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GRAPHICAL COMPUTATION OF STRESSES FROM STRAIN DATA

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ABSTRACT

The arithmetic involved in the use of the analytical solution for principal stresses in terms of strains on four intersecting gage lines 45° apart is very tedious if these stresses are computed at many locations. This paper presents graphs which materially reduce the time and labor necessary for such computations. There are presented, also, graphs by means of which normal and shearing stresses on oblique planes are readily obtained from principal stresses. The use of the various graphs is illustrated by numerical examples.

The time required to compute principal stresses, and other normal stresses and shearing stresses, from strain data, in the case of plane stress, may be considerably reduced through use of graphs. Such graphs are presented in this paper, and numerical examples illustrating their use are given.

Figure 1 shows a plate under the action of loads on its boundary. Suppose it is required to determine the state of stress at any location 0. The lines U and V are the directions of the maximum and of the minimum principal stresses, respectively. The lines 1, 2, 3, and 4 are the directions of gage lines intersecting at 45° as shown, along which strain readings are taken. Let ϵ_1, ϵ_2, ϵ_3, and ϵ_4 be the components of strain along these gage lines. Then with the value for Poisson's ratio of 1/3, which probably is adequate for most structural materials, the principal stresses are given by the formulas:[1]

$$\sigma_u=\frac{3}{8}E\left[\epsilon_1+\epsilon_2+\epsilon_3+\epsilon_4+\sqrt{(\epsilon_1-\epsilon_3)^2+(\epsilon_2-\epsilon_4)^2}\right] \tag{1}$$

for the maximum principal stress, and

$$\sigma_v=\frac{3}{8}E\left[\epsilon_1+\epsilon_2+\epsilon_3+\epsilon_4-\sqrt{(\epsilon_1-\epsilon_3)^2+(\epsilon_2-\epsilon_4)^2}\right] \tag{2}$$

for the minimum principal stress, where E is Young's modulus of elasticity.

These equations may be put in the form

$$\sigma_u=\frac{3}{8}E(A+B), \tag{3}$$

$$\sigma_v=\frac{3}{8}E(A-B), \tag{4}$$

where

$$A=\epsilon_1+\epsilon_2+\epsilon_3+\epsilon_4, \tag{5}$$

$$B=\sqrt{(\epsilon_1-\epsilon_3)^2+(\epsilon_2-\epsilon_4)^2}. \tag{6}$$

[1] W. R. Osgood, *Determination of principal stresses from strains on four intersecting gage lines 45° apart.* J. Research NBS **15**, 579 (1935) RP851.

The directions of the principal stresses are given by the relation [2]

$$\tan 2\theta = -\frac{\epsilon_2 - \epsilon_4}{\epsilon_1 - \epsilon_3}, \tag{7}$$

where θ (fig. 1) is the angle measured positive counterclockwise from the direction of σ_u to gage line 1.

The quantity A of equation 5 is obtained directly by addition of the measured strains ϵ_1, ϵ_2, ϵ_3, and ϵ_4.

The quantity B of equation 6 and the angle θ of equation 7 can be obtained graphically from figure 2 as follows. Find the point the rectangular coordinates of which are $|\epsilon_1 - \epsilon_3|$ and $|\epsilon_2 - \epsilon_4|$.[3] The polar coordinates of this point are B [4] and $|\theta|$. The value of B may be read on either of the scales for the rectangular coordinates by following a circle from the point to either of the axes. The value of $|\theta|$ may be read on one of the two scales for θ. The table at the top of figure 2, read vertically, indicates for the various combinations of signs of $\epsilon_1 - \epsilon_3$ and $\epsilon_2 - \epsilon_4$ whether the inside scale, θ_i, or the outside scale, θ_0, is to be used, and gives the sign of θ.

Figure 3 is a nomograph for determining the principal stresses, σ_u and σ_v, after the values A and B have been determined, for the case E=28,600. Similar graphs may be drawn corresponding to any value of E. The maximum principal stress, σ_u, is obtained in the following way. Locate $A \times 10^5$ on the left-hand scale, $B \times 10^5$ on the upper half (marked "*FOR* σ_u") of the right-hand scale, and connect the two points on the scales with a straight line. This line intersects the middle scale at the value of σ_u. The minimum principal stress, σ_v, is obtained in the same way, except that $B \times 10^5$ is located on the lower half (marked "*FOR* σ_v") of the right-hand scale.

There are several ways to construct one nomograph which will give σ_u and σ_v for all values of E, but it is more convenient to construct a separate nomograph, such as is shown in figure 3, for each value of E. A nomograph for a definite value of E can be made as follows. Axes for A, B, and σ are drawn parallel to each other, the axes of A and B on opposite sides of the axis of σ and equidistant from it. Zeros for the three axes are laid off on the same straight (horizontal) line. The graduations on all three scales are uniform and the same on the A and B scales. These scales are laid off with 100 graduations on each side of their zeros. Since the σ scale is also uniformly graduated, it is necessary to locate only one point other than the zero on the σ axis. This may be done by solving equation 3 for A, with $A=B$, E equal to the desired value of the modulus of elasticity, and σ_u chosen equal to any convenient value. A line connecting the values of A and B ($A=B$), thus computed, intersects the σ axis at the chosen value of σ_u and thus locates the necessary point for graduating the σ scale.

Suppose, for example, it were desired to regraduate the σ scale of figure 3 to adapt it for E=30,000. To locate the value $\sigma=10$ on the scale, we substitute $\sigma_u=10$, $A=B$, and E=30,000 in equation 3 and solve for A, getting $A=B=44.4 \times 10^{-5}$. A line connecting 44.4 on

[2] See footnote 1.

[3] $|\epsilon_1 - \epsilon_3|$ and $|\epsilon_2 - \epsilon_4|$ are first multiplied by some power of 10 (the same for each) to bring the values within the range of the scales. The value of B, subsequently read from the chart, must be divided by this same power of 10.

[4] See footnote 3.

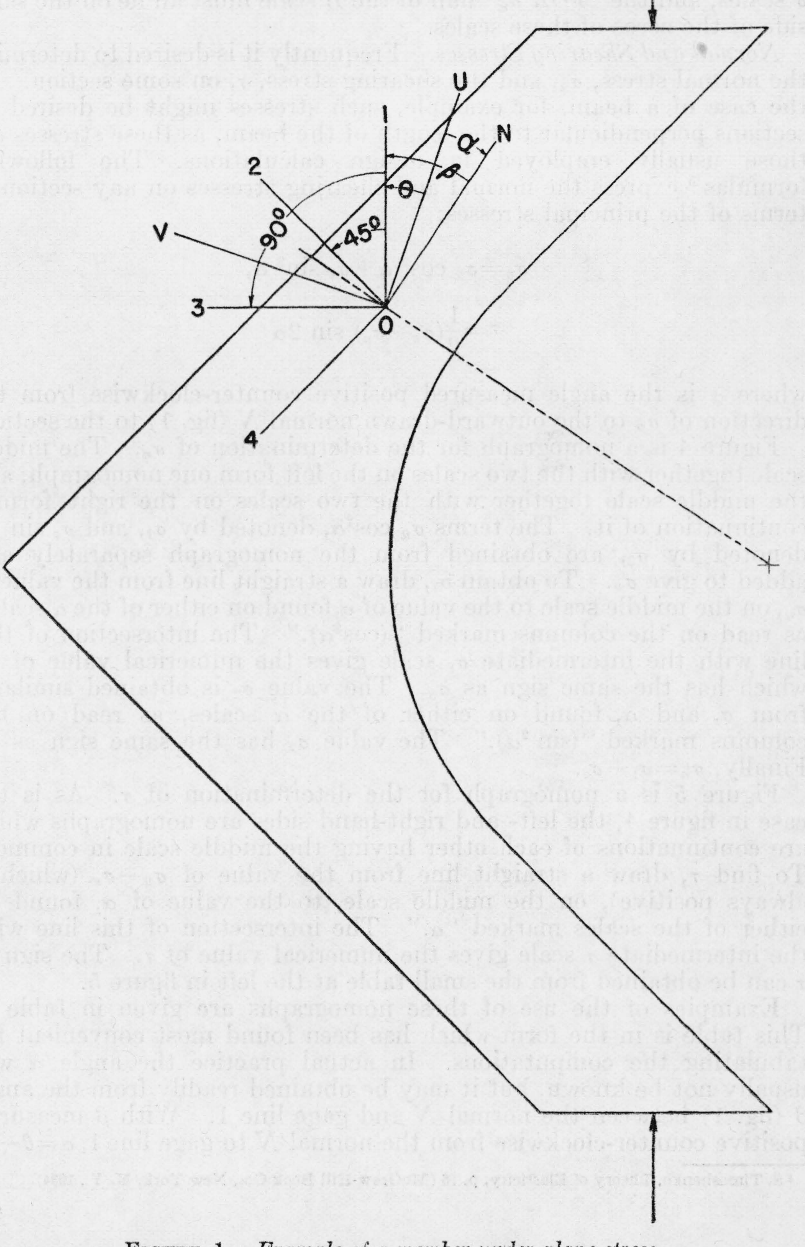


FIGURE 1.—*Example of a member under plane stress.*

Four gage lines (1, 2, 3, 4), intersecting at O. Directions (U, V) of principal stresses, and direction (N of normal to any section. As shown above, the angle α is negative, the angles β and θ, positive the A scale with 44.4 on the B scale intersects the σ scale at a point to be marked 10. The space between 0 and 10 on the σ axis is divided into 10 equal parts, and the graduations extended in both directions and subdivided as may be desired. The positive halves of the A and σ scales, and the "*FOR* σ_u" half of the B scale must all lie on the same side of the zeros of these scales.

Normal and Shearing Stresses.—Frequently it is desired to determine the normal stress, σ_n, and the shearing stress, τ, on some section. In the case of a beam, for example, such stresses might be desired on sections perpendicular to the length of the beam, as these stresses are those usually employed in design calculations. The following formulas [5] express the normal and shearing stresses on any section in terms of the principal stresses:

$$\sigma_n=\sigma_u \cos^2 \alpha+\sigma_v \sin^2 \alpha, \tag{8}$$

$$\tau=\frac{1}{2}(\sigma_v-\sigma_u) \sin 2\alpha \tag{9}$$

where α is the angle measured positive counter-clockwise from the direction of σ_u to the outward-drawn normal N (fig. 1) to the section.

Figure 4 is a nomograph for the determination of σ_n. The middle scale together with the two scales on the left form one nomograph, and the middle scale together with the two scales on the right form a continuation of it. The terms $\sigma_u \cos^2\alpha$, denoted by σ_1, and $\sigma_v \sin^2\alpha$, denoted by σ_2, are obtained from the nomograph separately and added to give σ_n. To obtain σ_1, draw a straight line from the value of σ_u, on the middle scale to the value of α,found on either of the α scales, as read on the columns marked "($\cos^2\alpha$)." The intersection of this line with the intermediate σ_1 scale gives the numerical value of σ_1, which has the same sign as σ_u. The value σ_2 is obtained similarly from σ_v and α, found on either of the α scales, as read on the columns marked "($\sin^2\alpha$)." The value σ_2 has the same sign as σ_v. Finally, $\sigma_n=\sigma_1+\sigma_2$.

Figure 5 is a nomograph for the determination of τ. As is the case in figure 4, the left- and right-hand sides are nomographs which are continuations of each other having the middle scale in common. To find τ, draw a straight line from the value of $\sigma_u-\sigma_v$ (which is always positive), on the middle scale, to the value of α, found on either of the scales marked "α." The intersection of this line with the intermediate τ scale gives the numerical value of τ. The sign of τ can be obtained from the small table at the left in figure 5.

Examples of the use of these nomographs are given in table 1. This table is in the form which has been found most convenient for tabulating the computations. In actual practice the angle α will usually not be known, but it may be obtained readily from the angle β (fig. 1) between the normal N and gage line 1. With β measured positive counter-clockwise from the normal N to gage line 1, $\alpha=\theta-\beta$.

[5] S. Timoshenko, Theory of Elasticity, p. 16 (McGraw-Hill Book Co., New York, N. Y., 1934).

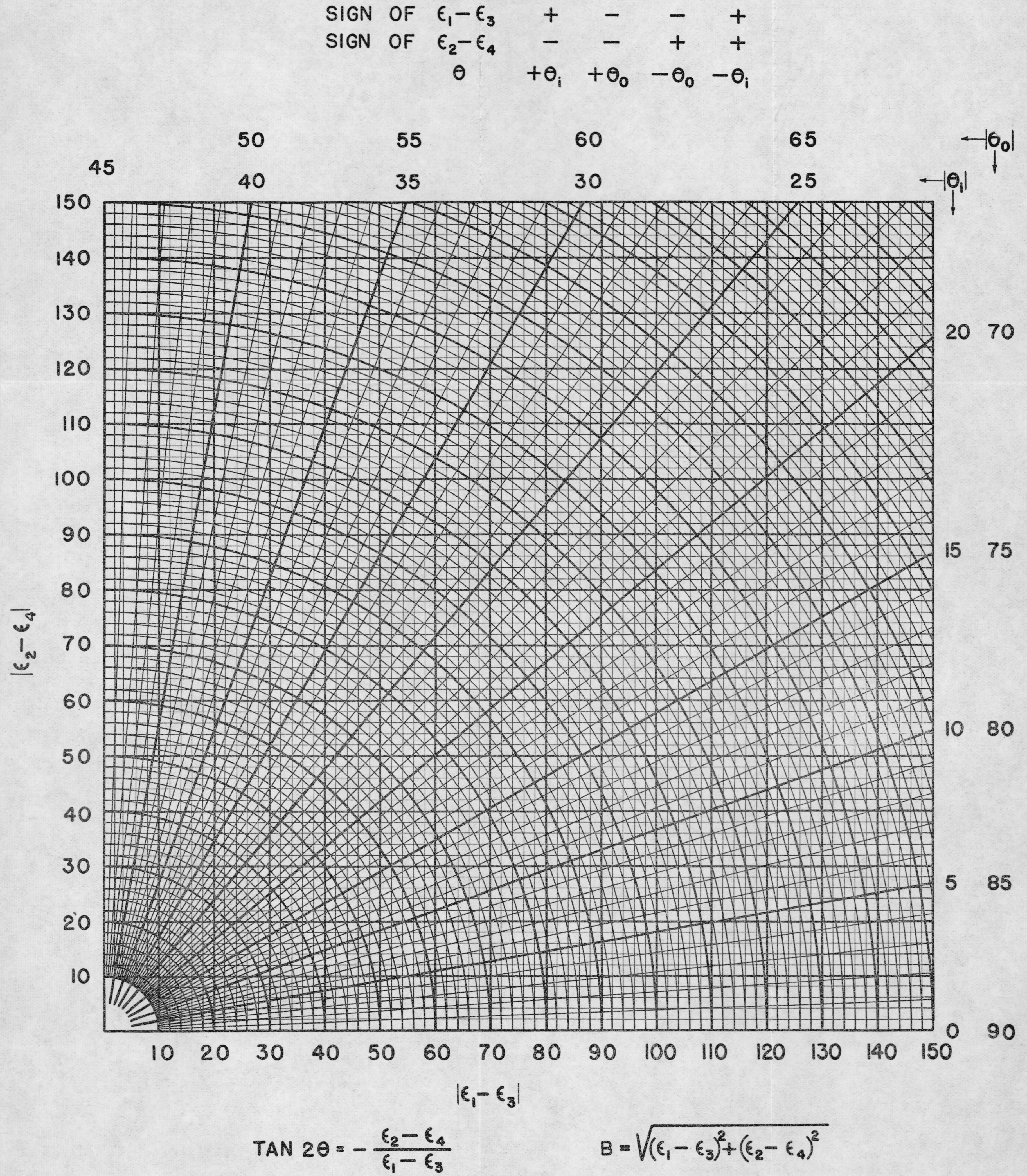


FIGURE 2.—*Graph for* $B=\sqrt{(\epsilon_1-\epsilon_3)^2+(\epsilon_2-\epsilon_4)^2}$ *and for* θ.

12341—37 (Face p. 440) No. 1

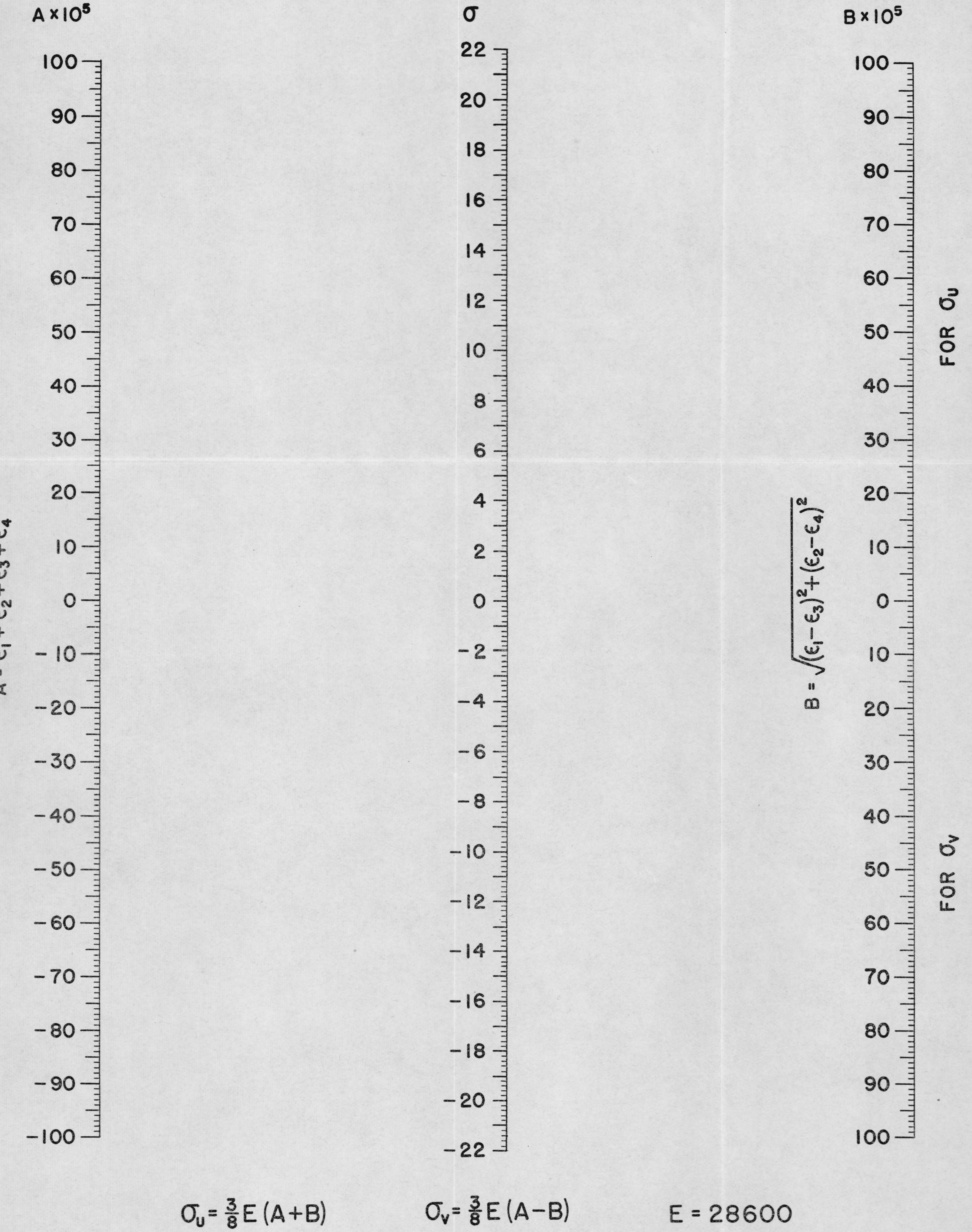
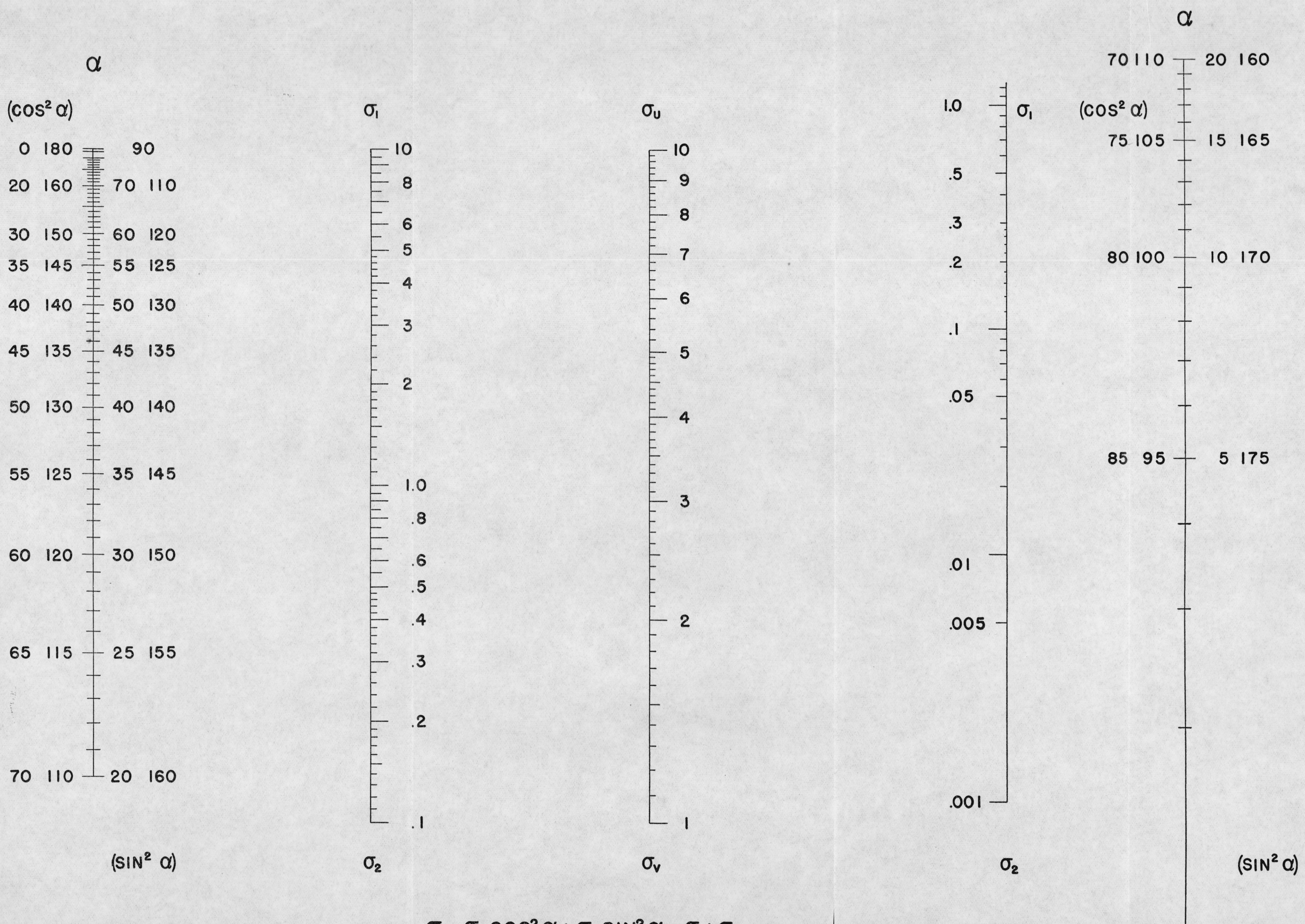


FIGURE 3.—*Graph for principal stresses, σ_u and σ_v*



$$\sigma_N = \sigma_U \cos^2\alpha + \sigma_V \sin^2\alpha = \sigma_1 + \sigma_2$$

FIGURE 4.—*Graph for normal stress σ_n.*

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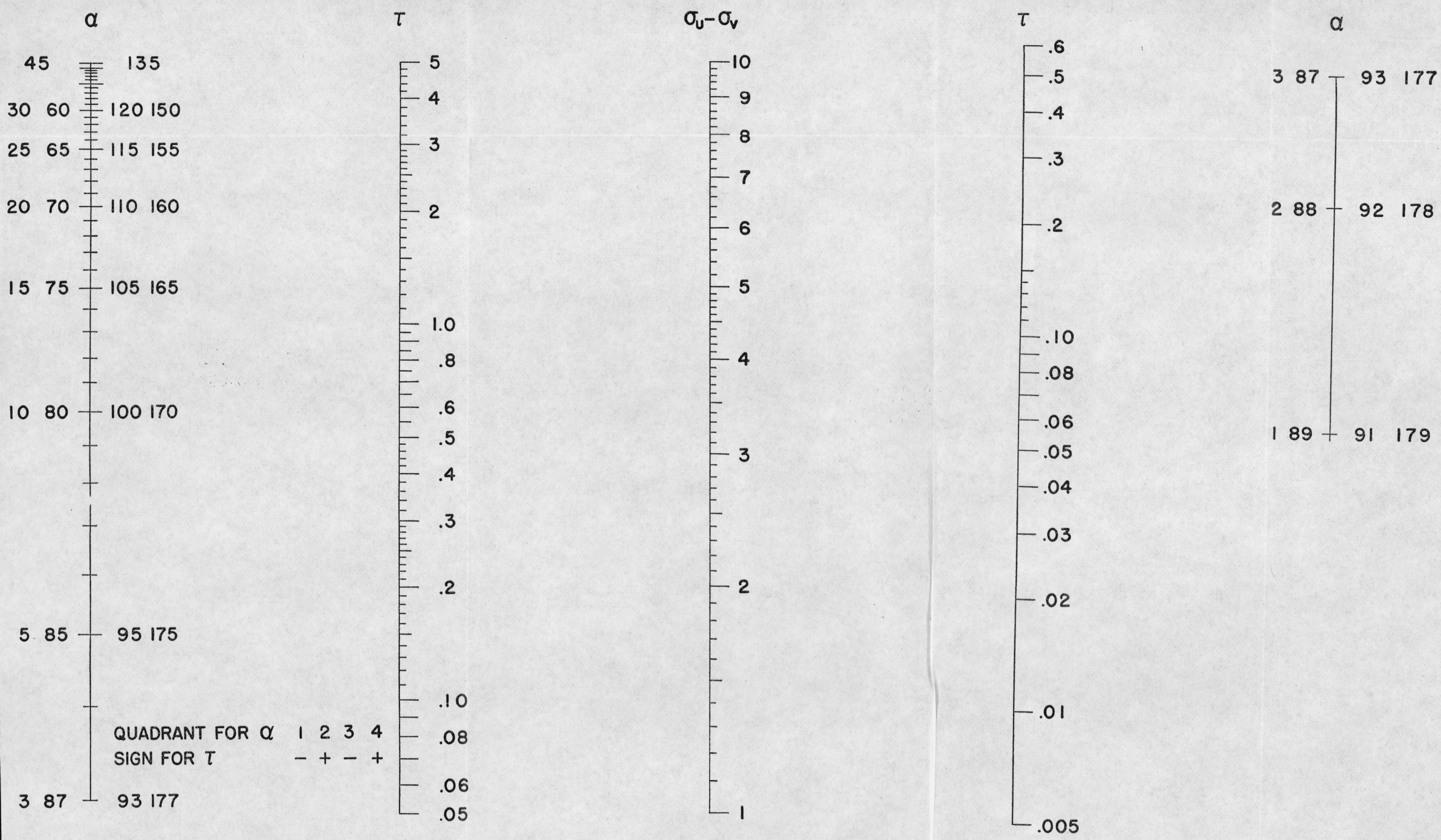

$$\tau = \frac{1}{2}(\sigma_V - \sigma_U)\ \mathrm{SIN}\ 2\alpha$$

FIGURE 5.—*Graph for shearing stress τ.*

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TABLE 1.—*Examples of calculations of stresses from strain data by means of the charts*

[E=28,600 kips/in.^{2}]

Quantity \ Example→	1	2	3	4
ϵ_1 (observed)	+0.00017	+0.00038	−0.00031	−0.00039
ϵ_2 (observed)	+.00008	−.00003	−.00016	+.00021
ϵ_3 (observed)	−.00027	−.00034	+.00044	−.00007
ϵ_4 (observed)	−.00017	+.00007	+.00028	−.00065
$\epsilon_1-\epsilon_3$	+.00044	+.00072	−.00075	−.00032
$\epsilon_2-\epsilon_4$	+.00025	−.00010	−.00044	+.00086
$A=\epsilon_1+\epsilon_2+\epsilon_3+\epsilon_4$	−.00019	+.00008	+.00025	−.00090
B (fig. 2)	.00051	.00073	.00087	.00092
σ_u, kips/in.^{2} (fig. 3)	+3.4	+8.7	+12.0	+.1
σ_v, kips/in.^{2} (fig. 3)	−7.5	−7.0	−6.7	−19.5
$\sigma_u-\sigma_v$	10.9	15.7	18.7	19.6
$\tau_{max}=\frac{1}{2}(\sigma_u-\sigma_v)$ kips/in.^{2}	5.4	7.8	9.3	9.8
θ, degrees (fig. 2)	−14.8	+4.0	+74.8	−55.2
β (from layout), degrees	+45.0	+45.0	+45.0	+122.0
$\alpha=\theta-\beta$, degrees	−59.8	−41.0	+29.8	−177.2
σ_1, kips/in.^{2} (fig. 4)	+.9	+5.0	+9.0	+.1
σ_2, kips/in.^{2} (fig. 4)	−5.6	−3.0	−1.7	0
$\sigma_n=\sigma_1+\sigma_2$, kips/in.^{2}	−4.7	+2.0	+7.3	+.1
τ, kips/in.^{2} (fig. 5)	+4.7	+7.7	−8.1	−.9

WASHINGTON, August 10, 1937