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NOTE ON THE EFFECT OF A COVER GLASS IN REFLECTANCE MEASUREMENTS

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ABSTRACT

It has been found from theoretical considerations that the use of a cover glass may result in an error of as much as 10 percent when two diffusing surfaces, one light and one dark, are compared for apparent reflectance. The error arises from multiple reflections between sample and cover glass, the lighter sample obtaining much more added illumination from this cause than the darker. The error depends on the angular distribution of the illuminant, the angle of view, and the diffusing characteristics of the samples as well as their reflectances.

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I. INTRODUCTION

When it is necessary to measure the reflectance of a material (such as a liquid or jelly, or one composed of powder, small crystals, or chopped-up fibers) whose free surface is stable only in a horizontal position, and there is available a reflectometer which requires the surface to be other than horizontal, the use of a cover glass suggests itself. Even when the surface can be tested in a horizontal position, there are materials out of which it is difficult to form a reproducible free surface. For these materials the use of a cover glass may improve reproducibility of surface to a marked extent. This discussion deals briefly with the effect of a cover glass on reflectance measurements of such materials.

II. THEORY

The following simplified analysis applies to the comparison of two perfectly diffusing surfaces, one of reflectance, R_1, the other of reflectance, R_2. One of these may be thought of as a standard of reflectance. Without a cover glass the measured reflectance of the first relative to the second will be simply R_1/R_2. Now let both be covered with identical cover glasses of transmission T and reflectance R, and so illuminated and viewed by the reflectometer that none of the light flux may be reflected from the cover glass directly into the photometer; that is, all light which is counted in the measurement must come from the two surfaces being compared. Let F be the flux which passes through the cover glass directly from the light source and is initially incident upon the sample. The total flux incident on the sample exceeds this initial flux because of multiple reflection between the cover glass and the sample. That is, a part, R_1, of the initial flux is diffusely reflected by the sample and strikes the cover glass which, itself, returns a fraction R, to the sample, and so on. The total incident flux, therefore, depends in addition on the reflectance R_1 of the sample, thus:

$$\begin{aligned}\text{Total flux} &= F+FR_1R+FR_1RR_1R+\ldots\ldots\ldots\ldots\\ &= F(1+R_1R+R_1^2R^2+\ldots\ldots\ldots).\\ &= F/(1-R_1R).\end{aligned}$$

The reflectometer measures the ratio of the flux reflected by the first surface to that reflected by the second, after both have been transmitted by the respective cover glasses; that is, it measures the ratio:

$$\frac{FR_1T/(1\text{-}R_1R)}{FR_2T/(1\text{-}R_2R)}=(1/Q)(R_1/R_2), \qquad (1)$$

where Q is a correction factor equal to $(1\text{-}R_1R)/(1\text{-}R_2R)$ to be applied to the measured ratio, thus:

$$R_1/R_2=Q\times\text{measured ratio}. \qquad (2)$$

This formula applies rigorously to a comparison of two perfectly diffusing surfaces regardless of the angular distribution of incident flux provided only that no light directly reflected from the source by the cover glass is counted in the measurement; that is, the distribution may be nearly completely diffused as in the Priest-Lange reflectometer, or it may be nearly unidirectional at, for example, about 45°. A similar formula should apply also to the measurement of apparent reflectances of nearly perfectly diffusing samples for these angular conditions of illumination, as in the Priest-Lange reflectometer, the Hunter reflectometer, the Appel-Hickson photometer, and as recommended by the International Commission on Illumination.

However, many real surfaces depart considerably from perfect diffusers. For such surfaces, therefore, we cannot get along with a single number, R_1, yielding the fraction of total incident light that is reflected independently of the angle of incidence. The reflected fraction of the flux incident by multiple reflection depends on the angular distribution of this flux. Also, since the angular distribution varies according to which multiple reflection is being considered, we cannot characterize the mirror reflection of the cover glass by a single number, R. The exact solution for real surfaces analogous to the simple solution for perfectly diffusing surfaces is thus seen to be very complicated and, even if complete information concerning the angular distribution of incident and reflected light were at hand for a given reflectometer and material, it is doubtful whether the exact solution would be of sufficient interest to justify the trouble of finding it. It is possible, however, to develop an argument showing that the amount of correction is reduced by an equal departure of both samples from a perfect diffuser. Formula 1 may be taken as a limiting case according to this argument.

An introduction to this argument is afforded by attempting an approximate evaluation of R for a cover glass of refractive index $n=1.5$. For perpendicular incidence, we have:

$$R=\rho\left[1+\frac{(1-\rho)^2t^2}{1-(\rho t)^2}\right],$$

where ρ is reflectance at each glass-air interface and t is transmittance, which, for a thin cover glass of good quality, may be safely taken as 1.000. For such a glass, and with index of refraction $n=1.5$, ρ is equal to 0.04 and R equals 0.077. This formula may also be used for an approximate computation of R_θ, the value of R as the angle of incidence, θ, is varied. The value of ρ_θ may readily be computed for θ between 0 and 90° from the general Fresnel formula. In the present case it will vary from 0.04 to 1.00. The approximation in the computation of R_θ by the above formula arises because (1)ρ changes in value with each multiple reflection (for any given value of θ) due to successive increase in the degree of polarization resulting from the transmission and reflection of light at a surface at other than normal incidence, (2) the angle of incidence at the second air-glass interface cannot exceed a certain critical value, much less than 90° and (3) the plate is not infinite in extent. However, neglecting these errors, we may evaluate approximately the reflectance of a glass plate for diffuse illumination as:

$$R_D=\frac{\sum_0^{\pi/2} R_\theta \sin 2\theta}{\sum_0^{\pi/2} \sin 2\theta}=0.16$$

according to the method given by McNicholas.[1]

The value of R_D thus computed is certainly too high, it may be considered as an upper limit. A lower limit for R_D is obtained by considering reflection between the sample and the first air-glass surface only; this gives about 0.10.

It is to be noted that, although for perpendicular incidence the cover glass reflects but 0.077, for grazing incidence it reflects nearly 1.00. The action of the cover glass is not only to build up the total flux incident on the sample by multiple reflection, but also to proportion it according to angle of incidence in such a way that a preponderance of the added flux is nearly at grazing incidence. If the sample, itself, by departure from a perfect diffuser also favors this distribution of reflected flux, it is evident that much of the light added to the sample by multiple reflection will not be counted in the measurement of the relative reflectance but will be lost by passage both through the edges of the cover glass and through the small gap between cover and sample. We may take, therefore, formula 1, which applies to perfect diffusers, also as a limiting case for the comparison of actual samples of identical imperfectly diffusing types; that is, two samples of similar diffusing characteristics will show a cover-glass effect similar to that represented by formula 1 but smaller. If the samples being compared are not identical in diffusing characteristics,

[1] H. J. McNicholas, *Absolute methods in reflectometry*, BS J. Research 1, 29 (1928) RP3. See equation 18, p. 50.

the direction of the cover-glass effect may, however, reverse; that is, it may be expected that a highly diffusing sample may obtain more effective illumination by multiple reflection than a light glossy sample.

III. EXPERIMENT

The approximate validity of these considerations has been checked in the following way. Two paper samples were compared on the Priest-Lange reflectometer [2] with and without a cover glass. The first sample was a gray paper diffusely reflecting about 0.42 and cut from the cover of a reprint from the JOSA of 1920. The second sample was mimeograph paper diffusely reflecting about 0.75.

According to formula 1 the ratio measured with the cover glass should be:

$$\frac{R_1}{R_2} \cdot \frac{1-0.75R}{1-0.42R},$$

where R is less than 0.16 and greater than 0.10; that is, the measured ratio should lie between $0.943(R_1/R_2)$ and $0.966\ (R_1/R_2)$. The actual experimental results were:

Observer	R_1/R_2	Ratio with cover glass
KSG	0. 556	0. 533=0. 958 (R_1/R_2)
DBJ	. 559	. 533= . 954 (R_1/R_2)

The agreement is seen to be good and it supports the assumption that these samples do not deviate from perfect diffusers enough to invalidate the analysis.

IV. CONCLUSION

For diffusing surfaces the use of a cover glass may result in an error of as much as 10 percent of the reflectance. A cover glass should, therefore, be avoided wherever possible because it may give results not comparable to those obtained without it. The effect of the cover glass cannot be easily calculated because it depends on the diffusing characteristics of the two samples being compared as well as on their reflectances. In such practical cases as the use of abaca fibers (manila fibers) or sisal fibers, relative to porcelain or magnesium oxide, the analysis may be in serious error. Experiment alone can tell what the magnitude and sign of the effect may be.

WASHINGTON, January 31, 1936.

[2] I. G. Priest, *The Priest-Lange reflectometer applied to nearly white porcelain enamels*, J. Research NBS **15**, 529 (1935) RP847.