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A FILTER FOR OBTAINING LIGHT AT WAVE LENGTH 560 mμ*

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ABSTRACT

A filter transmitting a narrow band of light at 560 mμ is of particular importance in the colorimetry of sugar solutions, in optical pyrometry, in abridged spectrophotometry, and in photometry. A new 4-component glass filter has been designed, which isolates and transmits light at 560 mμ more effectively than previous filters. Two components of this filter are of Corning glass and two of Jena glass.

The spectral transmissions of the filter and its components are illustrated. The spectral centroid of the light transmitted by the filter is at 560 mμ for both incandescent and daylight illuminants. Various other luminous characteristics of the filter are also defined and values given.

CONTENTS

I. INTRODUCTION

The wave length 560 mμ has been shown to have particular significance in the colorimetry of sugar solutions.[1] "Through a simple measurement of absorption or transmission at this wave length, it is possible to arrive at a nearly correct estimate of color in relation to the effective quantity of coloring material per gram of saccharine dry substance.[2]" Values at this wave length may be obtained either (1) by means of a spectrophotometer with the usual incandescent illuminant, (2) by use of the Hg-vapor illuminant, measurements being made at 546 and 578 mμ, and the value at 560 mμ being derived from the values obtained at the two Hg wave lengths,[3] or (3) with a photometer, incandescent illuminant, and filter properly isolating a narrow spectral region at 560 mμ.[4] The last method is the simplest

* This paper will appear also in J. Optical Soc. Am. **25**, **131** (1935).

[1] H. H. Peters and F. P. Phelps, Color in the sugar industry. Tech. Pap. BS **21**, 261(1926–27) T338; H. H. Peters and F. P. Phelps, A technical method of using the mercury arc to obtain data at wave length 560 mμ in the spectrophotometric analysis of sugar products. BS J. Research **2**, 335 (1929) RP38.
J. F. Brewster, Color filter for visual sugar colorimetry. Facts About Sugar **28**, 228 (1933).

[2] RP38, page 335; (see footnote 1).

[3] RP38 (see footnote 1).

[4] As proposed by Brewster (see footnote 1).

but depends for its reliability on the effectiveness of the filter used to obtain this wave length.

A filter transmitting only a narrow spectral region in the green or yellow-green is also of use in certain problems in optical pyrometry. Such a filter has been used (1) in the determination of the true temperature and total radiation of luminous gas flames,[5] and (2) in checking the accuracy with which the temperature scale can be realized in practice.[6] In these cases, the exact location of the isolated region is unimportant, although, of course, it must be accurately known; 560 mμ is as suitable as any other wave length. The quantity determined and used in such work is known as the "effective wave length";[7] and in order that this be as independent as possible of the visibility function, which varies from one observer to another, the filter must isolate a very narrow spectral region. The filter should, however, transmit as freely as possible in this region in order that sufficient light be available for measurement.

A third use for a 560 mμ filter is in abridged spectrophotometry. For certain types of work complete spectrophotometry is unnecessary; it has been found sufficient to use a photometer with filters [8] for isolating the four principal Hg lines and such additional spectral regions as may be desirable and are obtainable with an incandescent illuminant. A 560 mμ filter gives a point well spaced between the green and yellow lines of Hg.

Such a filter is also desired in certain photometric investigations.[9] The filter would furthermore be useful wherever "cold light" is desired, since the transmitted energy is so close to the spectral region of maximum visibility.

The present filter was made possible by the appearance of two new Jena glasses, described below. It was designed and constructed during the summer of 1934,[10] and was described at the October 1934 meeting of the Optical Society of America.[11]

II. SPECTRAL TRANSMISSION OF THE FILTER AND ITS COMPONENTS

The following glasses were used in combination, with thicknesses as noted: 1. Corning [12] 351, 4.55 mm; 2. Corning didymium, 5.82 mm; 3. Jena [13] VG 3, 1.99 mm; 4. Jena BG 18, 1.94 mm.

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[5] H. C. Hottel and F. P. Broughton, Determination of true temperature and total radiation from luminous gas flames—Use of special two-color optical pyrometer. Ind. Eng. Chem. Anal. Ed., **4**, 166 (1932).

[6] H. T. Wensel, National Bureau of Standards, unpublished work.

[7] P. D. Foote, "Center of gravity" and "Effective wave length" of transmission of pyrometer color screens, and the extrapolation of the high-temperature scale. Bul. BS **12**, 483 (1915–16) S260.

[8] W. D. Appel, A method for measuring the color of textiles., Am. Dyestuff Reptr. **17**, 49 (1928); Genevieve Becker and W. D. Appel, Evaluation of manila rope fibers for color. BS J. Research **11**, 811 (1933) RP627

[9] Robert Livingston, Note on the transmission characteristics of four green glass filters. J. Optical Soc. Am. **24**, 227 (1934).

[10] The author was not at that time aware of the work of Livingston, who used three of the four components of the present filter in one of the filters designed by him.

[11] J. Optical Soc. Am. **25**, 46 (1935).

[12] For Corning glasses see advertising booklet entitled, Glass Color Filters, issued by Corning Glass Works, Corning, N. Y.

[13] For Jena glasses see advertising booklet entitled, Jena Colored Optical Filter Glasses for Scientific and Technical Purposes, issued by Jena Glass Works, Schott and Gen., Jena, and obtainable from Fish-Schurman Corporation, 230 East Forty-fifth Street, New York, N. Y.

The spectral transmissions of these glasses were determined for the thicknesses given and are illustrated in figure 1. In this figure is

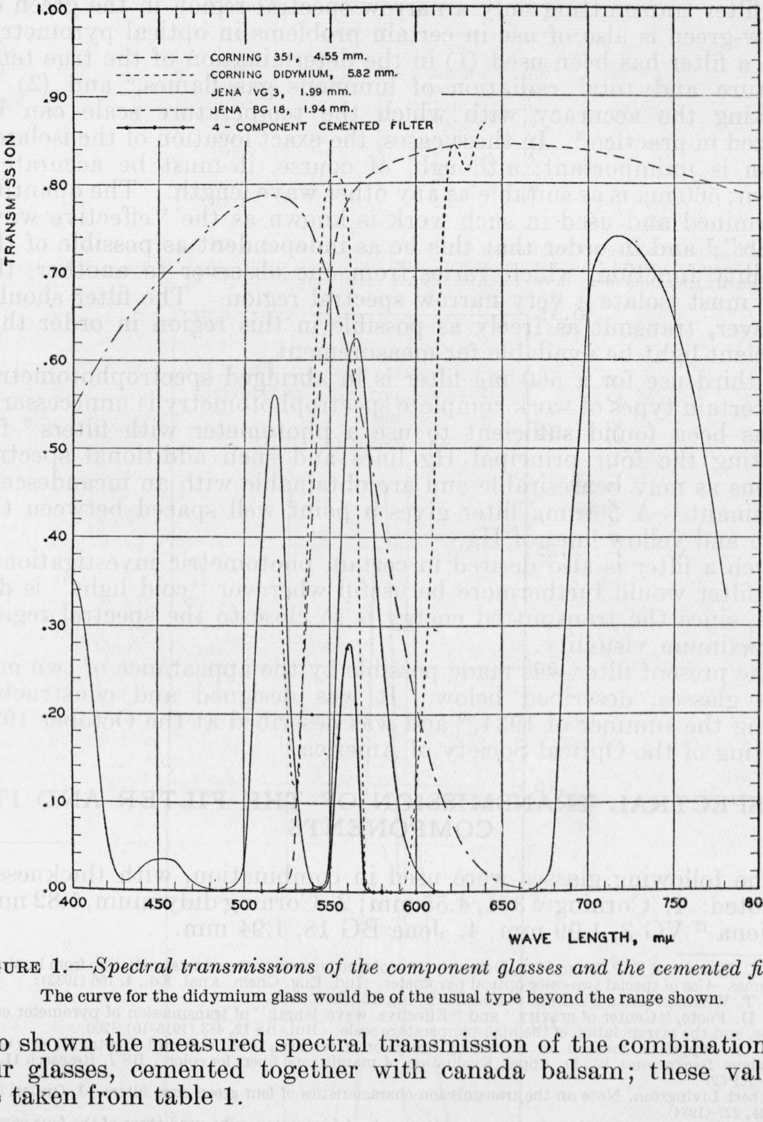

FIGURE 1.—*Spectral transmissions of the component glasses and the cemented filter.*

The curve for the didymium glass would be of the usual type beyond the range shown.

also shown the measured spectral transmission of the combination of four glasses, cemented together with canada balsam; these values are taken from table 1.

TABLE 1.—*Spectral transmission of 560 mμ filter*

Wave length	Transmission	Wave length	Transmission
mμ		mμ	
510	[a] 0.0000	570	.0095
20	.0004	75	.0003
30	.0011	80	.0004
40	.0019	85	.0001
545	.0029	90	.0001
50	.028	600	.0006
55	.174	10	.0003
60	.275	20	.0001
65	.212	30	[a] .0000

[a] The transmission at wave lengths less than 510 mμ and greater than 630 mμ is effectively zero, except that (1) there is a slight transmission, approximately 0.0002, in the red near 690 mμ, and (2) the filter undoubtedly transmits freely in the infrared at wave lengths greater than 1500 mμ.

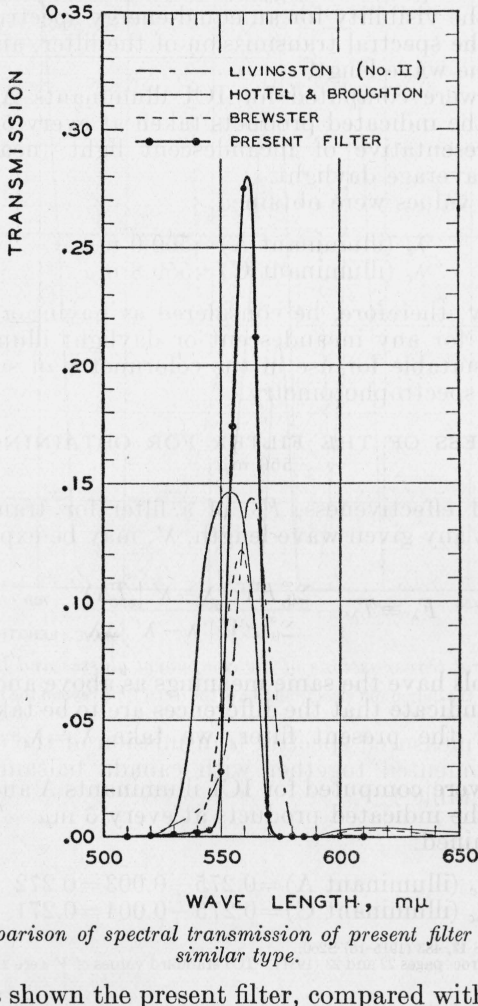

FIGURE 2.—*Comparison of spectral transmission of present filter with others of similar type.*

gure 2 is shown the present filter, compared with other filters [14] designed to obtain the same or closely adjacent regions. It may be

[14] The values for Livingston's filter have been multiplied by $(1/.91)^3$ to make them more nearly comparable to the other data. His published curve refers to an uncemented 4-component filter.

noted that the spectral transmission curve of the present filter has a ratio of height to width about twice that of the other filters. The decreased transmission in the region 590 to 650 mμ is also of importance.

III. LUMINOUS CHARACTERISTICS OF THE FILTER

1. SPECTRAL CENTROID OF THE TRANSMITTED LIGHT

The spectral centroid, or wave-length center of gravity of the transmitted light, is defined as:[15]

$$\lambda_c \equiv \frac{\int_0^\infty EVT\lambda d\lambda}{\int_0^\infty EVTd\lambda}$$

in which

E is the spectral energy of the illuminant,
V is the visibility for an equal-energy spectrum,
T is the spectral transmission of the filter, and
λ is the wave length.

Values of λ_c were computed for ICI illuminants A and C [16] by summations of the indicated products taken at every 5 mμ. Illuminant A is representative of incandescent light (near 2,840° K), illuminant C of average daylight.

The following values were obtained:

$$\lambda_c \text{ (illuminant A)} = 560.0 \text{ m}\mu$$
$$\lambda_c \text{ (illuminant C)} = 559.8 \text{ m}\mu$$

The filter may, therefore, be considered as having a spectral centroid of 560 mμ for any incandescent or daylight illuminant. It is thus especially suitable for use in the colorimetry of sugar solutions and in abridged spectrophotometry.

2. EFFECTIVENESS OF THE FILTER FOR OBTAINING LIGHT AT 560 mμ

The combined effectiveness, $F_{\lambda'}$, of a filter for transmitting and isolating light at any given wave length, λ', may be expressed as

$$F_{\lambda'} \equiv T_{\lambda'} - \frac{\Sigma_0^\infty EV \mid \lambda - \lambda' \mid T\Delta\lambda}{\Sigma_0^\infty EV \mid \lambda - \lambda' \mid \Delta\lambda}$$

where the symbols have the same meanings as above and the absolute value signs (| |) indicate that the differences are to be taken all greater than zero. For the present filter, we take $\lambda' = \lambda_c = 560$ mμ and $T_{\lambda c} = 0.275$.

Values of $F_{\lambda c}$ were computed for ICI illuminants A and C by taking summations of the indicated products at every 5 mμ. The following values were obtained:

$$F_{\lambda c} \text{ (illuminant A)} = 0.275 - 0.003 = 0.272$$
$$F_{\lambda c} \text{ (illuminant C)} = 0.275 - 0.004 = 0.271$$

[15] P. D. Foote, Bul. BS **12**, 483 (1915–16) S260.
[16] Int. Comm. Illum. Proc. pages 22 and 23 (1931). ICI standard values of V were also used.

The effectiveness of the other filters illustrated in figure 2 is much less than this. For example, the Hottel and Broughton filter has an effectiveness

$$F_{\lambda'}=0.146-0.009=0.137$$

for illuminant A with λ' taken at 555 mμ, which is close to the value of λ_c for this filter. For Livingston's filter $F_{\lambda c}$ will equal 0.14 approximately, and for Brewster's filter 0.12 approximately; this is apparent by inspection, since for filters of the type illustrated in figure 2 the value of $F_{\lambda c}$ is slightly less than the maximum transmission.

3. EFFECTIVE WAVE LENGTH FOR OPTICAL PYROMETRY

Within the visible spectrum, the energy distribution of a black body is represented closely by Wien's equation

$$E=C_1\lambda^{-5}e^{-\frac{C_2}{\lambda\theta}} \tag{1}$$

If E_1 and E_0 represent the spectral-energy distributions of a black body at the respective temperatures, θ_1 and θ_0, the ratio of E_1 to E_0 may be written as

$$E_1/E_0=e^{\frac{C_2}{\lambda}\left(\frac{1}{\theta_0}-\frac{1}{\theta_1}\right)} \tag{2}$$

For two given temperatures the value of the ratio is, therefore, simply a function of λ.

If V be the value of the visibility function at any wave length, λ, then

$$\frac{E_1V}{E_0V}=\frac{L_1}{L_0}=e^{\frac{C_2}{\lambda}\left(\frac{1}{\theta_0}-\frac{1}{\theta_1}\right)} \tag{3}$$

This is the principle of the optical pyrometer; by measuring the ratio of luminosities, L_1/L_0, for some known wave length, λ, the value of θ_1 may be computed from that of θ_0.

The type of optical pyrometer commonly employed does not, however, use light of a definite wave length, λ, but of a range of wave lengths transmitted by a suitable filter. The ratio of luminosities, R_L, measured with the pyrometer is in this case given by

$$R_L=\frac{\int_0^\infty E_1VTd\lambda}{\int_0^\infty E_0VTd\lambda} \tag{4}$$

where T is the spectral transmission of the filter used.

R_L may also be computed from this equation for any arbitrarily selected pair of temperatures, θ_1' and θ_0'. Substituting this computed value of R_L for E_1/E_0 in equation 2, or for L_1/L_0 in equation 3, and solving for λ, gives a wave length, λ_e, at which the ratio of spectral energies or luminosities is equal to the ratio of the luminosities which would be obtained with the filter for the two temperatures, θ_1' and θ_0'.[17] The quantity, λ_e, is called the effective wave length.

The computation of λ_e may be repeated for other pairs of temperatures. Since λ_e changes but slightly with changes in either θ_1' or θ_0', a

[17] This method of treatment is due to Hyde, Forsythe, and Cady, Astrophys. J. **42**, 294 (1915).

comparatively small number of values of λ_e need to be calculated to determine λ_e as a function of the upper and lower temperatures with sufficient accuracy throughout the useful range of measurement.

It can also be shown [18] that if the quantity

$$\lambda_\theta = \frac{\int_0^\infty E_\theta V T d\lambda}{\int_0^\infty E_\theta V T (1/\lambda)\, d\lambda}$$

is calculated for a series of temperatures, the value of λ_e for two temperatures, θ_1 and θ_0, is given by the equation

$$\frac{1}{\lambda_e} = \frac{1}{2}\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_0}\right)$$

The changes in λ_θ with temperature and observer depend primarily upon the width of the transmission band of the pyrometer filter. Such filters are often used by observers whose visibility function has never been determined, and in such cases the effective wave length used is that calculated for the ICI standard observer. It is a distinct advantage, therefore, to have a filter for which the effective wave length does not differ much for different observers. The quantity, λ_θ, for the present filter has been computed by Dr. H. T. Wensel,[19] by summation with wave-length intervals of 2.5 mμ. The following values were obtained for the ICI observer:

$$\lambda_{1336} \text{ (gold point)} = 560.5_7 \text{ m}\mu$$
$$\lambda_{2046} \text{ (platinum point)} = 560.2_3 \text{ m}\mu$$

Dr. Wensel has also calculated the values for that observer, otherwise normal, whose visibility differs most from the standard.[20] For this observer the values found are:

$$\lambda_{1336} = 560.7_4 \text{ m}\mu$$
$$\lambda_{2046} = 560.4_1 \text{ m}\mu$$

The difference of 0.3 mμ between values for the two selected temperatures and that of 0.2 mμ between the extreme and standard observer are approximately ⅛ and ⅕ of the respective differences for the red selenium glasses ordinarily used in optical pyrometers. The present filter, however, will have its greatest application at temperatures above 2,000° K, because in this range the rather low transmission of the filter is compensated by the increased brightness of the illuminant and accurate measurement becomes possible. It is estimated that temperature determinations can be made with this filter which will be uncertain by not more than 1 or 2° K, even though the observer's visibility is unknown and the effective wave length is assumed constant.

[18] P. D. Foote, Bul. BS 12, 483 (1915–16) S260. Foote, Fairchild, and Harrison, Pyrometric practice. Tech. Pap. BS 170, see p. 271 (1921).

[19] The author is further indebted to Dr. Wensel for advice regarding the preparation of this section.

[20] Out of the 52 observers tested. See BS Sci. Pap. 19, 131 (1923–24) S475.

4. LUMINOUS TRANSMISSION

Luminous transmission is defined as

$$T_L \equiv \frac{\int_0^\infty VETd\lambda}{\int_0^\infty VEd\lambda}.$$

Values of T_L were computed by summations of the indicated products taken at every 5 mμ.

For ICI illuminant A, $T_L=0.033$; for ICI illuminant C, $T_L=0.035$.

5. LUMINOUS EFFICIENCY OF THE TRANSMITTED ENERGY

Light is most efficiently produced by radiant energy at 555 mμ, the wave length of maximum visibility. At this wave length, the luminous efficiency is equal to the reciprocal of the least mechanical equivalent of light, M, the latest value for which is 0.001602 watt [21] per lumen. At any other wave length, the luminous efficiency is equal to V/M.

The luminous efficiency of the light transmitted by the filter is, therefore, equal to

$$e_L = \frac{\int_0^\infty ET(V/M)d\lambda}{\int_0^\infty ETd\lambda} = \frac{1}{M} \frac{\int_0^\infty ETVd\lambda}{\int_0^\infty ETd\lambda}$$

=616 lumens per watt for both ICI illuminant A and ICI illuminant C.

This value is about 99 percent of the maximum possible luminous efficiency, which is equal to 1/0.001602, or 624.2 lumens per watt. To make this value truly effective for the present filter, however, it must be used in combination with a water cell of sufficient thickness [22] to absorb the infrared energy otherwise transmitted by the filter.

IV. REPRODUCIBILITY AND PERMANENCE

It is well known that glasses of the type used in this filter are not precisely reproducible in spectral transmission from one melt to another. In this case, however, it is believed that little difficulty will be experienced in securing a satisfactory duplication of the present filter if the component parts are ordered by number and thickness from the respective dealers. A few hundredths of a millimeter deviation from the thicknesses given is of little consequence. The exact location of the transmitted light depends mostly on the Corning didymium and Jena VG 3 glasses, and by decreasing the thickness of one or the other of these glasses if necessary, the transmission band can be shifted slightly towards longer or shorter wave lengths. The other two glasses contribute little towards defining the exact position of this band, but are important in preventing the transmission of light at longer and shorter wave lengths.

Most glasses are considered permanent with ordinary usage. The surface of the Jena VG 3 glass tends to deteriorate somewhat upon ordinary exposure. It should, therefore, be mounted on the inside of the filter.

WASHINGTON, MARCH 1, 1935.

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[21] Wensel, Roeser, Barbrow, and Caldwell, J. Research NBS **13**, 161 (1934) RP699.

[22] From the published data on the Jena BG 18 glass (see footnote 13), it is estimated that 2 cm of water are sufficient to give complete absorption of infrared energy with the present filter.