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DETERMINATION OF THE BRINELL NUMBER OF METALS

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ABSTRACT

The procedure used in making Brinell tests must be closely controlled in order that two observers testing a given metal at different locations should obtain Brinell numbers that are in close accord. Small variations in testing procedure will be inevitable so that it becomes important to know the effect of these variations on the magnitude of the Brinell number obtained. The present paper considers the effect on the Brinell number of such variations with the help of data available in the literature supplemented by new tests wherever the existing data seemed deficient. Attention is given to the effect on the Brinell number of variations in testing procedure, i. e., rate of applying load, time under nominal load, error in load, and error in measuring the diameter of indentation. The effect of variables residing in the specimen is discussed next under the separate heads of nonuniform properties, curvature of surface, thickness, spacing of indentations, and angle between load line and normal to specimen. Variations in the type of ball used were considered last, particular attention being paid to differences in elastic deformation and in permanent compression of the ball under load. The paper concludes with recommendations for a test procedure which would lead to greater concordance in the Brinell numbers obtained by different observers using a ball of given diameter on a specimen of given metal.

CONTENTS

*Deceased.

I. INTRODUCTION

1. PURPOSE OF THIS PAPER

The Brinell test came into common use soon after its introduction by the Swedish engineer, J. A. Brinell,[1] in 1900. An extensive literature dealing with the test has grown up, but it has not yet been possible to arrive at an understanding of the test that will allow one to predict the Brinell number of a given material from its other physical properties. The chief value of the Brinell test, just as that of other indentation tests, lies in its simplicity, in the fact that it measures a combination of properties that has proved to be significant in the choice of metals, and that it can be used to check the uniformity of a given product by making a few indentations in that product and measuring their diameters.

It follows from the lack of a basic understanding of the Brinell test that it can be expected to give concordant results in the hands of different operators at different locations only if the test conditions are closely controlled and if the effect on the Brinell number of small changes in these conditions is understood. It is the purpose of this paper to bring together, and in places to supplement, present knowledge in regard to the effect on the Brinell number of small variations in the several variables entering into its determination. It is hoped that this will assist in further developing a standard procedure for Brinell testing. A short description of the Brinell test will aid in understanding the nature of these variables.

2. DESCRIPTION OF BRINELL TEST

(a) TEST FIXTURE

The apparatus necessary for a Brinell test consists of a machine for making an indentation with a sphere under a known load, and means for measuring the diameter of that indentation.

The machine that was used in making the tests at the National Bureau of Standards described in this paper is shown in figure 1. It consists of a heavy cast-iron frame A, an adjustable anvil B, and a hydraulic press C. The specimen is placed on the anvil B and is brought into contact with a 10-mm ball attached to plunger D, which, in turn, is connected to the ram of the hydraulic press. Hand pump E is used to force oil into the hydraulic press. The resulting compressive force set up between the ball and the specimen may be read off approximately on pressure gage F. With continued pumping the pressure will increase until it is just sufficient to lift the crossbar G and the dead weights H, which are connected to the pressure chamber by a ball piston without packing. The pressure in the cylinder and, therefore, the force exerted by the ball will remain practically con-

[1] Communications Congrès International des Méthodes d'Essai des Matériaux de Construction, Paris, 2, 83-94. (1900.)

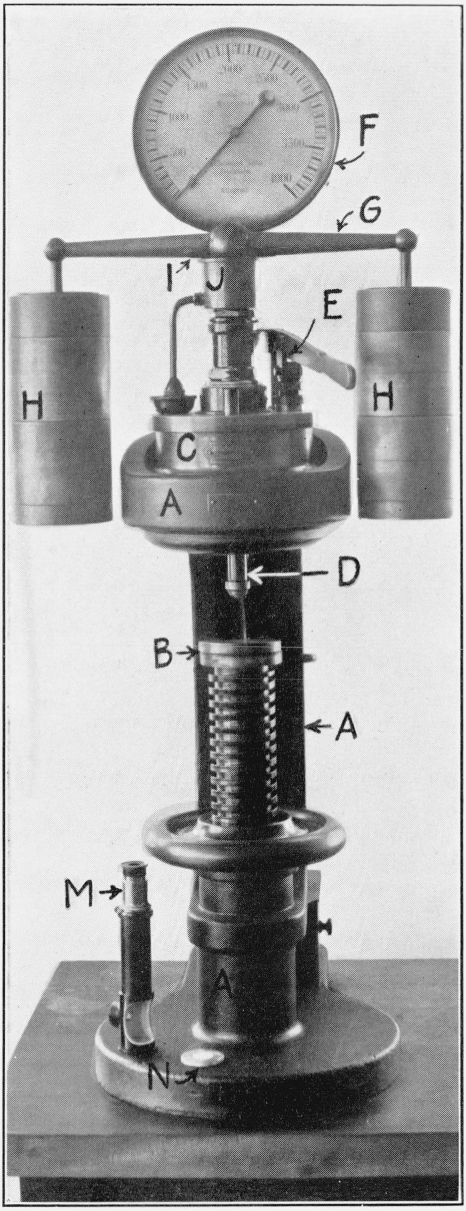

FIGURE 1.—*Machine for making Brinell test and microscope for measuring diameter of indentation.*

stant as long as the piston remains raised to a position of floating equilibrium.

This machine was chosen because investigation showed that the motion of the indenting tool had no appreciable lateral play and differences caused by frictional forces in the ball piston were too small to be observed in calibrating with a proving ring. Previous tests with an improvised dead-weight Brinell machine had shown that errors as great as 5 or 10 percent could be produced by very small rocking of the ball under load.

A load of 3,000 kg is commonly used for metals having a Brinell number greater than 100 and one of 500 kg for metals having a Brinell number less than 100.

Figure 1 also shows the Brinell microscope M, that was used in most of the tests at the National Bureau of Standards to measure the diameters of the Brinell indentations. This microscope has a fixed 7-mm scale, which may be read directly to 0.1 mm and to 0.01 mm, by estimation.

(b) BRINELL FORMULA

The Brinell number was defined by Brinell as the average axial stress over the surface A of the indentation produced by a steel ball, assuming that surface to be spherical, as it would be for an infinitely rigid spherical ball. This leads to the formula

$$H=\frac{P}{A}=\frac{2P}{\pi d^2}\left(1+\sqrt{1-\left(\frac{d}{D}\right)^2}\right), \tag{1}$$

where

P=load (kg),
D=ball diameter (mm), and
d=indentation diameter (mm).

Both P and D must be specified in giving the Brinell number, since it varies somewhat with each of these. Since the work of Brinell, suitable balls of other materials than steel have become available. As the indentation diameter, on the same metal, may differ for balls of different material (see p. 83) it is now necessary to specify also the material of the ball. Tables for H with P=3,000 kg, D=10 mm and with P=500 kg, D=10 mm have been computed from this formula, which give the Brinell number corresponding to the observed diameter d of the indentation.[2]

II. TEST SPECIMENS

The materials used in this investigation are listed and described in table 1. They are tabulated according to lot numbers, ranging from 10 to 90. Not all lots available in the laboratory were used for this investigation and only those used are listed.

[2] Misc. Pub. BS 62 (1924).

TABLE 1.—*Materials used in investigation*

Lot	Material	Approximate chemical composition (in percent)														Condition
		Al	Cu	Si	Fe	Mn	Mg	Sn	Pb	Zn	Ni	C	Cr	V	W	
10	Aluminum alloy SAE 30	90.80	7.95	0.58	0.59	0.08										Cast.
11	Aluminum alloy 2S	99.01	0.19	.35	.44	.01										Rolled.
12	Duralumin	94.36	3.71	.35	.49	.60	0.49									Heat treated.
13	Magnesium						([a])									Extruded at 300° C.
14	Magnesium alloy	4.00				.30	95.70									Cast at 680° C.
15	Do	4.00				.30	95.70									Heat treated at 450° C, quenched in water.
16	Do	4.00				.30	95.70									Rolled at 450° C.
17	Do	4.00				.30	95.70									Extruded at 290° C.
20	Phosphor bronze		89.35		.04	.05		10.23	0.01							Annealed.
21	Do		89.35		.04	.05		10.23	.01							Cold-rolled.
22	Nickel silver		63.70		.17	.21				18.00	17.92					Annealed.
23	Do		63.75		.16	.19			.015	17.77	18.13					Cold-rolled.
24	Brass		65.12		.01				.08	34.79						Annealed.
25	Do		65.12		.01				.08	34.79						Cold-rolled.
27	Nickel steel										3.50					As received from mill.
28	Carbon tool steel											0.90				Annealed.
33	Monel metal		30.00	.10	2.00	1.75					66.00	.20				Hot-rolled.
34	Nickel		.20	.10	.50	.25					99.00	.10				Do.
35	Aluminum bronze	7.92	91.90							.18						Annealed.
36	Do	7.55	92.16							.29						Cold-rolled.
37	Copper		99.97													Annealed.
38	Do		99.97													Cold-rolled.
53	Carbon steel											.09				As received from mill.
54	Do											.28				Do.
55	Do											.68				Do.
56	Nickel-chromium steel										2.14	.30	0.82			Do.
57	Chromium steel											1.01	1.33			Do.
58	Chromium-vanadium steel											.30	1.11	0.25		Do.
59	Tungsten steel											.60	3.50		14.00	Do.
81	Brass															Cold-rolled.
82	Do															Do.
83	Duralumin															Do.
84	Carbon tool steel															Do.
85	Carbon steel															Do.
86	Do															Do.
89	Do															Do.
90	Do															Do.

[a] Commercially pure.

III. CAUSES OF DISCREPANCIES IN THE DETERMINATION OF THE BRINELL NUMBER

The causes of discrepancies in the determination of the Brinell number may be grouped as variations in the apparatus and procedure, in the specimen, and in the shape and material of the ball.

1. APPARATUS AND PROCEDURE

(a) RATE OF APPLYING LOAD

A rapid rate of applying load will affect the diameter of the Brinell indentation in two ways. It will add an inertia load and a friction load to the nominal load and thus increase the size of the indentation, and it will allow less time for the plastic flow of the material, and, in that way, decrease the size of the indentation.

The magnitude of the first of these two effects will depend both on the method of applying the load and on the type of machine used. It is probably small as long as the load is applied slowly and without jerking and as long as the friction forces opposing the motion of parts in the machine are small compared to the forces effecting that motion.

This last condition is easily satisfied in a machine of the design shown in figure 1. The friction force between the lower piston and cylinder in the hydraulic press C is probably less than 0.5 percent of the impressed load of 3,000 kg, and the force between the ball piston lifting the balancing weights and its sleeve is certainly less than 0.5 percent of the 24–kg weight lifted at maximum load, since the piston is constantly covered with oil.

In the absence of appreciable friction the only forces that may lead to a pressure greater than that required to maintain the weights G and H in floating equilibrium are the inertia forces due to an upward acceleration of the weights G and H. The acceleration of the indenting plunger D is, in general, so small as to be entirely negligible and it is, in addition, in an upward direction, i. e., in a direction leading to a lessening of the load rather than an overload.

The relative error due to an upward acceleration a of G and H will be equal to the ratio of the resulting inertia force to the weight of the floating parts, i. e., equal to the ratio of its upward acceleration a to the downward acceleration g of gravity

$$\frac{\Delta P}{P}=\frac{a}{g}. \tag{2}$$

The acceleration a will be large under two conditions, first, when the weights G and H are accelerated from zero velocity to a finite upward velocity at the instant at which the 3,000-kg load is reached for the first time, and second, when during the maintenance of maximum load the downward drift of the weights is reversed into an upward motion by a stroke of the pump E, in order to maintain the weights in floating equilibrium.

The acceleration mentioned first was greatly reduced in the machine of figure 1 by attaching to the fixed cylinder J a leaf spring I, which starts to raise the balancing weights at half load, i. e., 1,500 kg on the indenting ball or 12 kg on the ball piston, and so imparts an upward motion to G and H, allowing them to come to floating equilibrium with a small upward velocity. The travel of this spring is 0.3 cm. The average acceleration for an interval as small as 1 second from half load to full load would, therefore, be

$$a=\frac{2\times 0.3}{1^2}=0.6 \text{ cm/sec}^2.$$

If this acceleration were maintained at the instant of coming to floating equilibrium the overload would be

$$\frac{\Delta P}{P}=\frac{0.6}{981}=0.0006,$$

which is an entirely negligible overload.

In estimating the magnitude of the acceleration due to pumping, the weights were assumed to drift down at a rate of 0.1 cm/sec. This approximates the observed rate of drift for the machine shown in figure 1. If the floating parts were then accelerated by a stroke of the pump E rapidly enough to gain an upward velocity of 2.9 cm/sec within 1 second, the average acceleration would be 3 cm/sec^2, and the corresponding relative error due to dynamic overload would be

$$\frac{\Delta P}{P}=\frac{3}{981}=0.003.$$

This also is a negligibly small overload. It is believed that average accelerations greater than 3 cm/sec^2 do not occur in careful testing with machines of the type shown in figure 1.

The overload due to inertia is therefore negligibly small with a machine of the type shown in figure 1, in all practical cases, as long as the loads are applied smoothly.

The second effect, that is, the decrease in indentation diameter with increasing rate of loading may, according to Guillery,[3] [4] become appreciable for average rates of loading of the order of 1,000 kg/sec, provided the maximum load is maintained a sufficiently short time (less than a minute). In the case of soft cast iron he obtained diameters that were 3 percent smaller for a loading interval of about 15 seconds than those for an interval of 4 minutes. C. Grard [5] states that there is no appreciable effect of the loading interval on the Brinell number provided the maximum load is maintained for more than 2 minutes.

In the absence of data covering different materials it was decided to make a short series of tests to provide further information on the subject. Two sets of five or more indentations each were made in 23 specimens of widely different materials which were selected from the group listed in table 1. The load was applied relatively slowly in making one set of indentations and rapidly in making the other; the relatively slow rate of loading was taken as 30 seconds from no load to full load for the 3,000-kg maximum load and 10 seconds for the 500-kg maximum load; for loads applied rapidly the loading interval was 6 seconds for the 3,000-kg load and 2 seconds for the 500-kg load. The rate of applying load was approximately uniform, i. e., the handle of pump E, figure 1, was operated at approximately a constant number of strokes per minute. The load was held at the maximum for 15

[3] Compt. Rend. **165**, 468–471 (1917).
[4] Rev. Met. **18**, 101–110 (1921).
[5] Trans. Sixth Int. Cong. Assn. Testing Materials, 1912, report III[2].

seconds in every test; actually it would have been desirable to make this interval zero, but since inaccuracies in timing are unavoidable it was felt that the value should not be too small, and 15 seconds was chosen in the belief that it would lead to comparable results.

The average diameters for each set of indentations are given in table 2. The specimens are identified in this table by lot numbers; their compositions may be found from table 1. It is seen that the average diameters of the indentations for all the steel specimens do not differ by as much as 0.01 mm; they agree within 0.02 mm for all remaining specimens, except those of nickel, copper, and brass. An examination of the individual readings for these metals showed that the individual readings differed more than the difference between the averages in all cases. The observed differences are in 9 cases positive, in 12 cases negative, and in 2 cases too small to detect. For all the specimens the effect of rate of loading up to 500 kg/sec appears to be smaller than the accidental variations in indentation diameter due to lack of homogeneity and due to other causes.

TABLE 2.—*Tests to determine the possible effect of the rate of application of load upon diameter of indentation*

Lot	Material	Load	Diameter of indentation[1]		Difference
			Load applied slowly	Load applied rapidly	
		kg	mm	mm	mm
38	Hard copper	3,000	6.794	6.744	−0.050
35	Soft aluminum bronze	3,000	6.426	6.410	−.016
20	Soft phosphor bronze	3,000	6.034	6.032	−.002
53	Low-carbon steel	3,000	5.932	5.935	+.003
22	Soft nickel silver	3,000	5.861	5.884	+.023
12	Duralumin	3,000	5.434	5.450	+.016
25	Hard brass	3,000	5.330	5.310	−.020
54	Medium carbon steel	3,000	5.331	5.332	+.001
86	Cold-rolled carbon steel	3,000	4.999	4.997	−.002
33	Monel metal	3,000	4.900	4.904	+.004
23	Hard nickel silver	3,000	4.618	4.608	−.010
11	Aluminum alloy 2S	500	4.594	4.604	+.010
28	Carbon tool steel	3,000	4.546	4.543	−.003
21	Hard phosphor bronze	3,000	4.156	4.154	−.002
55	High-carbon steel	3,000	4.002	4.009	+.007
59	Tungsten steel	3,000	3.973	3.968	−.005
37	Soft copper	500	3.802	3.776	−.026
56	Nickel-chromium steel	3,000	3.754	3.756	+.002
58	Chromium-vanadium steel	3,000	3.725	3.722	−.003
24	Soft brass	500	3.942	3.520	+.028
57	Chromium steel	3,000	3.298	3.296	−.002
27	Nickel steel	3,000	3.104	3.104	.000
84	Carbon tool steel	3,000	3.103	3.103	.000

[1] Each value is the average of at least 5 determinations.

(b) TIME UNDER NOMINAL LOAD

The effect on the diameter of the Brinell indentation of the time under maximum load has been investigated by W. N. Thomas [6], W. Deutsch [7], M. Guillery [8], and P. Lieber.[9] Each one of these

[6] J. Iron and Steel Inst. **93**, 255–269 (1916).
[7] Forsch. Gebiete Ingenieurw. **M1**, 7–23 (1919).
[8] See footnotes 3 and 4.
[9] Z. Metallkunde **16**, 128–131 (1934).

investigators found that a certain time interval was required to allow the ball to penetrate to its position of static equilibrium.

In the case of mild steel Thomas found that some 5 to 10 minutes were required to come within 1 percent of the Brinell number for loading intervals as long as 1 hour.

Deutsch recommended a duration under maximum load of not less than 3 minutes in testing soft bearing metals. Guillery concluded that 3 minutes are required to bring the diameters of indentation on mild-steel specimens within 1 percent of the final equilibrium value. Lieber found that an interval of 15 minutes under maximum load is required for some very soft bearing metals to bring the Brinell number within 1 percent of the final value; he found that the Brinell numbers after 3 minutes at maximum load may be as much as 12 percent above the final value.

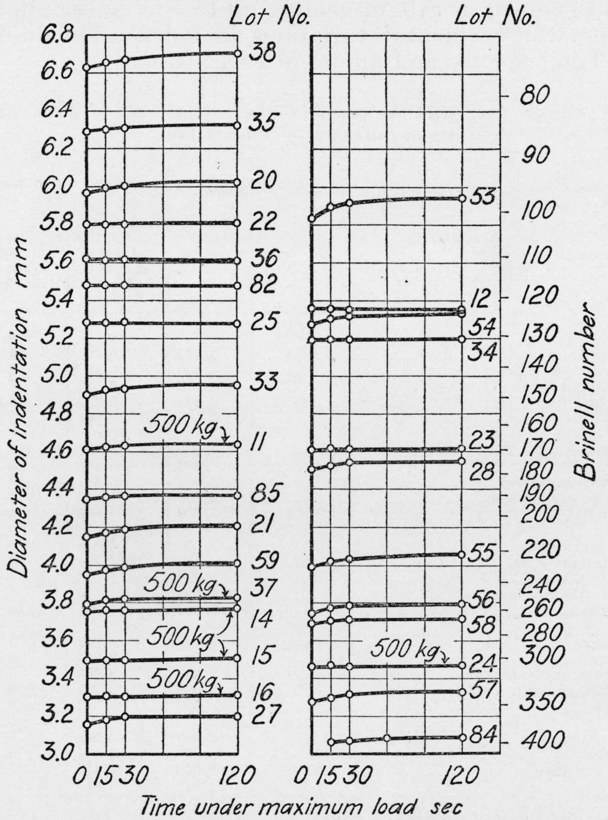

FIGURE 2.—*Variation in indentation diameter with time under maximum load.*

Brinell scale on right margin of figure applies to all curves except those marked 500 kg.

The investigators mentioned above confined their study of the time effect to mild steel, copper, and the soft bearing metals. It seemed worth-while to extend this research to a wider variety of materials selected from those listed in table 1.

The tests were made as follows: Four sets of five indentations each were made in each specimen, the time from no load to full load being 10 seconds and the time under maximum load being 0, 15, 30, and 120 seconds, respectively, for the successive sets of five indentations each. A maximum load of 3,000 kg was used on all specimens, except those from lots no. 11, 14, 15, 16, 24, and 37, which were tested using a maximum load of 500 kg. The average diameter of indentation for each set is plotted against time under maximum load in figure 2. The Brinell numbers of all specimens, except those tested under 500-kg load, may be read from the vertical scale at the right of the figure. It is seen that the creep for most materials is quite rapid during the first 30 seconds under maximum load; it is much less rapid in the interval from 30 to 120 seconds. The Brinell number in this interval decreased more than 1 percent for only two of the materials tested, i. e., soft copper (specimen 38, 1.34 percent) and carbon steel (specimen 55, 1.30 percent). The decrease in Brinell number was between 0.5 percent and 1 percent for 10 of the 29 specimens tested, and it was below 0.5 percent for the remaining 17.

For most materials, then, the Brinell number varies less than 1 percent for loading intervals between 30 and 120 seconds.

(c) ERROR IN LOAD

The effect on the Brinell number of a relative error $\Delta P/P$ in the applied load is given by the Brinell formula, page 61, if it is assumed that this formula gives a Brinell number P/A independent of P in the region considered. Differentiation gives

$$\frac{\Delta H}{H}=\frac{1}{H}\frac{\partial H}{\partial P}\,\Delta P=\frac{\Delta P}{P}, \tag{3}$$

that is, the relative error in the Brinell number is equal to the relative error in the applied load. The assumption, on which the correctness of this equation is based, that the Brinell number is independent of the load, is true in first approximation only. The closeness of this approximation may be computed by using the empirical relation between load P and indentation diameter d established by E. Meyer [10] for a large number of metals indented by steel balls:

$$P=ad^n, \tag{4}$$

where a is a constant depending on the material and the ball diameter and n is a constant depending on the material alone. Meyer found values of n ranging from $n=1.91$ to $n=2.4$. The Brinell number may be written in terms of Meyer's law as [11]

$$H=\frac{2}{\pi D}\,a^{2/n}\,P^{1-2/n}\left(D+\sqrt{D^2-\left(\frac{P}{a}\right)^{2/n}}\right). \tag{5}$$

The change of the Brinell number with load was computed from this by differentiating with respect to P and substituting in equation 3. The resulting expression involves n and P/a. P/a was replaced by d^n and $\Delta H/H$ was calculated for the most unfavorable pairs of values of n and d. For $n=1.91$ and $d=7$ mm, $\Delta H/H=-0.26\,\Delta P/P$, and

[10] Forsch. Gebiete Ingenieurw. **65** (1909).
[11] H. O'Neill, J. Iron and Steel Inst. **107**, I, 343-376 (1923).

for $n=2.4$ and $d=2$ mm, $\Delta H/H=+.16\ \Delta P/P$. It appears from this that $\Delta P/P$ on the right side of equation 3 must be multiplied by 0.74 in the first case and by 1.16 in the second to give a value of $\Delta H/H$ corrected for the variation of the Brinell number with load. This variation adds less than 20 percent to the correction as given by equation 3 in the most unfavorable cases that could be found. Assuming a 20-percent correction to equation 3, the applied load should be correct within 0.33 percent in order to keep the error in the Brinell number from that source within 0.4 percent.

(d) ERROR IN MEASURING THE DIAMETER OF INDENTATION

The area A entering in the expression P/A for the Brinell number is defined as the area of the surface of contact between the ball and the specimen under load. It is assumed in the derivation of the Brinell formula that A can be measured by the diameter of the indentation d left after removing the ball.

Actually there may be considerable uncertainty in the magnitude of this diameter, and hence in the value of the Brinell number P/A obtained. For some materials the edge of the indentation is very poorly defined, even when the surface finish is good. Sometimes there is a ridge around the indentation extending above the original surface of the specimen, and at other times the edge of the area of contact is below the original surface [12] as is illustrated in fig 3. In some cases there is no sharp line of demarcation between the indentation and the surrounding surface; one surface merely rounds off into the other. In all cases there is uncertainty as to the portion of the visible indentation which was actually in contact under load. At present no methods are known which will in all cases eliminate all uncertainty as to the actual contact area. The best that can be done is to insure that different observers will not secure too widely different results on the same indentation.

For some specimens, the indentations may be made more distinct by using balls etched with nitric acid, as suggested by Axel Hultgren.[13]

The borders of the indentation will be still more distinct if a ball of more rigid material than steel is used. Styri [14] found that indentations made with 5-mm Carboloy (tungsten-carbide) balls at 750-kg load have a remarkably clear outline, even on specimens having Brinell numbers as high as 780. This observation was confirmed by tests made at the National Bureau of Standards (p. 88). In these tests 10-mm Carboloy balls were used to indent specimens up to 750 Brinell at 3,000 kg. It must be borne in mind, however, that Carboloy balls will indicate considerably higher Brinell numbers than steel balls on a given specimen because of the difference in elastic properties; this is discussed on page 85.

In general, there will be an error Δd in reading the diameter d of the indentation. The relative error $\Delta H/H$ in the Brinell number due to a relative error $\Delta d/d$ may be computed from the Brinell formula 1 by differentiation

$$\frac{\Delta H}{H}=\frac{1}{H}\frac{\partial H}{\partial d}\Delta d=-\left[1+\frac{1}{\sqrt{1-\left(\frac{d}{D}\right)^2}}\right]\frac{\Delta d}{d} \tag{6}$$

[12] F. E. Ross and R. C. Brumfield, Proc. Am. Soc. Testing Materials **22**, part II, 312–334 (1922).
[13] Axel Hultgren, Mech. Eng. **43**, 445 (1921).
[14] Metals and Alloys **3**, 273–274 (1932).

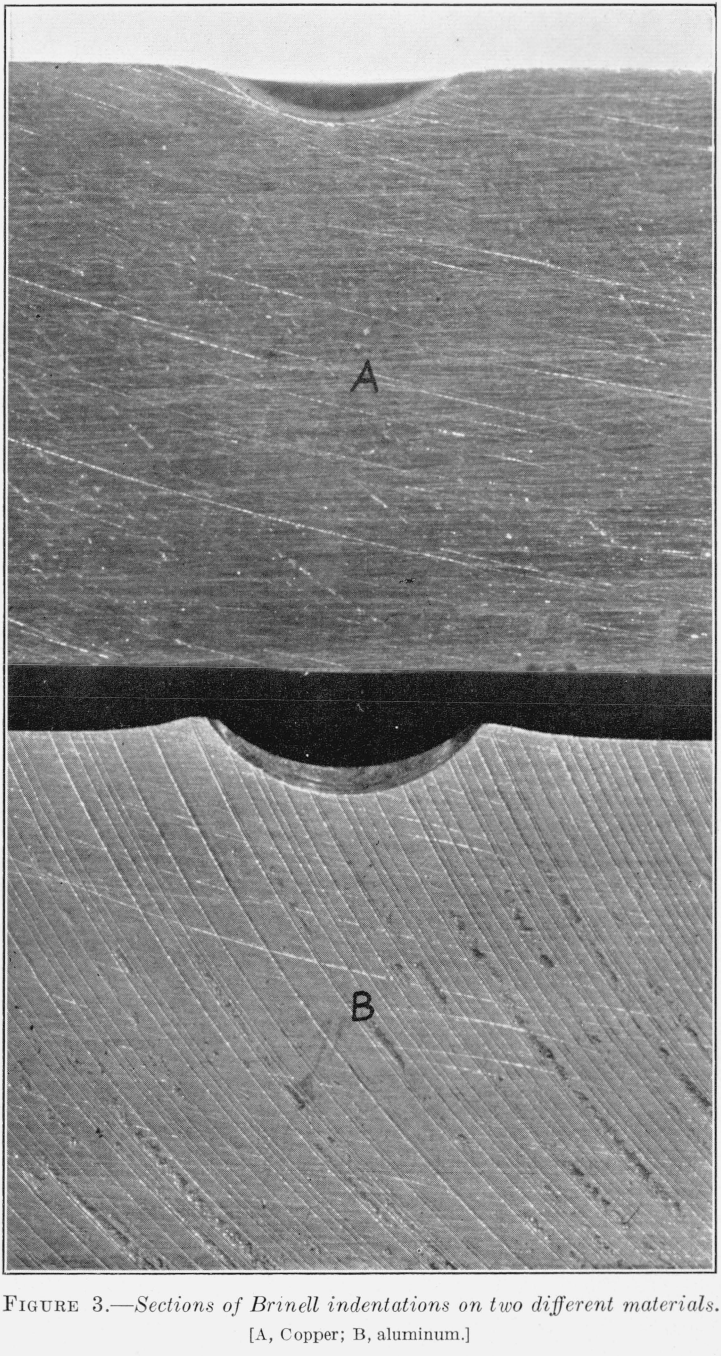

FIGURE 3.—*Sections of Brinell indentations on two different materials.*

[A, Copper; B, aluminum.]

The minus sign indicates that an increase in indentation diameter corresponds to a decrease in Brinell number.

Figure 4 shows curves for the percentage error 100 $\Delta H/H$ in the Brinell number H plotted against H for values of d ranging from 0.005 to 0.050 mm. The error in the Brinell number is less than 1 percent, as long as the error in diameter does not exceed 0.01 mm.

Errors in reading the diameter of the indentation may be ascribed to two causes, first, to an error in the reading of the instruments used for measuring the diameter, and second, to indefiniteness of the boundary of the indentation itself.

The error in reading a Brinell microscope of common design, such as the one shown in figure 1, should not exceed 0.01 mm over the entire 7-mm scale, if the microscope is in proper adjustment. The adjustment of the microscope may be checked easily by placing it on a calibrated 7-mm scale, such as the scale marked on disk N shown next to the microscope in figure 1, and verifying that the image of this scale and the scale on the reticule coincide within 0.01 mm; this corresponds to 0.1 of a scale division.

No discussion was found in the literature of the magnitude of the error due to indefiniteness of the boundary of the indentation, although this error is probably the greatest single factor contributing to the lack of concordance in Brinell numbers obtained by different observers testing a given material. The following series of tests was made to provide some information on this point.

TABLE 3.—*Average errors in reading Brinell indentation diameters*

Observer→		1	2	3	4	5	Mean value
Diameter ↓	H	Average error [1]					
mm		mm	mm	mm	mm	mm	mm
2.418	644	0.0095	0.0054	0.0134	0.0113	0.0201	0.0120
2.676	524	.0050	.0064	.0156	.0217	.0194	.0136
3.082	392	.0037	.0025	.0135	.0124	.0068	.0078
3.716	267	.0052	.0035	.0045	.0110	.0143	.0077
4.177	209	.0034	.0050	.0088	.0122	.0151	.0089
4.660	166	.0049	.0054	.0069	.0081	.0206	.0092
Mean value		0.0053	0.0047	0.0104	0.0128	0.0160	0.0099

[1] Each value given is the average for 6 indentations.

Six groups of six Brinell indentations each were made on steel specimens whose surfaces had been ground plane and the diameter of each indentation, taken as the average of two mutually perpendicular diameters, measured at an angle of about 45 degrees to the direction of grinding, was obtained by two methods. First the diameter of each indentation was obtained with a traveling microscope which was read by estimation to 0.001 mm. Then the diameter of each indentation was obtained by each of five observers with the regular Brinell microscope (M, fig. 1), which was read by estimation to 0.01 mm. The observers were chosen from the staff of the Bureau's Engineering Mechanics Section. Observers 1 and 2 had had considerable experience in measuring the diameters of Brinell indentations, observer 3 had had only a small amount of experience, while observers 4 and 5 were inexperienced.

The relatively more concordant results obtained with the traveling microscope were considered as correct, and the error for each observer for each indentation was computed by subtracting the diameter obtained with the traveling microscope from the diameter obtained with the regular microscope. These results are plotted in figure 5.

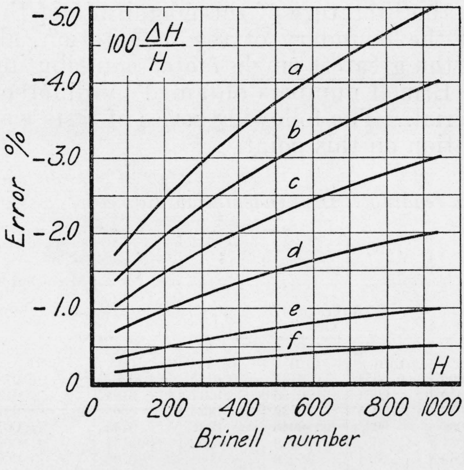

FIGURE 4.—*Error in Brinell number due to error in indentation diameter.*

	Error in indentation diameter (mm)
Curve a	0.05
Curve b	.04
Curve c	.03
Curve d	.02
Curve e	.01
Curve f	.005

The diameters of the six indentations of each group were nearly equal. The averages of the absolute values of the errors for each group were computed for each observer. They are given in table 3. The last line of table 3 lists mean values of the average errors for each observer, and the last column lists mean values for each group of indentations.

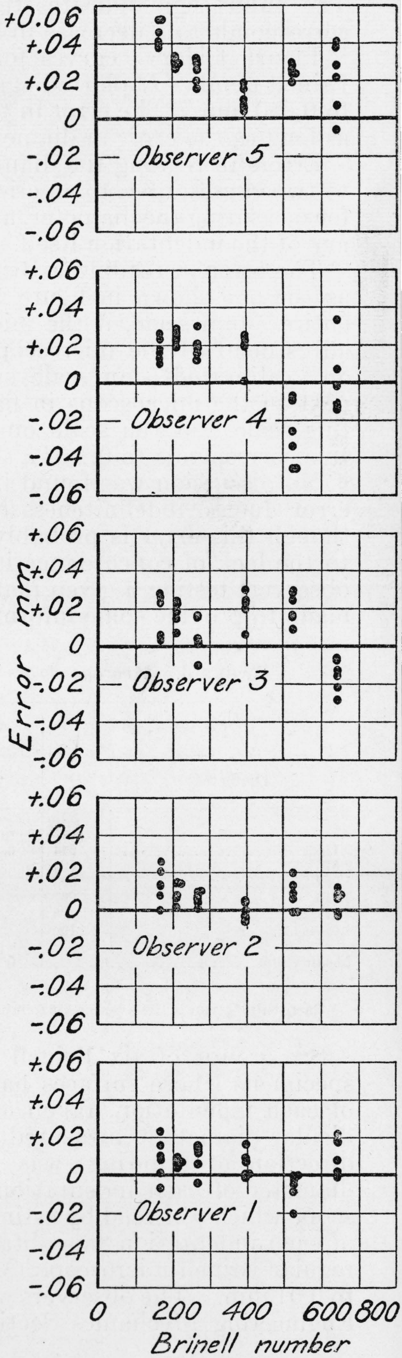

FIGURE 5.—*Errors in reading indentation diameters for five different observers.*

[Observers 1 and 2 with considerable experience observer 3 with a little experience; observers 4 and 5 with no experience.]

From these results the average percentage error in the Brinell number was computed for each observer for each group. These values are plotted in figure 6 as a function of the Brinell number.

The results for each observer shown in figure 5 indicate the presence of systematic as well as accidental errors. The systematic error changes with the Brinell number of the specimen in a different way for each observer. Both the systematic and the accidental errors were smaller for the experienced observers than for the remaining observers. Apparently a certain amount of experience improves an observer's ability to distinguish the boundary of the contact surface.

Figure 6 shows that the average percentage error in the Brinell number for the experienced observers was always less than 1 percent, while that for the inexperienced observers exceeded 2 percent in some cases.

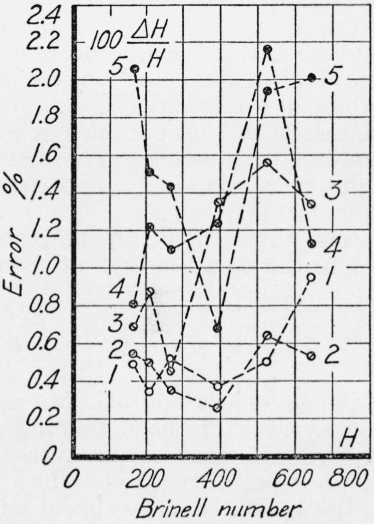

FIGURE 6.—*Average error in Brinell number due to uncertainty in reading indentation diameter.*

[Curves 1–1 and 2–2 represent results obtained by experienced observers; 3–3 by an observer with a little experience; 4–4 and 5–5 by observers with no previous experience.]

2. SPECIMEN

(a) NONUNIFORM PROPERTIES

If the compressive properties of a flat specimen are not uniform, e. g., if they change with the direction of rolling, this will be reflected in a noncircular shape of the indentation left after the Brinell test. Thus, if the material yields more easily under compressive stresses in the direction of rolling than at right angles to that direction, the indentation will be roughly elliptical with a maximum diameter in the direction of rolling and a minimum diameter at right angles to that direction. An average value of the Brinell number for an indentation with a noncircular boundary will be obtained if the diameter of the indentation is taken as the average of diameters in four directions, roughly 45 degrees apart.

It is assumed here that the surface of the specimen is finished by filing, machining, or grinding to such smoothness that the tool marks do not interfere with the measurement of the indentation diameter. For most materials there is no difficulty in finishing the specimen so that the error in the measured diameter caused by tool marks does not exceed 0.01 mm.

If the test used affects only a very small amount of the test material, there may be some question as to the proper interpretation of results because variations in indentation numbers may be due to local differences in the surface layer of the specimen and not to systematic variations in the body of the material. The standard Brinell test is probably comparatively free from uncertainties of this sort because in it a fairly large amount of the test material is affected. Variations in the Brinell number greater than can be accounted for by the variations in the test procedure and in the indenting ball will actually indicate corresponding variations in those properties of the material in which the user is interested.

(b) CURVATURE OF SURFACE

It is frequently necessary in practice to measure the Brinell number on a curved surface rather than a plane surface. An indentation on a curved surface of a specimen having uniform properties will not have a circular boundary unless the curvature is constant in all directions, as in the case of a sphere. The question arises as to which diameters to measure and how to average them so as to obtain the "equivalent diameter", which may then be substituted in the Brinell tables or in formula 1. A good approximation to the equivalent diameter will be obtained when the diameters measured and the method of averaging are such that the area computed from the equivalent diameter approximates closely the area of the actual indentation.

The relative error in the Brinell number corresponding to an error ΔA in the assumed area of the indentation is then from 1:

$$\frac{\Delta H}{H} = -\frac{\Delta A}{A}. \tag{7}$$

A convenient approximation to the equivalent diameter would be the average of the maximum and the minimum diameters of the indentation, that is, the average of the diameters in the two planes of principal curvature. The corresponding value of ΔA is derived in the appendix 1, page 92, as a function of the principal curvatures of the specimen (equation 17, page 94), and the distance from the center of the indenting ball to the point of intersection of the load line with the original surface of the specimen. The area A of the surface of the sphere embedded in the specimen is given by equation 14 of appendix, page 93. Knowing both A and ΔA, the relative error in the Brinell number can be computed for various values of the Brinell number P/A.

Figure 7 shows the result for the indentation produced by a 10-mm ball under 3,000-kg load on cylindrical specimens of 20 mm and of 50 mm diameter, as well as on specimens having a concave cylindrical curvature of 10 mm radius and 25 mm radius, respectively. For two of the surfaces considered, the maximum radius of curvature of the specimen is twice that of the indenting ball, while for the remaining two it is 5 times the radius of curvature of the ball. For the extremely high ratio of 1:2 the error in Brinell number is less than 3 percent for the concave cylindrical surface and less than 1 percent for the convex cylindrical surface; for the ratio 1:5 the error is less than 0.3 percent for both concave and convex cylindrical surfaces. The cylindrical specimen is a rather severe test of the approximation; the errors involved would be larger only in the case of specimens with anticlastic curvature, i. e., specimens whose principal radii of curvature have opposite signs. Two cases of anticlastic curvature were computed, one in which the specimen had radii of curvature of +10 mm and −10 mm and the other in which the radii were +25 mm and −25 mm (fig. 8). The error becomes as large as 6.5 percent in the first case, but it is less than 1 percent in the second case.

The error in the Brinell number due to curvature of the specimen may be reduced, in general, to less than 1 percent by using the average of the two principal diameters of the indentation as the equivalent diameter, provided the minimum radius of curvature of the specimen is equal to or greater than 5 times the radius of the indenting ball.

(c) THICKNESS

The material of the specimen is permanently deformed for an appreciable distance below the surface of the indentation. If this deformation extends to the lower surface of the specimen opposite the indentation, one of the following effects may result.

The support given by the hardened-steel anvil may effectively increase the resistance of the material directly under the ball, thereby causing the indentation to be smaller than one which would be produced in a thicker specimen of the same material.

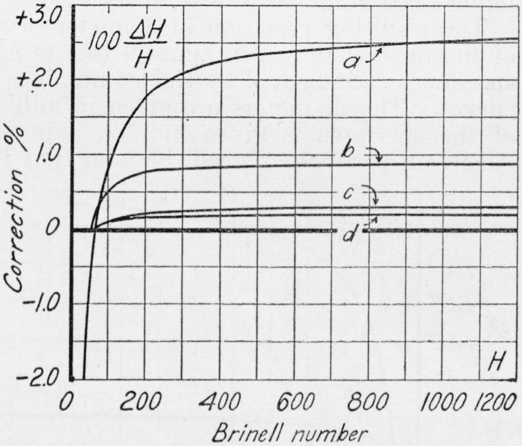

FIGURE 7.—*Correction due to cylindrical curvature to be added to Brinell number.*

[10-mm ball, under 3,000-kg load]

Curve	Cylindrical surface	Radius (mm)
a	Concave	10
b	Convex	10
c	Concave	25
d	Convex	25

The cohesion of the material directly under the ball may be insufficient to support the load and this portion may yield rapidly with increasing pressure. The diameter of the indentation may, therefore, be larger than one obtained on a thicker specimen.

The effect of thickness on the Brinell number of steel specimens has been investigated by H. Moore[15] and by W. N. Thomas.[16] Moore found an increase of about 3 percent if the depth of the indentation was one-third the thickness of the specimen. A ratio of 1:7 was considered safe by Moore to eliminate the effect of thickness. Thomas concluded from his tests that the effect of thickness was negligible for a 10-mm ball at 3,000-kg load, provided the specimen was at least 0.38 in. thick.

15 Trans. Fifth Int. Cong. Assn. Testing Materials 1909, report II_4.
16 J. Iron and Steel Inst. **93**, 255-269 (1916).

It seemed desirable to add to these results by carrying out a series of tests on specimens of various materials. The following procedure was adopted in carrying out these tests. One surface of each specimen was machined so that the thickness of the specimen decreased uniformly approximately 0.05 in. for each inch of length. The other surface was polished with emery paper, without previous machining, and indentations were made about 1 inch apart. This spacing gave the desired variation in thickness and eliminated at the same time the effect of any given indentation on an adjacent one. The time under maximum load was in every case 30 seconds.

The resulting variation of diameter of indentation with thickness of specimen is shown in figure 9 (see table 1 for composition of test specimens as indicated by the lot numbers near the right end of each curve). The diameters are given in millimeters while the thickness of the specimen is given in inches, in accordance with the usual American practice. It will be seen that in most cases the diameter decreased with increasing thickness indicating that the weakening in the cohesion of the material usually predominates over the strengthening due to the backing of the steel plate below the specimen. For specimens 14, 35, and 37 the second effect appears to be more important than the first, since the diameters of the indentations were found to increase with increasing thickness.

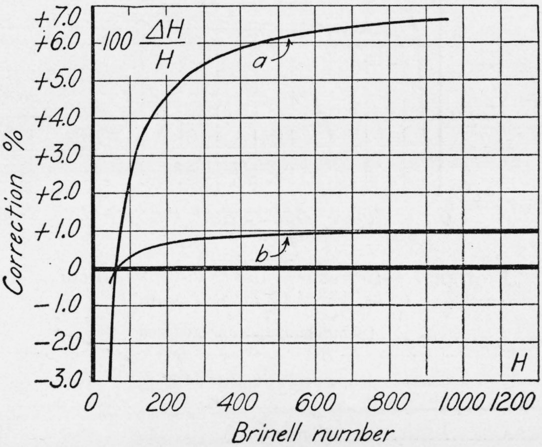

FIGURE 8.—*Corrections due to anticlastic curvature to be added to Brinell number*

[10-mm ball, under 3,000 kg load]

Curve a, anticlastic surface, principal radii 10 mm. Curve b, anticlastic surface, principal radii 25 mm.

The curves faired through the individual points have been used to determine a "critical" thickness, arbitrarily defined as the thickness at which the apparent Brinell number for the indentation differed by 1 percent from that for the thickest portion of the specimen. The corresponding thickness is indicated by a short vertical line on each curve; it varied between the limits of 0.08 in. and 0.32 in. For other metals, as well as for steel, a thickness of specimen of 0.4 in. may, therefore, be considered sufficient, in nearly all cases, to make negligible the effect of thickness on Brinell number. This thickness agrees closely with the thickness of 0.38 in. recommended by Thomas for steel specimens.[17]

It was noted that under each indentation made, where the thickness was less than the "critical" value, a spot of altered surface was visible on the under side of the specimen. As a large variety of engineering materials was used in this investigation, it seems safe to

[17] J. Iron and Steel Inst. **93**, 255–269 (1916).

assume that the absence of a visible spot on the under surface of the specimen indicates that the thickness of the specimen exceeds the critical thickness as defined above.

(d) SPACING OF INDENTATIONS

(1) *From Edge.*—If an indentation is made too near the edge of the specimen it may be both too large and too unsymmetrical.

H. Moore [18] concluded on the basis of tests on specimens of steel and of rolled brass that the center of the indentation should be at least 2½ times the diameter of the indentation distant from the edge to avoid errors from this source.

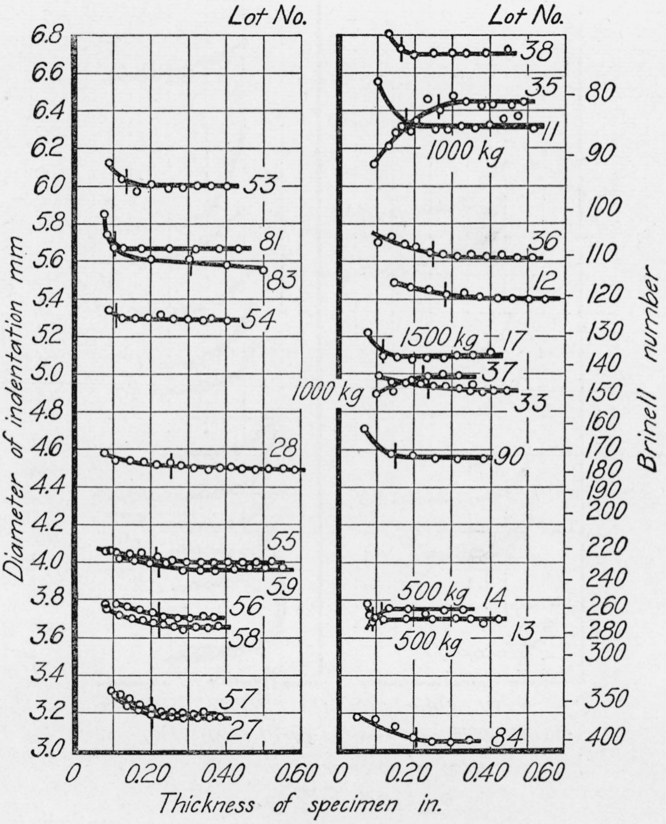

FIGURE 9.—*Variation in indentation diameter with thickness of specimen.*

[Brinell scale on right margin applies to all curves except those marked 500, 1,000, or 1,500 kg.]

It seemed desirable to extend these results to a wider variety of materials. Indentations were made on 23 different specimens selected from the materials listed in table 1. The thickness of each specimen was greater than 0.40 in. so as to make the effect of thickness negligible (see previous section). The time under load was in every case 30 seconds.

[18] See footnote 15.

Figure 10 shows the relation between the diameter of the indentation and the distance from the edge for each of the 23 specimens. As in figure 9 a value was found for the "critical" distance in each case, i. e., the distance at which the apparent Brinell number for the indentation differs by 1 percent from that obtained for the maximum distance. This value is indicated by a short vertical line. The ratio

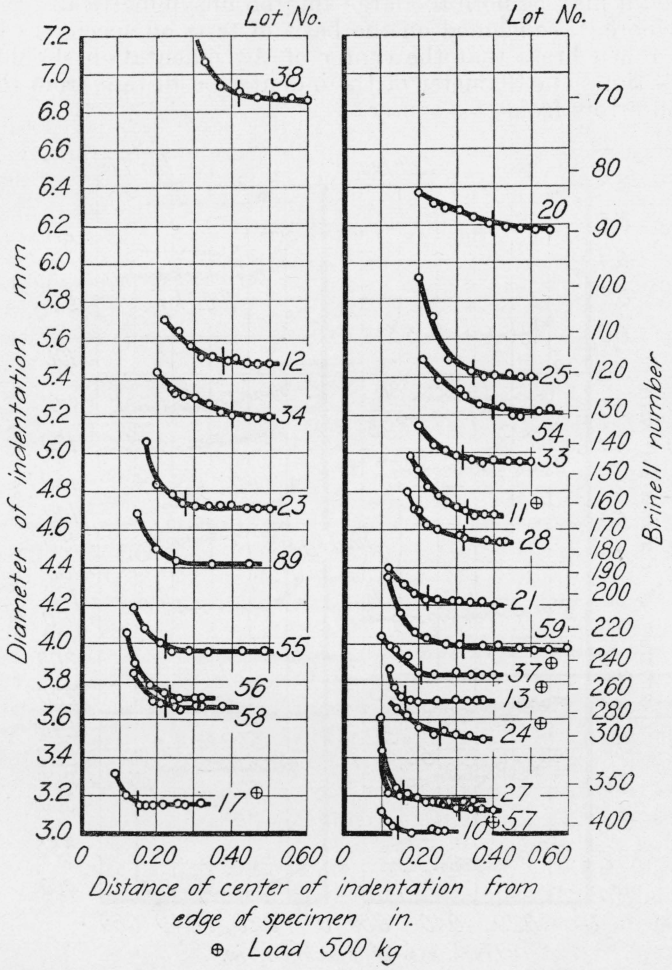

Figure 10.—*Variation in indentation diameter with distance of Brinell indentation from edge.*

[Brinell scale on right margin applies to all curves except those marked to indicate 500-kg load instead of 3,000 kg.]

of this critical distance to the diameter of the indentation was found to vary from 1.1 to 2.6.

The error in Brinell number due to edge spacing may be neglected if the distance of the center of the indentation from the edge of the specimen is equal to or greater than three times the diameter of the indentation.

(2) *From Adjacent Indentations.*—If an indentation is made too close to one made previously, at least three possibilities of error are introduced.

Deformation of the material resulting from a second indentation may extend into the one made first, decreasing its diameter along the line connecting their centers.

Lack of sufficient supporting material may make the second indentation too large.

Work-hardening of the material resulting from the first indentation may decrease the size of the second indentation.

The magnitude of the effect on the Brinell number of adjacent indentations was investigated for different specimens selected from the materials listed in table 1. The thickness of each specimen was greater than 0.40 in. and the time under load was in every case 30 seconds.

The following test procedure was adopted:

Six pairs of points A—A′, B—B′, C—C′, . . . were marked on the specimen, each pair being well separated from any neighboring pair and from the edges of the specimen. The points were so located that the distances AA′, BB′, CC′, . . . decreased progressively. Indentations were made with the points A, B, C, . . . as centers. The diameters parallel to AA′, BB′, CC′, . . . were then measured. Indentations were made at points A′, B′, C′, . . . and the diameters parallel to AA′, BB′, CC′, . . . were measured. The diameters of the indentations at A, B, C, . . . were again measured.

The results of the measurements are shown in figure 11. This is a plot of the distance between indentation centers as abscissas and $d_0 - \Delta d$ as ordinates, where d_0 denotes the diameter of the indentation before the second one was made close to it and Δd is the decrease in this diameter after the second indentation is made. In each case values for the distance between indentation centers were found for which there was no variation greater than 1 percent in the Brinell number for the indentation. The smallest of these values is indicated on the curve by a short vertical line and will hereafter be called the critical distance. For specimen 38 the ratio of the critical distance to the corresponding diameter of the indentation is 1.6. For all other specimens this ratio is less.

A comparison of diameters of indentations A, B, C, . . . made first, with those of indentations A′, B′, C′, . . . made last, showed the effect which the indentations made first had upon those made last. This effect was quite small and no critical distance larger than the largest one shown in figure 11 was found.

The error in the Brinell number due to indentation spacing will not exceed 1 percent if the distances between centers of adjacent indentations are equal to or greater than three times the diameter of the indentation.

(e) ANGLE BETWEEN LOAD LINE AND NORMAL TO SPECIMEN

It is not always practical to have the surface of the specimen at the point at which it is to be indented exactly normal to the load P producing the indentation. Usually there will be a small angle α between load line and normal to the surface; this will reduce the normal load from P to $P\cos\alpha$ and in addition will add a component of load $P\sin\alpha$ acting in a direction tangential to the surface of the specimen. The reduction in the normal load may lead to an increase in the observed Brinell number, while the addition of the tangential component may elongate the indentation in the direction in which it acts and may lead to an increase in area and consequent decrease in the observed Brinell number. The question arises within what limits α must be kept to make the error in the Brinell number from this cause negligibly small.

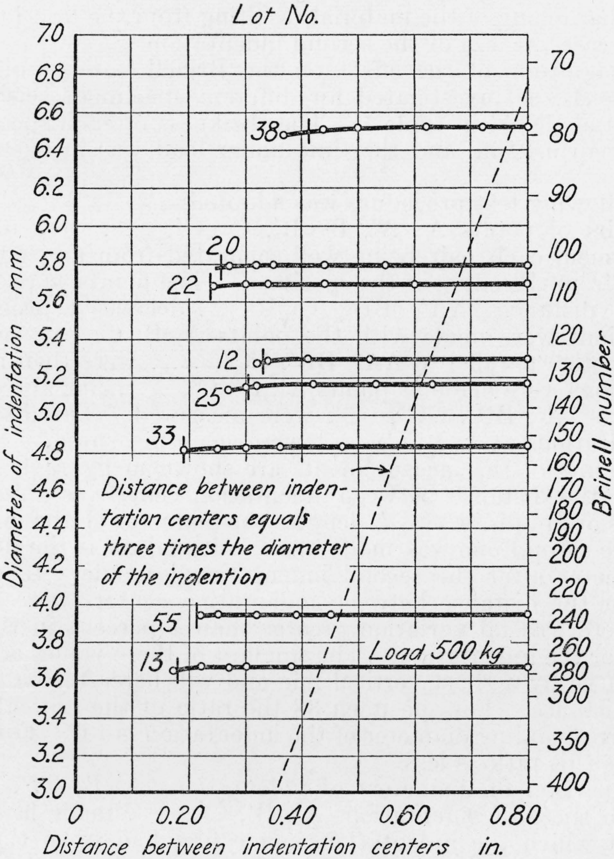

FIGURE 11.—*Variation in indentation diameter with distance of Brinell indentation from adjacent indentations.*

[Brinell scale on right margin applies to all curves except curve 13.]

An estimate of the error involved was obtained for a soft steel block (Brinell number 187) and a hard steel block (Brinell number 524) in the following manner.

The Brinell number of each steel block was determined from four or five indentations apiece under normal loading. A 2-degree wedge was then placed under the block and two further indentations were made; this was followed by the making of two indentations each with a 4-degree and a 6-degree wedge.

The resulting variation of the observed Brinell number with wedge angle is plotted in figure 12. Strictly speaking, the angle between the normal to the specimen and the load line will be larger than the wedge angle because of the play in the indenter D (fig. 1). Measurements showed that the maximum play possible was only about 0.5 degree. This is too small a variation to have any measurable effect on the Brinell number and it was accordingly neglected in plotting figure 12. Figure 12 also shows a plot of the percentage increase in the Brinell number due to the reduction in normal load computed from

$$\frac{\Delta H}{H}=\frac{\Delta P}{P}=1-\cos\alpha. \tag{17}$$

The percentage increase appears to be too small to be measured, and it is entirely overshadowed by the second effect which becomes appreciable for deviations from normal loading of 4 degrees or more. The error does not exceed 1 percent for angles of 2 degrees. The error exceeded 8 percent for the indentations on the hard-steel block with the 6-degree wedge.

The two specimens are sufficiently far apart in Brinell number to make it probable that the error will not exceed 1 percent for other steels also, as long as the deviation from normal loading does not exceed 2 degrees. The naked eye will suffice in most cases to check alignment within 2 degrees.

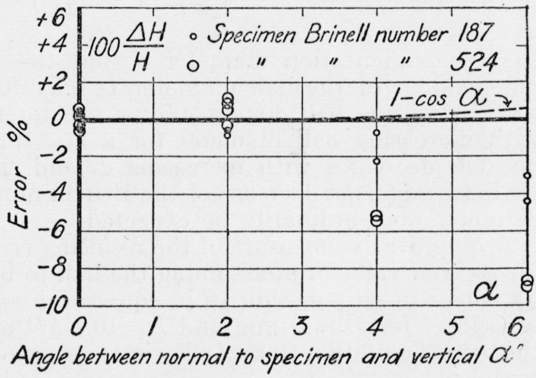

FIGURE 12.—*Error in Brinell number due to deviation from normal loading.*

3. INDENTING BALL

The Brinell formula assumes that the indenting ball remains a sphere of nominal diameter (e. g. 10 mm) during the test. Actually this condition is never satisfied. The ball will, in general, deviate from the nominal diameter, its shape will not be truly spherical, and, furthermore, it will deform under load elastically and may deform permanently. Each of these conditions results in a curvature of the contact surface different from the nominal curvature which, in turn, leads to a difference in contact area and a corresponding change in the computed Brinell number.

(a) ERROR IN DIAMETER

If the indenting ball is spherical when not under load but has a diameter $D+\Delta D$ instead of the nominal diameter D, the resulting

[19] Forsch. Gebiete Ingenieurw. **65** (1909).

indentation diameter d will vary, in general, with ΔD. The magnitude of the variation has been studied by Meyer [19] for a number of metals indented with steel balls of various diameters. He found that this variation could be expressed by a formula of the type

$$P = ad^n = a_D\left(\frac{D}{D+\Delta D}\right)^{n-2} d^n, \tag{18}$$

where a is a constant depending on the material of the specimen, and on the nominal diameter D of the ball used, and where n is a constant of the material alone. Meyer found values of n ranging from 1.91 (for lead) to 2.38 (for a certain type of cast iron). For most metals n lies in the neighborhood of 2.2. It is seen by solving equation 18 for d

$$d = \left(\frac{P}{a}\right)^{1/n}\left(1+\frac{\Delta D}{D}\right)^{1-2/n}, \tag{19}$$

that the indentation diameter d (not the Brinell number) will be independent of the ball diameter D only for $n=2$; it will increase with increasing ball diameter for n greater than 2 and will decrease with increasing ball diameter for n less than 2. Since the Brinell number decreases with increasing d, and since n is greater than 2 for most metals, a decrease of the Brinell number with increasing ball diameter may ordinarily be expected.

A quantitative measure of the resulting error in the Brinell number for a given value of n, assuming the ball to be rigid, may be obtained by substituting equation 19 in equation 5 and calculating the difference in H for $D=10$ mm and $D=10+\Delta D$ mm. Eliminating a with the help of equation 19 the following expression results for $\Delta D/D$ small compared to 1.

$$\frac{\Delta H}{H} = \frac{\partial H}{\partial D}\frac{\Delta D}{D} = \frac{2}{n}\left(1-n+\frac{1}{\sqrt{1-\left(\frac{d}{D}\right)^2}}\right)\frac{\Delta D}{D}. \tag{20}$$

This coincides with the corresponding expression which would be obtained from equation 1 for the special case $n=2$ upon holding everything constant except D.

Figure 13 shows the percentage error 100 $\Delta H/H$ calculated for the extreme cases $n=1.91$ and $n=2.4$, as well as for the ideal case $n=2$ for deviations $\Delta D=0.10$ mm, and 1.00 mm from a nominal diameter of 10 mm. The percentage error for $\Delta D=0.10$ mm does not exceed 0.5 percent for Brinell numbers between 67 and 945.

The diameters of steel balls produced by modern manufacturing methods are within 0.015 mm of the nominal diameter. The error due to variation in diameter of such balls for any value of n observed by Meyer stays below 0.1 percent, which is entirely negligible.

(b) NONSPHERICAL SHAPE

If the indenting ball is nonspherical in shape when not under load, there will be a resulting error in the Brinell number. This may be estimated from equation 20, provided the radii of curvature in the region of contact are known. ΔD may be taken as the difference between twice the average radius of curvature—i. e., the effective diameter of the ball in the region of contact—and the nominal diameter. Consider, for example, a ball ground to the shape of an ellipsoid of revolution with the minimum diameter of the ellipsoid coinciding with the line of action of the load on the ball. Such a ball will have an effective diameter at the point of loading larger than the average diameter by an amount $3\Delta D$, where $D+\Delta D$ is the diameter at the equator of the ellipsoid of revolution, and $D-\Delta D$ is its diameter along

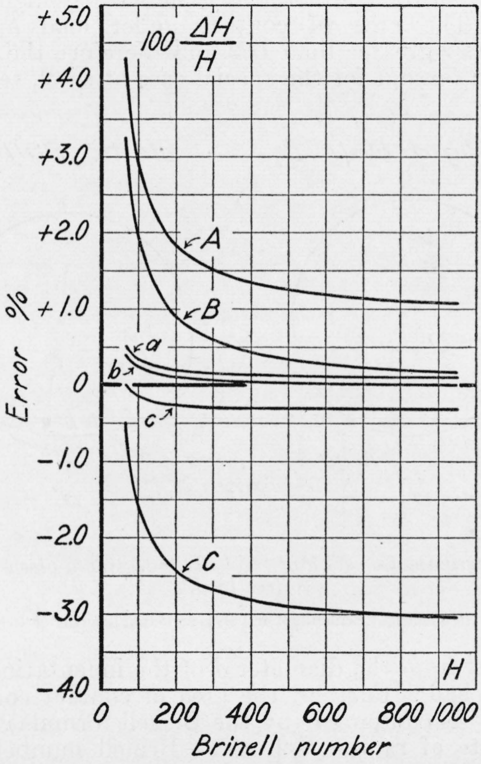

FIGURE 13.—*Error in Brinell number due to error in ball diameter.*

Curve	Error in ball diameter (mm)	n	Curve	Error in ball diameter (mm)	n
A	1.0	1.91	a	0.1	1.91
B	1.0	2.00	b	.1	2.00
C	1.0	2.40	c	.1	2.40

the load line (connecting the poles). The consequent error in the Brinell number will, according to equation 2, be less than $1.6\,\Delta D/D$ in practical cases (n between 1.91 and 2.4). The error would not exceed 0.24 percent if the tolerance in diameter is set at ± 0.015 mm.

(c) DEFORMATION OF BALL UNDER LOAD

The diagrammatic sketch of figure 14 will assist in a discussion of the effect of deformation of the ball on the Brinell number. Balls B_1 and B_2 are imagined to indent a given specimen under a given load, and both are taken to be spheres of the same diameter when not under load. Ball B_1 is an "ideal" Brinell ball; it is perfectly rigid and will not deform under load. If such a ball were possible, an "ideal" Brinell number could be determined by direct measurement of the indentation it produced. Ball B_2 is an actual Brinnell ball; it is made of deformable material and will yield under load elastically, and if the stress in the contact area is sufficiently high it will deform permanently. The area of contact under load approximates a sphere of radius r greater than $D/2$ and therefore the diameter d' of the indentation (except for the special case of $n=2$, see equation 19)

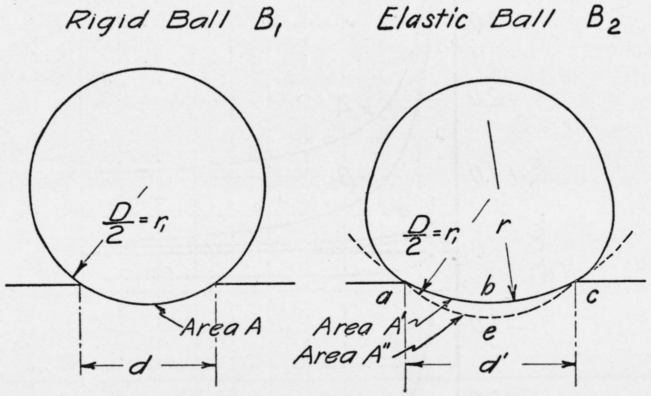

FIGURE 14.—*Diagrammatic sketch of two balls indenting a given specimen under a given load.*

[Ball B_1 is an ideal rigid Brinell ball and B_2 is an actual deformable Brinell ball.]

will be different from the diameter d of the indentation produced by the ideal rigid ball. Further, the area of contact corresponding to the diameter d' is computed (by the Brinell formula) as if it were a spherical calotte of radius $D/2$. The Brinell number so measured and computed, in contrast to the hypothetical ideal Brinell number, instead of being a characteristic mechanical constant of the specimen alone is also a function of the elastic and inelastic properties of the indenting ball which determine its shape under load.

The importance of this effect, especially for materials of high Brinell number, is shown clearly by tests made by Mailaender,[20] who found differences up to 10 percent between the Brinell number of a given specimen computed from indentations with a 5-mm diamond ball under 187.5-kg load and with a 5-mm steel ball under the same load. Styri [21] found differences of the same order between

[20] R. Mailaender, Stahl u. Eisen **45**, 1769–1773 (1925).
[21] H. Styri, Metals and Alloys **3**, 273–274 (1932).

the Brinell numbers computed from indentations with 5-mm Carboloy balls under 750-kg load and those with 5-mm steel balls under the same load.

Brinell defined his "hardness number" (nombre de dureté) in terms of the indentation produced by a ball of hardened steel (acier trempé).[22] According to this definition Brinell numbers computed from balls of other materials have "errors" depending upon the difference between their elastic moduli and the moduli of steel. In view of the increased use of Carboloy and diamond balls, it seems preferable to consider these effects not as errors but as differences between Brinell numbers measured under different conditions analogous to the difference between the 3,000-kg Brinell number and the 500-kg Brinell number of the same material.

Meyer's formula 18 could be used to evaluate this effect if Meyer had extended his work to very hard materials and had, in addition, obtained values of n for balls of other materials than steel, and if the value of the radius of curvature r of the loaded ball in the contact surface (fig. 14) were known. Knowing both r and n, the constant a of equation 18 could then be calculated from the diameter of the indentation d' and, in turn, the Brinell number for an ideal rigid ball from this constant a.

Even without knowing n, a rough estimate for r and d' may be obtained for the case of elastic deformation of the ball from Hertz's theory for the contact of two elastic bodies. Such an estimate is made below, followed by a consideration of the effect on the Brinell number of permanent deformation of the ball.

(1) *Elastic Deformation.*—The effect of elastic deformation of the ball would be measured by the difference between the Brinell number P/A' obtained for an elastic ball (B_2, fig. 14), and the Brinell number P/A obtained for an ideal rigid ball (B_1, fig. 14), where A' is the area of the actual surface of contact under load of the elastic ball (a b c in fig. 14) and A is the contact area of the ideal rigid ball. It is not possible to measure A' under load. To obtain an approximation to A', it is computed as if it were the area of a spherical indentation of diameter d' with the radius of curvature of the ball when not under load; that is, A' is replaced by A'', where A'' is the area of the contact surface intersecting the plane of the paper in figure 14 in a e c. The assumption of $A'=A''$ is convenient in that the Brinell number P/A'' corresponding to d' may be read directly from a Brinell table and may be compared with that corresponding to d. The error due to this assumption is less than 1 percent, as is shown in figure 19 in the appendix 2, p. 95.

In applying Hertz's theory it must be remembered that it is not strictly applicable to the case of the deformation of the indenting ball in the Brinell test for two reasons:

1. The specimen does not remain elastic during the test.

[22] Communications Congrès International des Méthodes d'Essai des Matériaux de Construction, Paris, 2, 85 (1900).

2. The diameter of the contact surface is not, in general, small compared to the diameter D of the indenting ball.

In spite of these limitations, however, the theory forms a useful basis for discussion of the effects of changes in ball diameter, load, and material of the ball on the Brinell number obtained on a given specimen. These effects become most pronounced for the case of indentations on very hard materials. This is just the case which is more closely approximated by the theory, since the indentation is small on hard materials and since the specimen is not as severely deformed as for softer materials.

According to Hertz's theory the radius a of the circle of contact is, for the special case of elastic contact between a ball and plane, given by[23]

$$a^3=\frac{3Pr_1}{16}(\theta_1+\theta_2), \tag{21}$$

where

P=normal load transmitted by the ball to the plane

r_1=radius of curvature ($D/2$) of the ball in its unloaded condition.

$\theta_1=\frac{4}{E_1}(1-\mu_1^2)$=elastic constant for the ball

$\theta_2=\frac{4}{E_2}(1-\mu_2^2)$=elastic constant for the plane

E_1=Young's modulus for the ball.

E_2=Young's modulus for the plane.

μ_1=Poisson's ratio for the ball.

μ_2=Poisson's ratio for the plane.

Assuming that a is the radius $\frac{d'}{2}$ of the actual indentation gives

$$d'^3=\frac{3}{2}Pr_1(\theta_1+\theta_2), \tag{22}$$

while the diameter of the indentation made by the ideal rigid ball ($\Theta_1=0$) would be given by

$$d^3=\frac{3}{2}Pr_1\theta_2. \tag{23}$$

Substituting this value in equation 22 gives

$$d^3=d'^3-\frac{3}{2}Pr_1\theta_1. \tag{24}$$

The Brinell number is obtained by substituting d' in the Brinell formula. The corrected Brinell number, which would be obtained with an ideal rigid ball, may be obtained by substituting d, computed from

[23] Gesammelte Werke, Barth, Leipzig, 1, 155–196 (1894/95).

equation 24 in the Brinell formula. This requires a knowledge of the elastic constant Θ_1 of the material of the Brinell ball. The elastic constants for some materials that have been used for Brinell balls are:

Steel. $\theta_1 = 1.770 \times 10^{-4}$ kg^{-1}mm^2

Carboloy. θ_1 lies between 0.770×10^{-4} and 0.834×10^{-4} kg^{-1}mm^2

Diamond. θ_1 lies between 0.2955×10^{-4} and 0.515×10^{-4} kg^{-1}mm^2.

The values for Carboloy were calculated from Young's modulus, as determined by tests at the National Bureau of Standards; Poisson's ratio was assumed to be between 0.16 and 0.32. The values for diamond were calculated from the bulk modulus given in International Critical Tables; Poisson's ratio was assumed to lie between 0.16 and 0.32.

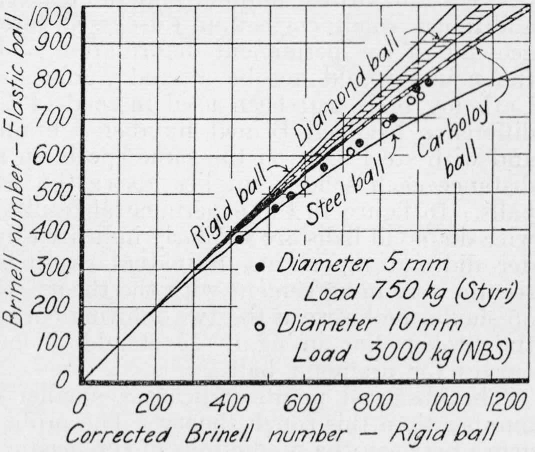

FIGURE 15.—*Theoretical and observed relations between Brinell numbers obtained with 10-mm balls of various materials under 3,000-kg load or 5-mm balls under 750-kg load.*

Figure 15 shows curves of Brinell number *vs* corrected Brinell number calculated from equations 23 and 24 for steel balls, Carboloy balls, and diamond balls of 10 mm diameter, under 3,000-kg load, or of 5 mm diameter, under 750-kg load. Figure 16 shows similar curves for steel balls and diamond balls of 5 mm diameter, under 187.5-kg load. The curves indicate roughly how the Brinell number should be expected to vary with the elastic properties of the ball. The variation increases from zero at low Brinell numbers to a large value at high Brinell numbers.

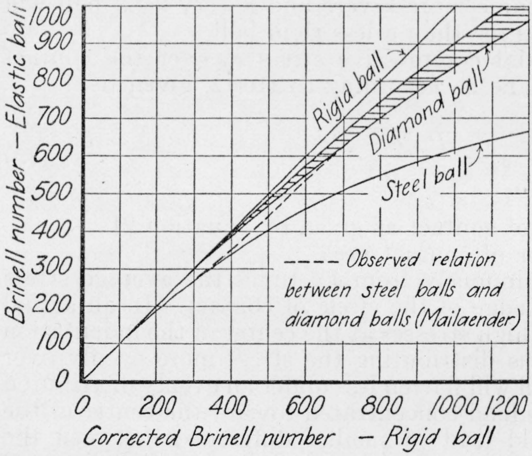

FIGURE 16.—*Theoretical and observed relations between Brinell numbers obtained with 5-mm balls of various materials under 187.5-kg load.*

Results taken from actual tests by Styri[24] using Carboloy balls and steel balls (fig. 15) and by Mailaender[25] using diamond balls and steel balls (fig. 16) have been replotted for comparison with this rough theory. Each point in figure 15 represents two determinations of the Brinell number on a given specimen, one made with a steel

[24] H. Styri, Metals and Alloys **3**, 273–274 (1932).
[25] R. Mailaender, Stahl u. Eisen **45**, 1769–1773 (1925).

ball and the other with a Carboloy ball. In plotting the points a corrected Brinell number was assigned to each specimen, on the assumption that the curve for the steel ball represented the relation between the Brinell number and the corrected Brinell number. An additional small correction (discussed later) was applied to take account of the permanent deformation of the balls. The trend of the points would not be altered significantly if the curve for the Carboloy balls had been used instead of that for steel balls. The difference between Brinell numbers obtained with Carboloy balls and with steel balls on the same specimen shown in figure 15 is the distance each solid point lies above the theoretical curve for steel balls. In figure 16 the experimental results obtained by Mailaender with diamond balls are similarly indicated by a broken line (Mailaender did not report his individual readings). If the experimental results were in agreement with the theory, the points shown in figure 15 should lie between the two limiting curves for Carboloy balls and the broken line in figure 16 should lie between the two limiting curves for diamond balls.

The observed points indicate a smaller difference in the Brinell number than this rough theory. The probable reason for this divergence between the predictions of the elastic theory and actual observations is to be sought in the difference between the actual stress distribution in the contact area and that assumed in Hertz's theory.

This difference in stress distribution has little effect on the deformation of balls indenting very soft material since the average stresses acting on the ball are so small in that case that there is no noticeable difference between the deformed shape of, say, a steel ball and a diamond ball. As the indentation becomes smaller with harder materials the stresses become more severe and a very rigid ball will be flattened to a lesser degree than a less rigid ball.

The distribution of axial compressive stress σ_z over the contact area between ball and plane is, according to Hertz, given by

$$\sigma_z = \frac{3P}{2\pi} \frac{\sqrt{a^2 - x^2}}{a^3}, \tag{25}$$

where

a=radius of the circle of contact as given by equation 21

x=distance from center of contact area.

The stress decreases continuously from 1½ times the average stress at the center to 0 at the edge of the circle of contact. In an actual Brinell test the relatively high stresses at the center of the indentation will produce yielding, thus distributing the stress more evenly over the contact area. The ball will flatten less under an evenly distributed load than under the same load concentrated toward the center of the contact area. This would lead to smaller differences between the diameters of indentations obtained with steel balls and with diamond balls than those predicted from Hertz's theory. As the hardness of the specimen increases further the indentation becomes smaller and flatter and the stress distribution assumed by Hertz is more nearly approached. The difference between theory and experiment should, therefore, decrease in going to the harder materials. This is verified by the plots of figures 15 and 16. The difference, as shown in figure 15, between the Brinell numbers reported by Styri [26] for a given

[26] H. Styri, Metals and Alloys 3, 273–274 (1932).

specimen with a 5-mm Carboloy ball and with a steel ball at 750-kg load comes close to that predicted by the theory at a "Carboloy" Brinell number of about 800. Figure 16 shows that the same agreement holds for the comparison of a diamond and a steel ball under 187.5-kg load at a "diamond" Brinell number of about 600.

Styri's results for the comparison of steel balls and Carboloy balls were checked at the National Bureau of Standards by tests on 10-mm balls. These tests will be described in detail later. The five points taken from these tests (fig. 15, open circles) fall close to the solid points taken from Styri's tests.

The small correction for the permanent compression of the balls is less than 5 in Brinell number for all of Styri's tests and less than 20 for the tests made with high-grade balls at the National Bureau of Standards. The observed difference in Brinell number on a given specimen made with a steel ball and a Carboloy ball was found to be as high as 70 in some cases. The differences in elastic deformation may, therefore, be several times greater than those caused by permanent deformation of the ball.

In the case of extremely hard specimens both effects will have to be considered, since the elastic theory in itself is not sufficient. Nevertheless, theory and tests show that the elastic deformation of the ball may lead to large differences in the measured Brinell number if balls with sufficiently different elastic properties are used. It is, therefore, necessary to specify the material of the ball in quoting Brinell numbers above 500.

(2) *Permanent Deformation.*—If the ball deforms permanently during the test the contact area will be flattened even more than in the case of elastic deformation, thus increasing still more the effective diameter of the ball. This effect produces an appreciable error in practical testing with high-grade steel balls only if the Brinell number of the specimen exceeds 500. For steels the diameters of the indentations increase with increase in ball diameter so that the greater the permanent deformation of the ball the lower will be the Brinell number computed from the resulting indentation.

It is important, for practical work, to specify the limits of Brinell number of the specimen within which a given Brinell ball may be used without introducing an uncertainty, caused by its permanent deformation, greater than a certain figure, e. g., 1 percent.

This problem has been considered by several workers in the field. Mailaender [27] carried out a comprehensive series of indentation tests with 10-mm steel balls of different hardness, indenting specimens from 110 to 680 Brinell under 3,000-kg load. The Brinell number of the ball, as estimated from the diameter of indentation in bringing one ball in contact with the other under 3,000-kg load, varied from 230 to 720.

Mailaender concluded from these tests that the diameter of the indentation remained independent of the Brinell number of the ball as long as the latter was at least 1.77 times the Brinell number of the specimen.

Hultgren [28] made an even more extended series of tests with 10-mm steel balls of various hardness under 3,000-kg load. His specimens

[27] R. Mailaender, Stahl u. Eisen **45**, 1769–1773 (1925).
[28] J. Iron and Steel Inst. **110**, II, 183–218 (1924).

ranged in Brinell number from 506 to 735. Instead of using either the Brinell number or the effective radius of curvature of the contact area of the ball as independent variable, he used the more easily measured shortening or permanent compression of the loaded diameter after the first test. Hultgren obtained a series of correction curves which show, for instance, that a permanent compression of 0.025 mm in the ball will lower the Brinell number computed from the indentation by about 25 and a compression of 0.010 mm by about 5.

Hultgren used five different makes of steel balls on each of five different specimens, four of which were chromium steel, while the fifth (specimen V) was high-speed steel. He made five or more indentations on each specimen with each make of ball and measured the permanent compression and the Brinell number for each indentation. From these individual values averages of the permanent compression of the ball after the first test and of the observed Brinell number H were obtained for each specimen and each type of ball.

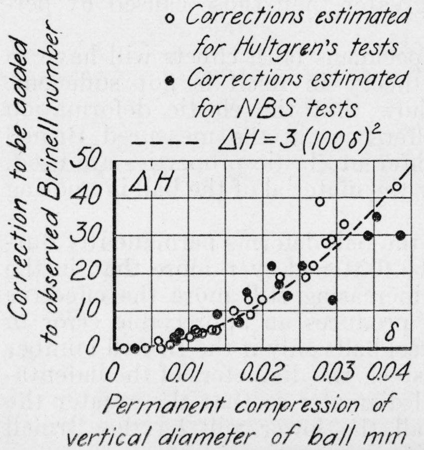

FIGURE 17.—*Estimated corrections in Brinell number due to permanent deformation of ball.*

[10-mm steel balls, 3,000-kg load]

A plot of Hultgren's corrections $\Delta H = H_c - H$ against δ, where H_c is a Brinell number corrected for the permanent compression δ of the ball, showed that all Hultgren's points, except those for specimen *V*, grouped themselves with considerable scatter about a parabola of the type $\Delta H = \alpha\delta^2$.

Obviously the scatter of the individual points depends on the value of this corrected Brinell number H_c. It was found that this scatter could be reduced and that at the same time the points for specimen *V* could be brought into closer agreement with the remaining points by extrapolating to new values of H_c in the following manner. Assume that the relation $\Delta H = \alpha\delta^2$ holds for each specimen, plot *H* against δ^2 for each individual test, and fair a straight line through the plotted points. H_c will then be the ordinate of the straight line for the abscissa $\delta = 0$. The values of H_c so determined were within 5 in Brinell number of those estimated by Hultgren, except for specimen *V*, for which 720 was obtained in place of Hultgren's value of 735. The differences ΔH between the average values of *H* for each type of ball for each specimen and the values H_c for each specimen, derived as described above, are plotted against δ as open circles in figure 17, for values of δ less than 0.04 mm. For larger deformations no relations even approximately consistent were found between ΔH and δ.

The solid points in figure 17 represent the results of a few check tests made at the National Bureau of Standards. Steel balls (10 mm in diameter) of three makes A, B, C, were used in these tests. A and B designated ordinary steel Brinell balls, while C designated steel balls cold-worked by the Hultgren process. In addition, a number of 10-mm Carboloy balls were used for comparison with steel balls. These also gave a further check on the effect of elastic deformation of the ball on the Brinell number, which was discussed in the previous section.

The test specimens consisted of five disks of chromium steel (SAE 52100) heat treated by the Division of Metallurgy of this Bureau to have Brinell numbers ranging from 500 to 700. Four indentations with each type of ball, A, B, and C, were made on each one of the five disks. A new steel ball was used for each indentation, and the diameter of each ball in the direction of loading was measured with a Zeiss optimeter before and after test. The permanent compressions corresponding to the difference of these two readings, together with the Brinell numbers corresponding to the observed indentation diameters, are listed in the first six columns of table 4. The last two columns give corresponding results obtained with Carboloy balls. Since only two Carboloy balls were available for each specimen each ball was used twice; the permanent compression listed is that measured after the first indentation. Some of the balls of group A showed compressions greater than 0.04 mm. The values for these balls (denoted by an "a") were not included in the averages given at the bottom of each group for the reason noted above.

TABLE 4.—*Results of tests to show effect of deformation of ball on Brinell number*

Type of ball→	A		B		C		Carboloy	
Specimen ↓	Brinell number	Compression of ball	Brinell number	Compression of ball	Brinell number	Compression of ball	Brinell number	Compression of ball
		mm		mm		mm		mm
1, H_c=511	508	0.0079	505	0.0094	512	0.0025	525	0.0000
	508	.0071	508	.0084	511	.0030	526	------
	[a]491	[a].0445	506	.0084	513	.0030	521	.0033
	510	.0086	507	.0089	509	.0025	522	------
Average	509	.0079	506	.0088	511	.0028	524	.0016
Correction	2	------	2	------	0	------	0	------
Corrected Brinell number	511	------	508	------	511	------	524	------
2, H_c=592	582	.0191	575	.0183	584	.0102	610	.0066
	590	.0168	579	.0180	588	.0076	613	------
	580	.0175	575	.0183	590	.0084	610	.0061
	588	.0137	580	.0168	593	.0066	612	------
Average	585	.0168	577	.0178	589	.0082	611	.0064
Correction	9	------	10	------	2	------	1	------
Corrected Brinell number	594	------	587	------	591	------	612	------
3, H_c=652	640	.0231	630	.0272	646	.0122	685	.0091
	639	.0234	628	.0239	647	.0130	684	------
	639	.0211	629	.0259	633	.0185	678	.0084
	636	.0259	632	.0246	647	.0119	679	------
Average	638	.0234	630	.0254	643	.0139	682	.0088
Correction	17	------	19	------	6	------	2	------
Corrected Brinell number	655	------	649	------	649	------	684	------
4, H_c=690	------	[a].2508	665	.0343	658	.0203	742	.0142
	678	.0290	656	.0338	670	.0203	737	------
	[a]628	.0533	652	.0353	672	.0216	741	.0142
	676	.0300	666	.0335	674	.0224	736	------
Average	677	.0295	660	.0342	668	.0212	739	.0142
Correction	26	------	35	------	13	------	6	------
Corrected Brinell number	703	------	695	------	681	------	745	------
5, H_c=697	[a]529	[a].1334	665	.0366	684	.0221	754	.0152
	667	.0386	661	.0345	674	.0221	757	------
	------	[a].3856	661	.0350	673	.0239	741	.0132
	[a]554	[a].1217	662	.0353	690	.0211	746	------
Average	667	.0386	662	.0354	680	.0223	750	.0142
Correction	45	------	38	------	15	------	6	------
Corrected Brinell number	712	------	700	------	695	------	756	------

[a] Values not included in average because the deformation of the ball was greater than 0.04 mm.

Values for the corrected Brinell numbers H_c for the five specimens were obtained from the individual test results by fairing a straight line through a plot of H versus δ^2 using the procedure already described above in the discussion of Hultgren's results. The values of $\Delta H = H_c - H$ corresponding to these corrected Brinell numbers are plotted as solid points in figure 17. It is seen that the points fall roughly about a common curve with the points computed from Hultgren's data. The scatter of all points increases with increasing compression. A large part of this scatter is, no doubt, due to nonuniform response to heat treatment. The resulting lack of uniformity may be expected to increase with the Brinell number of the specimen and hence with the permanent compression produced in a given ball. All of the observed results are approximated roughly by the simple empirical formula

$$\Delta H = 3(100\delta)^2, \tag{26}$$

which is shown as a dotted line in figure 17. This formula was used to correct the average Brinell numbers listed in table 4. The corrections are less than 1 percent for Carboloy balls, less than 3 percent for steel balls C, but they exceed 5 percent for steel balls B, indenting the hardest specimen, and 6 percent for steel balls A.

The effect of permanent deformation of the Carboloy balls would be similar to that of the steel balls, though not necessarily of exactly the same magnitude. The Carboloy balls used in the tests were so uniform that any difference was masked by the experimental error. For that reason the Brinell numbers obtained with the Carboloy balls have been corrected by the same formula, 26, used with the steel balls.

The Brinell numbers obtained with Carboloy balls are in every case higher than those obtained with steel balls; this is due primarily to the greater rigidity of Carboloy as compared to steel (see previous section). The correction for permanent deformation of the ball reduced the maximum difference in the average Brinell numbers obtained with the three types of steel balls in every case except specimen 4, in which it increased it from 17 to 22. Much of this scatter is, as already mentioned, due to nonuniform response of the specimen to heat treatment.

Figure 17 shows that the error due to permanent deformation of the ball is below 5 Brinell numbers for balls showing a permanent compression of less than 0.01 mm after the first loading, and below 20 Brinell numbers for balls showing a permanent compression of less than 0.025 mm after the first loading. Table 4 shows that high-grade steel balls are available which show permanent sets less than 0.01 mm at 500 Brinell and less than 0.025 mm at 700 Brinell. The corresponding permanent compressions for Carboloy balls were found to be even less.

IV. RECOMMENDATIONS FOR BRINELL TESTING

It is possible, after having discussed in detail the effect of small variations in the several variables that enter into the determination of Brinell numbers, to draw up a list of recommendations designed to keep the combined error due to these variations down to a small figure. Such recommendations may assist in further standardization of the Brinell test and may in that way lead to greater concordance between the Brinell numbers obtained by different observers using balls of given diameter on specimens of given material. These recommendations are based on tests of metal specimens having Brinell numbers greater than 70. They may not be sufficient for testing metals having Brinell numbers less than 70, such as soft bearing metals.

Grouping the individual factors in the order in which they are discussed above gives the following list of recommendations.

1. APPARATUS AND PROCEDURE

(a) The loading mechanism should be operated to give a uniform rate of loading not exceeding 500 kg/sec.

(b) The maximum load should be applied for 30 seconds.

(c) The error in the load applied by the machine should not exceed ⅓ percent. This should be checked by periodic calibration with a proving ring or other suitable device.

(d) The calibration of the apparatus used for measuring the diameter of the indentation should be checked frequently. The maximum error in the reading at any point on the scale should not exceed 0.01 mm. The indentation diameter should be read in two or more mutually perpendicular directions.

2. SPECIMEN

(a) The Brinell number should be computed from the average of diameter readings in at least four equally spaced directions if the indentation has a noncircular boundary. Care should be taken to polish the surface of the specimen to such a finish that the error in diameter reading due to tool marks does not exceed 0.01 mm.

(b) If the indentation is made on a curved specimen the minimum radius of curvature of the specimen should not be less than 25 mm for a 10-mm ball. The diameter of the indentation should be taken as the average of the two principal diameters.

(c) The specimen should be at least 0.4 in. thick.

(d) The distance of the center of the indentation from the edge of the specimen should be at least three times the diameter of the indentation.

(e) The distance between centers of adjacent indentations should be at least three times the diameter of the indentation.

(f) The angle between the load line and the normal to the specimen should not exceed 2 degrees.

3. INDENTING BALL

(a) The difference between the average diameter and the nominal (10 mm) diameter of the ball should not exceed 0.025 mm (0.001 in.). The average diameter should be the average of six or more different diameters of the ball.

(b) The difference between any individual diameter and the average diameter of new balls should not exceed 0.025 mm (0.001 in.).

(c) The material of the indenting ball (e. g. steel, Carboloy, diamond) must be specified in quoting Brinell numbers greater than 500.

The permanent compression of the loaded diameter of the ball after any indentation on a specimen having a Brinell number less than 500 should not exceed 0.01 mm. If, however, steel balls are used on specimens having Brinell numbers greater than 500, the permanent compression after any indentation should not exceed 0.025 mm.

The use of Carboloy balls is recommended for indentations on any specimen having a Brinell number greater than 500.

V. APPENDIX

1. ERROR IN THE BRINELL NUMBER DUE TO CURVATURE OF SPECIMEN

The Brinell formula:

$$H=\frac{P}{A}=\frac{2P}{\pi d^2}\left(1+\sqrt{1-\left(\frac{d}{D}\right)^2}\right) \tag{1}$$

assumes the surface of indentation to be a section of a sphere of diameter D bounded by a circle of diameter d. The surface of intersection between sphere and specimen will no longer have a plane boundary if the specimen has two different principal radii of curvature R_1, R_2. Instead of being a circle it will be a closed curve roughly elliptical in shape and having two principal axes, one of length d_1 and the other of length d_2. The error in the Brinell number assuming the equivalent diameter to be equal to

$$d_3=\frac{d_1+d_2}{2} \tag{2}$$

will be computed below.

The relative error in the Brinell number due to an error ΔA in the measurement of area, is from formula 1

$$\frac{\Delta H}{H}=-\frac{\Delta A}{A}. \tag{3}$$

The error in area may, in this case, be written

$$\Delta A=A-A_3, \tag{4}$$

where

A = area of surface of the sphere embedded in the specimen.
A_3 = area of equivalent section of sphere given by equation 1, that is,

$$A_3=\frac{\pi d_3^2}{2}\,\frac{1}{1+\sqrt{1-\left(\frac{d_3}{D}\right)^2}}. \tag{5}$$

The computation of the relative error $\Delta H/H$ involves the derivation of the three quantities A, d_1, d_2.

The surface of intersection A of a sphere with a specimen having principal curvatures of $1/R_1$ and $1/R_2$ will be computed first.

Let the origin of coordinates be at the center of the indenting sphere (fig. 18) and let any point on the surface of the sphere be described in terms of the latitude θ and the longitude ϕ. The curve of intersection ABCF of the sphere with the specimen may then be expressed as $\theta_i(\phi)$. The surface of indentation A will be a portion of a sphere of radius $r=D/2$ bounded by this curve, i. e.,

$$A=\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\theta i}(r\sin\theta d\phi)rd\theta=4r^2\int_{\phi=0}^{\pi/2}(1-\cos\theta_i)d\phi. \tag{6}$$

The integration, equation 6, can be carried out if the shape of the curve of intersection $\theta_i(\phi)$ is known, or more specifically, if $\cos\theta_i$ can be expressed as a function of ϕ.

The curve of intersection $\theta_i(\phi)$ is the locus of points common to the indenting spherical surface and the surface of the specimen. It is convenient to describe all points on the surface of the specimen in terms of cylindrical coordinates z, s, ϕ having their origin at the point E at which the load line (fig. 18) intersects the surface of the specimen. The z-axis is taken as coinciding with the load line and directed into the interior of the specimen, s is the radial coordinate (normal to

z) and ϕ is the longitude already used in describing points on the indenting sphere. z, s, will be points on the surface of the sphere also (see fig. 18) if

$$z=\epsilon r-r\cos\theta_i=r(\epsilon-\cos\theta_i). \tag{7}$$

$$s=r\sin\theta_i. \tag{8}$$

It is assumed in describing the surface $z(s,\phi)$ of the specimen, that only a small portion of this surface will be indented. The surface may then be approximated by a surface of the second degree with origin at E having a curvature at that point equal to the actual curvature of the specimen. This surface may be described mathematically by

$$z=\frac{1}{2}\,\frac{s^2}{R}, \tag{9}$$

where $1/R$ is the curvature at the longitude considered. The curvature at any longitude is related to the principal curvatures ($1/R_1$ at $\phi=0$, $1/R_2$ at $\phi=\pi/2$) by Euler's equation [29]

$$\frac{1}{R}=\frac{\cos^2\phi}{R_1}+\frac{\sin^2\phi}{R_2}. \tag{10}$$

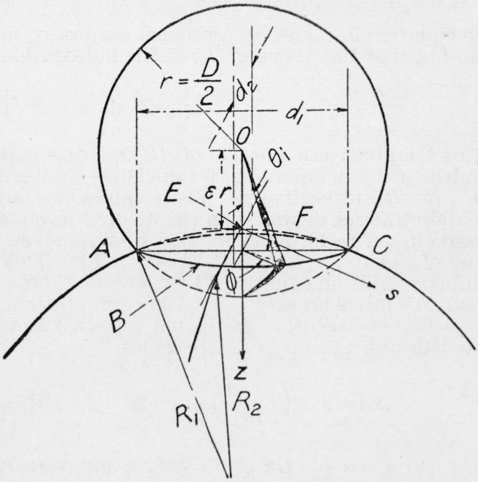

FIGURE 18.—*Diagrammatic sketch of intersection between a sphere and a curved surface.*

Solving equations 8, 9, and 10 for $\cos\theta_i$ gives

$$\cos\theta_i=\frac{R}{r}\left[1\pm\sqrt{1-2\frac{r}{R}\epsilon+\frac{r^2}{R^2}}\right]. \qquad (11$$

The negative sign in front of the radical applies here since $\cos\theta_i$ cannot be greater than 1 and since R/r is, in general, large compared to 1, while ϵ cannot be greater than 1.

Substituting equation 11 in equation 6 gives the following expression for the area of the surface of intersection of sphere and specimen

$$A=4r^2\int_{\varphi=0}^{\pi/2}\left[1-\frac{R}{r}\left(1-\sqrt{1-2\frac{r}{R}\epsilon+\frac{r^2}{R^2}}\right)\right]d\phi, \tag{12}$$

where the integrand is a known function of ϕ obtained by substituting in it equation 10 for 1/R. The resulting expression is not a simple integrable form but because $2\epsilon r/R+r^2/R^2$ are small it can be expanded in a series and integrated term by term.

It is convenient in carrying out this integration to substitute the ratios

$$\kappa=\frac{r}{\sqrt{R_1R_2}};\ \gamma=\frac{1}{2}\left(\frac{r}{R_1}+\frac{r}{R_2}\right), \tag{13}$$

which involve only the known radius r of the indenting sphere and the known principal curvatures $1/R_1$, $1/R_2$ of the specimen. The result of the integration in terms of these variables carried out to 5th order terms of κ, γ is equal to

$$\begin{aligned}A=2\pi r^2\Big\{(1-\epsilon)+(1-\epsilon^2)\Big[&\frac{\gamma}{2}+\frac{1}{4}\epsilon(3\gamma^2-\kappa^2)-\frac{5}{16}(1-5\epsilon^2)\left(\gamma^3-\frac{3}{5}\gamma\kappa^2\right)\\&-\frac{35}{64}\epsilon(3-7\epsilon^2)\left(\gamma^4-\frac{6}{7}\gamma^2\kappa^2+\frac{6}{70}\kappa^4\right)+\frac{63}{128}(1-14\epsilon^2\\&+21\epsilon^4)\left(\gamma^5-\frac{10}{9}\gamma^3\kappa^2+\frac{5}{21}\gamma\kappa^4\right)+\ldots\Big]\Big\},\end{aligned} \tag{14}$$

[29] Blaschke, Differentialgeometrie (Springer, Berlin, I, 57, 1921).

where ϵr again represents the distance from the center of the indenting sphere to the point of intersection of the load line with the original surface of the specimen (fig. 18).

The area A_3 may be computed next by substituting the average diameter

$$d_3 = 2r_3 = r_1 + r_2 \tag{15}$$

in equation 5. The two principal radii r_1, r_2, may be derived from equation 11 by noting that the diameter $2r_i$ of the indentation in any direction is equal to

$$2r_i = 2r \sin \theta_i = 2r\sqrt{1-\cos^2 \theta_i}. \tag{16}$$

Cos θ_i is given as a function of r/R and ϵ in equation 11. r_1 and r_2 were obtained by substituting in equation 16 the values obtained from equation 11, by letting $R = R_1$, $R = R_2$, respectively. These values of r_1, and r_2 were inserted in equation 15 to obtain d_3, and d_3 was then substituted in equation 5 to obtain A_3. The square roots in the resulting expression were removed by expansion into a series making use of the abbreviations in equation 13. This led to a formula which could be subtracted from equation 14 to get the difference between the area A and the assumed equivalent area A_3. The computation, while simple in principle, is cumbersome of execution and is not repeated here. The final expression carried out to fifth-order terms of κ, γ becomes

$$\begin{aligned}\Delta A = 2\pi r^2\left(\frac{1-\epsilon^2}{8\epsilon}\right)\Big\{&[(1-2\epsilon^2)(\gamma^2-\kappa^2)] + \frac{1}{2\epsilon}[(1+8\epsilon^2-15\epsilon^4)(\gamma^3-\gamma\kappa^2)]\\ &+\frac{1}{16\epsilon^2}[(3+5\epsilon^2+282\epsilon^4-406\epsilon^6)\gamma^4-(2+6\epsilon^2+332\epsilon^4-476\epsilon^6)\gamma^2\kappa^2\\ &-(1-\epsilon^2-50\epsilon^4+70\epsilon^6)\kappa^4]+\frac{1}{32\epsilon^3}[(1-6\epsilon^2-233\epsilon^4+2448\epsilon^6\\ &-2730\epsilon^8)\gamma^5+(2+16\epsilon^2+326\epsilon^4-3404\epsilon^6+3780\epsilon^8)\gamma^3\kappa^2\\ &-(3+10\epsilon^2+93\epsilon^4-956\epsilon^6+1050\epsilon^8)\gamma\kappa^4]+\ldots\Big\}\end{aligned} \tag{17}$$

This expression is not applicable to the case in which half of the ball is embedded in the specimen ($\epsilon = 0$). This, however, does not seriously detract from its usefulness. The diameter of a Brinell indentation does not exceed 70 percent of the diameter of the indenting ball in ordinary practice. It is apparent from this that $\epsilon > 0.7$ in practical work.

ΔA must be zero if the indentation is circular, i. e., if $R_1 = R_2$, as for a plane specimen or a spherical specimen; in that case $\kappa = \gamma$ in equation 13 and it is seen that the right side of equation 17 becomes equal to zero.

In the particular case of a cylinder of radius R_1, $\kappa = 0$ and equation 17 becomes

$$\begin{aligned}\Delta A = 2\pi r^2\left(\frac{1-\epsilon^2}{8\epsilon}\right)\Big[&(1-2\epsilon^2)\gamma^2+\frac{1}{2\epsilon}(1+8\epsilon^2-15\epsilon^4)\gamma^3+\frac{1}{16\epsilon^2}(3+5\epsilon^2+282\epsilon^4\\ &-406\epsilon^6)\gamma^4+\frac{1}{32\epsilon^3}(1-6\epsilon^2-233\epsilon^4+2448\epsilon^6-2730\epsilon^8)\gamma^5+\ldots\Big].\end{aligned} \tag{18}$$

2. ERROR IN THE BRINELL NUMBER DUE TO ERROR IN THE COMPUTED CONTACT AREA FOR A GIVEN DIAMETER OF INDENTATION

An error is introduced in the calculation of the Brinell number obtained with an elastic ball indenting a specimen to a diameter d' (fig. 14) by computing it from the Brinell formula as if it were a spherical calotte of radius $D/2$. (See p. 83.)

The error could be computed if the actual radius of curvature r (fig. 14) in the contact area of the loaded ball were known. An estimate of r may be obtained from Hertz's theory for the contact of an elastic ball and an elastic plane. The radius of curvature r of the ball in the contact area is, according to Hertz, given by

$$r=\frac{\theta_1+\theta_2}{\theta_2}r_1. \tag{1}$$

The radius of curvature is doubled, if the contact takes place between a ball and a plane of the same material $(\theta_1=\theta_2)$.

The elastic constant θ_2 may be eliminated from Hertz's equation 21, page 84, and from equation 1, and the following expression may be obtained for the curvature $1/r$ in the contact surface

$$\frac{1}{r}=\frac{1}{r_1}-\frac{3\theta_1}{16a^3}P. \tag{2}$$

In view of the limitations of Hertz's theory (p. 83), as applied to this case, it is not possible to calculate an exact value of r in figure 14. But it is possible to obtain an upper limit to r, and hence an upper limit to the difference between the Brinell number P/A' and P/A'' by substituting $d'/2$ for a in equation 23. The value obtained for r will be an upper limit because the substitution of $d'/2$ for a is equivalent to the replacement of the plastically deformed specimen by an elastically deformed specimen giving an indentation of the same diameter. It was shown on page 86 that the ball indenting a plastic specimen will be deformed less than the same ball indenting an elastic specimen to the same indentation diameter. Hence the radius of curvature will be less in the first case than in the second. For a 10-mm steel ball under 3,000-kg load ($r_1=5$ mm, $P=3{,}000$ kg, $E_1=2.1\ 10^4$ kg/mm^2, $\mu_1=0.25$) equation 2 above becomes

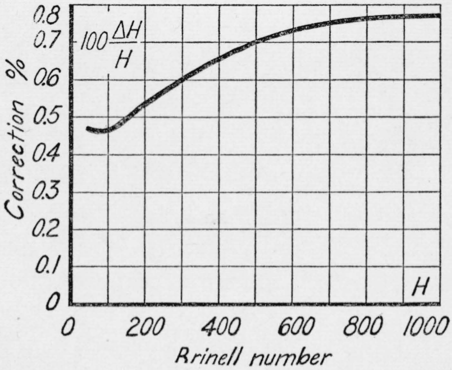

FIGURE 19.—*Upper limit to correction in Brinell number due to error in the computed contact area for a given indentation diameter.*

$$\frac{1}{r}=0.2-\frac{0.803}{d^3}\ \text{mm}^{-1}.$$

The average axial stress over the contact surface (equal to the corrected Brinell number) will then be obtained from the Brinell equation 1 by substituting in it $D=2r$ instead of $D=10$ mm. The error in the Brinell number can be computed by subtracting the Brinell number given by the tables from that just computed. Figure 19 shows the result of such a computation. The increase in the Brinell number computed on the assumption of a rigid 10-mm ball which would take account of this change in curvature, is found to be small; it ranges from 0.46 to 0.77 percent as the Brinell number is increased from 100 to 900. The actual percentage difference between P/A' and P/A'' is even less since the actual diminution in curvature of the ball must be less than that assumed in the derivation of figure 19.

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