U. S. Department of Commerce National Bureau of Standards

RESEARCH PAPER RP826

Part of Journal of Research of the National Bureau of Standards, Volume 15, September 1935

MECHANICAL PROPERTIES OF COTTON YARNS [1]

By Herbert F. Schiefer and Daniel H. Taft

ABSTRACT

The results of tests on three lots of cotton yarns are discussed. Two of the lots (80s and 10s) were spun in the experimental mill of the National Bureau of Standards. The 80s yarn was spun from 1$\frac{5}{16}$-inch staple Peeler cotton with 20 different twists. The 10s yarn was spun from 1-inch staple Upland cotton with 12 different twists. Both yarns were spun with right-hand (Z) and with left-hand (S) twist. The third lot was 2/160s mercerized yarn spun from 1$\frac{3}{4}$-inch staple Sea Island cotton. This yarn was considered to be of high commerical quality.

The breaking strength and elongation at rupture were determined by single-strand, multiple-strand, and skein tests. The variation in breaking strength, elongation at rupture, diameter, angle of twist, and irregularity of the 80s and 10s yarns with the amount of twist are shown. The average breaking strength per strand and the elongation by the multiple-strand test are less than those by the single-strand test and greater than those by the skein test. The data were studied statistically and an explanation is given of the differences which were obtained by the different test methods.

Twelve cones of the commercial yarn were tested for breaking strength by the single-strand test. A statistical study of the data indicates significant differences in the average breaking strengths between the different cones.

Some of the factors which affect the strength of a cotton yarn are discussed. The formulas $\tan \beta = \pi DT$ and $T = M\sqrt{C}$ where β, D, T, M, and C are angle of twist, diameter, turns of twist per inch, twist multiplier, and count respectively, are derived by assuming that the exposed fibers are arranged as helices on a circular cylinder and that 2 yarns of different counts have the same density when their angles of twist are equal. It was found that the values for the angle of twist computed by these formulas were consistently greater than the observed values. The empirical formula $\beta = \frac{\arctan \pi DT}{1 - 0.4\sqrt{C}}$, where $T = M\sqrt{C}$, appears to give results which are in good agreement with the observed values.

CONTENTS

[1] The authors express their appreciation to A. A. Mercier, who initiated part of this work, and to J. W. Porter, who assisted in spinning, for their assistance in this investigation before they left the National Bureau of Standards.

I. INTRODUCTION

A cotton yarn is a heterogeneous material. It consists of a large number of individual cotton fibers which are held together by twist. The fibers are not uniform in length, cross section, and strength. They are unequally distributed throughout the length of the yarn. The resulting yarn is irregular and contains hard and soft places, large and small cross sections, and strong and weak places.

A study of the irregularity of cotton yarn has recently been made by Lee.[2] His study is based upon a statistical analysis of the variations in the weight of specimens 6 inches long. The specimens were taken at the various stages during the yarn manufacturing process, from the card sliver to the spun yarn. His results are of great interest since they show the processes where the irregularity is increased or decreased and indicate the processes where improvement in yarn manufacture can be made.

In the experimental study of cotton fabrics at the National Bureau of Standards during the past 5 years a large amount of data has been obtained which shows the effect of twist on the strength, elongation, diameter, angle of twist, and irregularity of cotton yarns. These data are reported and discussed in this paper. The influence of these characteristics on the properties of fabrics woven from these yarns will be reported in 2 subsequent papers.

II. SAMPLES

Two lots of the cotton yarn discussed in this paper were spun in the experimental cotton mill of the Bureau. The third lot of yarn was a commercial product.

Information relative to the cotton and processing of the yarns spun at the Bureau is given in table 1.

TABLE 1.—*Manufacturing details of the cotton yarns spun at the Bureau*

Variety of cotton	Peeler	Upland
Grade	Good middling	Good ordinary
Staple length, inch	1 5/16	1
Breaker picker lap, oz/yd	15	18
Intermediate picker lap, oz/yd		17
Finisher picker lap, oz/yd	12	16
Card sliver, grains/yd	50	64
Sliver lap, grains/yd	484	
Ribbon lap, grains/yd	475	
Comber sliver, grains/yd	55	
First drawing sliver, grains/yd	52	64
Second drawing sliver, grains/yd		59
Finisher drawing sliver, grains/yd	50	53
Slubber roving, hanks/lb [a]	0.50	0.53
Intermediate roving, hanks/lb	1.26	1.26
Second intermediate roving, hanks/lb	4.04	2.10
Jack roving, hanks/lb	15.29	
Yarn, counts [b]	80s (67.2 typp)	10s (8.4 typp).

[a] A hank of cotton roving is equal to 840 yards.
[b] Count is the number of hanks per pound. "Typp" denotes the number of thousand yards per pound. The typp system provides a rational method of designating the size of all yarns, regardless of the kind of fiber composition. It was sponsored by E. D. Fowle in Textile World, **81**, 1,470 (Apr. 23, 1932).

The 80s yarn was spun with 20 different twists. The 10s yarn was spun with 12 different twists. The twist multipliers varied from 2.48 to 6.66 for the 80s yarn and from 2.25 to 9 for the 10s

[2] R. L. Lee, Jr. *A critical study of cotton manufacturing processes*, Textile Research **IV**, no. 5, 235-260 (March 1934); **V** no. 4, 167-183 (February 1935); **V**, no. 7, 326-336 (May 1935).

yarn. The number of turns of twist per inch, T, for each yarn may be calculated by using the formula

$$T = M\sqrt{C}$$

where M is the twist multiplier and C is the cotton count of the yarn. This formula becomes

$$T = M' \sqrt{\text{typp}}$$

for the typp system, where $M' = 1.091\,M$. Because of the familiarity of cotton spinners and weavers with the twist multipliers which are associated with the cotton count and because of the practical applications of the results to be presented, the cotton count is used in this paper instead of the typp system.

The 80s yarn was spun from double roving, using right-hand (Z) and left-hand (S) twist for each twist multiplier. The 10s yarn was spun from single and from double roving for each twist multiplier, using right-hand (Z) and left-hand (S) twist. The draft was increased in the spinning process as the twist was increased to keep the yarn counts as near 80s and 10s as practicable.

The relative humidity was maintained at approximately 65 percent during the manufacturing of these yarns.

The third type of yarn studied was a 2/160s commercial yarn spun from 1¾ inch staple Sea Island cotton. The 160s (134.4 typp) yarn was spun with a twist multiplier of 3.5 (44 turns of twist per inch). A twist multiplier of 4.4 (37 turns of twist per inch) was used in plying. The plied yarn was mercerized. This yarn was considered to be of high commercial quality.

III. TEST PROCEDURE

1. BREAKING STRENGTH AND ELONGATION AT RUPTURE

The breaking strength and elongation at rupture of the yarns were determined by the single-strand, multiple-strand, and skein tests. The tests were made on material which was exposed for at least 4 hours to an atmosphere of 65 percent relative humidity and 70° F.

In the single-strand test the yarn was clamped in the small jaws of a pendulum type machine, having 2 capacities of 18 and 36 ounces, respectively. The distance between the jaws was 4 inches and the jaws were separated at a rate of 4 inches per minute. In this test it is important to clamp the yarn with a small and uniform tension.[3] The procedure adopted was to apply sufficient tension on the yarn to cause the freely swinging pendulum to move slightly from its vertical position. The breaking strength and elongation at rupture were read from the scales of the machine. At least 50 determinations were made for each direction of twist and twist multiplier of the yarns.

[3] In some testing machines means are provided for clamping the pendulum in a vertical position while inserting the sample in the jaws of the machine. Clamping the pendulum in a vertical position while inserting the samples leads to erroneous results, especially in elongation. It has been found that the apparent elongation of the yarn is decreased by the considerable and variable tension under which the specimen is likely to be mounted under these conditions.

In the multiple-strand test [4] 100 strands were wound parallel on a split frame with a slight and uniform tension. The strands and frame were clamped in the jaws of a pendulum type of machine having a capacity of 150 pounds. The distance between the jaws was 4 inches and the jaws were separated at the rate of 12 inches per minute. The breaking strength and elongation at rupture were obtained from the curves drawn by the machine. At least 10 determinations were made for each direction of twist and twist multiplier of the yarns.

In the skein test 120 yards of yarn were wound on a reel having a perimeter of 1½ yards. This skein, consisting of 80 loops or 160 strands, was then placed on the 2 drums of a pendulum type of machine having a capacity of 150 pounds. The drums were 1 inch in diameter and were separated at the rate of 12 inches per minute. The breaking strength and elongation at rupture were obtained from the curves drawn by the machine. At least 10 determinations were made for each direction of twist and twist multiplier of the yarns.

2. DIAMETER AND ANGLE OF TWIST

The diameter of the yarn and the angle of twist, that is the angle at which the individual fibers appear to be inclined to the longitudinal axis of the yarn, were measured with a rotary stage microscope and micrometer. The yarn was wound without tension on a strip of black cardboard 1 inch wide. At least 10 determinations were made for each direction of twist and twist multiplier of the yarns.

3. FORMULAS [5]

The following formulas were used in computing the average, standard deviation, coefficient of variation, and their standard errors from the frequency distributions. N is the number of determinations, I is the class interval of the frequency distribution, f is the frequency in each class interval, m is the class interval midpoint taken as the assumed average, and d is the number of deviation steps a class interval is from m.

Average, A,

$$A = m + sI,$$

where m = an assumed average and $s = \frac{\Sigma\,(f\,d)}{N}$

Standard deviation, σ,

$$\sigma = I\sqrt{\frac{\Sigma(f\,d^2)}{N} - s^2}$$

[4] C. W. Schoffstall and H. A. Hamm, *A multiple-strand test for yarns*, BS J. Research **2**, 871 (1929) RP61. When this test method was developed it was customary to line the jaws of the testing machine with strips of rubber about 1/10 inch thick. This practice leads to erroneous results, especially in elongation. It was found that the apparent elongation of the yarn was greatly increased by an amount dependent upon the breaking load, the effect being due to the elongation of the rubber. The rubber was not used in the tests reported in this paper.

[5] The mathematical treatment may be found in the following texts. E. E. Day, Statistical Analysis (MacMillan & Co.); M. Ezekiel, Methods of Correlation Analysis (John Wiley & Sons); F. C. Mills, Statistical Methods (Henry Holt & Co.); C. H, Forsyth, Mathematical Analysis of Statistics (John Wiley & Sons); W. A. Shewhart, Economic Control of Quality of Manufactured Products (D. Van Nostrand Co.).

Coefficient of variation, V,

$$V=\frac{100\sigma}{A}$$

Standard error of the average, σ_A,

$$\sigma_A=\frac{\sigma}{\sqrt{N-1}}$$

Standard error of the coefficient of variation, σ_V,

$$\sigma_V=\frac{V\sqrt{1+2(0.01V)^2}}{\sqrt{2(N-1)}}$$

The coefficient of variation is taken as an indication of the irregularity of the yarn with respect to that property which is being considered. The standard error of the difference between two measures, σ_Δ, is the square root of the sum of the squares of the standard errors of the two measures. The difference between the two measures, Δ, divided by the standard error of the difference is taken as the significance ratio R. In statistical analysis a value of R greater than 3 is an indication of a significant difference between the two measures. This criterion has been adopted in this paper.

IV. RESULTS

1. BUREAU'S 80s (67.2 TYPP) COTTON YARN

There appeared to be no consistent differences in the data for the yarns spun with right-hand and left-hand twist for the various twist multipliers. No distinction is therefore made in the results presented.

The variations in breaking strength and in elongation at rupture by the single-strand test for the various twist multipliers are shown by frequency distributions in tables 2 and 3, respectively. The tables also contain the average, standard deviation, and coefficient of variation for the various twist multipliers.

The average breaking strength increases with an increase in twist multiplier to a maximum at a twist multiplier of about 3.33 and then decreases. The coefficient of variation decreases in general with an increase in twist multiplier to a minimum at a twist multiplier of about 3.67 and then increases. The standard error of the coefficient of variations, the difference between the coefficient of variation at twist multiplier 3.67 and the coefficient of variation for the other twist multipliers, the standard error of these differences, and the significance ratio of the differences are also given in table 2.

TABLE 2.—*Frequency distribution of breaking strength of 80s cotton yarn by the single-strand test*

Breaking strength (in oz.) at cell midpoint	Twist multiplier									
	2.48	2.71	2.93	3.19	3.26	3.33	3.41	3.49	3.58	3.67
2.8	4	------	------	------	------	------	------	------	------	------
3.1	7	------	------	6	------	------	------	------	------	------
3.4	26	11	------	8	3	------	------	------	------	4
3.7	20	16	20	11	14	------	7	7	14	10
4.0	16	23	20	11	21	5	15	12	13	14
4.3	18	18	29	22	14	22	17	27	25	26
4.6	9	15	17	13	10	26	22	19	14	23
4.9	------	9	8	10	12	22	18	10	19	14
5.2	------	7	5	11	14	10	12	15	7	6
5.5	------	1	1	6	6	5	5	10	2	3
5.8	------	------	------	2	6	7	2	------	4	------
6.1	------	------	------	------	------	2	2	------	2	------
6.4	------	------	------	------	------	1	------	------	------	------
Total	100	100	100	100	100	100	100	100	100	100
Average, *A*, oz	3.78	4.21	4.28	4.36	4.52	4.81	4.62	4.59	4.52	4.41
Standard deviation, oz	.477	.522	.435	.681	.660	.537	.546	.519	.579	.483
Coef of var., *V*, %	12.6± 0.91	12.4± 0.89	10.2± 0.74	15.6± 1.13	14.6± 1.06	11.2± 0.81	11.8± 0.85	11.3± 0.81	12.8± 0.92	11.0± 0.79
$\Delta = V - 11.0$, %	1.6± 1.21	1.4± 1.19	−0.8± 1.08	4.6± 1.38	3.6± 1.32	0.2± 1.13	0.8± 1.16	0.3± 1.13	1.8± 1.21	0.0± 1.12
Significance ratio, *R*	1.3	1.2	−0.7	3.2	2.7	0.2	0.7	0.3	1.5	0

Breaking strength (in oz.) at cell midpoint	Twist multiplier									
	3.76	3.86	3.96	4.07	4.58	5.05	5.64	6.11	6.37	6.66
2.5	------	------	------	------	------	------	------	4	6	9
2.8	------	------	------	------	6	2	------	13	4	17
3.1	2	1	6	5	4	17	5	11	18	22
3.4	7	7	6	7	9	9	14	14	22	20
3.7	16	14	7	19	16	15	14	12	10	20
4.0	16	19	29	18	16	17	18	7	6	6
4.3	11	18	16	13	14	8	16	7	12	5
4.6	19	21	11	16	7	8	10	12	12	1
4.9	12	10	16	10	11	3	8	4	4	------
5.2	12	8	4	8	5	9	8	8	4	------
5.5	5	2	3	4	4	8	6	4	0	------
5.8	------	------	2	------	6	4	1	3	2	------
6.1	------	------	------	------	2	------	------	1	------	------
Total	100	100	100	100	100	100	100	100	100	00
Average, *A*, oz	4.37	4.30	4.28	4.27	4.24	4.12	4.21	3.93	3.77	3.30
Standard deviation, oz	.618	.525	.612	.618	.825	.834	.675	.921	.772	.477
Coef of var., *V*, %	14.2± 1.03	12.2± 0.88	14.3± 1.04	14.6± 1.06	19.4± 1.43	20.2± 1.49	16.0± 1.16	23.5± 1.76	20.5± 1.52	14.4± 1.05
$\Delta = V - 11.0$, %	3.2± 1.30	1.2± 1.18	3.3± 1.31	3.6± 1.32	8.4± 1.63	9.2± 1.69	5.0± 1.40	12.5± 1.93	9.5± 1.71	3.4± 1.31
Significance ratio, *R*	2.5	1.0	2.5	2.7	5.2	5.4	3.6	6.5	5.6	2.6

The average elongation at rupture increases with an increase in twist multiplier. The coefficient of variation of the elongation, however, decreases slightly with an increase in twist multiplier to a minimum at twist multiplier 3.67 and then increases considerably for higher twist multipliers. The standard error of the coefficient of variations, the difference between the minimum coefficient of variation at twist multiplier 3.67 and the coefficient of variation for the other twist multipliers, the standard error of these differences, and the significance ratio of the differences are also given in table 3.

TABLE 3.—*Frequency distribution of elongation of 80s cotton yarn by the single-strand test*

Elongation (in %) at cell midpoint	Twist multiplier—									
	2.48	2.71	2.93	3.19	3.26	3.33	3.41	3.49	3.58	3.67
4.75	2									
5.15	11									
5.55	17	2	6	4						
5.95	40	34	19	15	5	1		1		
6.35	21	32	28	13	18	12	3	5		
6.75	9	11	19	32	21	21	9	12	6	7
7.15		19	21	24	26	22	35	17	28	22
7.55		2	7	6	18	23	25	27	28	42
7.95				4	9	17	25	29	28	20
8.35				2	3	4	3	6	6	9
8.75								3	4	
Total	100	100	100	100	100	100	100	100	100	100
Average, A, %	5.93	6.42	6.55	6.75	7.04	7.23	7.43	7.51	7.60	7.56
Standard deviation, %	.468	.480	.540	.588	.584	.568	.448	5.72	.472	.412
Coef of var., V, %	7.9± 0.56	7.5± 0.54	8.2± 0.59	8.7± 0.62	8.3± 0.59	7.9± 0.56	6.0± 0.43	7.6± 0.54	6.2± 0.44	5.4± 0.38
$\Delta = V - 5.4$, %	2.5± 0.68	2.1± 0.66	2.8± 0.70	3.3± 0.73	2.9± 0.70	2.5± 0.68	0.6± 0.57	2.2± 0.66	0.8± 0.59	0.0± 0.54
Significance ratio, R	3.7	3.2	4.0	4.5	4.1	3.7	1.1	3.3	1.4	0

Elongation (in %) at cell midpoint	Twist multiplier—									
	3.76	3.86	3.96	4.07	4.58	5.05	5.64	6.11	6.37	6.66
5.95	1	1	1							
6.35	0	2	2							
6.75	4	3	8	2		5				
7.15	23	14	18	14	12	9	5	6	3	
7.55	29	26	14	12	6	3	4	0	0	
7.95	21	28	28	41	12	8.	6	4	4	1
8.35	10	13	8	13	17	8	3	4	3	1
8.75	9	10	7	16	18	13	7	8.	6	3
9.15	3	3	12	2	15	11	12	7	12	6
9.55			2		10	9	10	9	8	6
9.95					8	9	13	14	20	10
10.35					2	11	13	10	12	9
10.75						6	15	7	11	6
11.15						7	7	3	1	10
11.55						1	4	7	4	9
11.95							1	11	8	15
12.35								8	6	5
12.75								0	0	5
13.15								2	2	7
13.55										2
13.95										2
14.35										0
14.75										3
Total	100	100	100	100	100	100	100	100	100	100
Average, A, %	7.73	7.81	7.87	7.97	8.59	9.09	9.67	10.17	10.12	11.23
Standard deviation, %	.600	.612	.788	.536	.852	1.276	1.188	1.500	1.304	1.484
Coef of var., V, %	7.8± 0.76	7.8± 0.76	10.0± 0.72	6.7± 0.48	10.1± 0.73	14.0± 1.01	12.3± 0.89	14.7± 1.06	12.9± 0.93	13.3± 0.96
$\Delta = V - 5.4$, %	2.4± 0.85	2.4± 0.85	4.6± 0.81	1.3± 0.61	4.7± 0.82	8.6± 1.08	6.9± 0.97	9.3± 1.13	7.5± 1.04	7.9± 1.03
Significance ratio, R	2.8	2.8	5.7	2.1	5.7	8.0	7.1	8.2	7.2	7.7

The strength, elongation, and irregularity of the 80s cotton yarn vary with the twist multiplier. The irregularity based upon strength and upon elongation is least for twist multiplier 3.67 and highest for twist multiplier 6.11. This change is shown very strikingly by the scatter diagrams shown in figure 1 where the strength is plotted against the elongation for the single-strand tests of the yarn of twist multipliers 2.48, 3.67, and 6.11. The coefficient of variation for strength and elongation are given in figure 1 for each twist multiplier and the average strength and elongation are indicated.

In the multiple-strand test, 100 strands, each 4 inches long, are subjected to tension simultaneously and all strands are elongated ally as tension is applied. The scatter diagrams of figure 1 show that there is a considerable variation in the elongation at rupture between the separate strands. In the multiple-strand test the strands having the lowest elongation at rupture are broken first and the load carried by them must then be assumed by the unbroken strands which break at higher elongations and which at this moment carry only a fraction of their ultimate breaking load. After several strands are broken the total load carried by the unbroken strands reaches a maximum value which is sufficient to break all of the remaining strands in rapid succession as soon as an additional strand is broken. This maximum load carried by the aggregate of the strands is taken as the breaking strength of the 100 strands. The average breaking strength per strand of 100 strands by the multiple-strand test is obviously less than the average breaking strength per strand obtained by testing 100 strands by the single-strand test because the maximum load is carried by less than 100 strands in the multiple-strand test and because the unbroken strands which have a

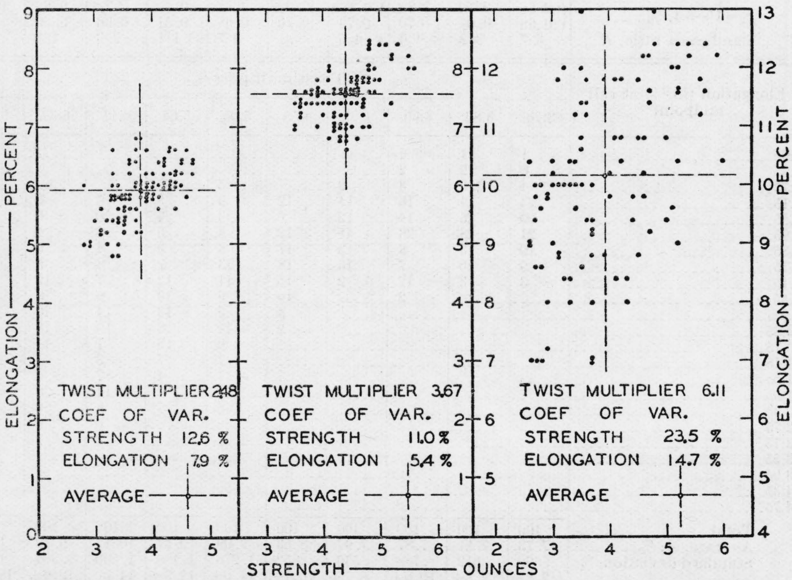

Figure 1.—*Scatter diagrams for three-twist multipliers of 80s cotton yarn.*

high elongation at rupture, carry a load less than their ultimate breaking strength when the load carried by the aggregate is a maximum. The ratio of the average breaking strength per strand by the multiple-strand test to the average breaking strength per strand by the single-strand test is a fraction which decreases, in general, with an increase in twist multiplier. This decrease is readily explained by the variation in the irregularity of the yarn with respect to elongation at rupture, which is measured by the coefficient of variation and takes into consideration the variations of both the magnitude and the spread of the elongation at rupture with an increase in twist multiplier. It should be noted that the single-strand tests were made at a lower speed than the multiple-strand tests. This factor may have some effect on the results.

Similar variations are also found for average elongation by the two methods of test, where elongation by the multiple-strand test refers to the elongation corresponding to the maximum load carried by the 100 strands.

In the regular skein test, 160 strands, 80 loops, are subjected to tension simultaneously. Each strand is in effect 27 inches long. On the basis of the discussion given above for the multiple-strand test the average breaking strength per strand and the elongation at the maximum load carried by the skein are expected to be lower than those

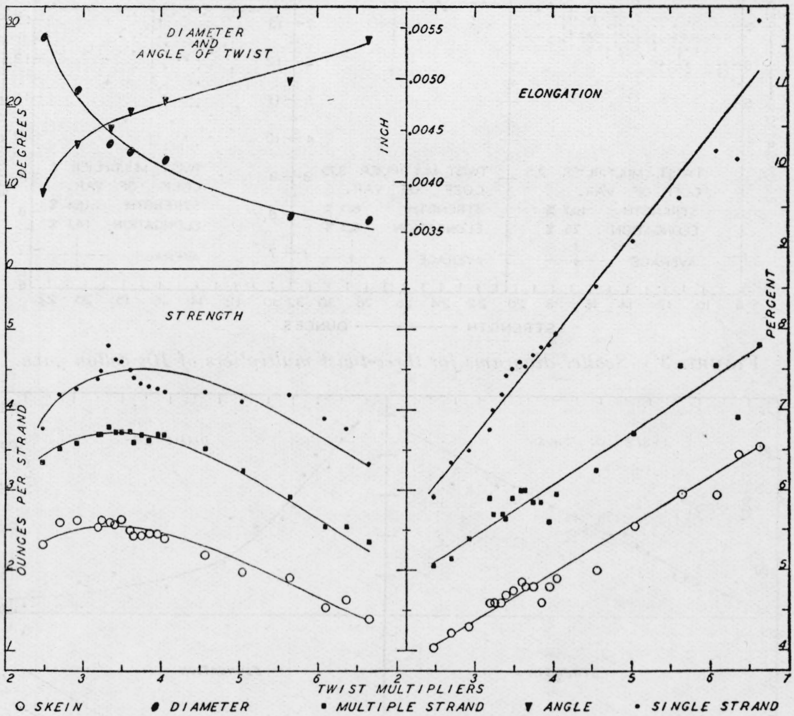

FIGURE 2.—*Effect of twist multiplier on the breaking strength, elongation at rupture, diameter, and angle of twist of the 80s cotton yarn.*

The differences in the results obtained by the three methods of determining the strength and elongation are also shown.

obtained by the single-strand test. An additional reduction is to be expected because the variations occurring in specimens 27 inches long are greater than those found in specimens only 4 inches long. The ratio of the average breaking strength per strand by the skein test to the average breaking strength per strand by the single-strand or by the multiple-strand test is a fraction which decreases in general with an increase in twist multiplier.

The results for strength and elongation by the three methods of test are plotted in figure 2 against twist multiplier. The graphs indicate the variation of strength and elongation with twist multiplier for each method of test as well as the relationship for strength and elongation between the three methods of test. The maximum strength by

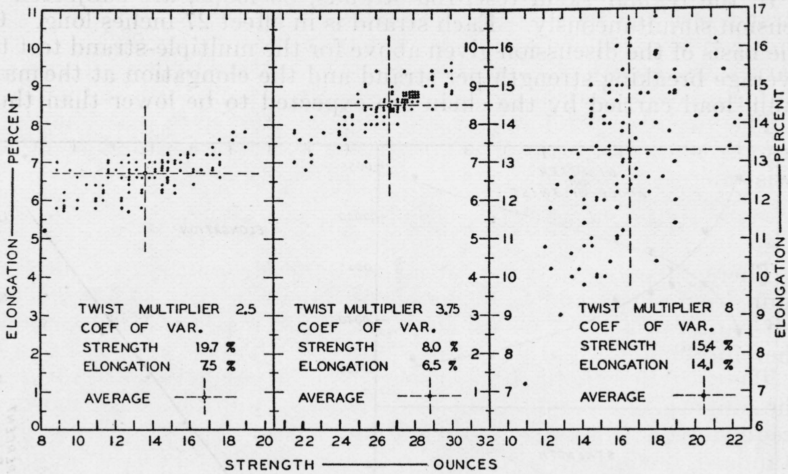

FIGURE 3.—*Scatter diagrams for three-twist multipliers of 10s cotton yarn.*

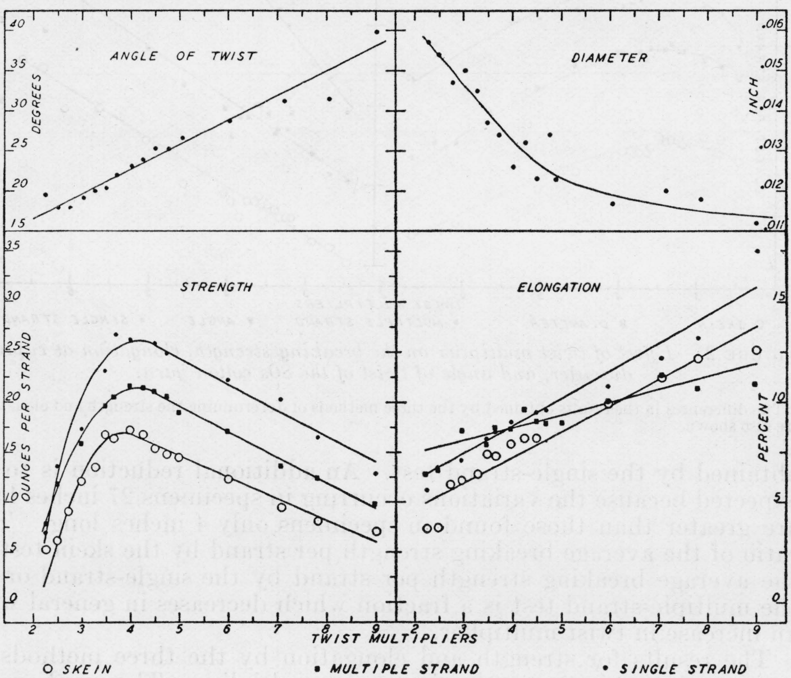

FIGURE 4.—*Effect of twist multiplier on the breaking strength, elongation at rupture, diameter, and angle of twist of the 10s cotton yarn.*

The differences in the results obtained by the three methods of determining the strength and elongation are also shown.

the multiple-strand and skein tests appears to occur at a slightly lower twist multiplier than for the single-strand test. This is in agreement with the shift noted by Mercier [6] for combed and carded cotton yarns. He reported maximum strength at twist multiplier 5.25 for the single-strand test and at 4.25 for the skein test of the same yarns. This shift can be explained on the basis of an increase in irregularity of the yarn with respect to elongation as the twist multiplier is increased. Unfortunately Mercier did not investigate this property in his study.

The variations of diameter and angle of twist with twist multiplier are also shown in figure 2. The diameter decreases and the angle of twist increases with an increase in twist multiplier.

2. BUREAU'S 10s (8.4 TYPP) COTTON YARN

There appeared to be no consistent difference in the data for yarns spun from single and from double roving with right-hand and with left-hand twist for the various twist multipliers. No distinction is therefore made in the results presented.

The variations in breaking strength and in elongation at rupture by the single-strand test for the various twist multipliers are shown by frequency distributions in tables 4 and 5, respectively. The average, standard deviation, coefficient of variation, standard error of the

TABLE 4.—*Frequency distribution of breaking strength of 10s cotton yarn by the single-strand test*

Breaking strength (in oz) at cell midpoint	Twist multiplier											
	2.25	2.5	3	3.5	3.75	4	4.5	4.75	6	7	8	9
4.5	5											
5.5	14											
6.5	34											
7.5	26											
8.5	15	4										1
9.5	6	9										6
10.5		5									1	19
11.5		9									0	13
12.5		15	2								4	10
13.5		9	1								6	24
14.5		20	12						1	4	18	10
15.5		10	19						4	12	19	6
16.5		6	21						3	11	18	8
17.5		9	9					1	5	11	6	1
18.5		2	11	2				0	3	15	9	1
19.5		2	12	7				1	10	12	9	1
20.5			4	10				6	10	10	1	
21.5			6	12	4	3	11	9	11	8	5	
22.5			3	17	5	7	10	13	13	3	4	
23.5				15	1	5	9	7	8	2		
24.5				13	10	15	10	15	14	6		
25.5				12	7	17	15	13	6	2		
26.5				7	20	16	13	13	5	2		
27.5				5	32	16	5	8	1	2		
28.5					6	10	7	4	3			
29.5					10	6	10	3	1			
30.5					5	3	5	4	0			
31.5						2	2	3	1			
32.5							3		1			
Total	100	100	100	100	100	100	100	100	100	100	100	100
Average, *A*, oz	7.0	13.6	17.3	23.2	26.7	26.2	25.9	24.9	22.2	19.3	16.6	12.9
Standard deviation, oz	1.22	2.68	2.30	2.26	2.13	2.26	3.04	2.93	3.50	3.08	2.55	2.24
Coef of var., *V*, %	17.4± 1.27	19.7± 1.45	13.4± 0.97	9.7± 0.69	8.0± 0.57	8.6± 0.62	11.8± 0.85	11.8± 0.85	15.8± 1.15	16.0± 1.16	15.4± 1.12	17.4± 1.27
$\Delta = V - 8.0$, %	9.4± 1.40	11.7± 1.56	5.4± 1.13	1.7± 0.89	0.0± 0.81	0.6± 0.84	3.8± 1.02	3.8± 1.02	7.8± 1.28	8.0± 1.29	7.4± 1.26	9.4± 1.40
Significance ratio, *R*	6.7	7.5	4.8	1.9	0	0.7	3.7	3.7	6.1	6.2	5.9	6.4

[6] A. A. Mercier and C. W. Schoffstall, *Effect of twist on cotton yarns.* BS J. Research **1**, 733 (1928) RP27.

TABLE 5.—*Frequency distribution of elongation of 10s cotton yarn by the single-strand test*

Elongation (in %) at cell midpoint	Twist multiplier											
	2.25	2.5	3	3.5	3.75	4	4.5	4.75	6	7	8	9
4.75	6											
5.25	30	1										
5.75	48	8	1									
6.25	14	23	14									
6.75	2	35	20	2	1							
7.25		31	41	6	5		1	1			1	
7.75		2	12	16	9	3	0	0			0	
8.25			12	49	33	20	9	4			0	
8.75				17	36	23	10	11			0	
9.25				9	15	27	31	31	2		0	
9.75				1	1	18	23	16	9		1	
10.25						9	20	23	26	7	8	
10.75							3	10	17	6	4	
11.25							3	3	24	15	6	
11.75								0	7	7	3	
12.25								1	14	27	11	
12.75									1	4	6	
13.25										17	11	
13.75										9	3	1
14.25										7	17	4
14.75										1	8	4
15.25											11	12
15.75											3	6
16.25											5	8
16.75											2	3
17.25												13
17.75												11
18.25												11
18.75												2
19.25												7
19.75												5
20.25												3
20.75												2
21.25												3
21.75												2
22.25												1
22.75												2
Total	100	100	100	100	100	100	100	100	100	100	100	100
Average, A, %	5.63	6.72	7.18	8.27	8.49	9.07	9.51	9.69	10.92	12.33	13.33	17.76
Standard deviation, %	0.42	0.51	0.60	0.55	0.55	0.65	0.73	0.79	0.81	1.14	1.88	2.09
Coef of var., V, %	7.4± 0.53	7.5± 0.54	8.3± 0.59	6.6± 0.47	6.5± 0.46	7.2± 0.51	7.7± 0.55	8.2± 0.59	7.4± 0.53	9.2± 0.66	14.1± 1.02	11.8± 0.85
$\Delta = V - 6.5$, %	0.9± 0.70	1.0± 0.71	1.8± 0.75	0.1± 0.66	0.0± 0.65	0.7± 0.69	1.2± 0.72	1.7± 0.75	0.9± 0.70	2.7± 0.80	7.6± 1.12	5.3± 0.97
Significance ratio, R	1.3	1.4	2.4	0.2	0	1.0	1.7	2.3	1.3	3.4	6.8	5.5

coefficient of variation, the difference between the minimum coefficient of variation at twist multiplier 3.75 and the coefficient of variation for the other twist multipliers, the standard error of these differences, and the significance ratio of these differences are also given in the tables for the various twist multipliers. The scatter diagrams shown in figure 3 are for twist multipliers 2.5, 3.75, and 8. The results for strength and elongation by the three methods of tests are plotted in figure 4 against twist multiplier. The variations in diameter and angle of twist with twist multiplier are also shown in figure 4.

In general, the same conclusions may be drawn from these results as were made for the 80s cotton yarns. They are not discussed individually for the sake of brevity.

3. COMMERCIAL COTTON YARN, 2/160s (2/134.4 TYPP)

Forty pounds of 2/160s mercerized cotton yarn of high commercial quality were purchased for a study of cotton fabrics for parachutes. One hundred breaking-strength determinations were made by the single-strand test on each of 12 cones of yarn selected at random from the entire lot. The variations in breaking strength of the yarn from the different cones and from the entire lot are shown by frequency distributions in table 6. The average, standard error of the average, standard deviation, coefficient of variation, difference between average of cone no. 1 and average of the other cones, and the standard error and significance ratio of these differences are also given in table 6.

TABLE 6.—*Frequency distribution of breaking strength for 12 cones of 2/160s mercerized cotton yarn of high commercial quality*

Breaking strength (in oz) at cell midpoint	Cone numbers												
	1	2	3	4	5	6	7	8	9	10	11	12	1 to 12, incl.
3.5	1												1
4.0	1								1				2
4.5	0	2				1			0				3
5.0	2	1	1			0	3	1	0		2		10
5.5	2	3	2	4	2	2	1	1	2	1	2	1	23
6.0	8	9	9	7	4	6	4	5	5	4	6	5	72
6.5	15	12	15	8	17	10	7	7	6	6	10	10	123
7.0	23	14	17	22	18	18	17	7	13	10	12	10	181
7.5	18	24	20	21	22	23	22	23	19	24	11	10	237
8.0	10	7	18	17	9	10	15	6	13	12	10	10	137
8.5	14	12	3	7	9	10	11	28	13	17	9	14	147
9.0	4	7	6	6	7	10	10	12	11	8	13	17	111
9.5	2	7	6	6	8	5	9	7	5	10	10	9	84
10.0		2	2	2	1	3	1	2	6	5	10	7	41
10.5			1		2	2		1	6	2	3	4	21
11.0					1					0	2	1	4
11.5										1		2	3
Total	100	100	100	100	100	100	100	100	100	100	100	100	1,200
Average, A, oz	7.24± 0.108	7.48± 0.120	7.48± 0.120	7.54± 0.104	7.66± 0.116	7.69± 0.117	7.73± 0.109	7.98± 0.107	8.02± 0.130	8.09± 0.111	8.11± 0.144	8.29± 0.135	7.774± 0.122
Standard deviation, oz	1.07	1.19	1.19	1.03	1.15	1.16	1.08	1.06	1.29	1.10	1.43	1.34	1.215
Coef of var., V, %	14.8	15.9	15.9	13.7	15.0	15.1	14.0	13.3	16.1	13.6	17.6	16.2	15.7
$\Delta = A - 7.24$, oz	0.00± 0.152	0.24± 0.161	0.24± 0.161	0.30± 0.149	0.42± 0.158	0.45± 0.158	0.49± 0.153	0.74± 0.151	0.78± 0.168	0.85± 0.154	0.87± 0.180	1.05± 0.172	0.534± 0.163
Significance ratio, R	0	1.5	1.5	2.0	2.7	2.9	3.2	4.9	4.7	5.5	4.8	6.1	3.3

The average breaking strength varies from 7.24 ounces for cone no. 1 to 8.29 ounces for cone no. 12. This variation is 13.5 percent of the average of 7.774 ounces for the entire lot and the difference between 7.24 and 8.29 ounces is 6.1 times its standard error, which is a significant difference. On the other hand the significance ratios of the differences between the average breaking strengths of cones nos. 1 and 12 and the average of the entire lot are 3.3 and 2.8, respectively. These values are close to the limiting value indicating a significance.

The coefficient of variation, which is taken as a measure of irregularity, varies from 13.3 to 17.6 percent between the different cones and is equal to 15.7 percent for the entire lot. This is somewhat higher than the irregularity of about 10 to 12 percent obtained for the 80s and 10s yarn for the twist multipliers yielding maximum strength. On the assumption that the single unmercerized yarn had the same irregularity as the Bureau's yarn it appears that plying and mercerization did not decrease the irregularity of this yarn materially. A more extensive study of the effect of these factors on the irregularity of the yarn must be made, however, to establish this fact.

V. FACTORS AFFECTING THE STRENGTH OF A COTTON YARN

The strength, length, and other characteristics of the cotton fibers have an effect on the strength of a cotton yarn. The relation of yarn strength to fiber strength has been discussed in considerable detail by Turner.[7] The strength of the yarn is related to the length of the cotton fiber. For example, it was found that a 10s yarn spun from the 1$\frac{5}{16}$-inch staple Peeler cotton had a breaking strength 63 percent greater than a 10s yarn spun with the same twist multiplier from the 1-inch staple Upland cotton. Similarly it is known that a combed yarn is stronger than a carded yarn spun from the same raw cotton because many of the shorter and inferior fibers are removed in the combing process.

The effect of twist on the strength of the yarn is indicated in this paper and confirms the results obtained in previous investigations. Imparting twist to a bundle of cotton fibers decreases the diameter of the bundle, that is, it brings the fibers in closer contact by a radial pressure, thereby increasing the frictional forces between fibers or the resistance of fiber slippage when tension is applied to the bundle. On the other hand twist introduces torsional forces or shearing stresses in the individual fibers thereby decreasing their breaking strength in tension. A moderate amount of twist reduces the diameter a considerable amount and therefore increases the frictional forces between the fibers, while it does not set up appreciable stresses in the fibers or produce an appreciable decrease in the breaking strength of the fibers in tension.[8] It is expected therefore that the strength of the yarn should increase very materially to a maximum with a moderate increase in twist. Additional increase in twist decreases the diameter slightly and the effects of the opposing strength factors balance each other approximately, producing relatively little change in the strength of the yarn from the maximum value. At high twist, however, the increase in the friction between fibers due to an increase in twist or a decrease in diameter is negligible while the decrease in the fiber strength due to internal stresses becomes appreciable. The strength of the yarn decreases rapidly with an increase in high twist and for excessively high twist the yarn may break during the twisting process without the addition of any external tension.

Two other factors are also operative in materially reducing the breaking strength of yarns at high twist relative to the breaking strength at low twist. They are (1) increase in the draft during spinning of yarns of high twist and (2) variation in twist between fibers near the axis and the surface of the yarn.

Increasing the draft for high twist to keep the count of the yarn at no tension constant has the same effect as reducing the weight per unit length of the yarn when it is under a tension substantially equal to its breaking strength. The decrease is proportional to the difference in elongation at rupture between a low-twisted and a high-twisted yarn.

The fibers located near the axis of the yarn are not twisted to the same degree as those near the surface. This difference increases

---

[7] A. J. Turner, *The foundation of yarn strength and yarn extension.* J. Textile Inst. **19**, T286–T314 (1928).

[8] H. A. Hamm and R. S. Cleveland, *Relation between the twist and certain properties of rayon yarns.* BS J. Research **7**, 617 (1931) RP361.

with twist. When tension is applied to yarn the fibers near the surface assume less of the load because of greater twist (greater elongation) than the fibers near the axis. For higher and higher twist this difference increases and the fibers near the surface become less and less effective.

The data in the literature seem to indicate that maximum strength of fine yarns is obtained at lower twist multipliers than in coarse yarn. Furthermore, a yarn appears to increase in strength more rapidly than its weight, that is, a 20s yarn has in general a breaking strength more than twice as great as that of a 40s yarn. These observations may be explained as follows: A fine yarn is usually spun from longer staple fiber and the ratio of the circumference of the yarn to the area of its cross section is greater in fine yarns than in coarse yarns. The longer fibers require less twist to increase the resistance of fiber slippage and the fibers retain more of their initial strength because the internal stresses due to twist are still small. Because of the greater ratio of the circumference to the cross-sectional area a smaller percentage of the total number of fibers are completely imbedded in the yarn or a greater percentage are exposed at the surface. The increase in twist cannot increase the frictional resistance of the fibers exposed at the surface. Both of these factors tend to produce maximum strength at lower twist multipliers. They explain why maximum strength is obtained at a lower twist multiplier in fine yarn than in coarse yarn.

The second factor explains why a 20s yarn, for example, has a breaking strength equal to more than twice that of a 40s yarn spun from the same raw cotton. The cross-sectional area of the 20s yarn is twice that of the 40s yarn, while the circumference of the 20s yarn is only 1.414 times that of the 40s yarn. Because a greater percentage of the total number of fibers are exposed on the surface in fine yarns and are therefore comparatively ineffective, it is apparent why the breaking strength increases at a greater rate than the rate of increase in weight, all other factors remaining constant.

VI. RELATIONS BETWEEN DIAMETER, ANGLE OF TWIST, COUNT, AND TWIST

A direct relationship between diameter and angle of twist with twist multiplier has been indicated for 2 counts in this paper. In fact the formula $T=M\sqrt{C}$, which has been used for many years and was presented by Joseph Köchlin on November 28, 1828, before the Société Industrielle de Mulhouse,[9] expresses a simple relationship between the count, twist multiplier, and number of turns of twist per inch. This formula can be readily derived by assuming that the exposed fibers are arranged as helices on a circular cylinder and that 2 yarns of different count have the same density when their angles of twist are equal. The projection of a helix on the xy-plane may be expressed by the equation

$$y=D/2 \sin 2\pi Tx$$

where the x-axis coincides with the axis of the cylinder, D is the diameter and T is the number of turns of twist per inch. The angle

[9] H. Brüggemann, Zwirne, p. 103 (1933). Published by R. Oldenbourg, Munich and Berlin, Germany,

of twist is by definition equal to the angle which a tangent line makes with the x-axis at the origin of the above curve. If β is the angle of twist, then it follows by differentiation that

$$dy/dx = \tan \beta = \pi DT \text{ when } x=0$$

Let D and T represent the diameter and number of turns of twist per inch of a yarn of count C and D' and T' similar values of a yarn of count C'. If these 2 yarns have the same angle of twist then

$$T = T'D'/D.$$

If w represents the weight per unit length of yarn of count C, δ the density, and k a constant, then

$$k/C = w = \pi \delta l D^2/4 \text{ and } k/C' = w' = \pi \delta' l D'^2/4,$$

since the count is inversely proportional to the weight. It follows therefore that

$$D'/D = \sqrt{\delta/\delta'} \cdot C/C' \text{ and } T = T'/\sqrt{C'} \cdot \sqrt{C}$$

on the assumption that $\delta = \delta'$ when $\beta = \beta'$. The expression $T'/\sqrt{C'}$ is ordinarily termed the twist multiplier M and represents the number of turns of twist per inch in a yarn of count unity, $C'=1$. It is interesting to note that in Köchlin's derivation the additional assumption is made that the coefficient of friction between fibers is the same in yarns of different count when the exposed fibers make the same angle of twist. Köchlin expressed the coefficient of friction as equal to the tangent of the angle of twist. This expression is analogous to the common way of expressing the coefficient of friction between two surfaces. This assumption is very interesting and probably applies to the surfaces of the exposed fibers. It is not necessary, however, in the derivation of the above formula.

In A of figure 5 are plotted the observed values of the angle of twist for 10s, 80s,[10] 100s,[11] and 120s[12] yarn against the values computed from the formula $\tan \beta = \pi DT$ in which D was taken equal to observed diameters and T, the number of turns of twist per inch, was computed by the formula $T = M\sqrt{C}$. It is seen that the computed values are consistently greater than the observed values and the difference appears to increase with the count of the yarn. This indicates that the assumptions made in the above formulas are not strictly fulfilled in spun yarns. From the data at hand it is not possible to determine which assumption is in error. In fact it would be very difficult to determine because the yarn is irregular in other properties and probably varies as much in helical arrangement of the fibers, form of the cross section, and density. On the basis of experimental data Wagner and Herzog[13] have derived the empirical formula $\tan \beta = 0.873\, \pi DT$. The computed values of β for 10s yarn by this formula are in excellent agreement with the observed values,

[10] [11] [12] These yarns were spun from the same cotton.

[13] P. Heermann und A. Herzog, Mikroskopische und mechanisch-technische Textiluntersuchungen, p. 233, Dritte Auflage (1931). Published by Julius Springer, Berlin, Germany.

but the agreement is not very good for yarns of higher counts (see B of fig. 5). The empirical formula

$$\beta = \frac{\arctan \pi DT}{1 - 0.4\sqrt{C}}$$

appears to give results which are in good agreement with the observed values for the yarns. The computed and observed values are plotted

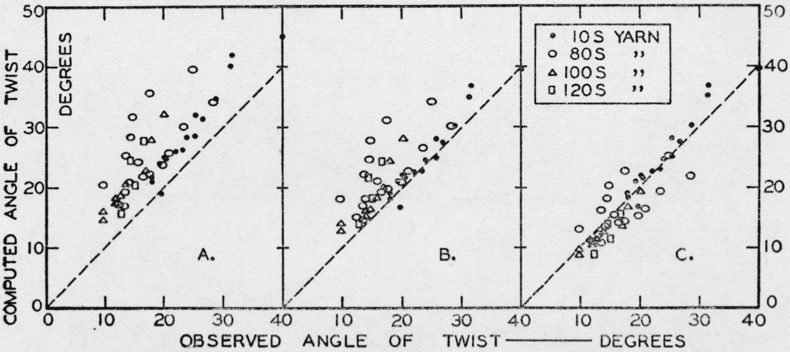

FIGURE 5.—*Relation between observed and computed values for the angle of twist.*

In A the angle of twist, β, was computed by using the formula $\tan \beta = \pi DT$.
In B the angle of twist, β, was computed by using the formula $\tan \beta = 0.873\,\pi DT$.
In C the angle of twist, β, was computed by using the formula $\beta = \frac{\arctan \pi DT}{1 - 0.4\sqrt{C}}$.

in C of figure 5. The scattering of the points is probably attributable to the variability in the yarn and to the uncertainty in the measurements.

WASHINGTON, July 2, 1935.