatively, therefore, each of the movable coils is completely determined. In what follows the subscripts m, F, and f will refer to the movable system and the fixed system respectively. The subscript f will refer to each of the separate fixed coils constituting a pair, and the subscript F will be applied to a pair of fixed coils connected in series and considered as a unit.

With the coil systems in the position of maximum inductance the effect of mutual inductance between the right-hand group and the left-hand group of coils (see fig. 1) contributes only about 3 percent to the total self inductance. For design purposes it is convenient to neglect this contribution and to compute each group of coils to have a maximum self inductance equal to one-half the value desired in the complete inductor.[12] On this basis we have to consider a pair of fixed coils in series, of total resistance R_F, and a movable coil of resistance R_m which is to be equal to R_F, while the inductance L_F of the two fixed coils in series is to be equal to L_m, the self inductance of the movable coil. The symbol L_f will denote the self inductance of each of the two fixed coils separately.

4. CONDITION FOR EQUALITY OF RESISTANCES OF THE TWO COIL SYSTEMS

Taking up first the requirement of equality of resistances, let us assume that the movable coil is to be wound with round wire of resistivity ρ. Let the diameter of the wire over the insulation be x_m, and the space factor be s; that is, s is the ratio of the cross section of the conductor to the area of the square which circumscribes the wire over the insulation. Then the resistance of each movable coil will be

$$R_m = 2\pi a\rho(N_m/sx^2{}_m) \tag{26}$$

Also, the number of turns in the movable coil is

$$N_m = b_m c/x^2{}_m \tag{27}$$

from which

$$x_m = (b_m c/N_m)^{\frac{1}{2}} \tag{28}$$

and

$$R_m = 2\pi a\rho(N_m{}^2/sb_m c) \tag{29}$$

Similarly, the resistance of the two fixed coils in series is

$$R_F = 2\pi a\rho(2N_f{}^2/sb_f c) \tag{30}$$

in which N_f is the number of turns in each of the two equal fixed coils.

[12] This true maximum value of self inductance, which is for the case of each movable coil coaxial with its fixed coils and with the magnetic fluxes additive, should be distinguished from the *useful* upper limit of self inductance L_u above which the manner of variation of the inductance with angular position ceases to be approximately linear. In the inductor described later in this paper the ratio L_u/L_l is about 8 percent lower than L_{max}/L_{min}. L_u, L_l, L_{max}, and L_{min} refer to relative angular positions of the coils of 175, 5, 180, and 0°. This 8 percent difference should be taken into account in the design of inductors of this type.

It is assumed that the space factor s is the same for all the coils. Then since the coils are to be wound so that $R_F=R_m$, it follows that

$$N_f=N_m(b_f/2b_m)^{\frac{1}{2}} \tag{31}$$

5. CONDITION FOR EQUALITY OF SELF INDUCTANCES OF THE TWO COIL SYSTEMS

The conditions for equality of the self inductances of the movable coil and of the pair of fixed coils are not determined so simply. The self inductance L_F of the pair of fixed coils consists of the sum of the equal self inductances L_f and L_f of the separate coils plus twice the mutual inductance between them, and may be determined by the device of imagining an intermediate fictitious coil ϕ, wound with wire of the same size (over the insulation) as that used for the two fixed coils and connected in series with them so as to form a coil (see fig. 5) whose cross section has a radial width c and an axial depth equal to the sum of b_m, $2b_f$, and twice the axial clearance δ between the fixed coils and the movable coil. Let the self inductance of the fictitious coil be denoted by L_ϕ. Let $L_{f,\phi,f}$ denote the self inductance of the large coil composed of the two fixed coils in series with the fictitious coil, and $L_{f,\phi}$ the self inductance of a coil composed of the fictitious coil and either of the two fixed coils. It may be shown [13] that the self inductance of the two fixed coils in series is

$$L_F=L_{f,\phi,f}+2L_f+L_\phi-2L_{f,\phi} \tag{32}$$

The four terms on the right in this expression are to be computed with the aid of eq 24.

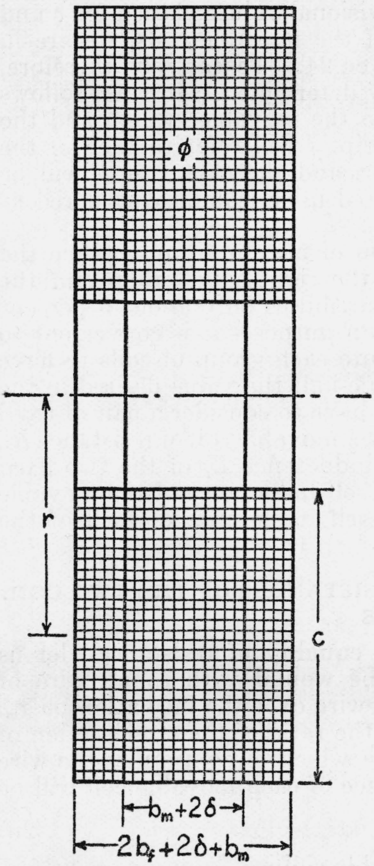

FIGURE 5.—*Cross-sectional view of two fixed coils and a fictitious coil ϕ.*

The fictitious coil is assumed to fill completely the space between the fixed coils, and to have the same number of conductors per unit of area of cross section as the latter. The assumption of such a fictitious coil is a mathematical expedient to facilitate the computation of the mutual inductance between the fixed coils.

In order that this formula shall apply, it is necessary to assume the same wire diameter over the insulation (and therefore the same number of turns per square centimeter of the cross section of the winding) for the fictitious coil as for the fixed coils. The number of turns in the fictitious coil (see fig. 5) will thus be

$$N_\phi=\frac{(b_m+2\delta)c}{b_f c}\cdot N_f$$

$$=\frac{b_m+2\delta}{b_f}\cdot N_f \tag{33}$$

[13] BS Sci. Pap. 8, 41 (1912) S169.

We already have the relation

$$N_f = N_m(b_f/2b_m)^{\frac{1}{2}} \tag{31}$$

which, combined with eq 33, gives

$$N_\phi^2 = \frac{(b_m+2\delta)^2}{2b_f b_m} \cdot N_m^2 \tag{34}$$

The square of the number of turns in each fixed coil is

$$N_f^2 = \frac{b_f}{2b_m} \cdot N_m^2 \tag{35}$$

For the fictitious coil plus either fixed coil

$$N^2_{f,\phi} = \frac{(b_m+b_f+2\delta)^2}{2b_f b_m} \cdot N_m^2 \tag{36}$$

and for the fictitious coil plus both the fixed coils

$$N^2_{f,\phi,f} = \frac{(b_m+2b_f+2\delta)^2}{2b_f b_m} \cdot N_m^2 \tag{37}$$

We can now write the equations for the inductances of these various coils as follows, using eq 24:

$$L_m = K\left[-2+\log_e \frac{8a}{0.2235(b_m+c)}\right] \tag{38}$$

in which, for brevity, the symbol K denotes the quantity $0.83\times 4\pi a N_m^2$. Similarly,

$$L_\phi = K\cdot\frac{(b_m+2\delta)^2}{2b_f b_m}\cdot\left[-2+\log_e \frac{8a}{0.2235(b_m+2\delta+c)}\right] \tag{39}$$

$$L_f = K\cdot\frac{b_f}{2b_m}\cdot\left[-2+\log_e \frac{8a}{0.2235(b_f+c)}\right] \tag{40}$$

$$L_{f,\phi} = K\cdot\frac{(b_m+b_f+2\delta)^2}{2b_f b_m}\cdot\left[-2+\log_e \frac{8a}{0.2235(b_m+b_f+2\delta+c)}\right] \tag{41}$$

$$L_{f,\phi,f} = K\cdot\frac{(b_m+2b_f+2\delta)^2}{2b_f b_m}\cdot\left[-2+\log_e \frac{8a}{0.2235(b_m+2b_f+2\delta+c)}\right] \tag{42}$$

Using these equations and eq 32, we are now able to set up explicitly an equation which expresses the condition that L_F must equal L_m. After a little obvious rearrangement, this equation takes the form

$$\begin{aligned} K\cdot\Bigg[&\left(-2+\log_e \frac{8a}{0.2235(b_m+c)}\right) - \frac{b_f}{b_m}\cdot\left(-2+\log_e \frac{8a}{0.2235(b_f+c)}\right) \\ &- \frac{(b_m+2b_f+2\delta)^2}{2b_f b_m}\cdot\left(-2+\log_e \frac{8a}{0.2235(b_m+2b_f+2\delta+c)}\right) \\ &- \frac{(b_m+2\delta)^2}{2b_f b_m}\cdot\left(-2+\log_e \frac{8a}{0.2235(b_m+2\delta+c)}\right) \\ &+ \frac{(b_m+b_f+2\delta)^2}{b_f b_m}\cdot\left(-2+\log_e \frac{8a}{0.2235(b_m+b_f+2\delta+c)}\right)\Bigg] = 0 \end{aligned} \tag{43}$$

The quantity for which K is an abbreviation cannot equal zero and hence may be ignored in solving this equation. Values having already been chosen for a, b_m, c, N_m, and δ, the only unknown is b_f. The solution is most readily made by numerical substitution. Values of the quantity in brackets are computed for a series of values of b_f, after which, by numerical or graphical interpolation, the value of b_f is found for which the quantity in brackets equals zero. The procedure is tedious but straightforward, and it will now be shown that the range of values of b_f to be tried is a narrow one.

In figure 6, (a) represents a cross section of the movable coil of N_m turns and self inductance L_m; (b) represents the same coil divided in

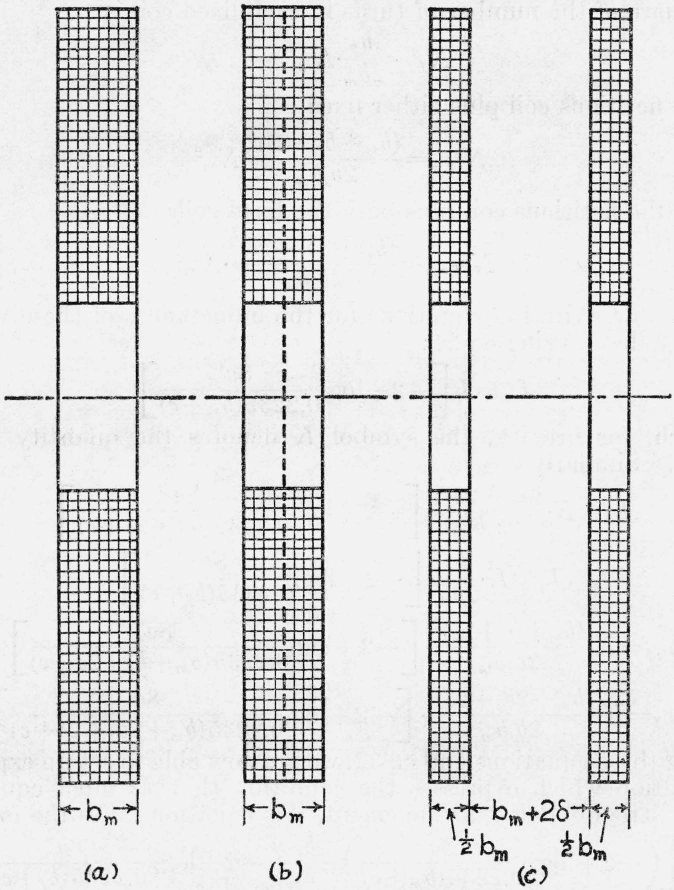

FIGURE 6.—*Development of a pair of fixed coils from a movable coil.*

In (a) is shown a cross section of a movable coil of the relative proportions used in the improved inductor. At (b) the movable coil is assumed to be divided into halves by a plane at right angles to the axis of the coil. At (c) these halves are shown as separated by the proper axial distance to admit placing between them, with suitable clearances, a movable coil of the form shown at (a). It is readily seen that the self inductance of the two halves in series and located as in (c) is less than the self inductance of the undivided coil as shown at (a). In the improved inductor the dimension marked $0.5b_m$ is increased to $0.6b_m$ and the coils are wound with more turns of somewhat larger wire.

halves by an imaginary plane normal to the axis of the coil; and (c) represents the halves separated axially by a distance $b_m+2\delta$ to constitute a pair of fixed coils. In either (b) or (c) the total self inductance of the halves (in series) is the sum of the two separate self inductances plus twice the mutual inductance between the halves. It is evident that the separation of the halves, in going from condition (b) to condition (c), reduces their mutual inductance and, therefore, the total self inductance of the pair. Such a pair of fixed coils complies with the requirement that its resistance should be equal to that of the movable coil, but its inductance will be less than that of the movable coil. To bring this inductance up to the desired value, leaving b_f unchanged, requires more turns of smaller wire. This, however, makes the resistance of the pair of fixed coils larger than that of the movable coil. It is evident that to make $L_F = L_m$ and also to keep $R_F = R_m$ we must make b_f somewhat greater than $b_m/2$, in order that each fixed coil may have more than $N_m/2$ turns of wire suitably larger in cross section than that in the movable coil. In beginning the solution of eq 43 we may, therefore, substitute $b_m/2$ for b_f, after which the substitution of values successively larger by small increments will make the value of the quantity in brackets in eq 43 approach and pass through zero. The value of b_f which makes this function zero, when substituted with N_m and b_m in eq 35, gives the number of turns N_f in each of the pair of fixed coils. The diameter x_f of the wire in these coils, over the insulation, is given by the formula, analogous to eq 28,

$$x_f = (b_f c/N_f)^{\frac{1}{2}} \tag{44}$$

The pair of fixed coils, wound with wire of this overall diameter having the same space factor s as the wire in the movable coil, will satisfy both of the requirements, namely, $R_F = R_m$ and $L_F = L_m$.

The length of a mean turn being the same for the movable coil as for the fixed coils, and the resistance of the movable coil being equal to that of the pair of fixed coils, it follows that the corresponding values of resistance per unit length, q_m and q_f, of the wires used, will have the simple relation

$$q_f/q_m = N_m/2N_f \tag{45}$$

Lest the impression should be given, however, that the fixed coils need to be of much larger wire than the movable coil and have an axial length b_f considerably exceeding $b_m/2$, the following data are given at this place for the inductor designed by the authors in accordance with the procedure outlined in this paper. The value found from eq 43 for b_f was 0.601 b_m, from which, by eq 31, $N_f = 0.548\ N_m$. The value found for the cross section of the wire in the fixed coils, by eq 44, was 1.097 times that of the movable coil; in other words, with wire of a given size in the movable coil, even one gage number larger would be much too large for the fixed coils. This difficulty was met (as will be detailed later) by winding the greater part of each of the fixed coils with wire of the size used for the movable coil, and finishing with wire of the next larger gage size.

6. CALCULATION OF THE MUTUAL INDUCTANCE BETWEEN THE TWO COIL SYSTEMS

It remains to be seen whether the coil proportions as determined by the process above outlined are such as to give a sufficiently high value of coupling coefficient and hence of the related quantities, namely, the mutual inductance between the movable coil and the fixed coils, and the ratio L_u/L_l. This mutual inductance may be computed by the device (see fig. 7) of imagining a fictitious coil ϕ' wound so as to fill completely the small clearance between one of the fixed coils and the movable coil. The calculation must be made in two stages. In the first stage it is assumed that a fictitious fixed coil f' and the fictitious coil ϕ' are wound with wire of the same size [14] as that in the movable coil. It may be shown[15] that the mutual inductance between the left-hand fixed coil in figure 7 and the movable coil is given by the expression

$$M_{f',m} = \tfrac{1}{2} \cdot (L_{f',\phi',m} - L_{f',\phi'} - L_{\phi',m} + L_{\phi'}) \tag{46}$$

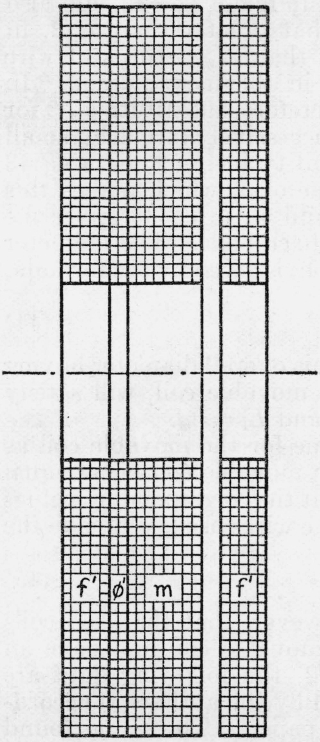

FIGURE 7.—*Cross-sectional view of group of link-shaped coils as used in the improved inductor, and a fictitious coil ϕ'.*

The fictitious coil is assumed to fill completely the space between the coils adjacent to it, and to have the same number of conductors per unit of area of cross section as the latter. As in the case of figure 5, the assumption of such a fictitious coil facilitates the computation of the mutual inductance between the two coils adjacent to it.

in which the terms in parenthesis represent respectively the self inductances of the groups of coils indicated by the subscripts; thus $L_{\phi',m}$ denotes the self inductance of the fictitious coil and the movable coil in series as one coil. The four terms in eq 46 are to be calculated by means of eq 24. The value of mutual inductance $M_{f',m}$ so obtained is higher than the desired value $M_{f,m}$ for the fixed coil wound with the somewhat larger wire which must be used, and must be reduced in the ratio of the number of turns in the actual fixed coil to the number computed for the fictitious fixed coil; that is

$$M_{f,m} = \frac{N_m (b_f/2b_m)^{\frac{1}{2}}}{N_m (b_f/b_m)} \cdot M_{f',m}$$

$$= \left(\frac{b_m}{2b_f}\right)^{\frac{1}{2}} \cdot M_{f',m} \tag{47}$$

This is the mutual inductance between one of the fixed coils and the movable coil, and it must be doubled to obtain the mutual inductance between the pair of fixed coils and the movable coil. Referring to eq 1, and remembering that by design $L_F = L_m$, the coupling coefficient is seen to be

$$k = 2M_{f,m}/L_F \tag{48}$$

If we change the connections so as to make the magnetic flux of the movable coil oppose that of the pair of fixed coils, the coupling coefficient k will have the same numerical value but the opposite sign. The ratio of the maximum self inductance of the coil group to its minimum, by eq 9, is thus

$$\frac{L_{max}}{L_{min}} = \frac{1+k}{1-k} \tag{49}$$

[14] By "same size" for the present purpose is meant the same diameter over the insulation. The significant thing here is that the number of turns per unit area of cross section of the winding shall be the same for all the coils involved in this kind of calculation.

[15] BS Sci. Pap. **8**, 41 (1912) S169; **18**, 469 (1922) S455.

This ratio should be, say 10 percent greater than the desired value of L_u/L_l. If it is not as large as desired, and if the clearance δ cannot be reduced, the only remedy is to use coils which are wider radially and shorter axially than those assumed. That is, with a given value of a, one must start with a movable coil having a larger value of c and a smaller value of b_m. The method is essentially one of trial and error.

7. EXPERIMENTAL CHECK OF THE LINEARITY OF THE SCALE LAW

Coil proportions and wire sizes having been determined for which $L_m=L_F$ and $R_m=R_F$, and for which the coupling coefficient has a sufficiently high value, it remains to be determined whether these coils can be used to form an inductor having a sufficiently good approximation to a linear scale law. This question must be answered by experiment. It is known that the linearity of the scale law is a function of the ratio S/r (see fig. 2). This matter was checked, in the design of the inductor to be described below, in the following manner.

Trial fixed coils and movable coils having the desired proportions were wound of copper strip of widths to give one turn per layer. These coils were mounted in a temporary wooden frame which permitted the dimension S to be varied. The scale law for this temporary instrument was then determined for a number of values of the radius S. The results of three such runs are shown in figure 8, in which the inductance scales of the various runs have been arbitrarily displaced vertically with respect to each other. Curves A and C represent conditions when the ratio S/r was respectively 2 percent greater and 4 percent less than its optimum value. Curve B represents the condition when S/r was within 1 percent of its optimum value. In these curves the circles represent the observed points and the solid lines represent the least-square straight lines which have been drawn, using all the experimental points between the 7.5° point and the 172.5° point, inclusive. It is obvious that curve B represents a better scale law than either of the other two. In order to have a more objective basis for judging the linearity of the scale law, the curve of figure 9 was drawn. A number of curves similar to those of figure 8 were drawn, corresponding to a number of values of the ratio S/r. For each of these curves the least-square deviation, ϵ, of the observed points from the corresponding straight line was computed. The magnitude of ϵ was taken to be a measure of the departure from linearity of the scale law under the given conditions. In figure 9 this quantity, ϵ, is plotted against the ratio S/r and is expressed as a fraction of a scale division, each scale division being 7.5° long. It is seen that ϵ goes through a decided minimum between the values of S/r of 2.24 and 2.26, a result which closely confirms the corresponding result of Brooks and Weaver.[16]

[16] BS Sci. Pap. **13**, 575 (1916–17) S290. The ratio designated as R/r in this earlier paper corresponds to that which is called S/r in the present paper.

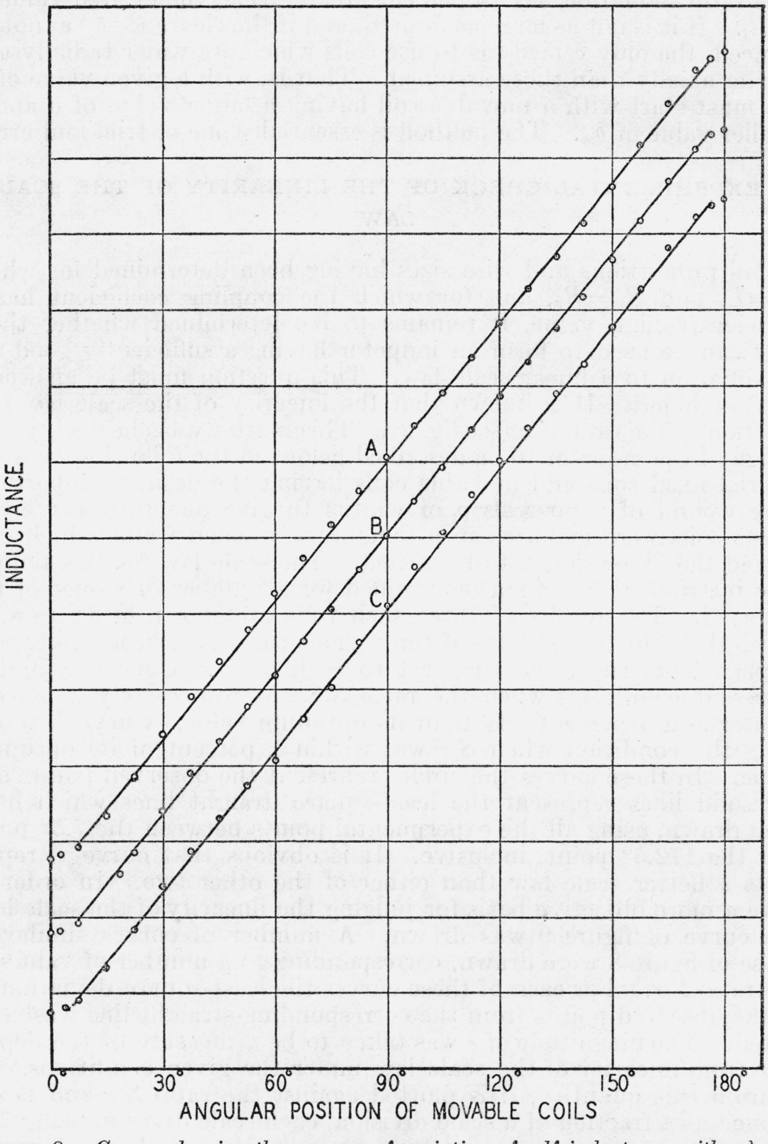

FIGURE 8.—*Curves showing the manner of variation of self inductance with relative angular position of fixed coils and movable coil, as affected by the relative spacing S/r of the two groups of coils.*

(See fig. 2 for the significance of S and r.)

In the case of curves A and C the ratio S/r was respectively 2 percent greater and 4 percent less than its optimum value. In the case of curve B this ratio was within 1 percent of its optimum value, namely, 2.25. See also figure 9.

The deviations from linearity, shown by the plotted points, are characteristic of systems of coils of the given form, and are not the result of errors in the measurements. Furthermore, it may readily be seen that the slope of all curves of this kind must become zero for the angular positions 0 and 180°.

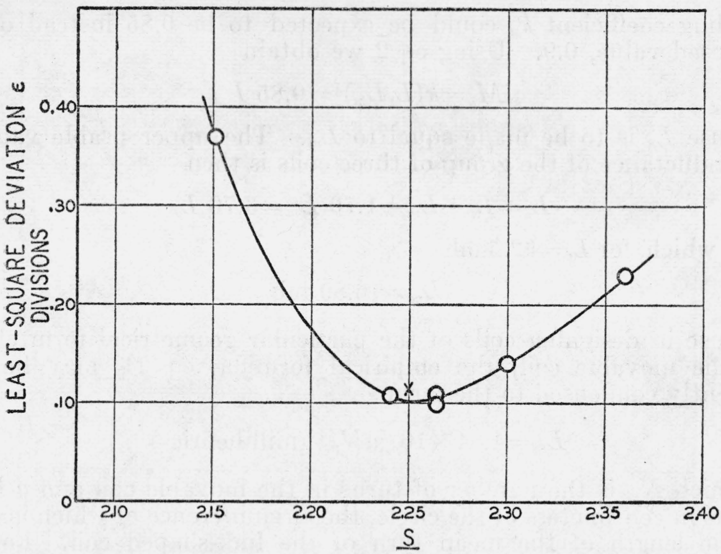

FIGURE 9.—*Curve showing the least-square deviation from linearity of curves similar to those in figure 8, as a function of the relative spacing S/r of the two groups of coils.*

The points plotted as circles were determined by measurements on the experimental inductor with coils wound with copper strip. The point marked *x* relates to the improved inductor, with wire coils, shown at the left in figure 10.

IV. DESIGN OF COILS FOR THE NEW INDUCTOR

It was proposed to construct a variable inductor to have a ratio L_u/L_l of at least 10. Because $L_{\max}$ is somewhat greater than L_u, and $L_{\min}$ somewhat smaller than L_l, $L_{\max}/L_{\min}$ would need to be at least 11. To provide a liberal allowance to cover unavoidable inaccuracies in winding the coils, failure to match exactly the inductances of the fixed system and the movable system, etc., coil proportions were determined, in the manner above outlined, which gave a calculated coupling coefficient of 0.9 at the point of maximum mutual inductance. By eq 7 this value of coupling coefficient would give $L_{\max}/L_{\min}=19$.

Referring to figure 2, a value of $r=3.14$ cm was chosen as a basic dimension, in terms of which the other significant dimensions were found to be as follows:

$$c=1.24r;\ d=2.06r;\ S=2.25r;\ b_m=0.318r;\ b_f=0.191r.$$

The separation which we planned to have between each fixed coil and the movable coil was 0.08 cm (1/32 inch).

The desired maximum usable value of self inductance, L_u, was 125 mh. For simplicity it was assumed initially that we had to design one movable coil and a pair of fixed coils to give a maximum usable value of 62.5 mh. Measurements made on the experimental copper-strip inductor indicated that the maximum usable value of the coupling coefficient k_u could be expected to be 0.85 instead of the proposed value, 0.9. Using eq 2 we obtain

$$M_u = k(L_F L_m)^{\frac{1}{2}} = 0.85\ L_m$$

because L_F is to be made equal to L_m. The upper usable value of self inductance of the group of three coils is then

$$L_u = L_F + L_m + 1.70\ L_m = 3.70\ L_m$$

from which, for $L_u = 62.5$ mh

$$L_m = 16.89 \text{ mh}$$

For use in designing coils of the particular geometrical form chosen for the movable coil, the empirical formula, eq 24, may be conveniently condensed to the form

$$L_m = 1.84 \times 10^{-5} a N_m^2 \text{ (millihenries)} \tag{50}$$

in which N_m is the number of turns in the movable coil and a is the radius in centimeters of the circle, the circumference of which is equal to the length of the mean turn of the link-shaped coil. For the proposed movable coil $a = 5.20$ cm, $L_m = 16.89$ mh. Substituting these values in the preceding formula, we get

$$N_m = 420$$

Using eq 31 we have, since $b_f/b_m = 0.601$,

$$N_f = 230$$

The cross-sectional area of the movable coil is $10 \times 39 = 390$ mm^2, from which the area of the square circumscribing the insulated wire must be $390/420 = 0.929$ mm^2, and hence the diameter of the insulated wire is 0.964 mm. Number 20 AWG [17] silk-enamel wire is enough smaller than this to allow for unavoidable irregularities of winding, and was used for the movable coil. The resistance per meter of no. 20 copper wire at 25° C is 0.034 ohm $= q_m$. By eq 45, the resistance per meter of the wire to be used for the fixed coils should be

$$\begin{aligned} q_f &= N_m q_m / 2N_f \\ &= (420 \times 0.034)/460 \\ &= 0.0391 \text{ ohm/meter.} \end{aligned}$$

This value calls for a wire having a cross-sectional area only 15 percent larger than no. 20. The next larger gage size, no. 19,[18] has a cross-sectional area 1.26 times that of no. 20. To avoid the necessity for drawing and insulating wire of a nonstandard intermediate size, each fixed coil was wound with no. 19 and no. 20 wires, with silk-enamel insulation, in such proportions as to fill the given cross section for winding and to have the desired resistance and number of turns. Each fixed coil was accordingly wound with 77 turns of no. 20 wire and 38 turns of no. 19.

[17] The diameter of no. 20 AWG bare wire is 0.032 in. (0.81 mm). The diameter over the insulation (enamel and one wrapping of silk) is 0.0363 in. (0.92 mm).

[18] The diameter of no. 19 AWG bare wire is 0.036 in. (0.91 mm). The diameter over the insulation (enamel and one wrapping of silk) is 0.0403 in. (1.02 mm).

V. DESCRIPTION OF THE NEW INDUCTOR

Coils as described above were wound and impregnated at this Bureau, after which, through the cooperation of the Leeds & Northrup Co., they were mounted in Bakelite plates and assembled to form a working instrument. It is shown at the left in figure 10, in which an inductor of the previous form is shown at the right for comparison. It may be seen that the general construction of the new inductor is identical with that of the older one. The principal structural data for the two inductors are as follows:

	Previous form	New form
Dimensions of fixed plates	245 mm square	281 mm square
Dimensions of movable plate	262 mm diameter	298 mm diameter
Weight of inductor, complete	4.35 kg (9.6 lb)	6.6 kg (14.6 lb).

The effect of increasing the dimension *c* of the coils (see fig. 2) is thus to increase the space occupied by the inductor by the factor 1.3 and its weight by the factor 1.5.

VI. PERFORMANCE OF THE NEW INDUCTOR

As measured in the completed instrument, the fixed system and the movable system of the new inductor have the following characteristics:

Inductance of four fixed coils in series = 35.10 mh.
Inductance of two movable coils in series = 35.57 mh.
Difference in inductance = 1.3 percent.

Resistance of four fixed coils in series = 9.52 ohms.
Resistance of two movable coils in series = 9.62 ohms.
Difference in resistance = 1.0+ percent.

$$L_{max} = 130.0_7 \text{ mh.}$$

$$L_{min} = 11.2_5 \text{ mh.}$$

These differences in inductance and reistance seem to indicate that there should have been a slightly greater proportion of no. 20 wire and hence more turns in the fixed coils. The values of L_{max} and L_{min} indicate a coupling coefficient at the positions of maximum and minimum inductance of 0.84, which is smaller than the value 0.9 predicted by calculation. This lower value is not the result of inaccuracy in the calculation but arises chiefly from the fact that in this experimental model the clearances between the coils are greater than we had planned to use. Study of the problem from the manufacturing standpoint will be necessary to determine the permissible amount of clearance in inductors as produced commercially.

The edge of the disk which carries the movable coils in the new inductor is graduated from 0 to 360° in 0.5° divisions. This kind of graduation was convenient for this experimental inductor but would not necessarily be applied to inductors manufactured for sale. The fiducial mark is so located on the upper (stationary) plate that the readings 90 and 270° correspond to zero mutual inductance between the two coil systems, and hence the points 0 and 180° correspond to the minimum and the maximum values respectively of the self inductance.

The plotted points of figure 11 show the self inductance of the new inductor, in the series connection, as a function of the relative position of the two coil systems. To bring out the close approach to linearity,

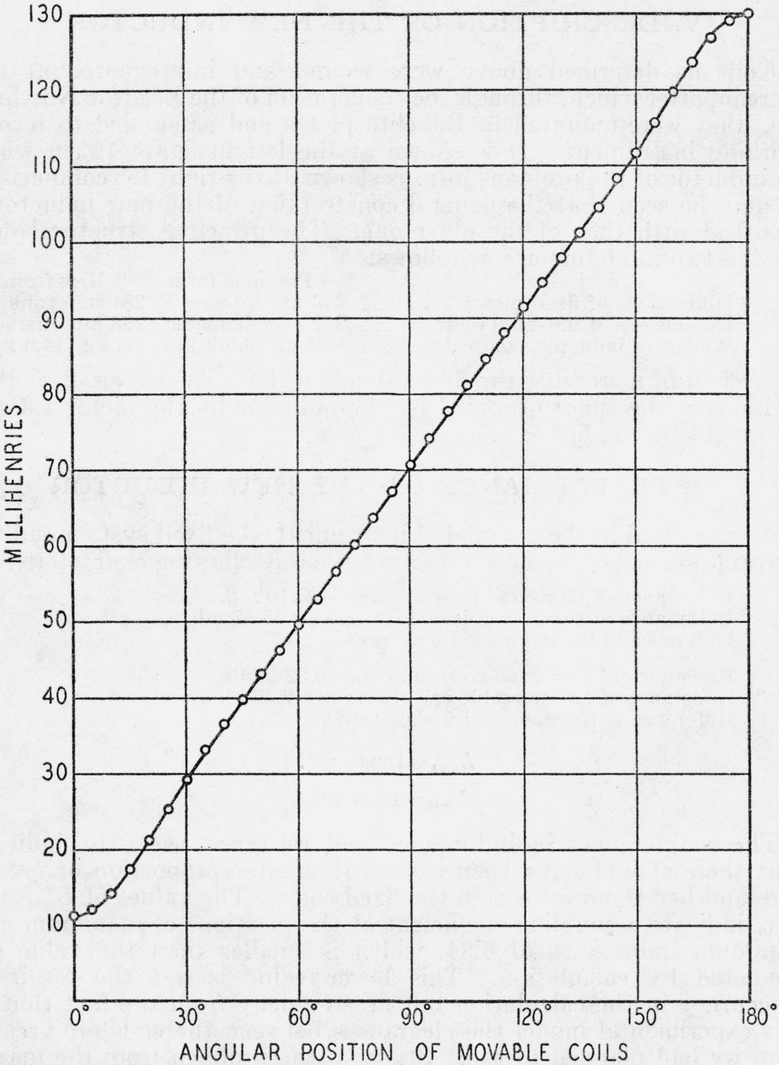

FIGURE 11.—*Curve showing the self inductance of the improved inductor as a function of the relative angular position of the movable coils with respect to the fixed coils.*

the part of the curve for abscissas from 30 to 150°, inclusive, has been determined by the method of least squares as the straight line which most nearly fits the observed points. The approach to linearity is seen to be very good. Below 30° and above 150° the rate of change in the slope is gradual enough to make it feasible to subdivide the scale down to 5° and up to 175°. The plotted points are those corresponding to bridge measurements in which the frequency of the current was 60 c/s. Measurements at 1,000 c/s were also made for the angular positions 5, 30, 60, 90, 120, 150, and 175°. The measured values of inductance for this higher frequency exceeded those made at the lower frequency by 0.00 to 0.11 mh; in other words, by amounts comparable

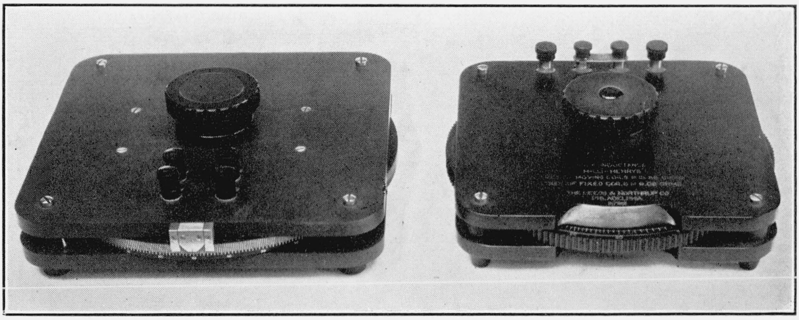

FIGURE 10.—*The improved inductor (at the left) and the older form.*

The latter is graduated to read directly in millihenries. The new inductor, being an experimental model, has a scale of angular degrees.

with those which might be caused by errors of 0.0 to 0.15° in setting the movable disk to a specified angular position with respect to the fixed coils. For practical purposes the effect of the change from 60 to 1,000 c/s may therefore be considered inappreciable.

Additional bridge measurements of the inductance, at the above-mentioned seven angular positions of the movable coils, were made at 60 and 1,000 c/s with the fixed system of coils in parallel with the movable system. The results, given in table 1, show that the change to the parallel connection reduces the inductance to one-fourth of the corresponding value for the series connection, within an amount which is less than the probable error of the bridge measurements. The last column gives the computed differences in inductance which arise from the fact that the inductance and the resistance of the movable system of coils are not exactly equal to those of the fixed system. These differences are far below the probable errors of the bridge measurements and the differences in inductance which would result from the inevitable small errors of setting the relative angular position of the two coil systems.

TABLE 1.—*Measured self inductance of improved variable inductor in series connection and in parallel connection*

Scale setting	Coils in series	$\frac{L_{series}}{4}$	Coils in parallel	Observed difference	Computed difference
Degrees	mh	mh	mh	mh	mh
0	11.26				
5	12.04	3.01	3.00	0.01	0.00_0
30	29.37	7.34	7.33	.01	$.00_0$
60	49.66	12.42−	12.41	.01−	$.00_1$
90	70.69	17.67	17.67	.00	$.00_1$
120	91.52	22.88	22.87	.01	$.00_1$
150	111.79	27.95	27.94+	.01−	$.00_2$
175	129.32	32.33	32.30	.03	$.00_5$
180	130.07				

Inspection of the curve of figure 11 shows that the scale law of the new inductor is sufficiently linear to be considered usable over the range 5 to 175°. From table 1 it appears that for these limiting positions L_u/L_l=129.32/12.04=10.7. The time-constant of the new inductor, for the 175° position of the coil systems, is 6.7 milliseconds. The use of a molded plate for supporting the movable coils, which would be feasible and economical for inductors in regular production, would probably permit a reduction of the clearances, and in consequence, a somewhat higher value of L_u/L_l and of the time-constant.

VII. CONCLUSION

The approach to linearity of the self inductance of the new inductor, as a function of the relative position of the coils, is as good as that of the older inductor. The ratio L_u/L_l in the new inductor is 10.7, as compared with 7.4 for the older model. The time-constant of the new inductor, for the upper limiting value of the self inductance, is 6.7 milliseconds as compared with 4.5 milliseconds for the older instrument. This increase in time-constant, however, has been obtained at the expense of an almost equal relative increase in the weight of the inductor. If it should be found feasible, in manufacture, to reduce the clearances to more nearly the values aimed at in the design, this reduction will increase both L_u/L_l and the time-constant without any increase in the weight.

The writers express their appreciation to Dr. Frederick W. Grover for valuable suggestions during their study of the problem; for his careful reading of the manuscript and his suggestions for its improvement. They are grateful to the Leeds and Northrup Co. for the construction of the framework of the instrument and the mounting of the coils, and to Miss C. M. Sparks for the very careful series of measurements which forms the basis of table 1 and of figure 11.

WASHINGTON, September 8, 1937.