U.S. Department of Commerce, Bureau of Standards

RESEARCH PAPER RP660

Part of Bureau of Standards Journal of Research, vol. 12, March 1934

SOURCES OF ERROR IN MEASURING OPACITY OF PAPER BY THE CONTRAST-RATIO METHOD

By Deane B. Judd

ABSTRACT

Contrast ratio of a paper sample is defined as the brightness of the illuminated sample when backed by a black-lined cavity divided by its brightness when backed by a surface of magnesium oxide. In practice other less fragile surfaces are substituted for the MgO surface, and to protect these substitute surfaces as well as the paper samples themselves the samples are kept from touching the surfaces. Both departures from the definition are potential sources of error. These errors have been investigated experimentally and a theoretical expression representing them is given.

CONTENTS

		Page
I.	Introduction	345
II.	Derivation of formulas	346
III.	Experimental results	348
IV.	Conclusions	351

I. INTRODUCTION

Contrast ratio of any thin sample is defined[1] as the brightness of the illuminated sample when backed by a black-lined cavity divided by its brightness when identically illuminated and backed by a surface of magnesium oxide. This ratio has been found of use in specifying tracing cloth, glassine paper, and envelope windows,[2] in which case a low value is desired, and in specifying printing paper[3] in which case a high value is desired. The present discussion deals particularly with the latter case, the experimental data referring to samples of contrast ratio greater than 0.50; furthermore, the derivations depend on the assumption that the light reflected and transmitted by the sample is completely diffused. Hence, the conclusions may not be

[1] Circular of the Bureau of Standards 63, Specification of the Transparency of Paper and Tracing Cloth, May 17, 1917.

[2] I. G. Priest, The Bureau of Standards Contrast Method for Measuring Transparency, Trans. Amer. Ceramic Soc., vol. 17, p. 150, 1915. R. E. Lofton, Study of the Windows of Window Envelopes for the Purpose of Developing Standard Specifications, B.S. Technologic Paper 343, vol. 21, pp. 385–399, 1926–27.

[3] Used by the Government Printing Office in the purchase of printing papers. See also C. F. Sammet, A Measurement of the Translucency of Papers, U.S. Bureau of Chemistry Circular no. 96, 1912. J. d'A. Clark, The Measurement of the Contrast Ratio of Opaque and Translucent Papers, Paper Trade Journal, vol. 89, November 14, 1929. M. N. Davis, Methods and Instruments for Opacity Measurement, Tech. Assoc. Papers, series 15, pp. 347–350, 1932. M. N. Davis, A Simple and Reliable Photo-Electric Opacity Tester, Tech. Assoc. Papers, series 16, pp. 277–279, 1933.

valid for tracing cloth, glassine paper, and the like, and should not be applied to them without further inquiry.

Although the definition of contrast ratio is in terms of a surface of magnesium oxide nearly in contact with the test specimen,[4] actual instruments for the measurement of opacity by the contrast-ratio method use a less fragile surface (usually magnesium carbonate) separated sometimes as much as 2 mm from the test specimen, and sometimes further protected from injury by interposition of a thin cover glass. It is the purpose of the present paper to investigate the errors in contrast ratio which may arise (1) from departure of the reflectance of the working standard white surface from that of the standard MgO surface, and (2) from separation of the white surface from the test specimen.

II. DERIVATION OF FORMULAS

The following symbols will be used:

$C_{R,t}$ ≡ ratio of brightness of illuminated sample backed by a cavity reflecting no light to its brightness when identically illuminated and backed by a surface of reflectance, R, at a distance, t.

C ≡ an abbreviation for $C_{1,0}$ which refers to contact with a perfectly reflecting surface ($R=1$, $t=0$).

f ≡ flux incident per unit area on the sample.

f_0 ≡ flux reflected per unit area of the sample when backed by the cavity of zero reflectance.

$f_{R,t}$ ≡ flux reflected per unit area of the sample backed by a surface of reflectance, R, at a distance, t.

First the connection will be found between the measured contrast ratio, $C_{R,t}$, and the reflectance, R, of the white backing whose distance from a thin, nonabsorbing, perfectly diffusing, test specimen is so small that further decrease makes no difference in the measured ratio ($t=0$); that is, $C_{R,0}$ will be found as a function of R. Since the test specimen does not absorb any light, we may write:

$$f \equiv f_{1,0} = f_0 + (f_{1,0} - f_0) \tag{1}$$

$$C \equiv f_0/f_{1,0} = f_0/f \tag{2}$$

It is seen from equation (1) that the incident flux may be thought of as divided into two parts, first, that reflected from the cavity-backed sample, f_0, and, second, that transmitted into the cavity, $f_{1,0}-f_0$. If now the surface of reflectance, R, be substituted for the black-lined cavity, the flux, $f_{1,0}-f_0$, illuminates the surface which returns an amount, $(f_{1,0}-f_0)R$, to the back side of the test specimen. Part of this flux passes through the test specimen which from equation (2) may be written $(f_{1,0}-f_0)R(1-C)$, while the rest, $(f_{1,0}-f_0)RC$ is reflected back to the surface of reflectance, R, and so on. The total flux passing through the sample due to these multiple reflections may be represented by an infinite series which, when added to

[4] Official Method of the Technical Association of the Pulp and Paper Industry for Determination of Opacity of Paper (no. T425m, obtainable from the association at 370 Lexington Ave., New York, N.Y.), states in this connection, "The white standard surface must not touch the surface of the test specimen but must be so near it that a further decrease in distance will not affect the test results."

the flux, f_0, reflected from the sample, makes up the whole flux leaving the surface, $f_{R,0}$:

$$f_{R,0}=f_0+(f_{1,0}-f_0)R(1-C)+(f_{1,0}-f_0)RCR(1-C)+\ldots$$
$$=f_0+(f_{1,0}-f_0)R(1-C)[1+RC+(RC)^2+(RC)^3+\ldots]$$
$$=f_0+(f_{1,0}-f_0)R(1-C)/(1-RC) \tag{3}$$

It will be noted that when $R=1$, equation (3) becomes identical with equation (1).

From the definitions of terms and equation (3):

$$C_{R,0}=f_0/f_{R,0}=\frac{f_0}{f_0+(f_{1,0}-f_0)R(1-C)/(1-RC)}$$
$$=\frac{C}{1-(1-C)[1-R(1-C)/(1-RC)]} \tag{4}$$

The derivation of $C_{R,t}$ as a function of R and t follows very similar lines. For simplicity the sample and white backing are considered to be circular of radius, r; and it is assumed that the white backing is drawn back in a tube of radius, r, whose inner walls have a reflectance, R_t. It is also assumed that the light flux is uniformly distributed over the walls of the cylindrical cavity formed between sample and white backing for t greater than zero. It is found that with these additional assumptions:

$$C_{R,t}=\frac{C}{1-(1-C)\left[1-\frac{R_c(1-C)}{(1-R_cC)}\frac{r}{r+t}\right]} \tag{5}$$

where $R_c=(rR+2tR_t)/(r+2t)$. Note that R_c in equation (5) replaces R in (4), it is the average reflectance of the walls of the cylindrical cavity exclusive of the front which is made up of the test specimen; when $t=0$, $R_c=R$, and equation (5) becomes identical with (4). The factor, $r/(r+t)$, appearing in the denominator takes account of the fact that the light flux in the cylindrical cavity is spread by multiple reflection over an increasing area as t increases, each reflection permitting a loss of flux by absorption not taken into account by R_c. The factor takes this simple form because of the assumption that the light flux is uniformly distributed over the walls. For very large values of t, the inaccuracy of this simplifying assumption clearly reveals itself because equation (5) then becomes: $C_{R,\infty}=1$; that is, the formula indicates that a white-lined cavity of infinite length would be equivalent to the black-lined cavity of finite length. This would be the case even for $R=1$ if the light flux could be forced to spread over the walls uniformly in accord with the assumption; actually, however, the sample would receive some light from the nearby illuminated inner wall of the tube which would make $C_{R,\infty}$ somewhat less than unity. This inaccuracy for t large is of no practical concern because in practice t is always considerably smaller than r.

III. EXPERIMENTAL RESULTS

Equation (4) depends only on the assumption that the test specimen be a thin, nonabsorbing, perfectly diffusing sample. Since most paper samples customarily measured for opacity by the contrast-ratio

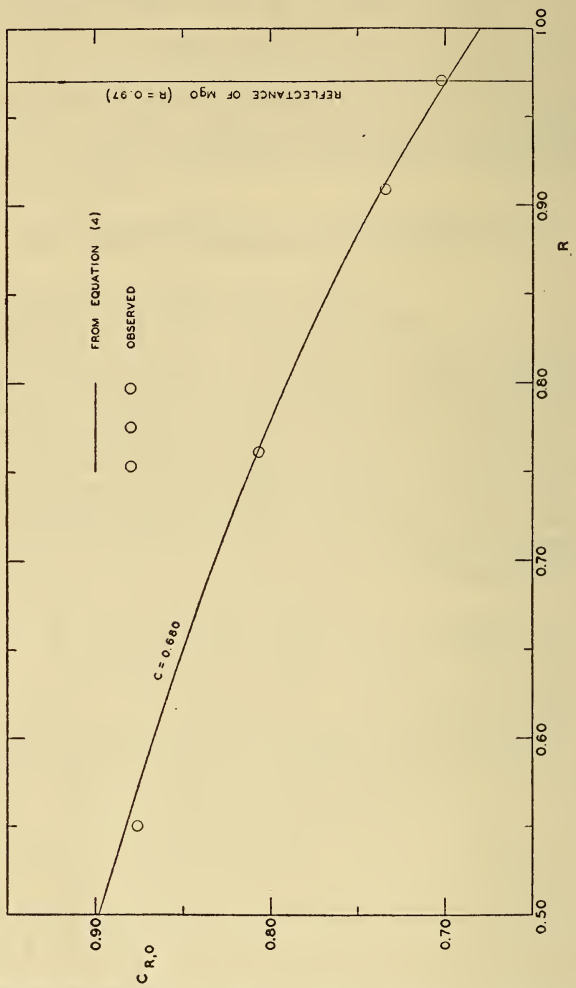

FIGURE 1.—*Measured values of contrast ratio, $C_{R,O}$, for a sample of depolished opal glass closely backed by surfaces of various reflectance, R, plotted against R.*

It is seen that equation (4) agreed well with the observed values. Note that the true value of contrast ratio, $C_{0.970,0}$, for this sample is about 0.70, while $C_{1,0}$ is about 0.68.

method satisfy these conditions fairly well, it will probably be sufficient to show that this equation adequately represents the actual relation for one fairly typical sample. The curve in figure 1 shows $C_{R,0}$ as a function of R according to equation (4) for $C=0.680$; the plotted points refer to a thin piece of opal glass depolished on both sides. The four surfaces used to back this sample were (1) light gray paper,

(2) white paper, (3) a freshly scraped magnesium carbonate block of unknown history and purity,[5] and (4) a freshly prepared surface of magnesium oxide. The reflectance of the MgO surface was taken as 0.97; [6] the reflectances of the other surfaces were measured relative to the MgO surface for diffused illumination and normal viewing. The distances between backing and sample were not zero but it is estimated that they were all less than 0.2 mm. Uncertainty as to this distance and the experimental uncertainty of setting is sufficient to account for the deviation between the plotted points and the curve representing equation (4). Note that the true contrast ratio defined in terms of MgO would be written according to the present notation $C_{0.970,0}$ not $C_{1,0}$; from the data of figure 1 it may be seen that $C_{0.970,0}$ for this sample is about 0.70, while $C_{1,0}$ is about 0.68.

Equation (5) involves the additional assumption that light flux within the cylindrical cavity is uniformly distributed. This is not strictly true for any nonspherical cavity even when the inner walls are perfectly diffusing, and for very flat cylinders such as are here considered as t approaches zero, the error may be considerable. It remains to be seen whether the relation derived from this simplifying assumption can represent experimental data on actual samples to any useful degree.

Figure 2 shows a comparison between equation (5) and experimental results on 4 samples obtained with the original model designed and used by Priest.[2] The magnesium carbonate block mentioned in connection with figure 1 was used as the white backing; the samples of highest and lowest opacity were paper, the samples of intermediate opacity were depolished opal glass. The constants used in equation (5) to represent the performance of this particular instrument are: $r=20$ mm, $R=0.91$, $R_t=0.50$. The radius of the circular sample and the reflectance of the $MgCO_3$ surface are, of course, accurately known, but the reflectance, R_t, of the inner walls of the imaginary cylinder was rather arbitrarily set at 0.5 because such a value gave a satisfactory fit for the data. The actual reflectance, R_t, is not a constant but decreases somewhat with increasing t.

It will be noted that equation (5) may be made to describe the performance of this particular instrument with fair accuracy. There is a tendency, which is probably significant, for the observed points to be high for t nearly zero; such an increase suggests that the assumption of equal distribution of light within the cylindrical cavity is producing appreciable error. There is also a significant tendency for the observed points to be high for large values of t; this may be ascribed to the decrease in the actual value of R_t as t increases which is not taken account of by equation (5).

[5] Although $MgCO_3$ blocks are widely used because of their convenience and cheapness to provide surfaces of constant reflectance nearly equal to unity, there is considerable doubt whether much reliance should be placed on such surfaces. It is known (Priest, Jour. Opt. Soc. Am. and Rev. Sci. Inst., vol. 20, p. 157, 1930; Pfund, Jour. Franklin Inst., vol. 189, p. 378, 1920; Hyde, Astrophys, Jour., vol. 35, p. 239, 1912) that commercial $MgCO_3$ is yellow. Furthermore, some of the commercial product has been found to be much yellower and darker than the usual product, exposure to strong light has been known to darken and yellow a block of $MgCO_3$ to a considerable depth beneath the exposed surface, and nothing is known of the permanence of the reflectance of pure samples kept shielded from light. The block of $MgCO_3$ used here is an example of the possible variation. It was found by comparison with MgO to have a reflectance of 0.91 (see fig. 1); this value is importantly lower than the accepted value, 0.98, given in the International Critical Tables.

[6] International Critical Tables, vol. 5, p. 262. Also Preston, The Reflection Factor of Magnesium Oxide, Trans. Opt. Soc., vol. 31, p. 29, 1929–30.

A similar, though less extensive, investigation of another design of opacimeter has revealed considerably larger variation in measured values of contrast ratio as t is varied. Equation (5), however, yields

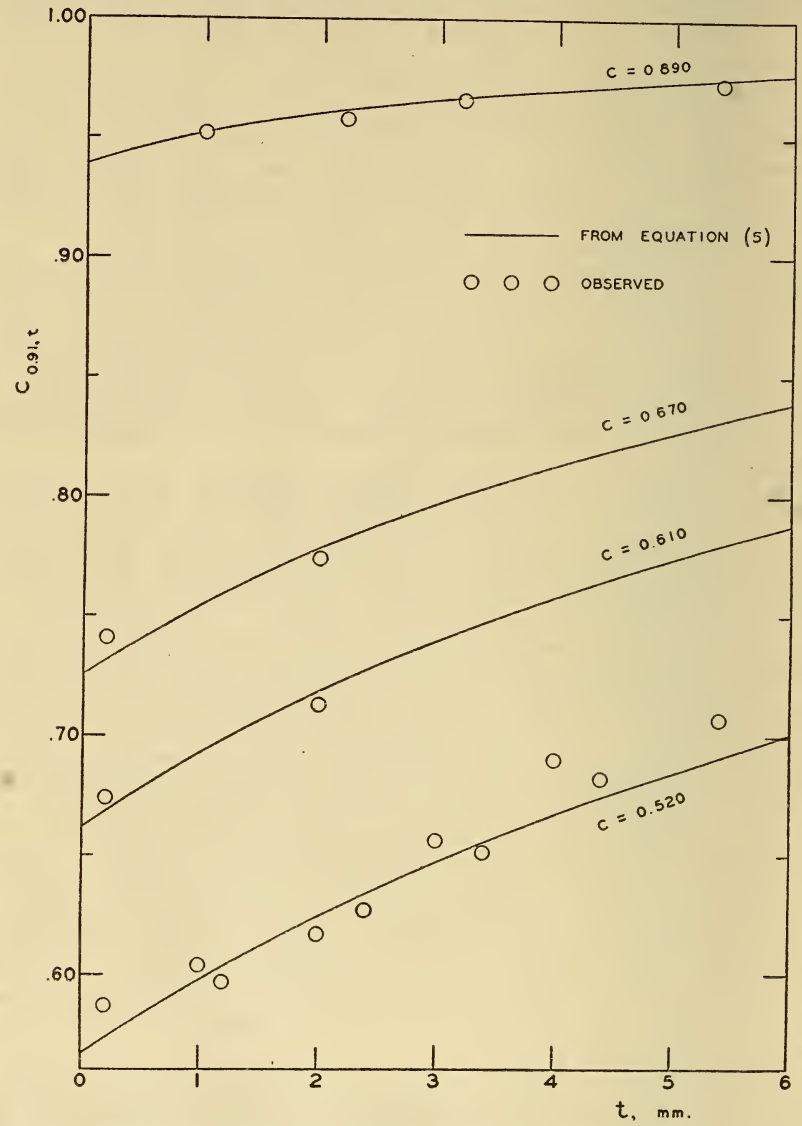

FIGURE 2.—*Measured values of contrast ratio, $C_{0.91,t}$, for four samples as a function of the distance, t, between sample and the working standard* $MgCO_3$ *surface whose reflectance, R, is 0.91.*

It is seen that equation (5) is in fair agreement with the observed values. The values of C (contrast ratio measured with the sample in contact with a perfectly reflecting white surface) substituted in equation (5) for comparison with the observed values are indicated on the graph. Note that the use of a magnesium carbonate surface of low reflectance (0.91) separated from the sample by 2 mm may result in measured values of contrast ratio too high by as much as 0.08.

this result also because of the smaller radius of sample effective in this instrument; and it may be made to represent the performance of this opacimeter with the same degree of accuracy.

IV. CONCLUSIONS

The following conclusions seem justified from the foregoing experimental and theoretical results.

1. Measured values of contrast ratio for paper depend to an important extent on the reflectance of the standard white surface which backs the test specimen. The use of magnesium carbonate as a substitute for magnesium oxide may introduce errors as large as 0.03 or 0.04 in measured values of contrast ratio for paper (see fig. 1, measured value, more than 0.73; true value, 0.70).

2. The derivation of equation (4) connecting measured contrast ratio with reflectance of the standard white surface makes feasible the use of permanent surfaces such as porcelain or opal glass as substitutes for the fragile MgO standard surface, or the impermanent $MgCO_3$ surface often used as a working standard. A measurement of reflectance relative to MgO combined with application of equation (4) would suffice to calibrate the instrument for samples which, like paper, absorb but little light.

3. Measured values of contrast ratio for paper depend to an important extent on the degree of separation between test specimen and white backing. A separation of 2 mm for a sample as large as 40 mm in diameter (see fig. 2) may yield measured values of contrast ratio higher by 0.03 or 0.04 than the values yielded by a separation of 0.2 mm.

4. If the distance between test specimen and white backing be accurately controlled at some definite value (say 2 mm) the readings may be approximately corrected for the error so introduced by application of equation (5),[7] but a more satisfactory solution is to be obtained by using a permanent working standard white backing such as porcelain or opal glass in which case the test specimen may be placed in contact with the white backing in exact conformity to the definition of contrast ratio.

WASHINGTON, October 1933.

[7] If the separation of sample and backing be controlled by interposition of a piece of glass, the effective separation (that is, the value of t to be substituted in equation (5)) is less than the actual separation because of refraction of light at the glass surface near the sample. Any appreciable absorption of light by the glass would also have to be taken into account.