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AN ANALYSIS OF CONTINUOUS RECORDS OF FIELD INTENSITY AT BROADCAST FREQUENCIES[1]

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ABSTRACT

Continuous records of the field intensities of most of the broadcasting stations in the United States have been made at the National Bureau of Standards receiving station near Washington, D. C. Typical records of received field intensities from several stations are presented. Maximum field intensities during 10-minute time intervals are analyzed in the following ways to illustrate sky-wave propagation phenomena at broadcast frequencies for distances up to 4,000 kilometers. (1) The diurnal variation of the 10-minute maxima is given for several stations. (2a) The variation of the 10-minute maxima is shown with respect to distance for night field intensities. (2b) These variations are also shown for sky waves received during the daytime. Empirical formulas are developed to represent the data of (2a) and (2b). The data are explained in terms of a theory of propagation of waves in the ionosphere.

CONTENTS

	Page
I. Experimental data	897
II. A theoretical explanation of the data	904
Appendix I. Reflection	906
Appendix II. Refraction	909

During the past 3 years graphical records of the field intensity of over 300 broadcasting stations in the United States and Territories have been made at the National Bureau of Standards receiving station at Meadows, Md., near Washington, D. C. The method used for recording the field intensity is described elsewhere.[2] It is the purpose of this paper to report these data and some of the conclusions reached from an analysis of these records.

I. EXPERIMENTAL DATA

In order to illustrate the type of field intensity record obtained and some of the general characteristics of the phenomena under consideration, some typical records are given of the received field intensity from broadcasting stations WLW in Cincinnati, Ohio, and WCKY in Covington, Ky. The upper record in figure 1 shows the received field intensity of WLW as measured at the Meadows field station (lat. 38°48′32″ N., long. 76°52′40″ W.). From 6 a.m. to 1 a.m. the power used was 50 kw and from 1 a.m. to 5:30 a.m. the power used was 500 kw. It is evident from the records that transmissions were stopped intermittently between 2 a.m. and 6 a.m. The

[1] Presented at the Ninth Annual Convention, Institute of Radio Engineers, Philadelphia, Pa., May 27 to 30, 1934, and at General Assembly, International Scientific Radio Union, London, England, Sept. 12 to 18, 1934.

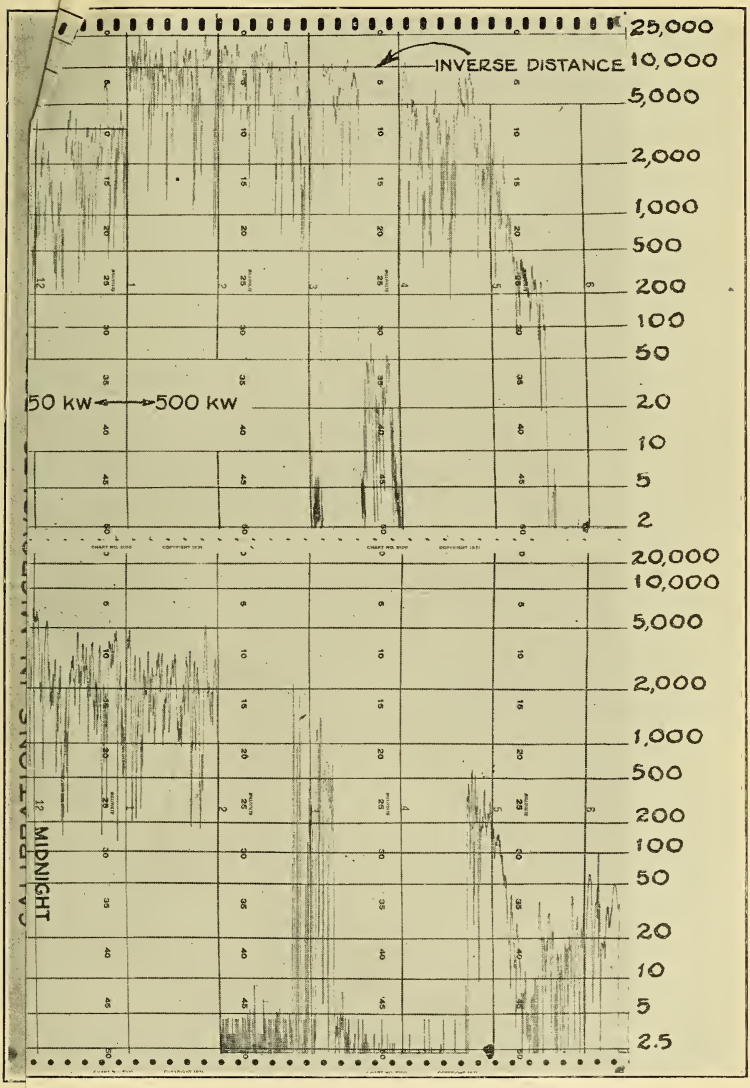
[2] K. A. Norton and S. E. Reymer. BS J. Research 11, 373 (1933) RP597.

inverse distance values were calculated on the assumption that these indicated amounts of power were actually radiated. The maximum received field intensities reach values 2½ times the inverse distance field intensities. The fading observed throughout the daytime indicates a received sky wave of intensity comparable to that of the ground wave since ground-wave field intensities are free from fading. In order to indicate the relative intensity of this station the intensities of the ground waves of the four local Washington stations are shown: WMAL, 630 kc/s, 500 w, 14 km; WRC, 950 kc/s, 500 w, 19 km; WJSV, 1,460 kc/s, 10 kw, 15 km behind directional antenna; WOL, 1,310 kc/s, 100 w, 17 km. On the lower record 500 kw was used throughout the day. The peak field intensity was only 17 mv/m on this day as compared to 25 mv/m a week earlier. Such a variation is typical of night field intensities at broadcast frequencies. The period of the night fading at this frequency is about 5 minutes.

Figure 2 shows two records of WCKY, the upper record for June 23 near the summer solstice and the lower record for December 20 near the winter solstice. The most conspicuous difference between the two records is the absence of any recordable field intensity during the summer day and the relatively strong sky wave present during the winter day. The peaks which may be seen during the summer day represent interference from other stations, only a weak beat note being audible for WCKY. The noise level at night is about eight times as strong in the summer. Also the night field intensities are about twice as strong in the winter record. Notice particularly the night fading at this frequency which has a period of about 1½ minutes. The record from midnight to 7 a.m. represents atmospherics.

In order to condense the enormous amount of data obtained, our records were analyzed in terms of the peak field intensities which were observed during 10-minute intervals. It may be mentioned that the average of six consecutive 10-minute maxima corresponds closely to the quasi-maximum field intensity for the hour, i. e., the field intensity which is exceeded during the hour only 5 percent of the time. In order to make some correction for the power it was assumed that each station radiated one-half of its rated power. Admittedly this was a very poor assumption but it was about the best that could be done in view of the fact that for most stations the amount of power which is radiated is not known. As an illustration of how poor this assumption regarding radiated power may be in some cases it may be mentioned that WLW with a rated power of 500 kw actually radiates at low angles power equivalent to 870 kw for the ordinary vertical distribution assumed for other stations. The usual formula for the radiated power for a vertical infinitesimal doublet is used in these calculations, i. e., $P_r = \frac{F^2 D^2}{C^2}$ kw, where F is the measured ground wave field intensity in microvolts per meter at a distance D in kilometers, and C is the velocity of light in kilometers/second. In the few cases where field intensity measurements had been made near the station, the radiated power was calculated by means of the above formula in terms of these measurements. Such cases were the exception rather than the rule and still do not take into account the radiation characteristics in the vertical and horizontal planes.

Figure 3 shows the diurnal variation of the 10-minute maxima for several clear-channel broadcasting stations. They are shown on a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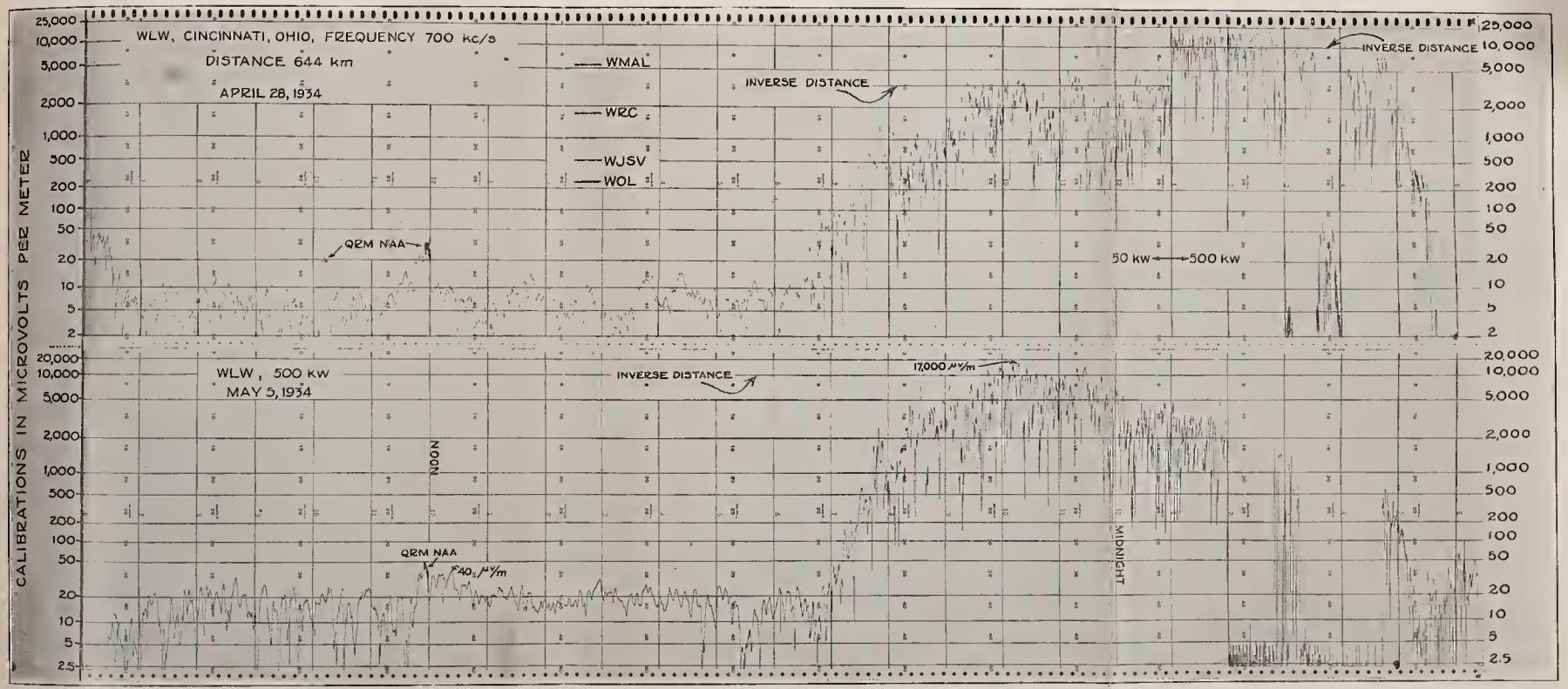
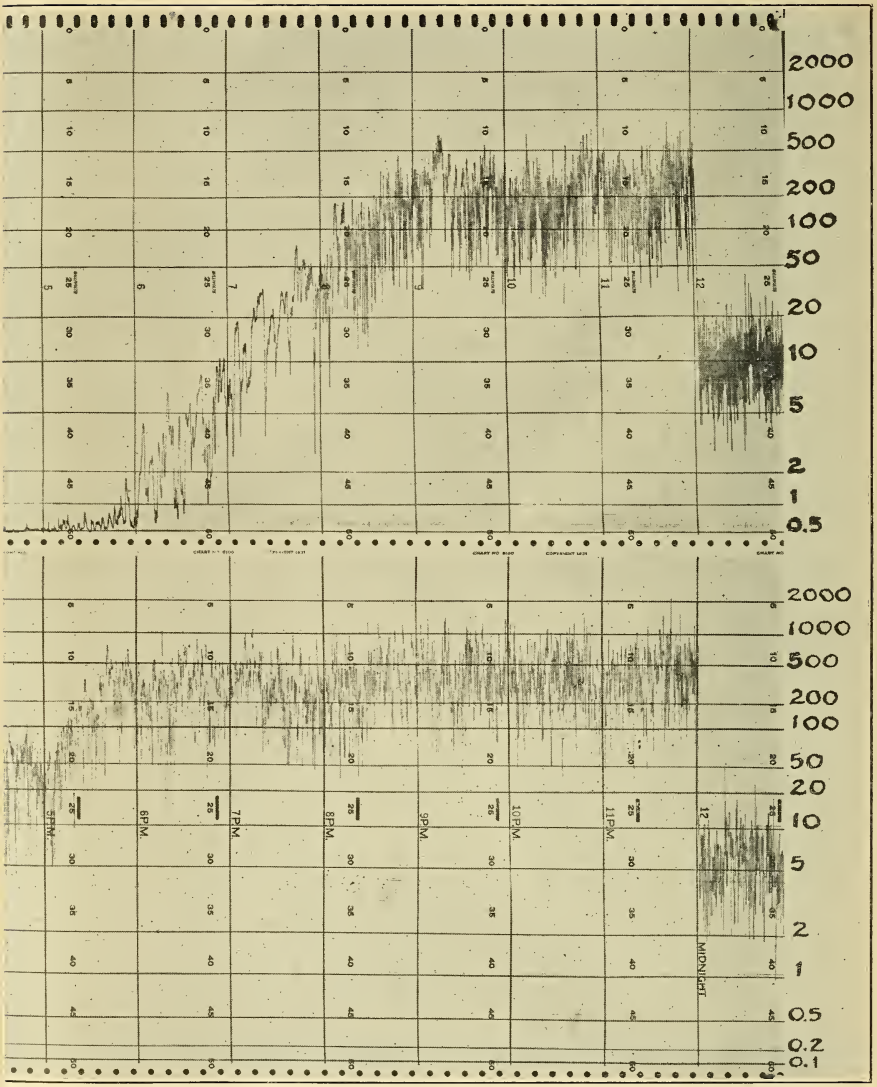


FIGURE 1.—*Graphical field intensity record of WLW, Cincinnati, Ohio.*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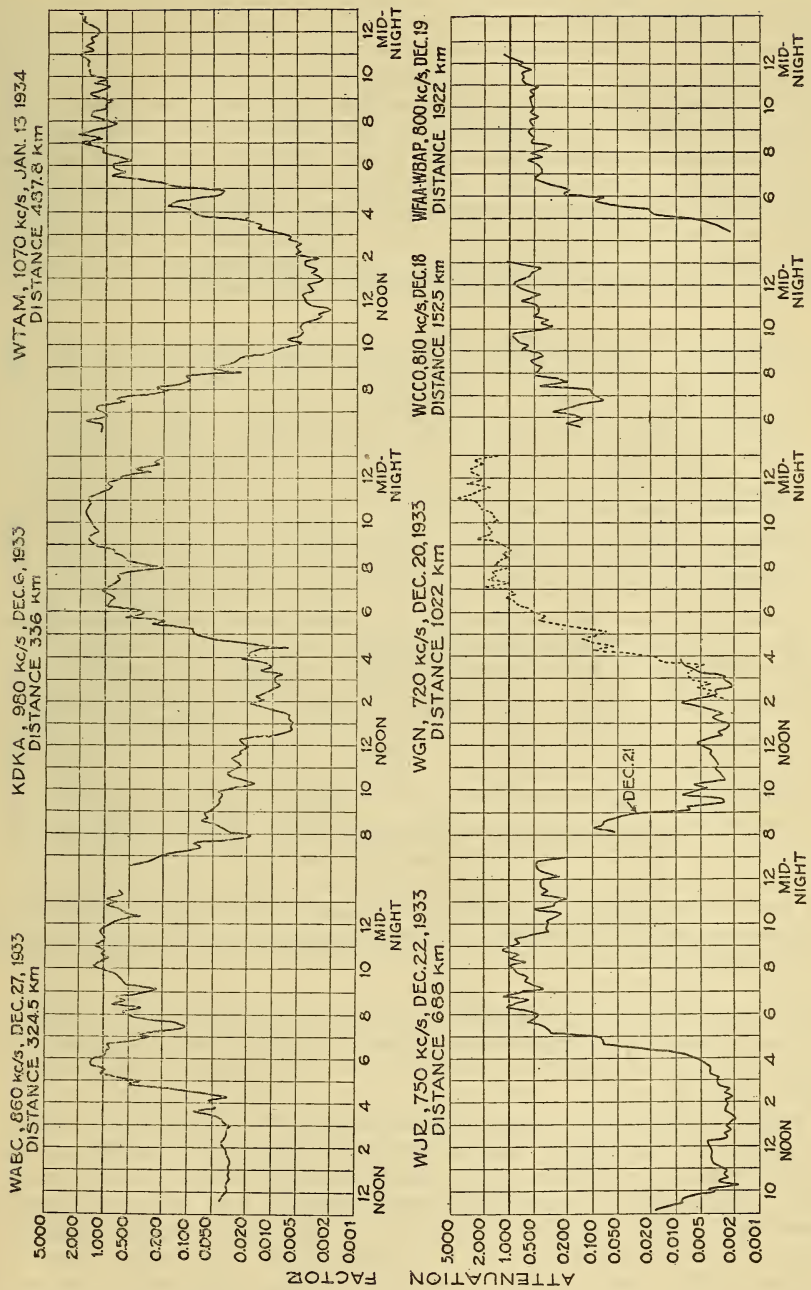

FIGURE 3.—*Sample diurnal variations of the attenuation factors of several broadcasting stations given as a function of distance.*

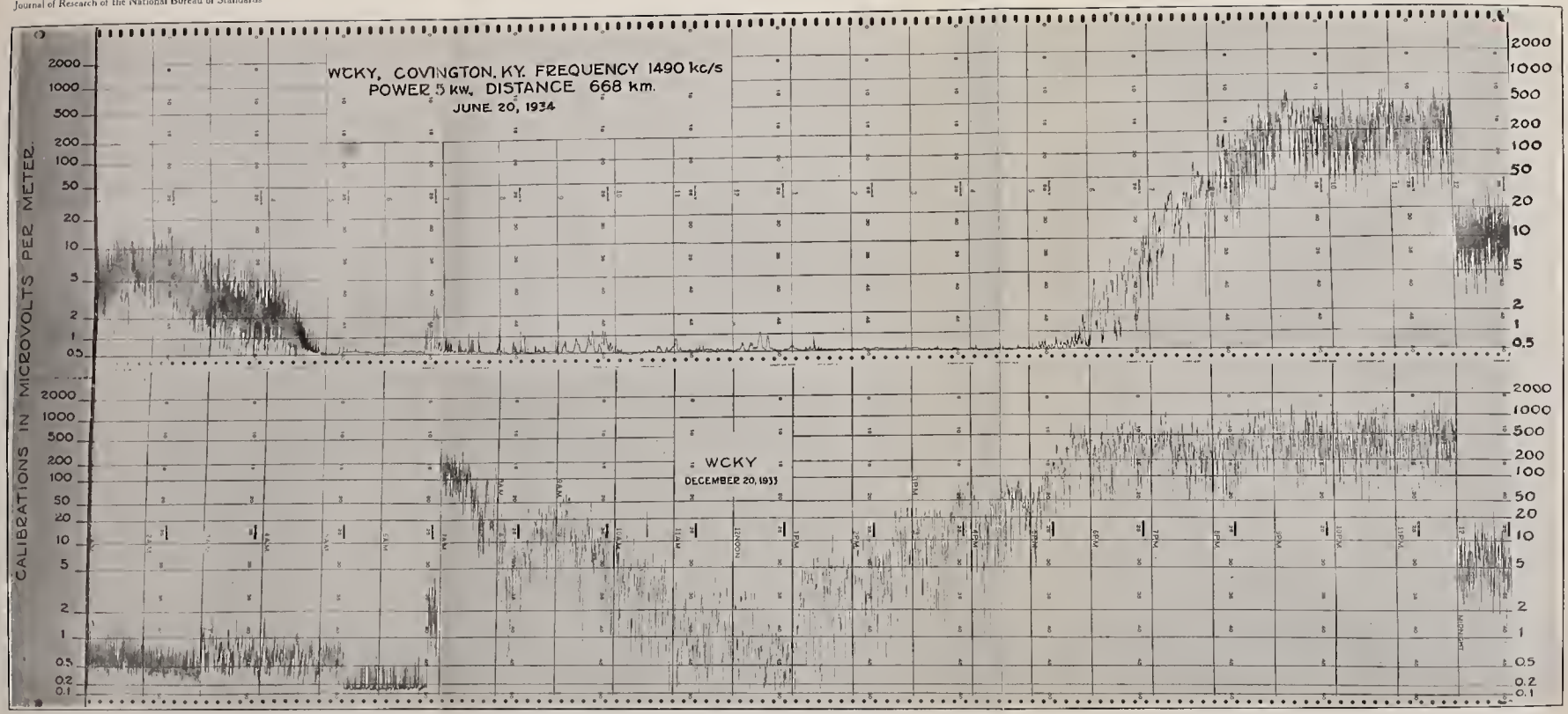

FIGURE 2.—*Graphical field intensity record of WCKY, Covington, Ky.*

logarithmic scale with the calculated inverse distance field intensity equal to one. If we write $F=AC\sqrt{P_r}/D$, then each measurement of field intensity determines an attenuation factor, A; these are the values plotted in figure 3. The principal thing to be noticed here is the increase in the ratio of the night field intensity to the day field intensity with distance. The increase in this ratio is evident in spite of the fact that it was impossible to keep all of the variables such as frequency and terrain constant. It may be seen that these attenuation factors are greater than 1 for all the stations at some time during the evening, the highest peaks occurring for WGN which reaches a peak of 4, corresponding to four times the calculated inverse distance field intensity. Of interest also is the fading observed at the greater distances during the daytime indicating the presence of a sky wave

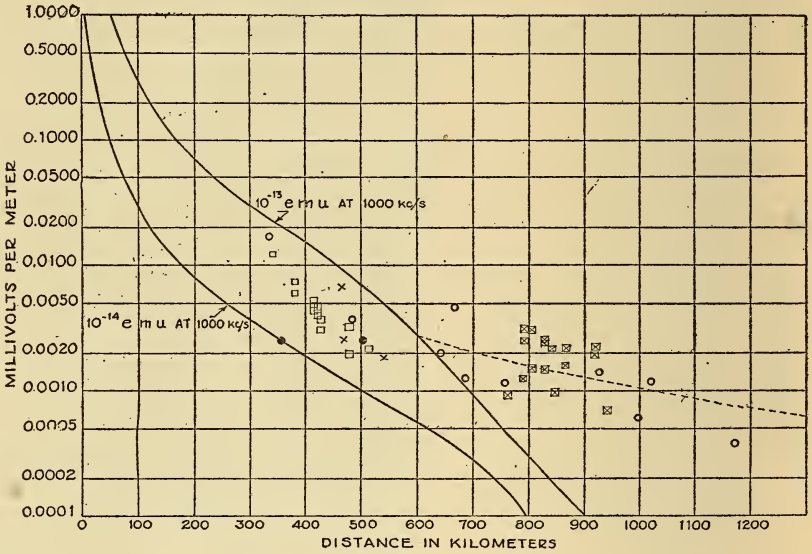

FIGURE 4.—*Daytime field intensity measurements reduced to 1* kw *of radiated power and given as a function of distance for broadcasting stations in the U. S. A.*

The solid curves were taken from the Report of the U. S. Committee on Radio Propagation Data and are theoretical ground-wave formulas. The dotted curve represents an empirical formula for distances greater than 600 km.

of intensity at least comparable to that of the ground wave. Evidence will now be given to show that this sky wave during the daytime is much stronger than the ground wave for distances greater than about 600 km.

Figure 4 shows the received daytime field intensities for a number of stations corrected to 1 kw of radiated power as a function of the distance at those distances for which there is fading throughout the day. Each point represents the average of the 10-minute peaks received between 10 a.m. and 2 p.m. The open circles represent our measurements made in December 1933, the crosses are for January 1934, and the solid circles are for April 1934. The solid lines indicate the theoretical variation of the received field intensity of the ground wave at 1,000 kc/s for 10^{-13} and for 10^{-14} electromagnetic units conductivity as taken from the Report of the U. S. Committee

on Radio Propagation Data.[3] The boxes represent measurements made on WLW by their engineers. The open boxes are for measurements east of WLW and the boxes with crosses are for measurements northeast of WLW. All of the measurements of WLW were made in February and March 1934, when snow was on the ground. It is probable that the received field intensities at distances greater than about 600 km are primarily due to sky waves since the variation with distance departs from the theoretical ground wave curves at these distances.

It was found that the empirical formula:

$$F=\frac{1{,}000\sqrt{P_r}}{D^2}(D>600 \text{ km, winter day}) \qquad (1).$$

fits the winter data satisfactorily for distances greater than 600 km This formula is shown dotted in figure 4. It may be mentioned that the daytime field intensity of WLW at 644 km and on 700 kc/s does not change much with season, while the daytime field intensity of WCKY at 688 km and on 1,490 kc/s is at least 100 times as strong during the winter day as for the summer day. It may be mentioned that the sky wave formula 1 agrees with the Sommerfeld ground wave formula at great distances for values of conductivity $\sigma_{emu}=4f^2\cdot 10^{-20}$ where f is the frequency in kc/s, i. e., for a conductivity of 10^{-14} electromagnetic units at 500 kc/s or a conductivity of 10^{-13} electromagnetic units at 1,600 kc/s.

In figure 5 the measured night field intensities are shown for about 300 stations as a function of the distance from the transmitter. Most of these measurements were made after 2 a. m. during the Federal Radio Commission's frequency monitoring schedules. Each point represents the maximum during a 10-minute time interval or the average of several such maxima but never more than six corresponding to an hour. There is one point for each separate night that the station was observed. They were corrected for power so as to correspond to 1 kw of radiated power. It will be observed that many of the maxima exceed the calculated inverse distance field as was evident also on the diurnal characteristics. Some of this excess is possibly due to too low an estimate of the radiated power but it is believed that part of it is real and may be explained on the assumption that several sky waves combine to give the observed maxima. Measurements were also made on two stations in Honolulu, Hawaii, at a distance of 7,775 km: KGU, 750 kc/s, 0.0011 mv/m per kilowatt on April 3, 1934; KGMB, 1,320 kc/s, 0.0037 mv/m per kilowatt on April 3, 1934, and 0.0076 mv/m per kilowatt on January 2, 1934. These points lie about the same distance below inverse distance as those near 4,000 km. The irregular curve shows the variation of night field intensity with distance as determined by a running average [4] of the data for distances less than 4,000 km. The general shape of this curve with a rise between 400 and 1,000 km is characteristic of broadcast transmission at night. The data obtained during the months of December 1933, January, February, and April 1934, were analyzed by months and no

[3] Proc. I. R. E., **21**, 1419 (Oct. 1933).

[4] The data were ordered according to distance. The field intensities for the first 10 distances were averaged to give an average field intensity for the distance corresponding to the average of the first 10 distances. The next average value of field intensity was determined by averaging the 4th to the 13th measured value of field intensity, etc. Since one application of this moving average process did not result in a sufficiently smooth curve, the process was repeated once to give the curve plotted in figure 5.

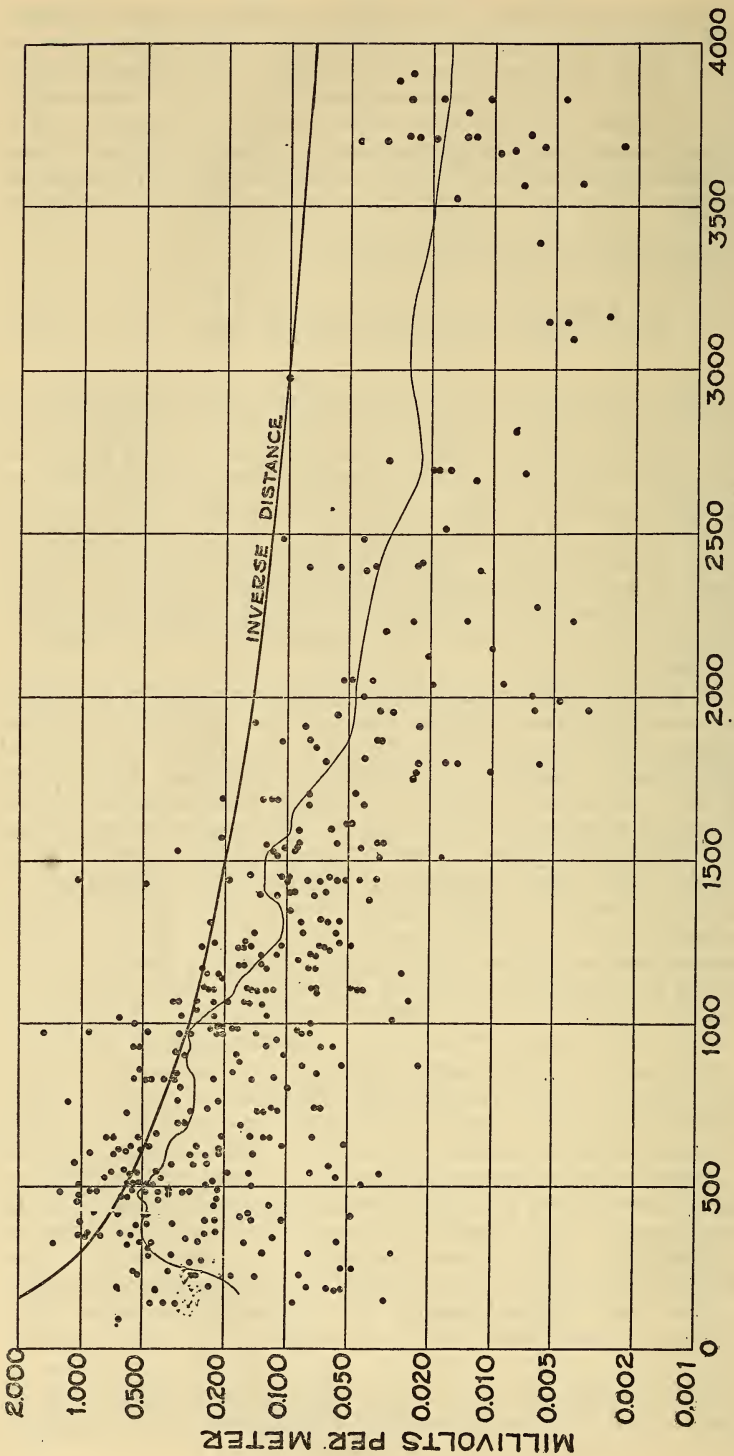

FIGURE 5.—*Night field intensity measurements reduced to 1 kw of radiated power and given as a function of distance for the broadcasting stations in the U. S. A.*

The irregular curve represents a running average of the data.

significant difference was found. However, as summer approached, the measurements were less reliable on account of the high atmospheric noise level, consequently small differences could not be determined. Continuous 24-hour records of the field intensity as received at Meadows, Maryland, of stations WEAF in New York, are available for a period of over a year. Similarly, records are available for WCKY in Covington, Ky., for over a year and for WLW for a period of greater than 6 months. These records are now being analyzed to determine samples of the seasonal variations at broadcast frequencies. Similarly, the data were analyzed for four different frequency ranges in the broadcasting band and no significant changes with frequency were found. Thus it may be concluded that there is no large variation of night field intensity with season or frequency for the range from 550 to 1,500 kc/s.

It was found that the empirical formula:

$$F = C\sqrt{P}\,\frac{1{,}600 + D}{4{,}410{,}000 - 11{,}000D + 10D^2} \text{ (night sky wave)} \qquad (2)$$

fits the averaged data satisfactorily. This formula, together with the averaged data, is shown in figure 6. The maximum sky wave field intensity predicted by formula 2 occurs at 580 km.

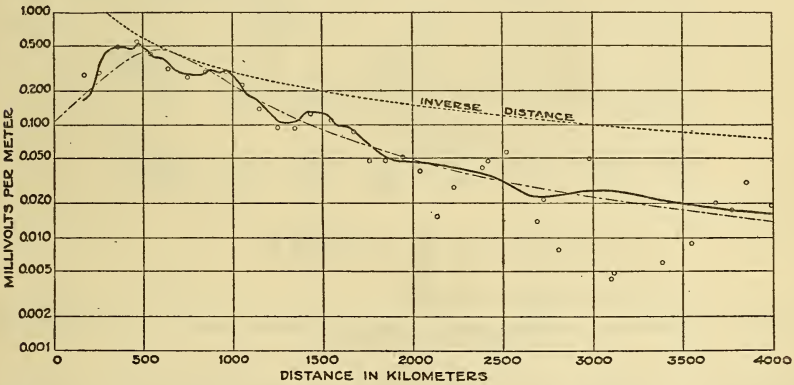

FIGURE 6.—*Two averages of night field intensity measurements together with the graph of an empirical formula which was designed to represent the data.*

The points plotted in figure 6 were obtained by an independent method of averaging and give credence to the major humps at 500, 1,000, and 1,500 km which were exhibited by the running average of the data. Each point represents the average of the measured field intensities in a 100-km interval of distance plotted at the average distance in the interval. The 0 to 100, 3,200 to 3,300, and 3,400 to 3,500 km intervals contained no measurements; the maximum number of measurements per 100-km interval was 30 which were made in the 1,100 to 1,200 km interval.

II. A THEORETICAL EXPLANATION OF THE DATA

Two simple theories of the attenuation of radio waves in the ionosphere were developed. The details of these theories are given in the appendix. The attenuation was calculated from these theories and an average polar diagram (i. e., $\sqrt{P_r(\psi_1)}$ see eq 23) was then calculated from the averaged field intensity data given in figures 5 and 6. Figure 7 shows two polar diagrams thus calculated. It should be emphasized that these represent the average polar characteristic of all the transmitting antennas multiplied by the polar characteristic of the receiving antenna. The receiving antenna was a vertical wire less than a quarter wave length long. The dotted graph was determined by setting $k=0$ in eq 23 on the assumption that the waves were reflected without absorption at a layer 100 km high; i. e., on the

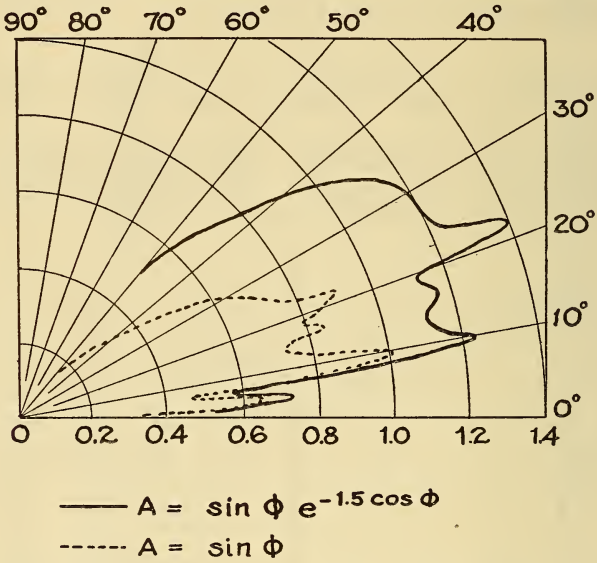

FIGURE 7.—*Theoretical average polar radiation characteristics for broadcasting stations in the U. S. A.*

assumption that the attenuation was due only to the fact that the wave traveled to a 100-km layer and back. This assumption corresponds for the purpose of our analysis to that used by P. P. Eckersley [5] and later by Stuart Ballantine [6] and gives an attenuation factor $A=\sin\phi_1$ where ϕ_1, denotes the angle of incidence at the layer. (See Appendix I.) The amount of radiation above 45 degrees (see dotted curve, fig. 7) which is calculated on this assumption is hardly as much as might be expected from the average transmitting antenna in use by the stations measured. The solid curve in figure 7 was calculated from the average data on the assumption that the waves were refracted back to earth at a 100-km layer in which the ion density varied exponentially with the height. This assumption gives an attenuation factor independent of the frequency and equal to $\sin\phi_1 e^{-k\cos\phi_1}$. (See Appendix II.) The constant k was arbitrarily

[5] Proc. I. R. E., 18, 1160 (July 1930).
[6] Proc. I. R. E., 22, 619 (May 1934).

taken to be 1.5. It is evident that a larger amount of high-angle radiation is obtained with an attenuation factor of this form and the relative amount may be changed further by an adjustment of the constant k. Since the solid curve in figure 7 has approximately the general shape which might be expected for $\sqrt{P_r(\psi_1)}$, we are led to believe that formula 23 properly relates the variables influencing sky-wave propagation in the broadcasting band at night. The constant k, being theoretically proportional to the average collision frequency of the ions along the ray path, may very reasonably be expected to vary in a large measure from night to night since small changes in k might cause large changes in the pressure and thus in the collision frequency. This will explain the fact that field intensity observations on the same station at a fixed distance vary enormously from night to night. This probable variation in k may also be offered as a partial explanation of

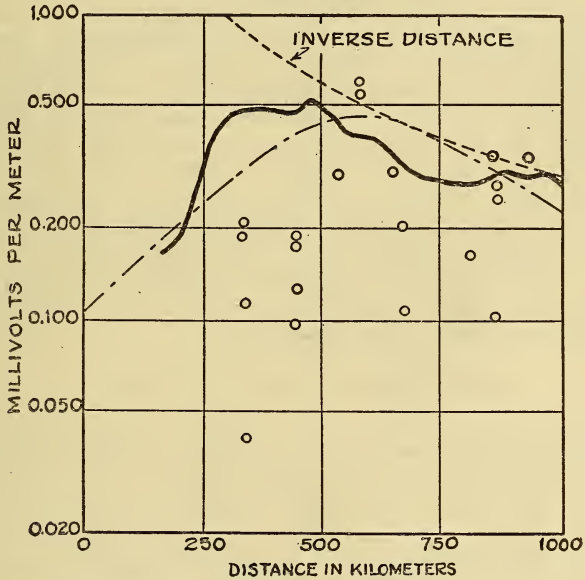

FIGURE 8.—*Night field intensity measurements of* WLW *compared to averaged data for other broadcasting stations in the U. S. A.*

the large variation of the data in figure 5 above and below the averaged data; the remainder of this variation is probably due to a variation of $P_r(\psi_1)$ for the different stations measured.

For distances greater than about 2,000 km the field intensities are received after two or more reflections since the curvature of the earth shields the receiver from the first reflection. At these distances also the refraction theory gives a more adequate explanation of the data, the simple reflection theory tending to predict too large values of received field intensity.

After the recent installation of the new 500-kw transmitter for WLW, their engineers made many field-intensity measurements. Figure 8 shows the results of several sets of measurements after they were corrected for radiated power. Each point represents the maximum field intensity observed at a given location. The period of observa-

tion for each of these points varied from about 5 min to about 5 hr. A loop antenna was used for reception. These data are in good agreement with our own at the greater distances. The low values observed between 300 and 500 km are probably caused by the low-angle radiator (0.58λ vertical antenna) used by this station. Thus in the single case where we have measurements at various distances on a station for which we know the radiated power we obtain good agreement quantitatively with our averaged data or empirical formula 2 and qualitatively with our theory.

In conclusion we may enumerate the deductions made from the analysis of the data. Received night field intensities are often greater than inverse distance, reaching their maximum values at about 600 km. The average night field intensity for the average broadcasting station may be predicted by means of formula 2, but the individual measurements on a station may be expected to vary by a factor of 10 above or below its average. There are no large variations of received field intensity with season or frequency although, in general, the received field intensity is slightly weaker in the summer. The variation of the received field intensity at night with distance appears to be determined primarily by the directional characteristics in the vertical plane of the transmitting and receiving antennas. The theoretical formula 23 seems to give an adequate explanation of the data, the variation of $P_r(\psi_n)$ giving an adequate explanation of the variation of received field intensity with distance and the variation of k giving a plausible explanation of the night-to-night and diurnal variations. Field intensities received during the daytime at distances greater than about 600 km are sky waves and may be predicted during the winter time by formula 1.

APPENDIX I. REFLECTION

It is assumed in deriving this theory for the attenuation of sky waves at broadcast frequencies that the waves are reflected or refracted back to earth from a layer in the ionosphere of virtual height 100 km. The term virtual height is defined in papers by G. Breit and M. A. Tuve,[7] and T. R. Gilliland, G. W. Kenrick, and K. A. Norton.[8] The theory is applied by making the further assumption that the primary portion of the energy reflected back to earth reaches the receiver after only a single reflection at the layer for distances less than about 2,000 km and after only two reflections for distances greater than 2,000 but less than 4,000 km.

Figure 9 shows the geometry for one of the n hops between the transmitter and receiver. Let ϕ_n denote the angle of incidence of the waves at the layer, $2n\,\theta_n$ the angular distance between the transmitter and the receiver, and ψ_n the angle above the horizon with which the waves leave the transmitter or arrive at the receiver. Here n denotes the number of reflections at the layer which the waves make before arriving at the receiver. Let $R=6{,}371$ km denote the radius of the earth, $h=100$ km denote the virtual height of the layer and l the distance from the transmitter to the layer. The geometry of figure 9 gives:

$$\tan\phi_n=\frac{\sin\theta_n}{1-\cos\theta_n+\frac{h}{R}} \tag{3}$$

$$\psi_n=\frac{\pi}{2}-\phi_n-\theta_n \tag{4}$$

If, as before, D denotes the distance in kilometers between transmitter and receiver we have:

$$D=222.1\ n\theta_n \text{ degrees} \tag{5}$$

[7] Phys. Rev., 21, 554 (1926).
[8] BS J. Research 7, 1083 (1931) RP390, or Proc. I. R. E., 20, 286 (Feb. 1932).

The greatest distance for which waves may be reflected back to earth after a single reflection is determined by setting $\psi_1=0$ in eq 4 and substituting the resulting value of ϕ_1 in eq 3, whence:

$$\theta_{\max}=\cos^{-1}\frac{R}{R+h}=10°5' \tag{6}$$

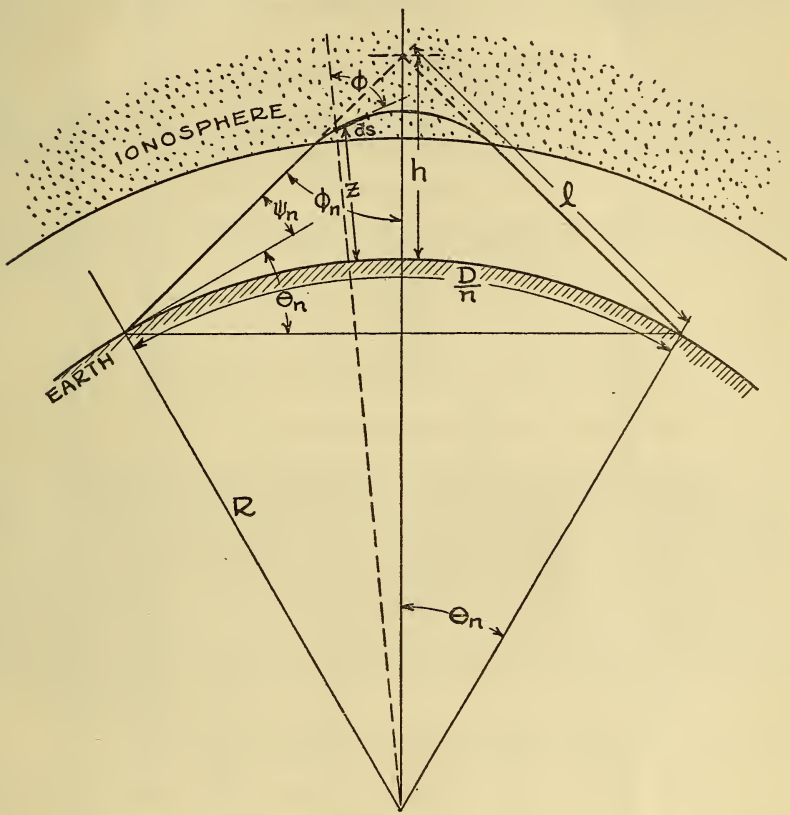

FIGURE 9.—*Geometry and the parameters concerned in theory of sky-wave propagation.*

In figure 10 ϕ_n and ψ_n are given graphically for one and for two reflections at the layer as a function of D. In general for n reflections we have the relations:

$$\phi_n(D)=\phi_1\left(\frac{D}{n}\right) \tag{7}$$

$$\psi_n(D)=\psi_1\left(\frac{D}{n}\right) \tag{8}$$

If we assume that the waves are reflected at the layer without absorption, then they will be received with an intensity:

$$F=\frac{C}{D}\sqrt{P_r(\psi_n)}\,A_n=\frac{C}{2nl}\sqrt{P_r(\psi_n)} \tag{9}$$

so that:

$$A_n=\frac{D}{2nl}=\frac{\theta_n}{\sin\theta_n}\sin\phi_n\cong\sin\phi_n \tag{10}$$

where the attenuation factor A_n simply gives the reduction in field intensity due to traveling the inverse path distance as compared to the inverse great circle distance.

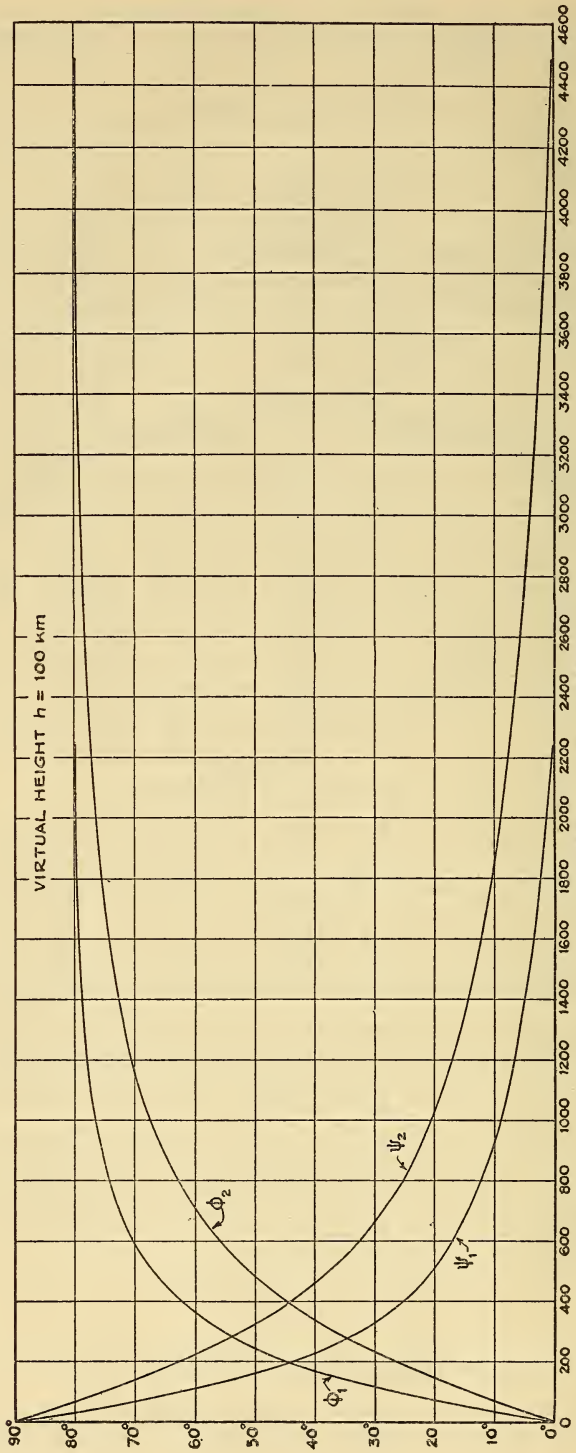

FIGURE 10.—*Variation with distance of two important parameters in the theory of sky-wave propagation.*

ϕ, the angle of incidence at the layer, and ψ, the angle above the horizon at which the radiation from the transmitter takes place.

It is possible to neglect the ratio $\frac{\theta_n}{\sin \theta_n}$ since θ_{max} for a 100-km layer is only 10°5′. Equation 10 then represents the attenuation factor for a single wave at a reflecting layer and is applicable for any finite distance between transmitter and receiver and for any number of reflections. It is evident that the succeeding reflections after the first will be weaker in the ratio $\sin \phi_n/\sin \phi_1$ and will be neglected in this discussion.

APPENDIX II. REFRACTION

The formula 10 was determined on the assumption that the waves were reflected without absorption at the layer. If we set Γ_n equal to the total attenuation along the ray path and γ equal to the attenutation per unit distance, then:

$$\Gamma_n = \int \gamma dS \tag{11}$$

where S denotes the distance along the ray path and the integration extends along the entire path. Now from the classical electromagnetic theory[9] we have the relations:

$$\gamma \sim \frac{Nv}{\mu\omega^2} \tag{12}$$

$$\mu^2 = 1 - \frac{CN}{\omega^2} \tag{13}$$

$$\cos \psi_n = \sin \phi \mu \left(1 + \frac{Z}{R}\right) \tag{14}$$

where N denotes the ionic density, v the collision frequency of the ions, μ the index of refraction, ω the angular frequency, Z the distance above the surface of the earth at any point along the ray path and C a constant. Results of ionization measurements during the solar eclipse of 1932 as reported by S. S. Kirby, L. V. Berkner, T. R. Gilliland, and K. A. Norton[10] indicate that the ionization during the daytime in the E layer consists mostly of heavy ions so that the modification of eq 13 due to the magnetic field of the earth may be neglected. At night this is probably no longer strictly true so that this constitutes one of the approximations of the theory. Equation 14 denotes the invariant for the ray path around a curved earth with ϕ denoting the angle which the ray makes with the vertical. From eq 3 and 4 we obtain:

$$\cos \psi_n = \cos\left(\frac{\pi}{2} - \phi_n - \theta_n\right) = \sin \phi_n \left(1 + \frac{h}{R}\right) \tag{15}$$

Combining eq 14 and 15 we obtain:

$$\sin \phi_n = \mu \frac{R+Z}{R+h} \sin \phi \cong \mu \sin \phi \tag{16}$$

Since there will be no absorption except in the layer, Z will not be much less than h throughout the range of integration so that we may safely make the approximation of setting the ratio $\frac{R+Z}{R+h} = 1$. The ray reaches the highest point and the greatest ion density, say N_0, when $\sin \phi = 1$ so that we obtain from eq 13 and 16:

$$N_0 = \frac{\omega^2}{C} \cos^2 \phi_n \tag{17}$$

At any point along the ray path we have:

$$dS = \frac{dZ}{\cos \phi} = \frac{\mu dZ}{\sqrt{\cos^2 \phi_n - \frac{CN}{\omega^2}}} = \frac{\mu dZ}{\cos \phi_n \sqrt{1 - \frac{N}{N_0}}} \tag{18}$$

[9] See e. g., P. O. Pedersen, The Propagation of Radio Waves, Copenhagen (1927).
[10] BS J. Research **11**, 829 (1933) RP629, and Proc. I. R. E. **22**, 247 (1934).

Substituting eq 12 and 18 into eq 10 we obtain:

$$\Gamma_n \sim \frac{n}{\omega^2 \cos \phi_n} \int \frac{N\, v\, d\, Z}{\sqrt{1-\frac{N}{N_0}}} = \frac{n}{\omega^2 \cos \phi_n} \int_0^{N_0} \frac{N\, v\, d\, N}{\frac{dN}{dZ}\sqrt{1-\frac{N}{N_0}}} \tag{19}$$

the first integration extending from the height for which $N=0$ to the height for which $N=N_0$. We will use the theorem of the mean to replace v by its mean value $\bar{v}$ and assume that N varies exponentially with the height so that:

$$\Gamma_n \sim \frac{n\bar{v}}{\omega^2 \cos \phi_n} \int_0^{N_0} \frac{dN}{\sqrt{1-\frac{N}{N_0}}} \tag{20}$$

Using the new variable $X=\frac{N}{N_0}$ we obtain:

$$\Gamma_n \sim \frac{n\ \bar{v}\ N_0}{\omega^2 \cos \phi_n} \int_0^1 \frac{dX}{\sqrt{1-X}} \sim n\ \bar{v} \cos \phi_n \tag{21}$$

Thus we obtain the result that the attenuation along the ray path [11] is proportional to the product of the number of reflections, the average collision frequency of the ions and the cosine of the angle of incidence at the layer. It is important to notice that the attenuation for the assumed exponential distribution of ion density is independent of the frequency. Since our results indicated no variation of received field intensity with frequency and since an exponential distribution of ion density is very probable because of the exponential absorption in the ionosphere of ultraviolet radiation from the sun, eq 21 very likely has the correct form. Combining the attenuation effects due to eq 10 and 21 we obtain the general expression for the attenuation factor for a ray after n reflections at the layer:

$$A_n = \sin \phi_n e^{-nk \cos \phi_n} \tag{22}$$

and the field intensity of this wave will be

$$F_n = \frac{C}{D}\sqrt{P_r(\psi_n)} \sin \phi_n e^{-nk \cos \phi_n} \tag{23}$$

where the radiated power is for the angle $\psi_n(D)$ and k is a constant independent of D and f but proportional to the average collision frequency of the ions along the ray path.

WASHINGTON, October 11, 1934.

[11] This same problem was solved in terms of somewhat less convenient variables in a paper by Shogo Namba, Proc. I. R. E., *21*, 238 (1933).