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TESTS ON A REINFORCED-CONCRETE ARCH OF THE ARLINGTON MEMORIAL BRIDGE

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ABSTRACT

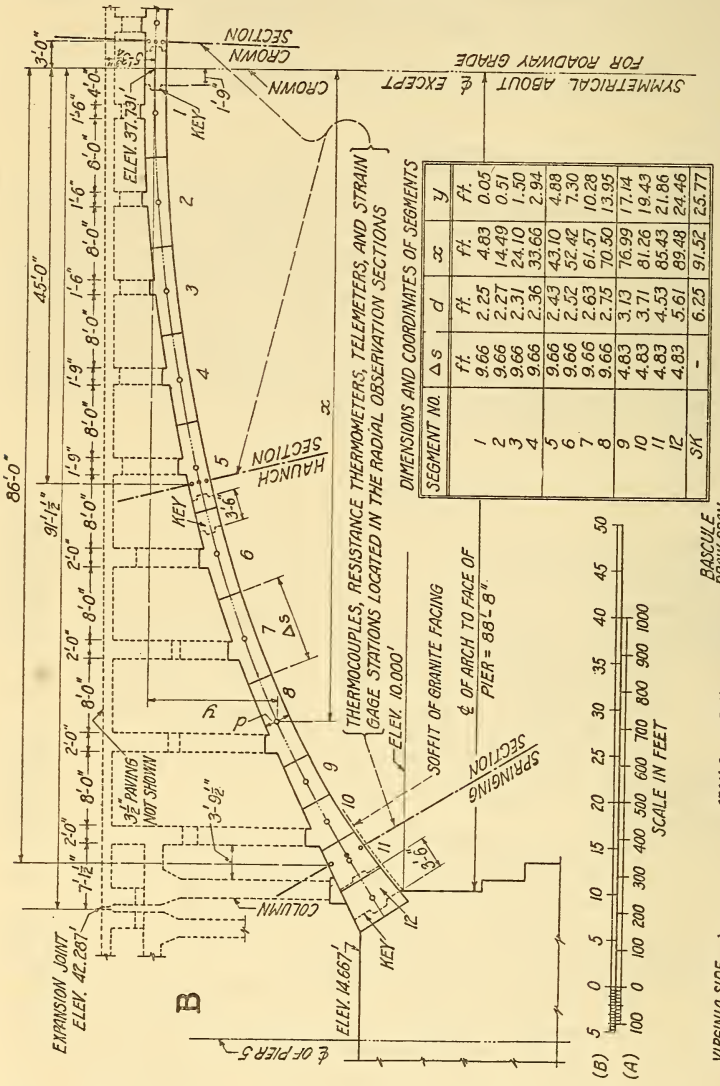
Tests were made on one of the arches of the Arlington Memorial Bridge to determine, during its construction and after its completion, changes in the temperature of the concrete, deflections, and deformations with changes in temperature and in load. The effect of the restraint of the superstructure on the deflection caused by changes of temperature was determined by comparing the vertical deflections of the arch before and after the construction of the superstructure. The effects of removing the centering from a portion of the arch barrel, the addition of the superstructure loads, shrinkage, and time-yield on the vertical deflections of the arch were shown and discussed. A transverse deflection of the arch was produced by differences in physical properties between the concrete barrel and the granite voussoir facing arches. The coefficient of thermal expansion of the arch was determined.

CONTENTS

	Page
I. Introduction	568
II. Description of arch span no. 7	569
III. Testing equipment and its location	573
IV. Testing procedure	575
V. Method of reduction and estimated precision of the observations	576
1. Temperature measurements	576
(*a*) Temperature measurements on radial observation sections	576
(*b*) Calculation of the average temperature and the temperature gradient	576
(*c*) Precision of the temperature measurements	577
2. Clinometer observations	578
3. Strain-gage and telemeter observations	579
VI. Results of the tests	580
1. Temperatures and temperature changes of the arch	580
(*a*) Initial temperature rise of the arch barrel blocks during hardening of the concrete	580
(*b*) Variation of the arch temperature with seasonal temperature changes	580
2. Coefficient of thermal expansion of the arch barrel	583
3. Vertical deflections of the arch barrel	584
(*a*) Deflection resulting from decentering the downstream inner strip	584
(*b*) Deflection produced by the addition of the superstructure walls and deck	584
(*c*) Effect of superstructure restraint on deflection produced by temperature changes	787
(*d*) Deflection occurring with time	589
(*e*) Vertical deflections in transverse sections	592
4. Strains at the principal sections due to thrust and bending	593
5. Pier movements	595
6. Movements of the expansion joints	596
VII. Summary	597
VIII. Acknowledgments	598

I. INTRODUCTION

The Bureau of Standards cooperated with the Arlington Memorial Bridge Commission in an investigation of the structural behavior of one of the reinforced-concrete arch spans of the Arlington Memorial Bridge. Information was obtained on the temperature changes of



SEGMENT NO.	Δs	d	x	y
	ft.	ft.	ft.	ft.
1	9.66	2.25	4.83	0.05
2	9.66	2.27	14.49	0.51
3	9.66	2.31	24.10	1.50
4	9.66	2.36	33.66	2.94
5	9.66	2.43	43.10	4.88
6	9.66	2.52	52.42	7.30
7	9.66	2.63	61.57	10.28
8	9.66	2.75	70.50	13.95
9	4.83	3.13	76.99	17.14
10	4.83	3.71	81.26	19.43
11	4.83	4.53	85.43	21.86
12	4.83	5.61	89.48	24.46
SK	-	6.25	91.52	25.77

DIMENSIONS AND COORDINATES OF SEGMENTS

FIGURE 1.—*A*, *Elevation of the Arlington Memorial Bridge, looking upstream, B, elevation of half span, arch no. 7.*

the concrete, the coefficient of thermal expansion of the arch, the deflections and deformations of the arch, and movements of the piers and expansion joints due to changes in temperature and the addition of the superstructure.

The Arlington Memorial Bridge [1] extends over the Potomac River from the Lincoln Memorial in Washington, D.C., to Columbia Island,

[1] For a further description of the Arlington Memorial Bridge see Nagle, John L., The Arlington Memorial Bridge, Military Engineer (Society of American Military Engineers), vol. 20, p. 154, 1928.

near the Virginia shore. The structure consists of two small approach spans, eight reinforced-concrete arches, and a bascule draw span in the center of the bridge. The concrete arches diminish gradually and symmetrically in rise and span from the bascule bridge toward the shore abutments. All exposed portions of the concrete except the interior of the arch soffits are faced with white granite ashlar. The facing on the up and down-stream edges of the arch barrels consists of heavy granite voussoir blocks dovetailed into the concrete of the barrel. The piers supporting the arches are of mass concrete.

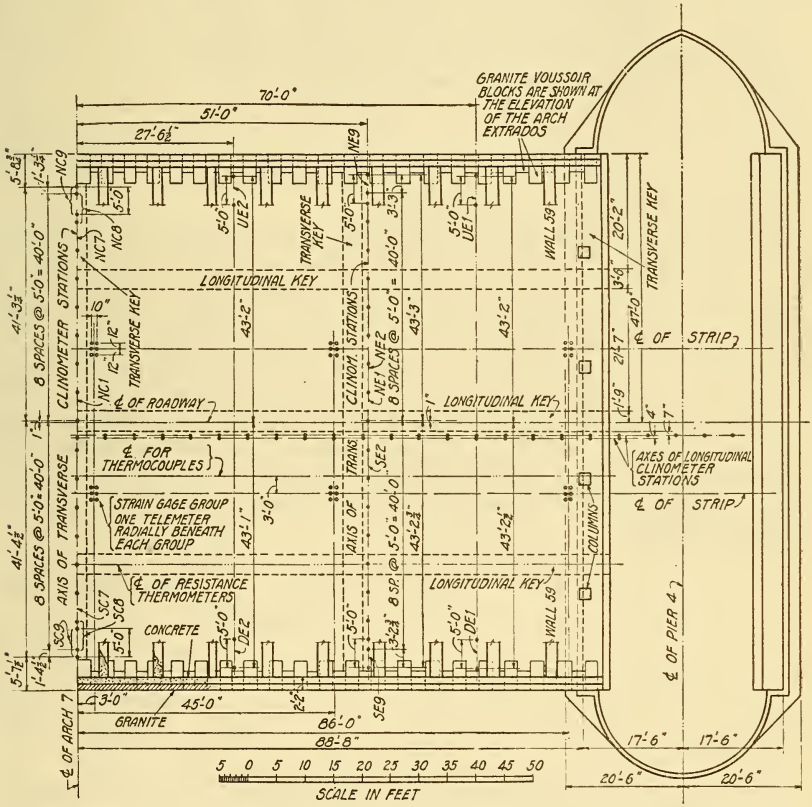


FIGURE 2.—*Plan of half span, arch no. 7, showing location of testing equipment.*
The granite voussoir blocks are cut on radial lines and extend upwards above the extrados of the barrel.

The over-all length of the bridge (including the abutments and the small abutment bypass arches) is 2,163 feet. The 60-foot roadway is flanked by two 15-foot sidewalks and heavy granite balustrades. The distance between the granite faces of the spandrel walls is 94 feet. A small scale elevation of the bridge is shown in figure 1 (A).

II. DESCRIPTION OF ARCH SPAN NO. 7

The design and assembly of testing apparatus and equipment was completed about the time the construction of the arch span designated as no. 7 was started, and this arch was, therefore, selected for

testing. The structural details of this arch are shown in figures 1 and 2.

Pier no. 4 at the east end of arch no. 7 is 41 feet wide and 130 feet long. The average rock elevation at the bottom is −36.6 feet (see figure 1 (A). Pier no. 5 at the west end of the span is 39 feet wide and 129 feet long. The elevation of rock under this pier is −33.3 feet.

The centering support for the arch barrels consisted of steel trusses connected with a hinge beneath the crown of the arch and supported on concrete ledges extending from the sides of the piers beneath the water line. The arch barrels were built up of four longitudinal strips of approximately equal width which are identified as the upstream inner and outer and the downstream inner and outer strips. Each strip was further divided into 4 portions, known as the 2 haunch blocks and the 2 crown blocks. The entire arch barrel was therefore divided into 16 portions separated by transverse and longitudinal keys 42 inches wide. The construction record and a summary of the principal test data are given in table 1.

The concrete in the blocks was mixed in the proportions by volume of 1 part portland cement, 1¾ parts Potomac River sand, and 3½ parts Potomac River gravel of 1-inch nominal size. The 1:2:4 concrete in the keys was made with high early strength cement. Sand was measured in an inundator, and the slump of the concrete was maintained at about 6 inches. Three 6- by 12-inch cylinders, representative of each day's pour, were tested at the age of 28 days. The average strength of these cylinders for the haunch and crown blocks was 3,900 lb./in.^{2} In addition, two groups of three 2- by 2- by 4-foot prisms were cast, one group being tested when 1 month old and the other at the age of 25 months. The averaged results of the tests on these prisms are:

Age	Compressive strength	Secant modulus of elasticity at—	
		200 lb./in.^{2}	2,000 lb./in.^{2}
	lb./in.^{2}	*lb./in.*^{2}	*lb./in.*^{2}
1 month	3,000	3,700,000	2,700,000
25 months	5,400	4,200,000	4,200,000

The longitudinal reinforcement at the intrados and the extrados of the arch consisted of 1-inch square bars 8 inches apart, resulting in slightly less than 1 percent of longitudinal reinforcement at the crown of the arch. The distance from the surface of the concrete to the face of the bars was 2 inches. The transverse reinforcement consisted of ½-inch square bars wired to the inside of the longitudinal bars on 30-inch centers. Shear reinforcement consisted of ¼-inch round bars running normal to the plane of the intrados and bent around the longitudinal bars at alternate intersections of the transverse and longitudinal reinforcement.

TABLE 1.—*Summary of construction data for arches 6, 7, and 8 with principal test data for arch 7*

[The sign of t is (+) when the intrados temperature is higher than the extrados temperature]

Date	Arch no.	Test no.	Relative crown elevation	Average temperature—T				Temperature gradient—t				Deck temperature	Remarks
				Springing	Haunch	Crown	Average	Springing	Haunch	Crown	Average		
			Inch	° *F.*	° *F.*	° *F.*	° *F.*	° *F./ft.*	° *F./ft.*	° *F./ft.*	° *F./ft.*	° *F.*	
July 17, 1928	8												Decentered i.u.s.[1]
Sept. 28, 1928	6												Do.
Oct. 6, 1928	7												Poured haunch blocks i.u.s.
Oct. 8, 1928	7												Poured crown blocks i.u.s.
Oct. 11, 1928	7												Poured cross keys i.u.s.
Oct. 17, 1928	7												Decentered i.u.s.
Oct. 19, 1928	6												Decentered i.d.s.[2]
Oct. 19, 1928	7												Poured haunch blocks i.d.s.
Oct. 20, 1928	7												Poured crown blocks i.d.s.
Oct. 23, 1928	7												Poured cross keys i.d.s.
Nov. 1, 1928	7	1	0	52.2	44.9	44.6	46.7	+1.5	+0.7	0	+0.7		Before decentering i.d.s.
Nov. 2, 1928	7												Decentered i.d.s.
Nov. 5, 1928	7	2	−.525	53.2	51.6	51.3	51.9	−.1	+.2	+.5	+.2		After decentering i.d.s.
Nov. 6, 1928	7	3	−.577	54.0	50.1	50.7	51.2	+.4	+1.6	+1.3	+1.2		Do.
Nov. 10, 1928	8												Decentered i.d.s.
Nov. 21, 1928	7	4	−.913	54.2	50.5	49.9	51.3	+.3	+2.3	+2.1	+1.8		Construction loads on i.d.s.
Nov. 22, 1928	7												Poured haunch blocks o.u.s.[3]
Nov. 23, 1928	6												Decentered o.u.s.
Dec. 3, 1928	7												Poured crown blocks o.u.s.
Dec. 10, 1928	7												Poured cross keys o.u.s.
Dec. 29, 1928	7												Poured center longitudinal key.
Jan. 2, 1929	7												Decentered o.u.s.
Jan. 3, 1929	7												Poured center longitudinal key blocks.
Jan. 18, 1929	8												Decentered o.u.s.
Feb. 5, 1929	7	5	−1.740	25.3	23.4	24.1	24.0	+.1	+1.3	+.4	+.8		No loads on two center strips.
Feb. 16, 1929	7	6	−1.465	32.0	30.9	31.5	31.2	−.1	+.5	+.1	+.3		No change in load.
Mar. 2, 1929	7	7	−1.220	38.6	40.1	40.0	39.7	0	+.2	+.5	+.2		Do.
Mar. 9, 1929	7	8	−1.368	37.4	34.7	34.6	35.3	0	+.7	−.7	+.2		Do.
Mar. 21, 1929	7	10	−.899	50.5	51.5	51.9	51.3	−.4	−.7	−.1	−.5		Do.
Mar. 27, 1929	7												Poured upstream longitudinal key.
Apr. 12, 1929	6												Decentered o.d.s.[4]
Apr. 17, 1929	7												Poured haunch blocks o.d.s.
Apr. 23, 1929	6												Started pouring cross walls.

Footnotes at end of table.

TABLE 1.—*Summary of construction data for arches 6, 7, and 8 with principal test data for arch 7*—Continued

[The sign of t is (+) when the intrados temperature is higher than the extrados temperature]

Date	Arch no.	Test no.	Relative crown elevation	Average temperature—T				Temperature gradient—t				Deck temperature	Remarks
				Springing	Haunch	Crown	Average	Springing	Haunch	Crown	Average		
			Inch	° *F.*	° *F.*	° *F.*	° *F.*	° *F./ft.*	° *F./ft.*	° *F./ft.*	° *F./ft.*	° *F.*	
Apr. 24, 1929	7												Poured crown blocks o.d.s.
Apr. 30, 1929	7												Poured cross keys o.d.s.
May 2, 1929	6												Deck slab concreted.
May 6, 1929	8												Decentered o.d.s.
May 8, 1929	7												Do.
May 11, 1929	7												Poured downstream longitudinal key.
May 22, 1929	7	11	−0.821	60.2	56.6	55.1	57.1	+0.3	+0.6	+1.2	+0.7		Wall forms and reinforcing steel in place.
May 22, 1929	7												Started concreting cross walls.
May 24, 1929	7	12	−.767	61.3	58.4	60.1	59.55	0	+1.4	+.5	+.8		Some of the cross walls concreted.
May 26, 1929	7	13	−.560	64.2	64.8	66.4	65.05	−.1	+.2	−.4	0		Additional cross walls concreted.
May 29, 1929	7												All cross walls concreted.
June 1, 1929	7	14	−.149	72.6	73.8	75.1	73.8	+.2	+.7	+.3	+.5		Resistance thermometers used.
June 3, 1929	7												Deck slab concreted.
June 5, 1929	7	15	−.715	64.9	60.4	61.0	61.7	+.8	+1.0	+.2	+.8		
June 6, 1929	7												Started construction of spandrel walls.
June 6, 1929	8												Started pouring cross walls.
June 20, 1929	8												Deck slab concreted.
June 22, 1929	7	16	−.339	76.1	78.1	77.3	77.4	+.3	−.7	+.3	−.2		Deck and wall forms partially wrecked.
Aug. 15, 1929	7												Spandrel walls completed.
Sept. 4, 1929	7												All debris removed from the arch.
Sept. 6, 1929	7	17	−.426	74.7	75.8	75.7	75.5	−.4	−1.2	−1.2	−1.0		Superstructure loads only.
Nov. 5, 1929	7	18	−.990	56.1	56.4	56.3	56.3	−1.4	−3.4	−4.2	−3.1		No change in load.
Jan. 7, 1930	7	19	−1.445	38.6	38.3	38.3	38.4	0	0	0	0	36.5	Do.
Jan. 25, 1930	7	20	−1.698	30.7	27.1	26.2	27.8	−1.5	−2.0	−2.2	−1.9	21.5	Do.
Feb. 8, 1930	7	21	−1.585	32.2	31.9	32.1	32.0	−.3	−.9	−1.2	−.8	30.7	Do.
Feb. 28, 1930	7	22	−1.260	44.5	46.9	47.8	46.5	−1.0	−4.0	−4.7	−3.4	43.6	Do.
Mar. 25, 1930	7	23	−1.261	46.2	47.5	48.0	47.3	+1.0	+1.6	+2.2	+1.6	54.5	Do.
Apr. 15, 1930	7	24	−.996	54.4	59.0	60.7	58.3	+1.1	−.7	−1.3	−.4	63.9	Do.
Apr. 19, 1930	7	25	−1.082	53.7	55.4	55.6	55.0	+.7	+.7	+1.4	+.9	60.3	Do.
May 5, 1930	7	26	−.733	68.0	69.8	69.4	69.25	−.9	−3.3	−3.0	−2.6	66.0	Do.
June 12, 1930	7	27	−.681	70.3	71.3	71.4	71.1	−1.0	−3.4	−3.7	−2.9	72.6	Do.
July 23, 1930	7	28	−.382	81.7	84.2	85.1	83.8	−.7	−3.8	−4.0	−3.1	82.1	Do.
Aug. 8, 1930	7	29	−.353	84.1	85.5	86.0	85.3	−1.0	−3.4	−3.8	−2.9	85.9	Do.
Oct. 10, 1930	7	30	−.993	65.7	63.8	63.7	64.25	−1.1	−2.0	−1.9	−1.8	63.6	Do.
Jan. 6, 1931	7	31	−1.737	34.4	35.1	35.5	35.0	+.9	+1.6	+1.6	+1.4	37.6	Do.

[1] I.u.s.: Abbreviated from inner upstream strip.
[2] I.d.s.: Abbreviated from inner downstream strip.
[3] O.u.s.: Abbreviated from outer upstream strip.
[4] O.d.s.: Abbreviated from outer downstream strip.

III. TESTING EQUIPMENT [2] AND ITS LOCATION

The temperature-measuring devices consisted of mercury in glass thermometers, thermocouples, and resistance thermometers. Strains were measured with telemeters and hand strain gages, and the deflection of the arch was observed by means of clinometers.

The thermocouples were of the single-junction copper-constantan type, connected to a 16-point potentiometer recorder. The recorder and samples of the thermocouple wire were calibrated at this Bureau.

The copper-coil resistance thermometers, including those within the telemeter cartridges, had a resistance of approximately 500 ohms. The resistance measurements were made with a portable wheatstone bridge which was connected directly to the leads of the thermometers at the extrados of the arch. Changes in resistance of about 0.5 ohm corresponding to changes in temperature as small as 0.5° F. were considered significant.

The clinometer,[3] figure 4, assembled at the navy yard, Washington, D.C., consisted of a tubular steel frame on which was mounted a spirit level and a micrometer-caliper head. The longitudinal spirit level was of such a sensitivity that a rotation of about 0.000015 radian caused the bubble to move one division (2 mm). The size of the bubble could be controlled by the use of an auxiliary reservoir, and the bubble came to rest quickly at all temperatures. The micrometer head could be read to the nearest 0.0001 inch by means of a vernier, corresponding to a rotation of 0.0000017 radian.

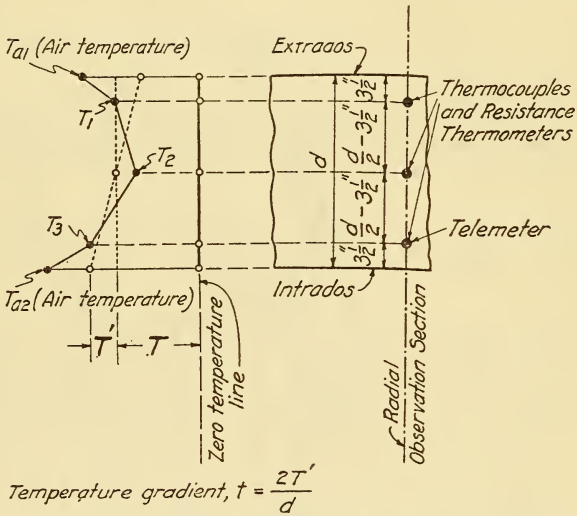


FIGURE 3.—*Detail of a radial observation section.*

The clinometer supports at the ends of each clinometer station consisted of steel and bronze bars set vertically in the concrete. They were alternately drilled and slotted to provide for small changes in length of the instruments or of the clinometer stations.

The 10 telemeters were of the single-resistor cartridge type.[4] During the construction of the superstructure water was ponded at the

[2] For a partial description of the testing equipment and the earlier tests see: Fishburn, C. C., Test measurements on the Arlington Memorial Bridge, at Washington, D.C., First International Congress for Concrete and Reinforced Concrete, Liége, Belgium (La Technique des Travaux—196 Rue Gretry, Liége), 1930.

[3] A clinometer was previously used in an investigation by the Special Committee on Concrete and Reinforced Concrete Arches of the American Society of Civil Engineers. See Bull. No. 174, Engineering Experiment Station, University of Illinois, Urbana, Ill.

[4] For a description of the telemeter see: McCullom, B., and Peters, O. S., A New Electric Telemeter, B.S.Tech. Paper No. 247, 1924, and Report on Arch Dam Investigation, Proc. Am. Soc. of Civil Eng., part 4, May 1928.

crown between the cross walls and evidently penetrated through the leads of the two telemeters at the crown, rendering them useless.

The deformations of the extrados of the arch were measured with 10-inch Whittemore strain gages.[5] The strain-gage stations were

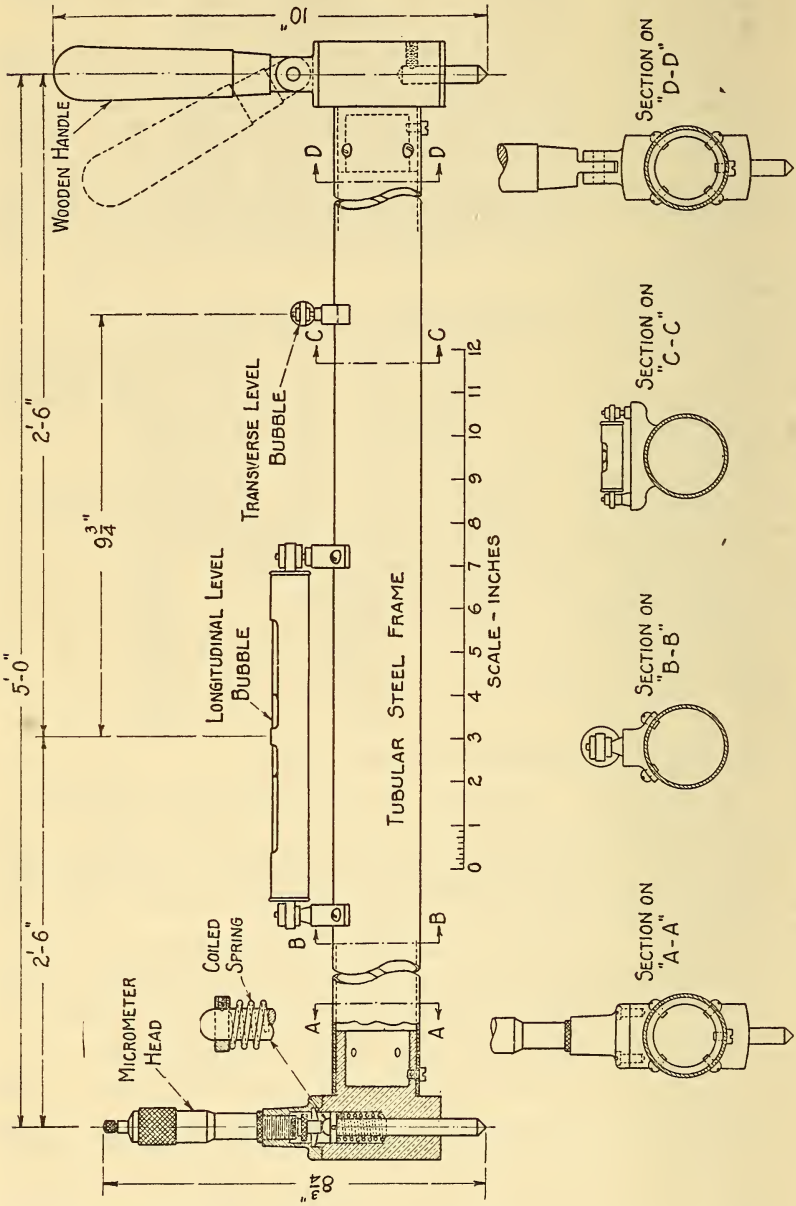


Figure 4.—*The clinometer.*

divided into 10 groups of 3 gage lines each, located as shown in figure 2, in a direction tangential to the curve of the extrados. Each gage

[5] For a description of a similar gage used on the Engineering Foundation Arch Dam Investigation, see Report of Arch Dam Investigation, Proc. Am. Soc. Civil Eng., May 1928.

length was marked by two bronze plugs flush with the surface of the concrete.

The location of the observation stations and of that part of the testing equipment which was fixed in position is shown in figures 1, 2, and 3. The thermocouples, resistance thermometers, telemeters, and the strain-gage stations were embedded in the concrete at five radial observation sections defined as the springing, haunch, and crown sections, according to their location as shown in figure 1. The two haunch sections and the two springing sections were symmetrically located about the crown. There were 9 thermocouples (not shown in the figures) located in the deck slab over the crown of the arch in 3 groups—1 group at the center of the roadway, 1 near the curb, and 1 midway between. The distance between the surfaces of the roadway slab and the upper and lower thermocouples in each group was 1½ inches.

IV. TESTING PROCEDURE

The thermocouples were placed in position before concreting the haunch and crown blocks in the downstream inner strip. Observations were made on these thermocouples as soon as the concrete was placed and were continued for a year after the concrete had cooled to air temperature.

The effect of decentering the downstream inner strip of the arch barrel was determined from 3 tests, 1 made before and 2 made immediately after decentering this strip. All types of instruments were used on these tests.

A temporary cessation of construction after the two center strips were joined by the concreting of the center longitudinal key afforded an opportunity to determine the effects of temperature changes on the unloaded arch. In the interval between February 5 and March 21, 1929, six tests were made during which the average concrete temperature varied between 24° and 51° F. The data from one of these (no. 9) were not used in this report because the test was made during the daylight hours and the effect of sunlight on the arch and clinometers affected the readings. Ninety percent of all observations were made during the late night or early morning hours when the fluctuations in air temperature usually were small.

Six tests to determine the effects of loads were made during the construction of the superstructure. Since the form work and timber bracing for the walls and deck interfered with the making of observations and the rapid addition and shifting of materials made it difficult to estimate loads, the study was confined to a comparison of the condition of the arch immediately before and after the construction of the superstructure, all debris having been removed from the span.

Fifteen tests were made on the completed structure. They included, in addition to the observations relating to the temperatures and deformations of the arch barrel, measurements of the deck temperatures, the widths of the expansion joints, and the difference in elevation between the ends of the transverse clinometer stations.

V. METHOD OF REDUCTION AND ESTIMATED PRECISION OF THE OBSERVATIONS

1. TEMPERATURE MEASUREMENTS

(a) TEMPERATURE MEASUREMENTS ON RADIAL OBSERVATION SECTIONS

The temperature of the arch barrel was determined from measurements made upon the radial observation sections shown in figures 1 and 3. Since, in radial sections located symmetrically about the crown the difference between the temperatures was small, the temperatures at such stations were averaged.

During tests 1 to 13, thermocouples were used to measure the temperatures T_1, T_2, and T_3 (fig. 3). The temperature T_3 was also measured with resistance thermometers located within each telemeter cartridge. Both the temperatures of the concrete extrados (T_{a1}) and the intrados (T_{a2}) were assumed equal to the air temperature measured at the base of the clinometer support. In the determinations of temperature with the thermocouples the average of a number of readings from each thermocouple was used. Because of slight maladjustment of the potentiometer recorder and a temperature gradient at the lead terminals, the readings from the group of thermocouples at each radial section contained constant errors which were not eliminated by averaging the readings. Since temperatures were measured more precisely with resistance thermometers and a portable bridge than with thermocouples and recorder, the temperature at a couple was considered to be that indicated by the couple plus the difference between the average temperature of the two resistance thermometers in the same radial plane and that of couple T_3 in the same group.

After test 13 the 15 resistance thermometers located in the downstream longitudinal key (fig. 2) were used for the determination of the temperatures T_1, T_2, and T_3, and thermocouples were used only for measuring the temperature of the deck slab. Enclosing the air space above the extrados by the completion of the superstructure resulted in differences in the temperature of the air at the intrados and at the extrados so that the temperature T_{a1} was determined as stated in the preceding paragraph and the temperature T_{a2} was assumed equal to the observed air temperature either on shore or above the roadway deck.

(b) CALCULATION OF THE AVERAGE TEMPERATURE AND THE TEMPERATURE GRADIENT

The equivalent linear temperature distribution which would have caused the arch to assume the same shape as that produced by the observed temperatures T_{a1}, T_1, T_2, T_3, and T_{a2}, is shown on the left half of figure 3. This equivalent temperature distribution was expressed for convenience by two components, the average temperature T and the radial increment of temperature T'. The component temperatures were computed from the observed temperatures under the assumption that the temperature gradient was constant between any two adjacent stations in a group T_{a1}, T_1, T_2, T_3, and T_{a2}.

The value of T may be written:

$$T=\frac{7(T_{a1}+T_1+T_3+T_{a2})+(d-7)\ (T_1+2T_2+T_3)}{4d},$$

where d is measured in inches.

The radial increment of temperature T' is equal to

$$T' = \frac{6S}{d^2},$$

in which S is the statical moment (about the gravity axis of the section) of the area between the broken line $T_{a1}T_1T_2T_3T_{a2}$ and the zero temperature line. The effect of differences in the temperature of the longitudinal reinforcement on the value of T' was included. The temperature gradient t given in table 1 is used in the text instead of the value T'. This temperature gradient is equal to:

$$t = \frac{2T'}{d},$$

and is taken as positive when the temperature of the intrados is higher than that of the extrados.

In the deflection calculations the temperatures of the segments (fig. 1) between radial observation sections were computed on the assumption that the increments in temperature of the segments varied as the increments of their radial thickness. If the temperatures and lengths of these segments be represented respectively by T and Δs then the average arch temperature $\frac{\Sigma T \Delta s}{\Sigma \Delta s}$ was found to be closely approximated by:

$$T_{Av} = 0.25T_s + 0.5T_h + 0.25T_c,$$

in which T_s, T_h, and T_c represent respectively the average temperature at the springing, haunch, and crown.

The changes in the values of T and t, denoted by ΔT and Δt may be obtained from the data in table 1.

(c) PRECISION OF THE TEMPERATURE MEASUREMENTS

A study was made of the temperature data from eight tests taken at random to determine the precision of the measurements of the average temperatures and temperature changes. In the study it was assumed that points within the arch barrel, symmetrically located relative to the crown and midplane of the barrel, were at the same temperatures during the tests. The standard deviations[6] of the temperature measurements T_s, T_h, and T_c on radial observation sections were computed from the standard deviations of observations made with the several types of instruments as indicated in table 2, cases I, II, and III. Each of the radial observation sections contained three resistance thermometers at the location T_3 (fig. 3), so that for both the springing and the haunch there were six like observations of temperature. The mean and the deviation from the mean of the temperature measurements made on these groups of six stations during a test were calculated, and the standard deviation of a resistance-thermometer observation for m measurements on n stations was taken as

$$\sigma = \frac{1}{m}\left[\sqrt{\frac{\Sigma V_a^2}{nc_1^2}} + \sqrt{\frac{\Sigma V_b^2}{nc_1^2}} + \cdots \sqrt{\frac{\Sigma V_m^2}{nc_1^2}}\right]$$

[6] See Mellor, J. W., "Higher Mathematics" (Longmans, Green and Company), p. 530 (1909 edition.

in which v is the deviation from the mean of n observations and c_1 is the coefficient given by Shewhart.[7]

TABLE 2.—*Precision of temperature measurements*

Case description	Standard deviation of the average temperature—T				Standard deviation of the changes in average temperature—ΔT			
	Springing	Haunch	Crown	Average [1]	Springing	Haunch	Crown	Average [1]
	°F.	°F.	°F.	°F.	°F.	°F.	°F.	°F.
I Thermocouples only. Early hardening period	0.8	0.8	0.8	0.5	1.1	1.1	1.1	0.7
II. Tests nos. 1 to 13. Thermocouples corrected by reference to telemeter temperature	.4	.4	.4	.2	.6	.6	.6	.3
III. Tests nos. 14 to 31. Resistance thermometers and telemeters	.15	.15	.22	.10	.2	.2	.3	.1

[1] The average for the arch.

The standard deviations of measurements made with thermocouples (table 2, cases I and II) were calculated similarly by comparing the thermocouple temperature with the average temperature of those resistance thermometers in like locations. The deviation was taken as the difference between the thermocouple and the corresponding resistance-thermometer temperature for each of 15 thermocouples during a test. The temperature observations made with resistance thermometers were assumed to be correct, and the standard deviation of the resistance-thermometer observations was neglected except for the measurement at T_3 in case II.

The standard deviation of all observations made with mercury in glass thermometers (stations T_{a1} and T_{a2}) was taken as 0.7° F. It can be seen that for those tests in which the 15 resistance thermometers and the 10 telemeters were used, the standard deviation for the average temperature changes occurring between tests was 0.3° F. for the crown section and 0.2° F. for the springing and haunch sections. The temperature gradients, observed before both the construction of the superstructure and the use of the resistance thermometers in the downstream longitudinal key, were too small compared with their standard deviations to be considered significant.

2. CLINOMETER OBSERVATIONS

The original clinometer data consisted (for each test) of two (or more) readings on each station. The second of these readings was made with the instrument reversed on the supports so that errors due to misalignment of the longitudinal level bubble with the frame were eliminated. A check on the accuracy of the observations and the seating of the instrument on the supports was made by adding the two readings on each station. This sum should be constant at all stations if the observations were correctly made.

The difference in elevation between the two ends of a station during the tests was equal to one half the difference between the readings on the station with clinometer reversed. Corrections were made for the effect of temperature on the length of the supports.

[7] Shewart, W. A., Basis for analysis of test results of die casting alloy investigation, Proc.A.S.T.M. vol. 29, part I, p. 200, 1929.

The crown deflections of the arch were calculated by adding algebraically the changes in the differences of elevations between the ends of the longitudinal stations between the center lines of the piers and the crown. Since the summation, if made from both piers, usually resulted in a small apparent change in the elevation of one pier relative to the other, the observed crown deflection was corrected by an amount equal to one half the change in relative pier elevation.

Changes in curvature of the arch barrel occurring between tests were determined by applying to the clinometer observations a correction for the slope of the arch. The change in curvature, λ, at a clinometer support is given by:

$$\lambda = (\beta_1 - \beta_2)\frac{1}{m},$$

in which β_1 and β_2 are the changes in the slope of two adjacent stations and m is the average chord length of the arch axis at these stations. For a horizontal station (crown) m was 60 inches. The strain at the extrados due to change in curvature is equal to $\frac{\lambda d}{2}$, in which d is the radial depth of the arch.

The precision of the clinometer observations was calculated from the data obtained during a special test. A total of 120 observations was made with 2 clinometers on 3 longitudinal clinometer stations. The standard deviation of a measurement representing the actual difference in elevation between the two ends of a station was found to be 0.00018 inch, and the resulting standard deviation for a measurement of a change in the difference in elevation between the two ends of a station was 0.00025 inch.

Using these values, the standard deviation of the measurement of a crown deflection would be 0.0012 inch, and the standard deviation of the change in the relative elevation of the piers would be 0.0016 inch. This special test was free from errors due to the rotation of the supports and changes in length of the instrument, as well as possible errors in the temperature correction for the length of the clinometer supports. It is reasonable, however, to suppose that indicated changes in relative pier elevation greater than 0.01 inch are due to the settlement of one pier relative to the other rather than to errors in observation.

3. STRAIN-GAGE AND TELEMETER OBSERVATIONS

The original strain-gage data consisted of two or more readings at each of the gage lines. Additional readings were made whenever the difference between the first two observations on a gage line exceeded 0.00001 inch per inch. Two readings were made on each telemeter during a test, one at the start and one near the finish of the test. The average value of the observations was used in the determination of strain and temperature.

A study was made of the standard deviation of the strain gage and the telemeter readings. For this purpose the observed deformations occurring consecutively between tests 17 to 31 were used. Assuming that the deformations at stations located symmetrically about the crown were equal, it was found that the standard deviation of observations made from 12 gage lines was roughly 3 times the standard

deviation of similar observations made from 4 telemeters. The standard deviations calculated from the telemeter observations did not include the effects of possible changes in calibration of these instruments.

The standard deviations for strain-gage and telemeter observations are as follows:

	Strain × 10^6
For single strain-gage station	35
For average of 12 strain-gage stations	10
For single telemeter station	7
For average of 4 telemeter stations	3

VI. RESULTS OF THE TESTS

1. TEMPERATURES AND TEMPERATURE CHANGES OF THE ARCH

(a) INITIAL TEMPERATURE RISE OF THE ARCH BARREL BLOCKS DURING HARDENING OF THE CONCRETE

The temperature of the concrete at the west springing in the downstream inner strip after placing and until air temperature had been attained is shown in figure 5. The initial temperature of the concrete when pouring was begun (about 8 a.m.) was approximately 75° F. A maximum temperature of 143° F. was observed about 25 hours after placing. The evolution of heat accompanying the hardening of the concrete continued for some time after the maximum temperature was reached, and it was nearly 15 days before the slowly dropping temperature of the concrete approximated the temperature of the air. Unless the construction of transverse keys in arches of this type be delayed more than a few days, there will be little advantage in pouring the keys separately from the main body of the arch.

The thermal insulating effect of the bottom forms and the wetting down of the concrete extrados caused a positive temperature gradient between the intrados and the extrados that lasted until thermal equilibrium with the air was established. The existence of a positive temperature gradient near the piers was of advantage in compensating for the strains caused by cooling of the arch concrete after the keys were placed.

The maximum observed temperature attained in the thinner portions of the arch barrel blocks varied between 90° and 100° F. As in the thicker portions of the arch, the maximum temperature was attained within 25 hours after placing the concrete. Positive temperature gradients were found to exist for about 8 days after pouring when thermal equilibrium with the air was reached.

(b) VARIATION OF THE ARCH TEMPERATURE WITH SEASONAL TEMPERATURE CHANGES

The maximum, minimum, and average air temperatures for 5-day periods between October 1928 and February 1931 are shown in figure 6. The temperatures of the arch barrel during the tests are also shown by numbered points and may be seen to approximate closely the mean air temperature for corresponding 5-day periods.

The minimum observed average temperature of the arch barrel, 24° F., was measured on February 5, 1929. After the completion of the superstructure the minimum observed average temperature was 28° F. on January 25, 1930. At this time the temperature of the

enclosed air beneath the deck was 27° F. and that of the outside air was 19° F.

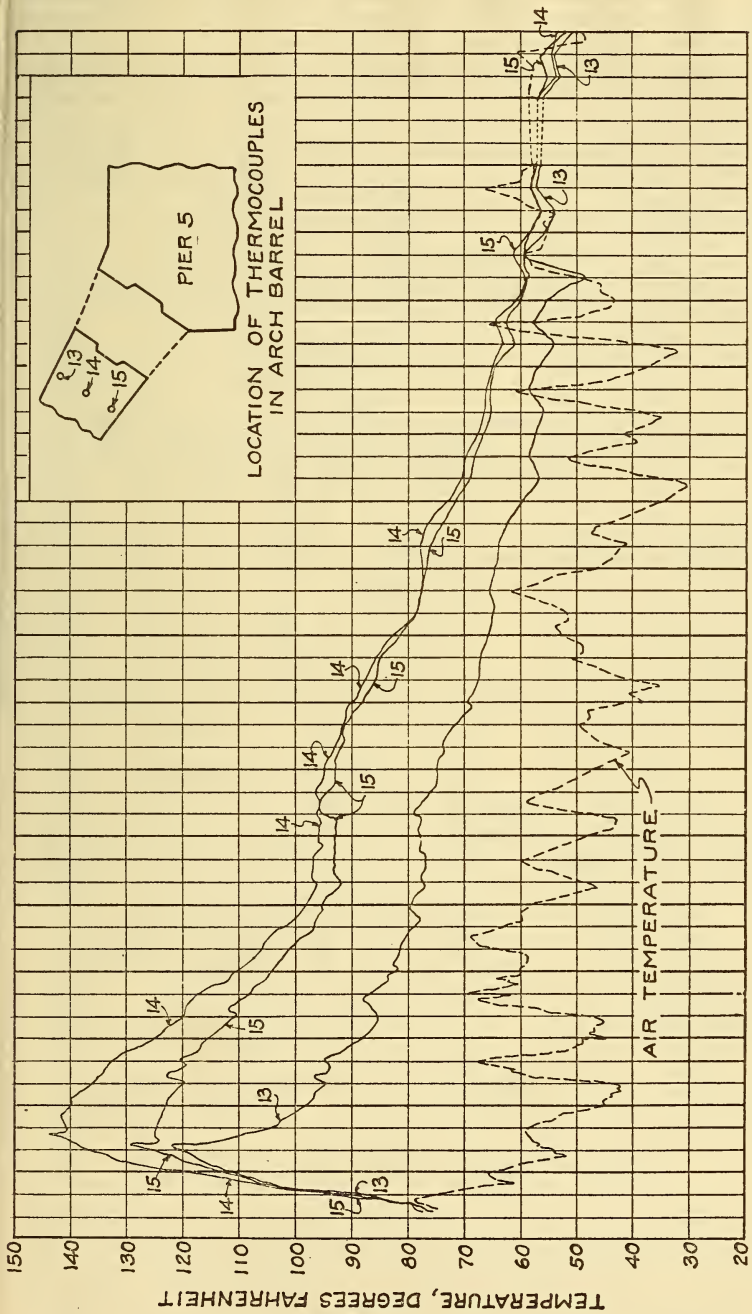


FIGURE 5.—*Initial temperature rise in the haunch block near pier 5.*

The thermal insulation of the bottom forms is responsible for the relatively high temperature indicated by thermocouple no. 15, as compared with that indicated by thermocouple no. 13.

The maximum observed temperature of the arch barrel was 85° F. at 6 a.m. on August 8, 1930 (test 29). The temperature of the

enclosed air beneath the deck was 89° F. and that of the outside air was 75° F. It is probable that the average temperature of the arch would have been greater had the observations been made in the afternoon.

After the completion of the superstructure the temperature gradients at the springing sections were usually smaller than those existing

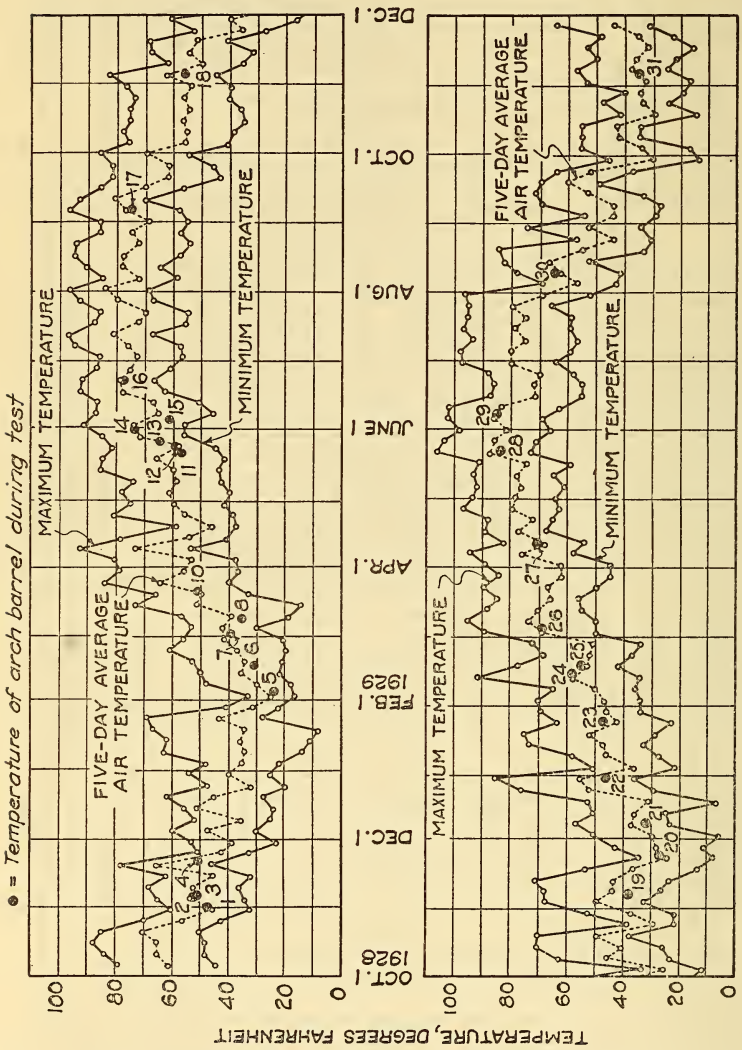


FIGURE 6.—*Temperature record—Fluctuation in maximum and minimum air temperature and the average air temperature for 5-day intervals.*

The average temperature of the arch-barrel concrete during the tests closely approximated the average air temperature for the previous 5 days.

at other portions of the arch, but the difference in temperature between the extrados and the intrados was generally constant throughout the span. The maximum positive temperature gradient was +2.2° F./ft. (at the crown on March 25, 1930) and the maximum negative temperature gradient was −4.7° F./ft. (at the crown on February 28, 1930). The temperature gradients at the springing rarely exceeded 1° F./ft.

During test 29 the difference between the temperature of the enclosed air space and that of the outside air caused a temperature gradient of $-3°$ F./ft. The tests made upon the completed structure during hot weather invariably showed temperature gradients of negative sign, that is, extrados temperatures higher than the intrados temperatures. During the colder months the sign of these temperature gradients varied. The observations indicate that for arches of this type the temperature gradients are negative at both extremely low and high values of average arch temperature.

2. COEFFICIENT OF THERMAL EXPANSION OF THE ARCH BARREL

The coefficient of thermal expansion, α, of the arch barrel, without superstructure loads was determined from the data obtained in tests 5, 6, 7, 8, and 10. The tests were made during the early morning hours and the effect of direct sunlight on the temperature of the arch extrados was so small that changes in temperature gradients were neglected.

In calculating α, temperature changes were considered to be symmetrical about the crown of the arch. The values of α for the two inner strips of the arch barrel were as follows:

Test no.	Change in average arch temperature	Crown deflection	Coefficient of linear thermal expansion, α
	°F.	*Inch*	*Millionths per °F.*
5-6	+7.8	0.275	7.5
6-7	+8.4	0.245	6.3
7-8	−4.4	[a] −0.148	7.0
8-10	+16.0	0.469	6.3
Average (weighted)			6.7

[a] Downward deflection.

The standard deviation of α calculated from weighted values of each test, depending upon the amount of the change in average temperature, was about 1.5×10^{-6}. It is possible that the difference between separate determinations of α may have been partially caused by changes in the moisture content of the concrete.

A study of volume changes was made on seven prisms of concrete which were cut from the extrados of the arch after the completion of test 31. These prisms were about 5 by 5 inches in cross section and 20 inches long. They had been cast in paraffined metal U-shaped forms open at both ends, the forms having been placed in position before the concreting of the arch blocks. Each prism contained a single strain-gage line, which had been used previously as the center gage line of a strain-gage group. The length of the gage lines in the prisms was measured while the prisms were being subjected to different storage conditions. The computed values of α are shown in table 3.

The value of α computed from the independent observations made on the air-dry prisms and that value obtained from the arch tests checked very closely. For the saturated prisms the coefficient was found to be slightly less. In all calculations applying to the arch, the coefficient of thermal expansion of the arch barrel was taken as 65×10^{-7} per ° F.

TABLE 3.—*Prism tests*

Test no.	Storage conditions	Temperature			Change in length per unit of length		α Coefficient of linear thermal expansion
		Duration	Average	Change between consecutive tests	Change between consecutive tests	Cumulative change	
		Days	° *F.*	° *F.*	*Millionths*	*Millionths*	*Millionths per °F.*
1	Air dry, uncontrolled humidity	0	69.0			0	
				+59.4	+370		
2	do	24	128.4			+370	6.2
				−59.4	−410		
3	do	50	69.0			−40	6.9
				+0.2	+380		
4	Immersed in water	82	69.2			+340	
				+43.5	[1] +345		
5	do	52	112.7			+685	[1] 7.9
				−44.0	−260		
6	do	46	68.7			+425	5.8
				+40.5	+250		
7	do	44	109.2			+675	6.2
				−40.9	−235		
8	do	30	68.3			+440	5.8
				+2.0	−320		
9	Air dry, uncontrolled humidity	388	70.3			+120	

[1] This comparatively large expansion was measured the first time the prisms were immersed in hot water, and was not used in the determination of the average of α.

3. VERTICAL DEFLECTIONS OF THE ARCH BARREL

(a) DEFLECTION RESULTING FROM DECENTERING THE DOWNSTREAM INNER STRIP

A vertical settlement of pier no. 4 relative to pier no. 5 was found to have occurred after decentering the arch strip. Since the clinometer data do not indicate an appreciable rotation of either pier, the horizontal displacement of the top of pier no. 4 was neglected. The possible effects of the differences in temperature gradients were not determined because the accuracy of these temperature measurements was relatively poor.

The theoretical deflection of the arch due to changes in average temperature and to the settlement of pier no. 4 as well as the deflection due to decentering are shown in figure 7. The deflection (effect of decentering) is equal to the difference between the observed deflection and the theoretical deflection.

(b) DEFLECTION PRODUCED BY THE ADDITION OF THE SUPERSTRUCTURE WALLS AND DECK

Tests 10 and 17 were compared to determine the deflection produced by the addition of the superstructure walls and deck. Test 10 was made before work on the superstructure was started and test 17 was made after this work was completed. The results of this study are illustrated in figure 8, which shows the observed deflection of the arch (curve *A*), the deflection produced by movements of Pier No. 4 together with that theoretically due to temperature changes (curve *B*), the load deflection (curve *C*), and, on the assumption of a modulus of elasticity of 4,000,000 lb./in.^{2}, the theoretical load deflection produced by the superstructure loads (curve *D*). Throughout the paper the effects of superstructure restraint on theoretical deflections have been neglected unless otherwise stated.

The longitudinal clinometer data indicated that the top of Pier No. 4 had rotated through an angle of about 30 millionths of a radian

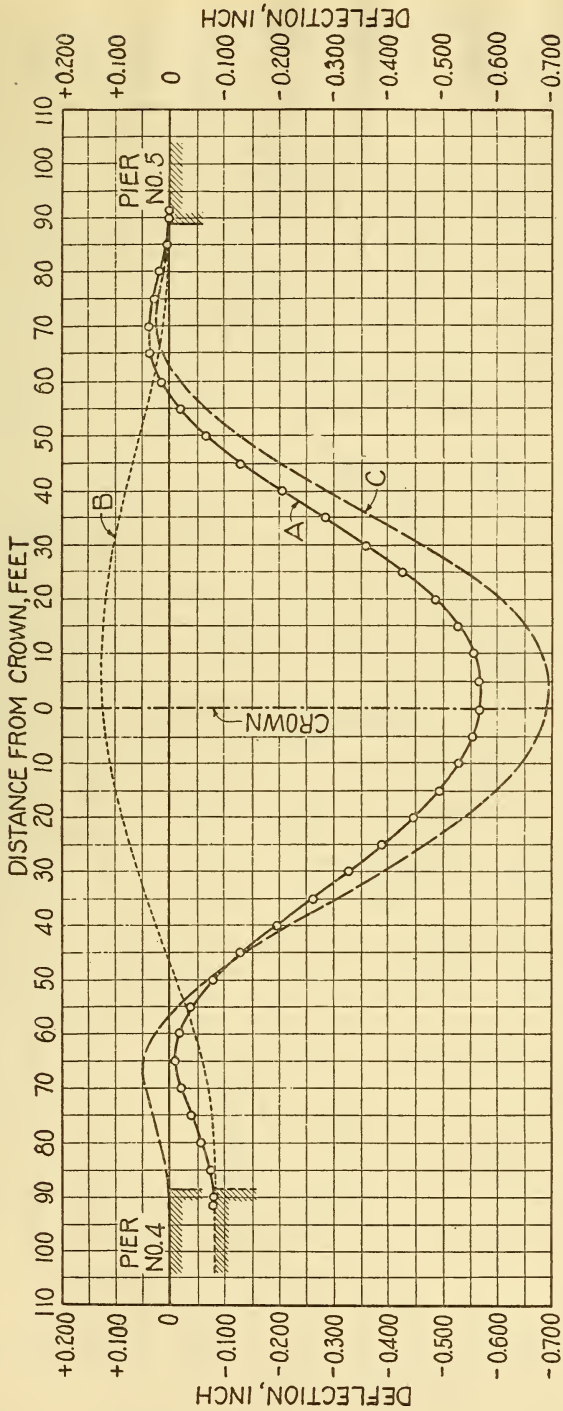


FIGURE 7.—*Deflection produced by decentering the downstream inner strip of the arch barrel.*

A, the observed deflection occurring between tests 1 and 2; *B*, the theoretical deflection produced by temperature changes and the vertical settlement of pier no. 4; *C*, the deflection caused by decentering.

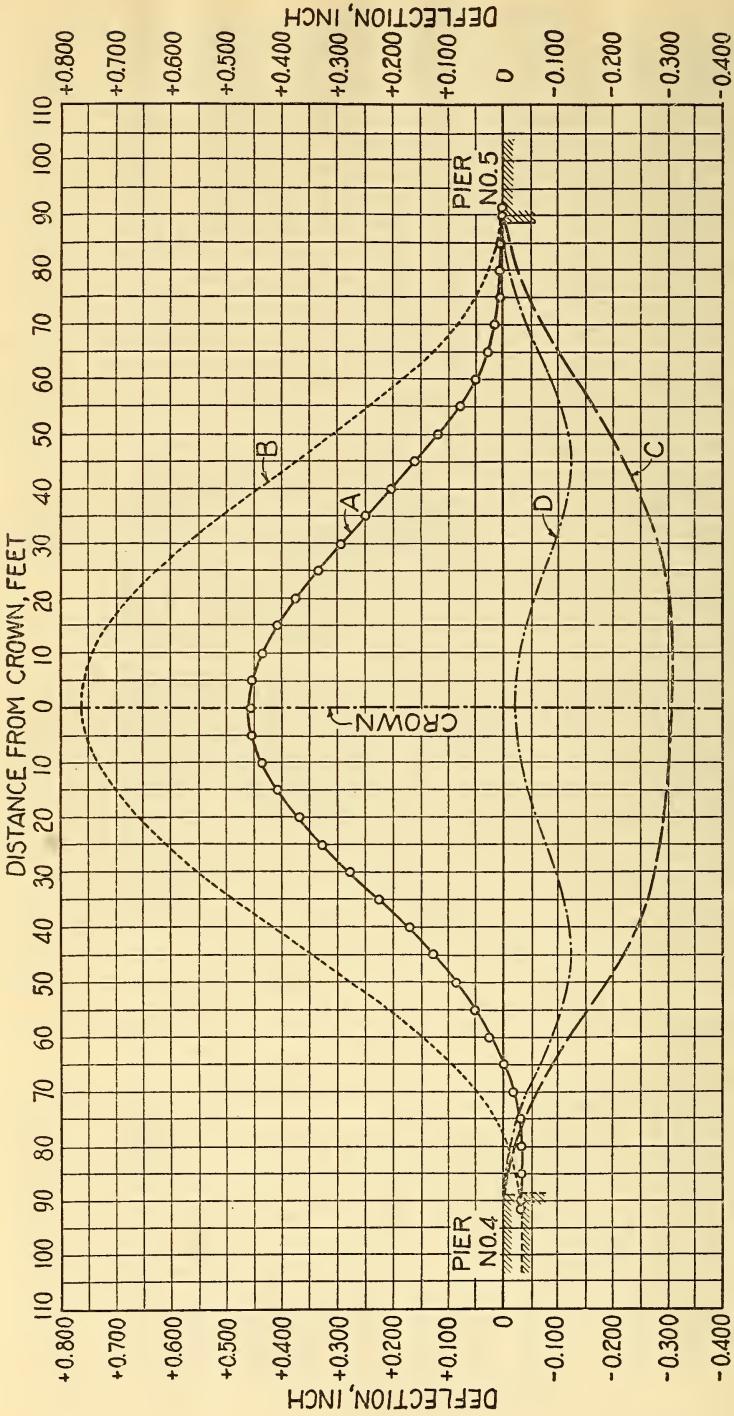


FIGURE 8.—*Deflection produced by the addition of the superstructure walls and deck.*

A, the observed deflection occurring between tests 10 and 17.
B, the theoretical deflection produced by temperature changes and pier movements.
C, the load deflection. (The difference between curves *A* and *B*.)
D, the theoretical load deflection for E_c=4,000,000 lb./in.^{2}

toward the crown of the arch and had settled 0.022 inch relative to Pier No. 5. The horizontal displacement of the pier was not measured. From the data it was noted that most of the rotation and settlement of Pier No. 4 occurred during the time between tests 10 and 11 when the superstructure of arch no. 6, to the east of Pier No. 4, was built. The superstructure load on arch no. 7 during test 11 included the major portions of the deck and cross-wall forms. Since the observed rotation of the top of Pier No. 4 was much greater than could be caused by deformation due to the applied forces from arch no. 6, it is probable that deformations in the foundation were responsible for most of the observed settlement and rotation of the pier. The horizontal movement of the top of Pier No. 4 was, therefore, assumed to be 0.02 inch (30 millionths of a radian times pier height) toward the crown, and the total settlement near the west side of the pier, at the end of the arch, was about 0.03 inch, below Pier No. 5.

The observed temperature changes occurring between tests 10 and 17 are:

Change in	Springing	Haunch	Crown	Average
Average temperature ΔT, °F	+24.2	+24.3	+23.8	+24.2
Temperature gradient Δt, °F./ft	0	−.5	−1.1	−.5

The superstructure loads per foot of width of arch barrel varied from 2,500 lb. at the crown to 10,500 lb. at the springing line.

The load deflection (*C*) shown in figure 8 is the difference between the measured deflection (*A*) and the calculated deflection (*B*) caused by temperature changes and pier movements. This load deflection (*C*) was a maximum at the crown and the shape of the deflection curve was similar, except that it was very much flatter near the crown, to one caused by a uniform temperature change. The theoretical load deflection (*D*) is a maximum 45 feet on either side of the crown, and the theoretical crown deflection was only about 20 percent of this maximum. The difference between the load deflections (*D*) and (*C*) may be due to shrinkage or flow which occurred during the 170 days between tests 10 and 17. This difference was sufficient to cause a reversal in the sign of the moments near the crown.

The study of the load deflection was originally made for the purpose of determining the modulus of elasticity of the arch barrel. It is evident that the long period of time between tests 10 and 17, and the large deflections produced by temperature changes and pier movements so affected the measured deflection that the load deflection, as determined from these tests, is an approximation which includes the indefinite effects of flow and shrinkage of the concrete.

The load deflection determined from the observed deflections on the assumption that there was no horizontal displacement of Pier No. 4, gave a crown deflection that was 13 percent less than shown in curve (*C*), but the shape of the curve was substantially the same.

(c) EFFECT OF SUPERSTRUCTURE RESTRAINT ON DEFLECTION PRODUCED BY TEMPERATURE CHANGES

The vertical deflections produced by a change in average arch temperature (p. 577) were affected by the restraint of the superstructure wall and deck as shown in figure 9. The deflection produced by a 1° change in average temperature before the superstructure was built (curve *A*) was plotted from data obtained on tests 5, 6, 7, 8,

and 10 and closely approximates the theoretical temperature deflection for the coefficient of expansion 65×10^{-7} per °F. The curve (*B*) was plotted from data obtained in tests 17 to 31, which were made after the superstructure was completed, except that the comparison between test 22 and test 23 was not included because the changes in average temperature occurring between these tests were much less relative to the changes in temperature gradients than between other tests. The effects of changes in temperature gradients were neglected because theoretically the deflection produced by these changes is much less than that caused by changes in average tempera-

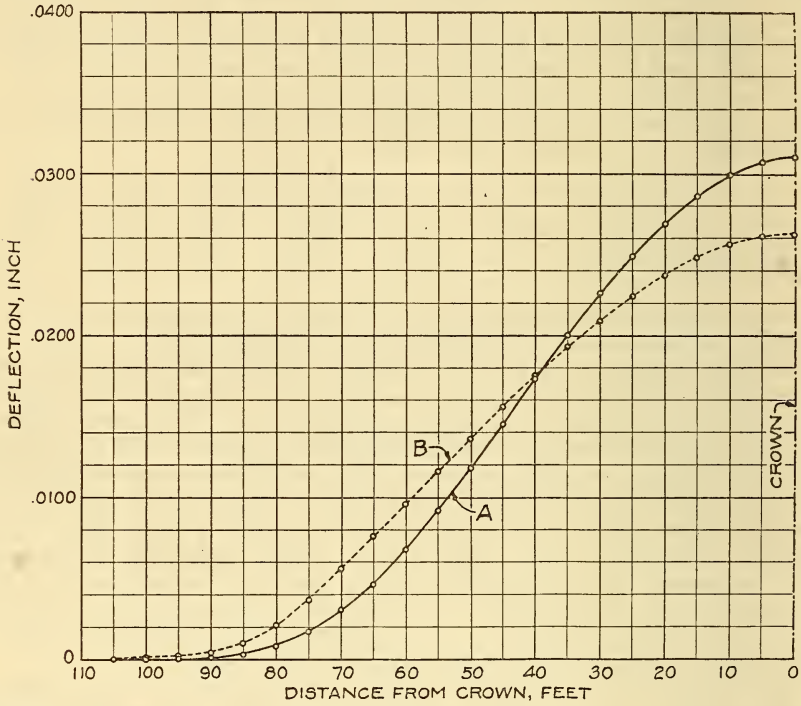


FIGURE 9.—*Observed deflection produced by a change in average arch temperature.*
A, before the construction of the superstructure; *B*, after the construction of the superstructure.

ture. From figure 9 it is evident that the superstructure restraint reduced the crown deflection and the curvature near the crown of the arch and increased the deflections near the piers.

The effect of changes in temperature gradients, Δt, and in average temperature, ΔT, on the crown deflection after the construction of the superstructure was determined also. The method of least squares was applied to data representing the crown deflection and temperature changes occurring between 15 consecutive tests (17 to 31). Since the crown deflections were small compared to the dimensions of the arch, they could be expressed linearly, thus:

Crown deflection (inch) $= +0.027\,\Delta T - 0.015\,\Delta t_s + 0.033\,\Delta t_h - 0.020\,\Delta t_c$

in which ΔT is the change in average arch temperature (°F.), and Δt_s, Δt_h, and Δt_c represent respectively the changes in the temperature gradients (°F./ft.) at the springing, haunch, and crown

sections. The standard deviations of the coefficients of ΔT, Δt_s, Δt_h, and Δt_c are respectively 4, 190, 145, and 155 percent of the coefficients.

For a change in the temperature gradient throughout the arch such that the extrados temperature dropped 1° lower than the intrados temperature (positive change), the crown deflection computed from the equation is +0.001 inch. Theoretically (assuming no superstructure restraint) the crown deflection produced by such a change in gradient is −0.002 inch. The deflections caused by changes in the temperature gradients were so small in comparison with those caused by change in average temperature that the data do not furnish a reliable measure of the effects of temperature gradients. The value of the coefficient +0.027 (inch per °F.) which gives the crown deflection caused by a change of 1° F. in average temperature of the arch, compares favorably with the value of 0.0262 shown on curve (*B*) in figure 9.

(d) DEFLECTION OCCURRING WITH TIME

The deflections due to shrinkage and flow are computed for periods of approximately 1 and 2 years.

The deflection for the 2-year period, shown in figure 10, was obtained by comparing test 2 (when the age of the inner strips was about 18 days) with test 30. In this figure the observed deflection is shown in curve *A*; the theoretical deflection produced by temperature changes, superstructure loads, and movements of the piers is shown in curve *B*; and the time-deflection (difference between *A* and *B*) is shown in curve *C*. The change in average temperature was about +12° F. and the vertical settlement of Pier No. 4 relative to Pier No. 5 was 0.07 inch. The theoretical load deflection (included in the ordinate of curve *B*) is shown in figure 8 (D). The deflections due to the slight horizontal and angular pier displacements and to changes in temperature gradients were negligible compared to the other relatively large deflections. The time-deflection (curve *C*) closely approximates at all points the deflection computed for a temperature drop of about 27° F.

The deflection for the 1-year period between test 19 and 31 (starting when the age of the two inner strips of the arch barrel was about 15 months) was determined from the data obtained in those tests and is shown in figure 11 (*A*). The theoretical deflection produced by the temperature changes is shown by curve *B*, and the deflection shown by curve *C* (the difference between *A* and *B*) represents the cumulative effect of all other changes during the period. The temperature changes which occurred at the radial observation sections and in the deck slab are.

	Temperature change at—				
	Springing	Haunch	Crown	Average	Deck
Average temperature ΔT, °F	−4.2	−3.2	−2.8	−3.4	+0.9
Temperature gradient Δt, °F./ft	+.9	+1.6	+1.6	+1.4	----------

There was no change in dead load on the arch, and the pier displacements were negligible.

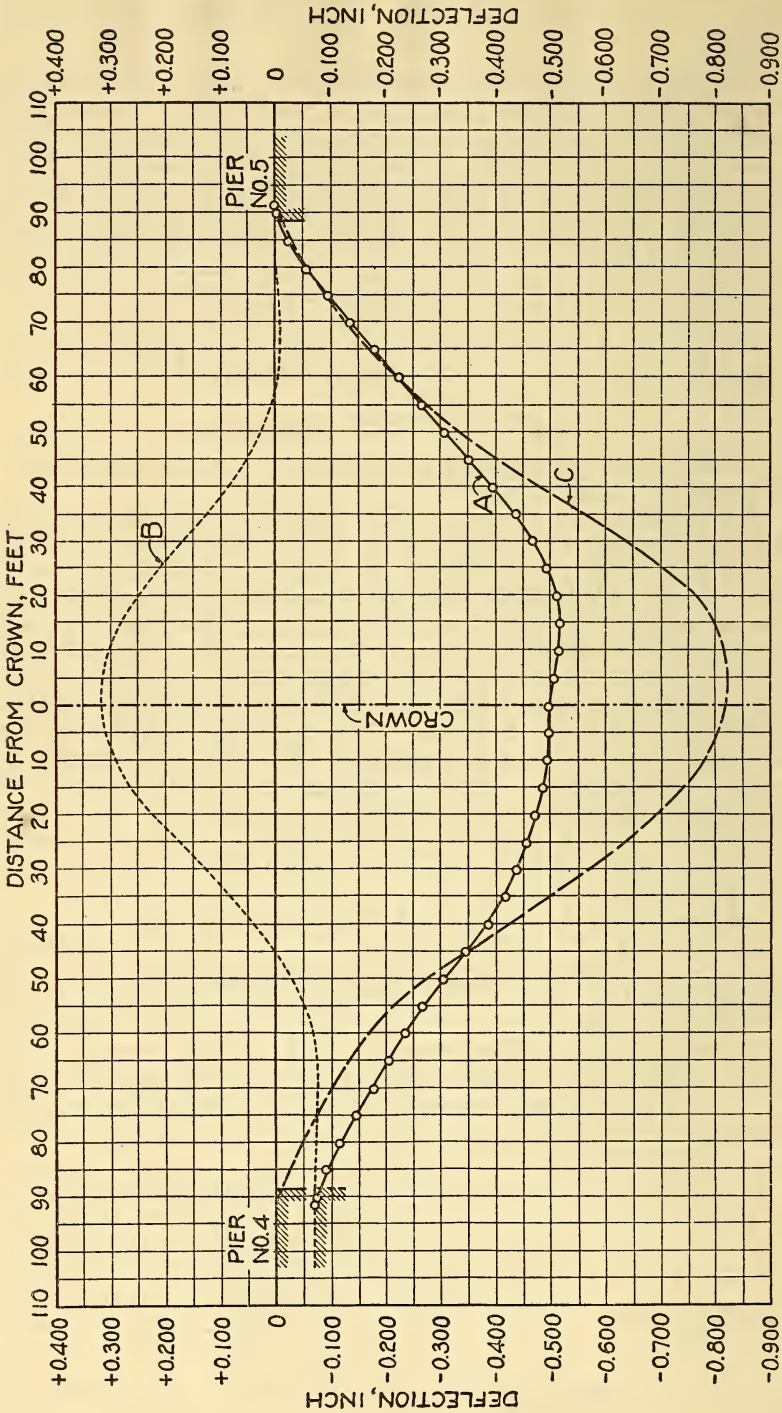


FIGURE 10.—*Deflection 2 years after decentering the downstream inner strip of the arch barrel.*

A, the observed deflection occurring between tests 2 and 30; *B*, the theoretical deflection produced by temperature changes, pier displacement, and the addition of the superstructure walls and deck; *C*, the deflection occurring with time.

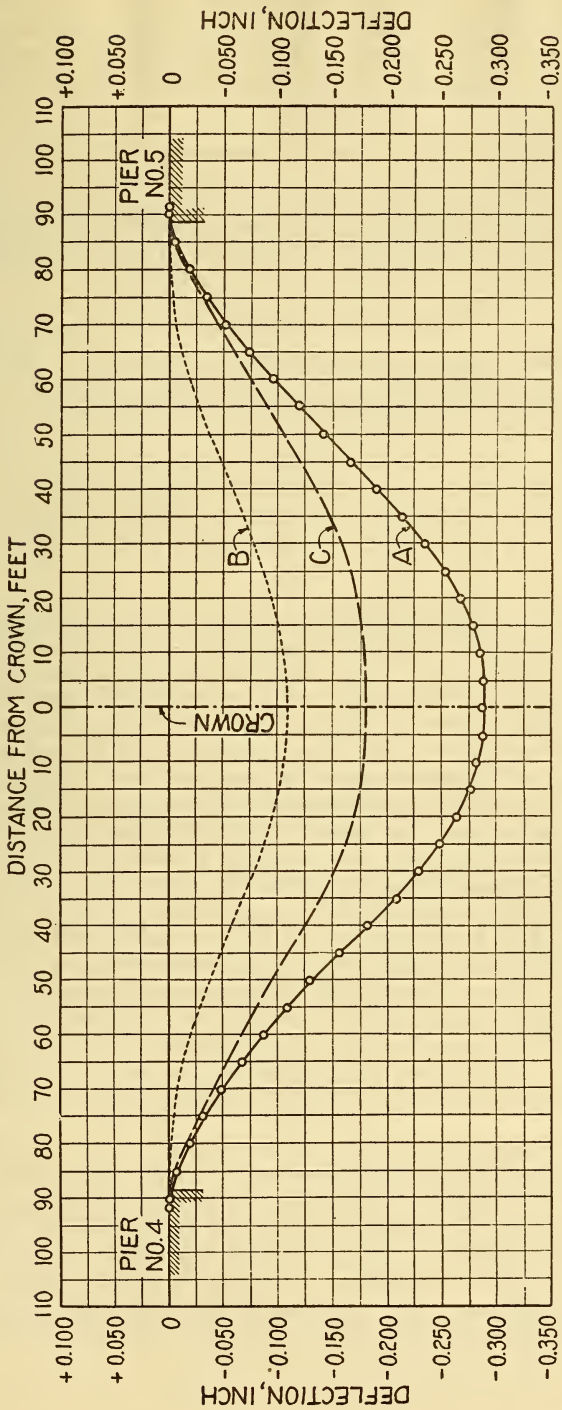


FIGURE 11.—*The deflection between January 7, 1930, and January 6, 1931.*

A, the observed deflection between tests 19 and 31; *B*, the theoretical deflection produced by temperature changes (no superstructure restraint); *C*, the deflection caused by flow and shrinkage of the concrete.

The time-deflection, figure 11 (*C*), is equivalent to that produced by a temperature drop of about 6° F. If the effects of superstructure restraint on the crown deflection were included in the deflection calculations, the equivalent temperature drop (based on crown deflection only) would be about 7° F.

(e) VERTICAL DEFLECTIONS IN TRANSVERSE SECTIONS

Since the reinforced concrete in the major portion of the arch barrel has a higher coefficient of thermal expansion than the granite voussoir facing arches, transverse deflections of the arch barrel were produced by changes in temperature. It is probable that temperature rises in the arches resulted in a transfer of dead load through the cross walls from the outer to inner portions of the barrel, and that temperature drops tended to reverse this load distribution. The actual proportion of the dead load carried by the four longitudinal concrete strips and the two granite facing arches depended upon the temperatures at which the inner and outer strips were constructed and decentered as well as the structural condition of these strips when they were bonded together by the longitudinal keys. The effect of shrinkage and flow in the concrete was probably responsible for a gradual increase of the amount of dead load carried by the facing arches.

Figure 12 shows the relative elevation of a transverse section through the crown during tests 20, 29, and 31 and illustrates the deflection of the section due to temperature, flow, and shrinkage. The elevation of each point on the section during test 29 is the datum for that point in tests 20 and 31. Tests 20 and 31 were both low-temperature tests while the highest observed temperature of the concrete in the completed structure was measured during test 29.

The deflections of the crown section between tests 20–29 and 29–31 are shown respectively by the ordinates between the curves *A–B* and *A–C*. These deflections are similar to those produced in a simple beam uniformly loaded over the central portion of the span and having a slight increase in load intensity near the ends. It is evident that with a rise in average arch temperature of 57.5° F. occurring between tests 20 and 29, the center of the section had the greatest upward deflection and probably carried an increased proportion of the dead load. The average arch temperature during test 31 was about 7° F. higher than during test 20 but the elevation of the crown section was lower during the latter test because of shrinkage and flow in the concrete. This drop in elevation of the section was probably accompanied by an increase in the dead load carried by the granite facing arches.

The center of the crown section deflected about 0.03 inch above or below the granite facing arches for each 10° F. rise or fall in arch temperature. The deflection of other transverse sections was of the same general shape but smaller because of the increased stiffness of the bridge and the closer proximity of the sections to the restraining influence of the piers. Therefore, the granite facing arches affected the vertical deflections of the arch barrel along the axis of the longitudinal clinometer stations but the magnitude of the effect was not determined.

4. STRAINS AT THE PRINCIPAL SECTIONS DUE TO THRUST AND BENDING

Thrust and bending strains at the principal observation stations obtained from the strain-gage and telemeter data corrected for temperature are given in table 4. These strains are the strains due to elastic deformations produced by loads and temperature changes plus the effects of shrinkage and flow. On the assumption that the distribution of bending strains across a radial section was linear, the thrust strain was equal to the average of the strains on the extreme fibers (extrados and intrados) and the bending strain was equal to one half their difference. Table 4 also includes the bending strains

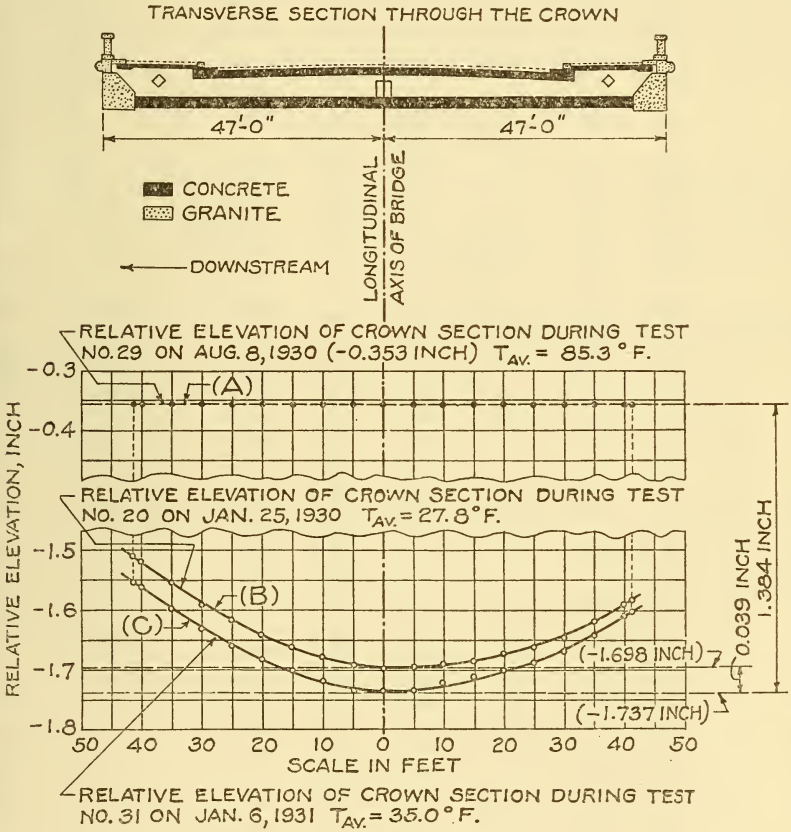


FIGURE 12.—*Vertical deflection of a transverse section through the crown.*

A, datum for crown section (test 29); *B* and *C*, deflection from datum, tests 20 and 31, respectively.

calculated from the clinometer data, corrected for changes in temperature gradients. The differences between the bending strains calculated by these two independent methods are in most cases comparatively small.

The changes in thrust strain between tests 19 and 31 were respectively $+67\times10^{-6}$ at the haunch section and $+66\times10^{-6}$ at the springing (table 4). Since there was no change in loads on the arch, and since the actual change in thrust due to temperature differences

TABLE 4.—*Extrados thrust and bending strains, corrected for temperature, at the radial observation sections*

NOTE.—Sign + = compressive strain, sign − = tensile strain.

Test no.	Springing (average)			Haunch (average)			Crown		
	Strain gage and telemeter		Clinometer bending	Strain gage and telemeter		Clinometer bending	Strain gage and telemeter		Clinometer bending
	Thrust	Bending		Thrust	Bending		Thrust	Bending	
	Millionths	*Millionths*	*Millionths*	*Millionths*	*Millionths*	*Millionths*	*Millionths*	*Millionths*	*Millionths*
5-6	+17	+23	+22	−2	+12	+7	+5	+11	−5
6-7	−8	0	+12	+1	−2	+4	−5	−21	−24
5-7	+9	+23	+34	−1	+10	+11	0	−10	−29
7-8	0	−10	−8	−11	−7	−8	−4	+4	+23
5-8	+9	+13	+26	−12	+3	+3	−4	−6	−6
8-10	+16	+35	+26	+24	+20	+14	+9	−14	−44
5-10	+25	+48	+52	+12	+23	+17	+5	−20	−50
10-11	+29	+21	+18	+20	−18	−18	+26	−14	+13
5-11	+54	+69	+70	+32	+5	−1	+31	−34	−37
11-12	+12	−4	−3	+17	+7	+13	+18	−4	−12
5-12	+66	+65	+67	+49	+12	+12	+49	−38	−49
12-13	+4	−5	−7	+10	+20	+7	----------	(1)	−20
5-13	+70	+60	+60	+59	+32	+19	----------	(1)	−69
13-14	−3	−16	−11	+14	+4	+7	----------	(1)	−49
5-14	+67	+44	+49	+73	+36	+26	----------	(1)	−118
14-15	+16	−26	−32	+5	−7	−11	----------	(1)	+49
5-15	+83	+18	+17	+78	+29	+15	----------	(1)	−69
15-16	+20	+44	+43	+41	+5	+9	----------	(1)	−20
5-16	+103	+62	+60	+119	+34	+24	----------	(1)	−89
16-17	+18	−16	−5	+19	−9	+5	----------	(1)	+10
5-17	+121	+46	+55	+138	+25	+29	----------	(1)	−79
57-18	0	−67	−62	+4	+17	+29	----------	(1)	+50
-18	+121	−21	−7	+142	+42	+58	----------	(1)	−29
18-19	−11	−58	−75	−26	−26	−18	----------	(1)	−15
5-19	+110	−79	−82	+116	+16	+40	----------	(1)	−44
19-20	−3	−3	−11	−10	+22	+22	----------	(1)	+27
5-20	+107	−82	−93	+106	+38	+62	----------	(1)	−17
20-21	0	−1	−4	−3	−18	−10	----------	(1)	−15
5-21	+107	−83	−97	+103	+20	+52	----------	(1)	−32
21-22	+12	+40	+56	+20	+24	+17	----------	(1)	+19
5-22	+119	−43	−41	+123	+44	+69	----------	(1)	−13
22-23	+2	−28	−17	+10	−42	−42	----------	(1)	−60
5-23	+121	−71	−58	+133	+2	+27	----------	(1)	−73
23-24	−19	−2	+32	−3	−5	+13	----------	(1)	+16
5-24	+102	−73	−26	+130	−3	+40	----------	(1)	−57
24-25	+42	+37	−5	+32	+26	−12	----------	(1)	−18
5-25	+144	−36	−31	+162	+23	+28	----------	(1)	−75
25-26	+2	+56	+56	+2	+14	+28	----------	(1)	+13
5-26	+146	+20	+25	+164	+37	+56	----------	(1)	−62
26-27	+9	+10	+13	0	−4	−1	----------	(1)	+4
5-27	+155	+30	+38	+164	+33	+55	----------	(1)	−58
27-28	+14	+22	+32	+26	−12	−1	----------	(1)	−10
5-28	+169	+52	+70	+190	+21	+54	----------	(1)	−68
28-29	−6	−4	+10	+4	−8	0	----------	(1)	−5
5-29	+163	+48	+80	+194	+13	+54	----------	(1)	−73
29-30	+34	−80	−77	+18	−7	−2	----------	(1)	+8
5-30	+197	−32	+3	+212	+6	+52	----------	(1)	−65
30-31	−23	−93	−133	−29	−20	−17	----------	(1)	+2
5-31	+174	−125	−130	+183	−14	+35	----------	(1)	−63

[1] Crown telemeters out of adjustment

was negligible (3 lb. per sq. in. at the crown), the above thrust strains may be attributed to shrinkage and flow, and are equivalent to a temperature drop of about 10° F. The equivalent temperature drop as determined from the crown deflection between these tests was about 7° F.

5. PIER MOVEMENTS

The pier movements, calculated from the longitudinal clinometer data, were usually so small that it was often difficult to determine whether they were significant or were the result of errors in observation. The rotation of the top surface of pier no. 4, and the vertical movements of the center of the area relative to the center of pier no. 5, are given in table 5. Most of the vertical displacement of pier no. 4 relative to pier no. 5 occurred between the earlier tests, when the loads on the piers were constantly increasing as the construction of the bridge progressed.

TABLE 5.—*Pier movement*

Test no.	Rotation [1] of pier 4		Deflection [2] of pier 4 relative to pier 5	
	Increment between tests	Total	Increment between tests	Total
	Millionths radian	*Millionths radian*	*Thousandths inch*	*Thousandths inch*
1–2			−83	−83
2–3			−6	−89
3–4			−53	−142
4–5			+34	−108
5–6			+6	−102
6–7			−2	−104
7–8			−2	−106
8–10			−3	−109
10–11	−43	−43	−16	−125
11–12	+10	−33	+1	−124
12–13	−1	−34	+3	−121
13–14	−19	−53	−5	−126
14–15	+17	−36	+1	−125
15–16	−1	−37	−5	−130
16–17	+8	−29	−1	−131
17–18	+29	0	+12	−119
18–19	+23	+23	−33	−152
19–20	0	+23	0	−152
20–21	+1	+24	−6	−158
21–22	−7	+17	+4	−154
22–23	−11	+6	+3	−151
23–24	−5	+1	+2	−149
24–25	−6	−5	+5	−144
25–26	−6	−11	−9	−153
26–27	−4	−15	+2	−151
27–28	−3	−18	−12	−163
28–29	−9	−27	+1	−162
29–30	+17	−10	+9	−153
30–31	+41	+31	+12	−141

[1] Sign: Positive rotation is away from the crown of arch 7.
[2] Sign: Positive deflection indicates a rise in pier 4 relative to pier 5.

The rotation of the clinometer stations on one side of the longitudinal axis of the pier surface was often greater than the rotation of the stations on the other side of this axis, indicating a warping of the

pier surface in addition to the rotation of the pier as a whole. This warping was concave for temperature rises and was probably caused by relative changes in temperature between the inner and outer portions of the pier.

The average rotation of the clinometer stations on pier no. 4 was toward the crown of arch no. 7 for rise in the temperature of the arch barrel. After the construction of the superstructure, and for temperature changes exceeding 10° or 15° F., the rotation of pier no. 4 was approximately proportional to the temperature change in the arch and equal to about 12×10^{-6} radian for a change of 10° F. The draw span abutment east of arch no. 7 was 50 percent wider than pier no. 4 and its bottom elevation was 3.5 feet below that of pier no. 4. The direction of rotation of the pier appeared to be influenced by changes in the temperature thrust from this abutment.

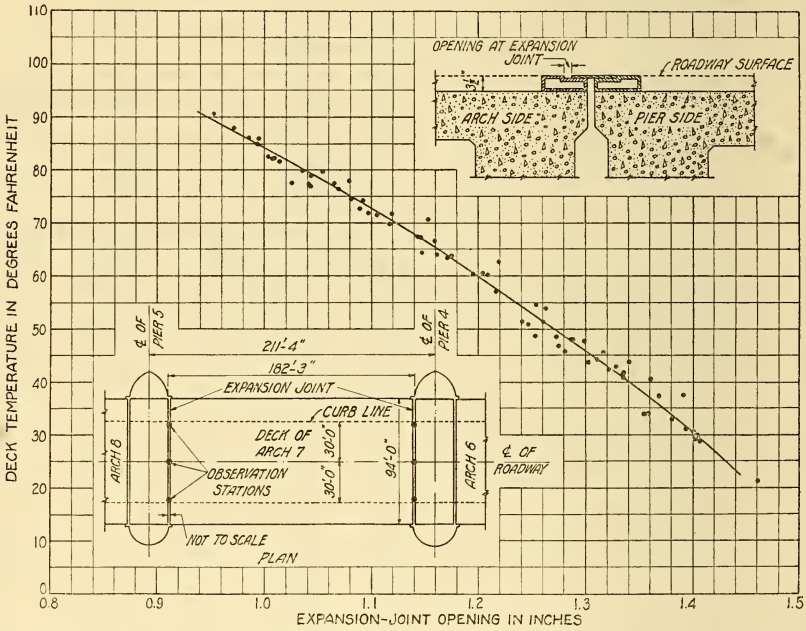


FIGURE 13.—*Changes in width of expansion joints; each point represents the average of 1 observation on each of the 6 stations.*

Location and essential details of the joints and observation stations are also shown.

6. MOVEMENTS OF THE EXPANSION JOINTS

Observations on the width of the expansion joints and the temperature of the deck slab were started in January 1930 and continued until January 1931. Figure 13 shows the results of the measurements, all of which were made in the early morning hours. The observed temperature of the deck slab varied between 20° and 90° F. and the coefficient of thermal expansion of the deck computed from the joint movement was 66×10^{-7} per ° F. This value agrees closely with that previously determined for the arch-barrel concrete, showing that the joints had functioned. The rate of movement of the joints showed a slight increase with increase in deck temperature.

VII. SUMMARY

1. A maximum temperature of approximately 143° F. (62° C.) was attained in the thicker portions of the arch barrel about 25 hours after the concrete was placed in the forms. The temperature of the air during the same period was about 73° F. (23° C.).

2. The temperature of the concrete of the crown section of the arch barrel (2.25 ft. thick) reached air temperature about 8 days, and that of the springing section (4.83 ft. thick) reached air temperature about 15 days after being placed.

3. After the concrete had cooled, the average temperature of the arch barrel closely approximated the average of the air temperature of the previous 5 days.

4. During a period of 27 months after the concrete had cooled, the maximum observed average temperature of the arch barrel was 85° F. (29° C.), and the minimum was 24° F. (−4° C.). The maximum and minimum air temperatures during the same period were 106° F. (41° C.) and 7° F. (−14° C.), respectively.

5. The linear coefficient of thermal expansion of the arch barrel, without superstructure restraint, as determined from the crown deflection and from the arch-barrel concrete cast in prisms was about 65×10^{-7} per ° F. (117×10^{-7} per ° C.), and that of the deck slab, calculated from measurements of the expansion joint movement, was approximately the same.

6. The restraint of the superstructure caused a reduction of about 13 percent in the crown deflection which would have been caused by changes in the average concrete temperature. For unit change in average temperature the deflection of and the bending moment in the arch barrel near the piers increased after the construction of the superstructure.

7. Unequal vertical deflections in transverse sections of the arch barrel occurred with changes in temperature because of differences in physical properties of the granite facing arches and the concrete arch barrel. On the completed structure the center of a transverse section through the crown deflected upward about 0.03 inch above the granite facing arches for a temperature rise of 10° F.

8. The vertical deflections of the arch barrel, produced by the addition of the superstructure walls and deck, were much greater than the theoretical (elastic) deflections. The shapes of the theoretical and observed load deflection curves were dissimilar and the theoretical and observed bending moments at the crown were different in sign. These differences in the deflections were probably caused by the shrinkage and flow in the concrete during the 170 days required for construction of the superstructure.

9. The effect of shrinkage and flow on the deflection of the arch during the period from the eighteenth day to the twenty-fourth month after the construction of the two inner arch-barrel strips was equivalent to a drop of 27° F. in the average arch temperature. The effect of shrinkage and flow after the superstructure was completed and during the period from the fifteenth to the twenty-seventh month was equivalent to a temperature drop of about 7° F.

13317—33——3

VIII. ACKNOWLEDGMENTS

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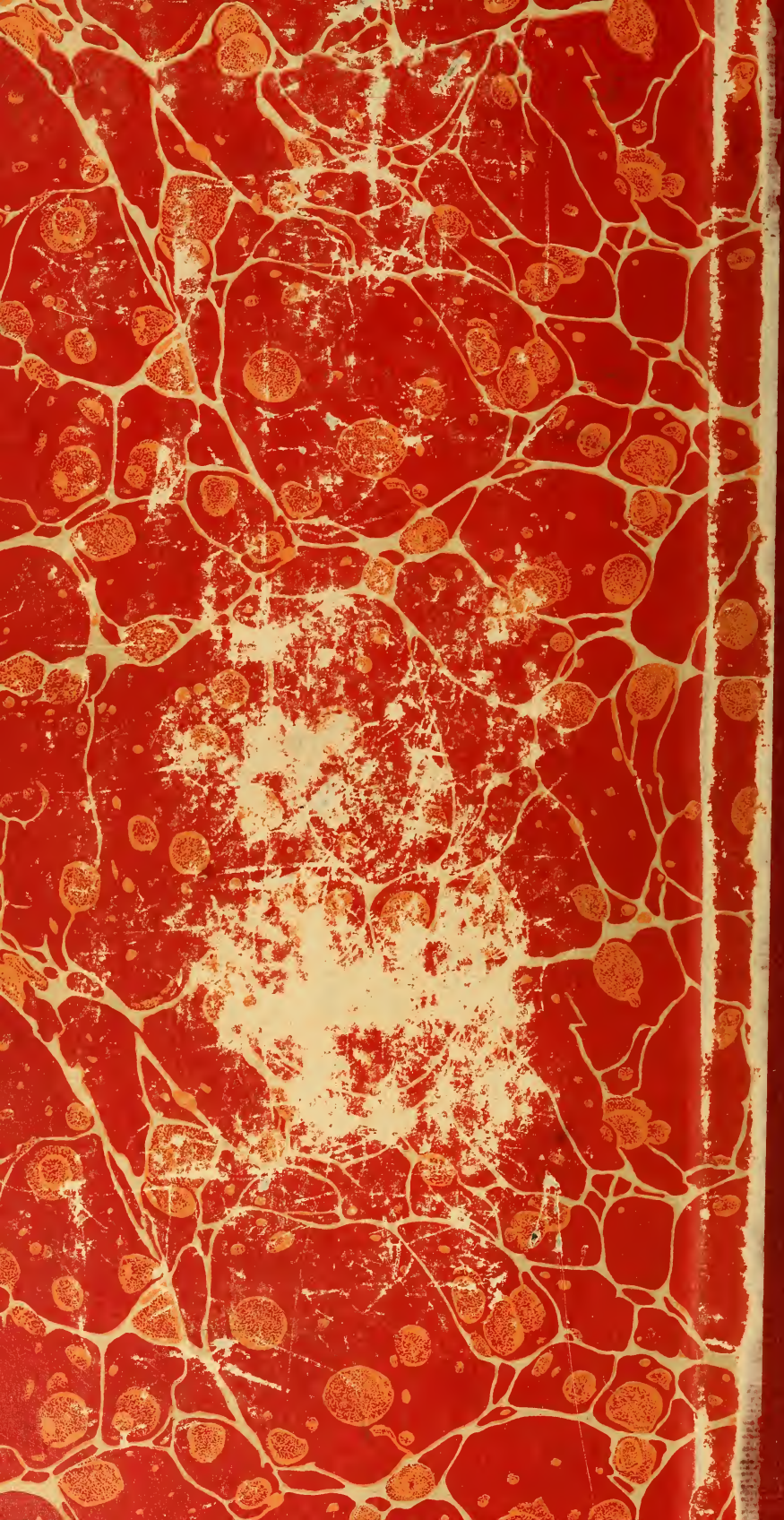
WASHINGTON, July 29, 1933.