SCATTERING OF ELECTRONS BY IONS AND THE MOBILITY OF ELECTRONS IN A CÆSIUM DISCHARGE

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ABSTRACT

The mobilities of electrons in the positive column of a cæsium discharge are derived from measurements of the electrical gradient and the electron temperature and concentration. The value of the cæsium atom cross section, estimated from the mobilities at small electron concentrations, is 3.3×10^{-14}cm^2 for electron energies of about 0.33 volt. The cross section apparently increases linearly with the concentrations of ions in the discharge. The variation is found to be due to the scattering of the electrons by ions. The value of the ion cross section can be evaluated from the rate of variation; it is 75×10^{-14}cm^2 for 0.33 = volt electrons. This magnitude can be explained by the electrostatic interaction between the ions and electrons.

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I. INTRODUCTION

Some recent work by T. J. Killian [1] shows that atomic electron scattering areas can be evaluated from the electrical characteristics of the positive column of a gas discharge. The method involves the measurement of the electron temperature, concentration, and the electrical gradient by means of the familiar Langmuir probe technique. Some measurements of this type by one of the authors have shown that the scattering area of the cæsium atom apparently increases with the discharge current. The present investigation was undertaken in order to find the reason for this unexpected behavior. The explanation was found to lie in the unusually large scattering area of the cæsium ion, due to its Coulomb force field. In a cæsium discharge at low pressures and high currents, the ions play a more important rôle in electron scattering than the atoms.

II. METHOD

1. THEORY

The mobility of a cloud of electrons drifting through a gas under the influence of an electric field of intensity X is defined by the relation

$$K = \frac{u}{X} \tag{1}$$

[1] T. J. Killian, Phys. Rev., vol. 35, p. 1238, 1930.

where K is the mobility and u is the average velocity of the electrons in the direction of the field. The current density I of the charge carried by the electrons is equal to Neu where N is the number of electrons in a cubic centimeter and e is the electronic charge.

Langevin [2] has derived the following expression for the mobility. It is correct if the atom can be assumed to behave as an elastic sphere in collisions with an electron.

$$K=\frac{I}{NeX}=\frac{3}{8}\pi^{\frac{1}{2}}\frac{e}{(2mkT)^{\frac{1}{2}}}\frac{1}{\overline{A}} \tag{2}$$

Here m is the mass of an electron and T is the electron temperature; k is Boltzman's constant. $\overline{A}$ is the total cross-sectional area in unit volume of the atoms obstructing the diffusion. If there is only one type of atom, then $\overline{A}=N_0A_0$ where N_0 is the number in the unit volume and A_0 is the "cross-section area" of the atom. If there are in addition N_+ ions of area A_+, $\overline{A}$ is given by the relation [3]

$$\overline{A}=N_0A_0+N_+A_+ \tag{3}$$

It is seen from equations 2 that $\overline{A}$ may be evaluated from measurements of I, X, N, and T.

In deriving the above expression for the mobility, it is assumed that the concentrations of ions and electrons are constant in space. In the discharge tube this concentration varies considerably over the cross section of the tube. It can be shown, however, that the values of the atomic areas computed will not be greatly in error if the average values of the current and electron and ion concentration over the tube cross sec tion are used in the equations.

2. THE DISCHARGE TUBE

The discharge tubes employed for the measurements were about 30 cm long and 1.8 cm in diameter. An oxide coated platinum ribbon which could be heated electrically was used as cathode. The anode was a platinum cylinder. The tubes were baked out at 400° C. and the cæsium distilled into the tube. A cæsium vapor discharge was maintained for several hours or longer before the tubes were sealed off from the pumps. During some of the work the tubes were connected to the pumps while the measurements were being made.

3. THE MEASUREMENTS

The probes used in the electrical measurements were platinum wires 0.4 and 1.0 mm in diameter; the sides were insulated by glass tubing and the flat ends were exposed to the discharge.

The electron temperature was derived from the slope of the straight line obtained by plotting the logarithm of the electron current to the probe against the probe potential. The temperature is given by the relations

$$T/11{,}600=V_0=(V_2-V_1)\log. e/(\log I_1-\log I_2)$$

[2] Thomson, "Conduction of Electricity through Gases," 3 ed., p. 170.
[3] See reference 2, p. 171.

I_1 and I_2 are the electron currents at potentials V_1 and V_2 and V_0 is two-thirds the average electron energy in volts.

The electron concentration was derived from the magnitude of the saturation electron current density to the probe when the latter was positive to the discharge.

$$N = 4.03 \times 10^{13} i_0 / T^{1/2}$$

i_0 is the saturation electron current divided by the prove area.

The axial electrical gradient was obtained from the difference in potential between two similar probes placed about 10 cm apart along the axis when they were drawing equal electron currents.

Most of the probe surfaces were located at the axis of the discharge tube. The average value of the electron concentration over a cylindrical discharge tube is, according to Tonks and Langmuir,[4] equal to 0.70 times the value at the axis. At high pressures this factor drops to 0.42. A few measurements of the distribution of the electron concentration across the discharge tube showed, however, that the former factor was more nearly correct even at the higher of the pressures used. The factor 0.70 was therefore used in all of the computations to convert the measured concentration at the axis to the average value over the discharge.

The current density, I, of equation (2) was taken to be the discharge current divided by the tube area.

The ion concentration is difficult to measure directly. Its value was taken to be that of the electrons, a relation which must be true to a high degree of accuracy.

The cæsium vapor pressure was computed from the temperature of the cæsium by means of the equation.[5]

$$\log P = -3{,}966/T + 7.1650$$

The number of atoms in unit volume was derived from the vapor pressure and the temperature of the discharge tube wall as determined by a thermocouple in contact with the glass.

For further details concerning the probe measurements and characteristics of the cæsium positive column, a recent paper by one of the authors should be consulted.[6]

III. RESULTS

Figure 1 shows a plot of the total cross section of atoms and ions divided by the number of atoms against the ion concentration, N_+. The ordinates are therefore from equation (3)

$$\frac{\overline{A}}{N_0} = A_0 + N_+ \frac{A_+}{N_0}$$

The curves obtained should be straight lines as is observed. It is obvious from the equation that the intercept of the line on the vertical axis is A_0, the cross section of the atom. The cross section of the ion can be derived from the slope of the line, A_+/N_0. Table 1 gives values of A_+ and A_0 derived in this way at various cæsium pressures.

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[4] Tonks and Langmuir, Phys. Rev., vol. 34, p. 876, 1929.
[5] Rowe, Phil. Mag., vol. 3, p. 534, 1927.
[6] F. L. Mohler, Collisions of the First and Second Kind in the Positive Column of a Cæsium Discharge, B. S. Jour. Research, vol. 9 (RP484), p. 493, October, 1932.

Column 3 gives the value of the electron energy in volts, V_0. Column 2 gives the number of cæsium atoms in unit volume at the pressure and temperature of the measurements.

It is seen that the ion cross section decreases with the electron energy while the cross section of the atom increases. The value of 3.3×10^{-14} for the atom at the higher electron temperatures does not differ greatly from the value given by Brode of 3.1×10^{-14}cm² for

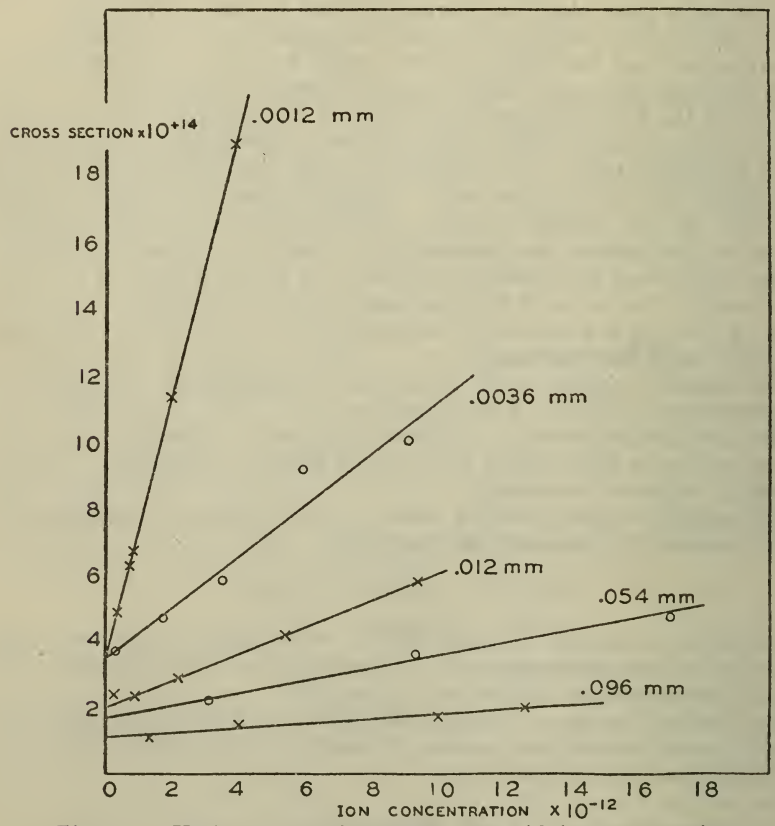

Figure 1.—Variation of total scattering area with ion concentration

Ordinates: Total scattering area per cm³ divided by the number of atoms; abscissæ, average ion concentration in ions per cm³.

electrons having the uniform energy of 0.6 volts. This latter value was obtained by a direct type of measurement.[7]

TABLE 1

Cs pressure	Number of atoms in cm³	Electron energy	Ion cross section	Cs atom cross section
mm	$\times 10^{14}$	*Volts*	$\times 10^{-14}$ *cm²*	$\times 10^{-14}$ *cm²*
0.065	12	0.222	134	1.4
.027	5.0	.235	117	2.0
.013	2.4	.263	98	2.4
.0060	1.1	.282	87	3.3
.0036	.67	.333	75	3.3
.0012	.22	.400	60	2.9

[7] Brode, Phys. Rev., vol. 34, p. 673, 1929.

Results at higher pressures than those tabulated show that the atom cross section falls off very rapidly with decreasing electron speed. Though measurements at these pressures are perhaps not trustworthy, it is interesting to note that such a rapid decrease in cross section with electron speed at low electron energies is predicted by the quantum mechanics.[8]

IV. SOURCES OF ERROR

An examination of equations (2) and (3) will show that the magnitude of the ion cross section is independent of a knowledge of the absolute value of the cæsium pressure or of the ion and electron concentration. It depends only upon measurements of the relative values of the electron concentrations. It should also be independent of the presence of a foreign gas in the discharge tube.

The value of the cæsium atom area on the other hand depends critically on all of these quantities. The values given in the table are the means of results obtained from a number of discharge tubes. One of the tubes yielded values of A_0 at low cæsium pressures which were almost twice as high as those given in the table. This was ascribed to the presence of foreign gas and the results were rejected. The values obtained from the measurements with the discharge tube on the pumps were in agreement with the tabulated results, though the accuracy was less because of the difficulty of knowing the cæsium temperature and vapor pressure under such conditions. Neglecting the rejected values, the mean deviation of the mean for all of the measurements was about 10 per cent.

The values of the ion cross section were, for unknown reasons, subject to larger fluctuations; the mean deviation from the mean of the results from a large number of discharge tubes was 20 per cent.

Another source of error involves the fact that the mobility equations of Maxwell and Langevin assume that the electrons have a Maxwellian velocity distribution in a system of coordinates moving with a velocity equal to the average electron velocity. The fact that the velocity distribution is actually Maxwellian is proven by the "straightness" of the plots of the logarithm of the electron probe current against the probe potential. Whether or not the perturbation of the velocity distribution due to the electrical field has the theoretical form is impossible to ascertain. The magnitude of the deviations is too small.

As mentioned above, an important source of error is the radial variation of ion and electron concentration across the discharge. Simple considerations show, however, that the values of the atom cross section will be correct if the average values are used in the mobility equation. The average of the electron concentration was assumed to be 0.7, the value at the center for all pressures. The values of A_0 at the higher pressures should, therefore, be reduced somewhat (equation(2)), if this factor becomes lower as the pressure is increased. The values of the ion area are, however, in error due to the use of average values. Detailed consideration shows that the curves of Figure 1 should not be straight, but should have a slight curvature.

[8] Quantum Mechanics of Collision Processes, P. M. Morse, Reviews of Modern Physics, vol. 4, p. 577, 1932.

The ion area, derived from the slope at zero ion current, should be multiplied by the Langmuir-Tonks factor, 0.70. Due to various uncertainties such a correction was not applied.

V. DISCUSSION

Maxwell and Langevin have given expressions for the mobility for laws of the force between the particles which vary as some inverse integral power of their mutual distance. The equations are supposedly exact if the drifting particles (electrons) have a small mass compared to those obstructing the drift. This is of course the case of interest here. Another requirement is that the effects of multiple collisions be negligible.

The Langevin equation for the inverse square law of attraction is as follows: [9]

$$K=\frac{6}{32\pi^{1/2}}\frac{(2kT)^{3/2}}{N_+e^3m^{1/2}\int_0^\infty \sin^2\theta a da} \tag{5a}$$

Here $a=mv^2b/e^2$, v is the electron velocity, and θ is the half of the angle of deflection of an electron whose initial line of approach passes within a distance, b, of the ion. For the inverse square law of force [10] $\sin\theta=1/(1+a^2)^{1/2}$, so that the integral in equation (5a) becomes $0.5\log_e(1+a^2)$. The definite integral therefor becomes infinite and the mobility is zero. A definite value of the mobility can be obtained if all collisions are neglected in which the electron is at a greater distance from the ion than the mean distance between the ions; that is, $1/N_+^{1/3}$. This is a reasonable procedure since the effects of all of the ions on an electron at distances greater than this will mutually annul each other. The assumption is equivalent to changing the upper limit of the integral to a value of a given by the relation

$$a_t=mv^2/e^2N_+^{1/3} \tag{4}$$

The mobility then becomes

$$K=\frac{12}{32\pi^{1/2}}\frac{(2kT)^{3/2}}{N_+e^3m^{1/2}\log_e[1+(mv^2/e^2N_+^{1/3})^2]} \tag{5}$$

The relation given by Maxwell [11] is almost identical, the numerical coefficient, $2/\pi^{3/2}$, being however about 1.7 times that of Langevin. It should be remarked that the mobility is very insensitive to changes in the value of a_t of equation (4); a change of tenfold producing only a 40 per cent change in K when $N_+=10^{13}$ (a value representative of conditions in the present experiment).

To compare the theoretical mobility with the observed, it is convenient to obtain an expression for the cross section area of a sphere which would yield the same value of the mobility according to equation (2) as results from the Coulomb law of interaction according to equation (5). This area is the value of A_+ of equation (3). It is easily found to have the form

$$A_+=\frac{\pi}{4}\frac{e^2}{V_0^2}\log_e[1+(mv^2/e^2N_+^{1/3})^2] \tag{6}$$

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[9] See reference 2, p. 170.
[10] See reference 2, p. 167.
[11] Richardson, The Electron Theory of Matter, p. 420, 1914.

where kT is replaced by eV_0. The quantity mv^2 in the logarithmic term may be replaced without appreciable error by $2eV_0$. The Maxwell theory yields an expression with a smaller numerical coefficient, $3\pi^2/64$.

With N_+ put equal to 10^{13} and V_0 having a value equivalent to 0.3 volt, one obtains for A_+ a value of 170×10^{-14} cm². The value corresponding to the numerical factor of Maxwell is 110×10^{-14}. Both are larger than the observed value 84×10^{-14} cm². The agreement is perhaps as good as can be expected from the uncertainties in both the measurements and the theory.[12]

From equation (6) it is seen that the ion area should vary inversely as the square of the electron temperature or V_0. The observed variation with temperature is less rapid than that predicted by this relation though the results at higher pressures obey it approximately.

The variation with temperature makes it apparent that the effect of ions will be difficult to observe in discharges in most other gases. The electron temperature in other gas discharges is usually some 10 times greater than in a cæsium vapor discharge; the ion area should therefore be 100 times smaller. The scattering due to ions should only be observable for very low pressures and high currents or when the gas atoms have a small scattering area.

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[12] The theory given above is obviously defective in that it fails to take into account rigorously distant collisions of an electron with two or more ions. L. H. Thomas, Proc. Roy. Soc., vol. 121, p. 478, 1928, has obtained an expression for the mean loss of momentum of an electron traveling through an ion gas in which an attempt has been made to allow for such distant collisions. If one follows Langevin in assuming that the electrons have a Maxwellian velocity distribution in a coordinate system moving with the average electron velocity it is possible to evaluate from the Thomas expression the total loss of momentum of the electrons in unit time. If this is equated to the momentum gained from the field, an expression is obtained for the mobility. It differs from that of equation (5) in that the numerical coefficient is smaller and that the logarithmic term is replaced by another which happens, however, to have the same numerical value to within a few per cent. The ion area obtained is larger than any given above, 380×10^{-14} cm².