RP—9

TESTS OF THE EFFECT OF BRACKETS IN REINFORCED CONCRETE RIGID FRAMES

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ABSTRACT

This paper contains results of analyses and tests of rigid frames of reinforced concrete with and without enlargements (termed "brackets") at the intersection of the horizontal top member with the vertical legs. All the frames tested were of inverted U form hinged at the lower end of the vertical legs. All had a span of 14 feet and a height of 7 feet; the length of the bracket used varied from 0 to 42 inches with different frames. It is shown that the relations between moments and size of brackets, which are determined exactly only by tedious and long-drawn-out computations, may be expressed by a simple empirical equation with an accuracy sufficient for many cases met with in practice. The application of formulas derived for the freely supported frames to the case of inverted U frames with legs fixed at the bottom or of closed frames is also made possible by showing that the vertical distance from the horizontal member to the point of inflection of the leg may be substituted for the total height of the freely supported U frame with nearly correct results. An empirical expression for the moment of inertia for use in determining the distribution of moments (but not for use in computing stress) has been developed which should assist greatly in preliminary or even final design of a frame. Throughout the paper comparisons of the test results with the simplified equation have been shown. In general, it appears that the use of properly designed brackets should result in economy of design.

CONTENTS

I. INTRODUCTION

1. PRELIMINARY

This paper is based upon the results of one of a number of investigations conducted in 1918 by the concrete ship section of the Emergency Fleet Corporation in developing the design and construction of reinforced concrete ships. The immediate object of the investigation described herein was to determine the effect produced upon the distribution of bending moments in a transverse ship frame by using brackets or haunches at the inside corners of the frame. The results of the investigation have a wider significance, however, as a contribution to present knowledge of the behavior of framed structures under stress, and their principal value doubtless lies in their application to the general field of reinforced concrete construction.

In recent years considerable attention has been paid to methods of analysis of rigidly connected frames, and this theoretical treatment has been supplemented by a comparatively few tests of reinforced concrete bents. However, there is little information available as to the correct analysis of frames with haunches or brackets at the intersections of members and apparently no test data bearing upon this form of construction. A well-known method of analysis has been used here, and although its application requires a number of assumptions it seems to give satisfactory results.

The value of definite knowledge regarding the effect of brackets on stress distribution follows from the fact that through their use the bending moment at any section may be made to vary considerably, and that the material used in brackets is placed where it can resist the increased bending moments which the brackets attract to the corners of the frame. Properly designed brackets will produce a considerable economy of weight.

The tests were made in 1918 at the John Fritz Civil Engineering Laboratory, Lehigh University, as a part of the structural laboratory investigation initiated by R. J. Wig, head of the concrete ship section, Emergency Fleet Corporation. This work was under the direction of W. A. Slater, engineer physicist of the National Bureau of Standards, to whom acknowledgment is made for his hearty support and generous assistance throughout the investigation and the preparation of this paper. Acknowledgments for assistance in the planning and performance of the tests are also due to G. A. Maney, designing engineer in the concrete ship section, and to Maj. W. M. Wilson and Maj. A. R. Lord, successively in charge of the laboratory for the Emergency Fleet Corporation. R. L. Brown, of the Engineering Experiment Station, University of Illinois, assisted in the supplementary tests of paper models described in Appendix II.

2. NATURE OF INVESTIGATION

An important use of brackets in ship construction is at the corners of transverse frames. Since a sharp corner at the intersection of horizontal and vertical members produces a section of highly concentrated fiber stress combined with a large negative bending moment, a bracket is used primarily to reduce the compressive stress at this place. In addition, since, in all types of statically indeterminate frames, the distribution of bending moments depends upon the relative stiffness of the members or parts of which the structure is composed, the bracket produces two other effects—(1) it affects the distribution of moments throughout the frame because of the local variation in stiffness that it produces, and (2) it affects the magnitude of the negative moment as well as the distribution of moments because it changes the shape of the axis of the members at the corners of the frame. The effect of this change in shape is sometimes regarded as a shortening of the span of the members in which the brackets are used.

In choosing the type of specimen for these tests the object was to secure a frame similar in form and loading to a ship frame and of fairly large size. A rectangular two-legged bent was chosen as approximating, in the inverted position, the lower part of a transverse ship frame. The columns were made hinged at the base in order to simplify the interpretation of the results. The brackets used were similar to those used in ship design.

In testing, all specimens were loaded to failure and observations of deformation and deflection were made at regular increments of load. The experimental data have been compared with analytical deductions. and, in general, a satisfactory agreement has been found.

II. ANALYTICAL TREATMENT

1. ANALYSIS OF THE EFFECT OF BRACKETS

Mathematical analyses of rigidly connected frames are usually based upon the assumption that the members of the frame are uniform in section throughout their length, and there is little information on the proper way of analyzing frames in which abrupt changes in cross section occur. However, in the analysis which follows, it was considered sufficiently accurate for frames containing brackets to sketch in the approximate axis of the frame (a line midway between the tension and compression faces) and to consider as fully effective the depth of the cross section measured normal to this axis. A semigraphical method often used in arch analysis was then applied to the frame.

In the analysis of the two-hinged frame of Figure 1, which shows one of the types tested, the outline of the frame was first drawn to scale and the axis divided into a number of sections approximately equal in length. The length of a section was denoted by ds, the depth of cross section by d, and the vertical distance of the centroid of the section above the hinges by y. Values of ds, d, and y were scaled from the drawing where necessary for each section of the frame. Values of d were used to calculate the moment of inertia, I, which, for reasons to be discussed in Section IV, was considered to vary as $d^{5/2}$. Letting M represent the bending moment at the centroid of a section due to vertical loads and reactions only, the following general formula [1] for the horizontal reaction of a two-hinged

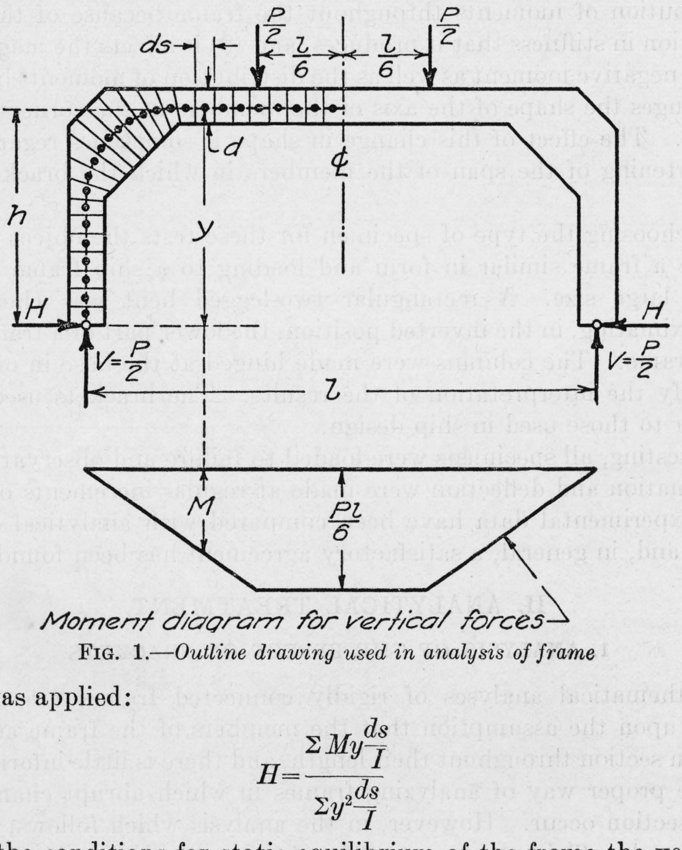

FIG. 1.—*Outline drawing used in analysis of frame*

arch was applied:

$$H=\frac{\Sigma My\frac{ds}{I}}{\Sigma y^2\frac{ds}{I}} \tag{1}$$

From the conditions for static equilibrium of the frame the value of the moment at mid span M_c for one-third-point loading was found to be

$$M_c=\frac{Pl}{6}-Hh \tag{2}$$

[1] For derivation of this formula see Johnson, Bryan, and Turneaure, "Modern Framed Structures," Pt. II, pp. 138 and 158. In this derivation the effect of deformations due to internal shearing and direct stresses are neglected. While these effects are usually negligible, they may be included in equation (1) by measuring y to a point other than the centroid of each section. Such points may be determined by use of the theory of the ellipse of elasticity.

Substituting values of H from equation (1) in equation (2), values of M_c were obtained and for convenience have been expressed in terms of the maximum moment, $\frac{Pl}{6}$, for a simple beam loaded as the top member was loaded. The latter quantity was used for the reason that it simplifies the application of values of M_c to designing.

To illustrate the relations expressed by equation (2), let it be assumed that the value of M_c is known for the rectangular frame of Figure 2 (a). Since the horizontal member is a straight beam, the numerical sum of the maximum negative and maximum positive moments is equal to $\frac{Pl}{6}$, and the trapezoid $ABCD$ represents the moment diagram for a single beam. Hence, laying off M_c fixes the

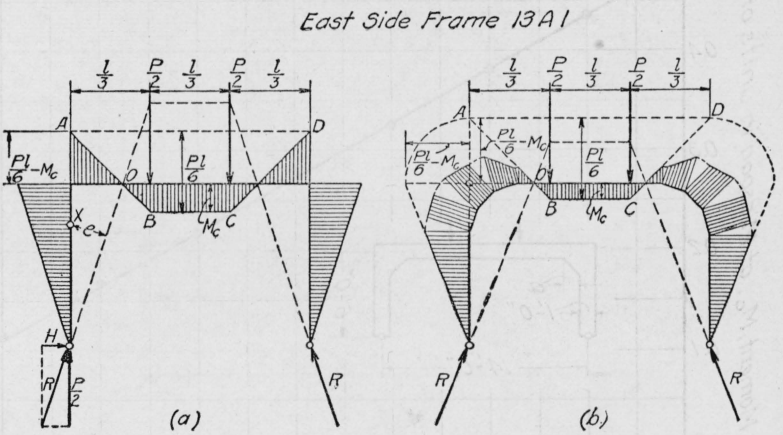

FIG. 2.—*Relations between moments and reactions in frames*

value of the negative moments at the corners, and the entire moment diagram for the frame is easily drawn. It is further evident that if R is the resultant of the horizontal and vertical reactions at the base, the moment about any point, x, lying in the axis of the frame and to the left of B is Re. This relation is particularly useful in treating a frame having brackets in which the axes of the horizontal and vertical members do not meet at right angles, as in Figure 2 (b). Here the top member is not straight and the numerical sum of the maximum positive and the maximum negative moments is not equal to $\frac{Pl}{6}$. However, if the simple beam moment diagram $ABCD$ and the known moment M_c be laid off, the moment diagram for the horizontal and vertical portions of the frame will be determined, and the point of inflection, O, will be located as shown in Figure 2 (b).

Figure 3 shows the results of analyses made by use of equation (1) for the purpose of designing test specimens. Values of the bending moment at mid span are plotted as ordinates against horizontal lengths of brackets as abscissas. All the brackets were assumed in the computations to make an angle of 45° with the horizontal. The points representing the calculated moments are seen to lie nearly on a straight line. The variation in moment at mid span is due almost entirely to the variation in stiffness produced by varying the length of the bracket. It is rather surprising, however, that the moment should vary so nearly as a linear function of the bracket length, and hence of the clear span for these particular frames. From the difficulty of analyzing such a frame a straight-line relation would not be expected to obtain, and it is evident that the line does

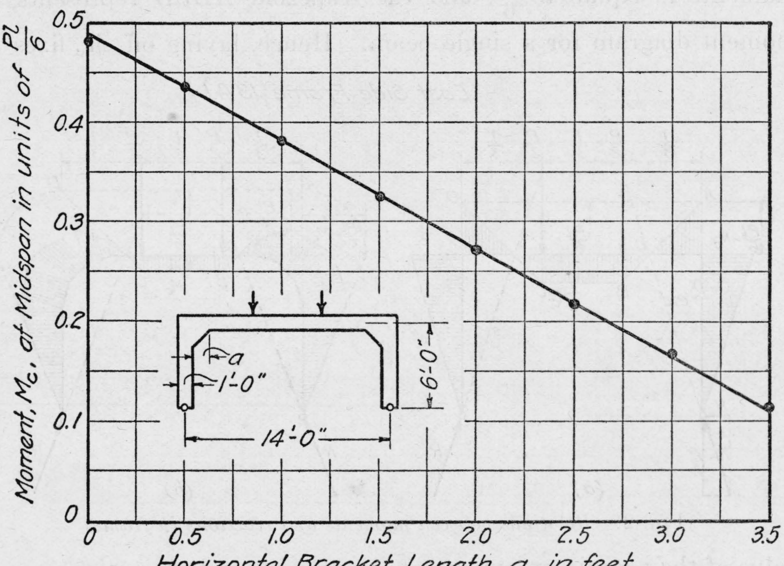

FIG. 3.—*Relations between calculated moments and size of brackets*

not apply exactly at the two extremities of the diagram where the bracket is either very small or very large.

To determine whether similar curves may be drawn for frames of other proportions, further calculations have been made. Figure 4 indicates the relative effectiveness of brackets when used in frames having different ratios of height to span. It is seen that the use of a given bracket has the greatest effect in changing the value of the moment at mid span when the ratio of height to span of the frame is small. This is apparently due to the fact that as the ratio of height to span decreases the bracket occupies a larger portion of the region of high negative moment. The principle involved here is sufficiently important to warrant further elaboration. For illustration, if in any frame the moment of inertia for a short portion of length be varied, there will result a change in the bending moment at all points in the frame. This change will be approximately proportional to the original bending moment at the section where the variation in moment of inertia is introduced. It will be seen, therefore, that an increase in the moment of inertia, such as that produced by a bracket or haunch, will be most effective if made where the original bending moment is largest. Referring again to Figure 4, when the ratio of height to span is 1.0, the negative moment at the corner of a frame without brackets is $0.40 \frac{Pl}{6}$, and a 24-inch bracket reduces the moment at mid span only 20 per cent. On the other hand, when the

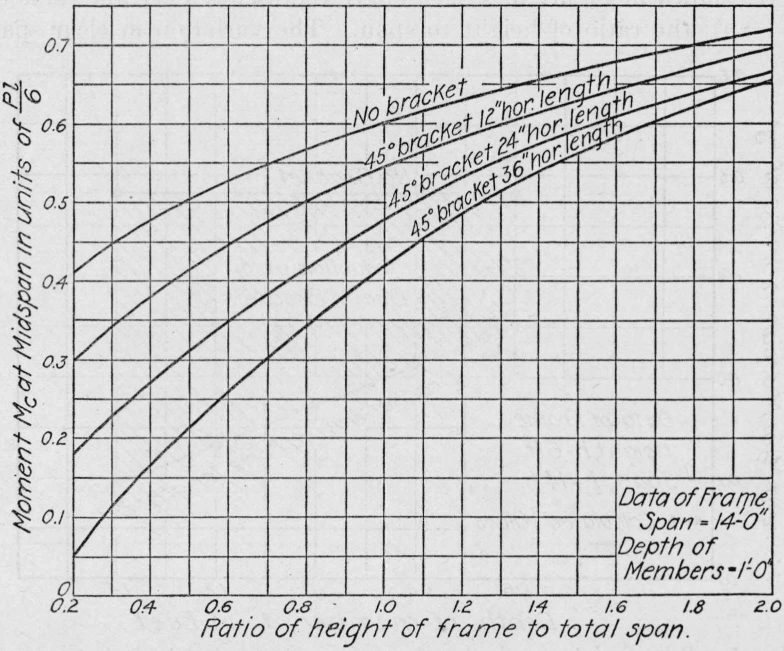

FIG. 4.—*Calculated effect of bracket with varying proportions of frame*

ratio of height to span is 0.3, the negative moment at the corner of the frame without brackets is $0.56 \frac{Pl}{6}$, and the 24-inch bracket reduces the moment at mid span by 51 per cent. Hence, for brackets to be most effective both in changing moment distribution and in reducing stresses [2] they must be used at points of high bending moments.

Another series of calculations was made to investigate the effect of slenderness of members on the effectiveness of haunches. For two frames which are alike, except that one had depths of members

[2] Wherever in this paper the word "stress" is used it designates the internal force per unit of area. This usage is consistent with the recommendations of Committee E-1 of the American Society for Testing Materials. See Proceedings A. S. T. M., **23**, Pt. I, p. 937; 1923; also **25**, Pt. I, p. 879; 1925.

and lengths of brackets greater than the other by a fixed percentage, Figure 5 shows that the effectiveness of the brackets in reducing the moment at mid span is less with the smaller than with the greater depth of member. This might be expected because the lighter frame is stiffened along a smaller portion of its length than the heavier one, and because the axis of the lighter frame at the corner lies farther from the resultant reaction than that of the heavier one. The information of Figures 4 and 5 has been replotted, using values of the moment at mid span, M_c, as ordinates and the ratio of the clear span to the total span, s/l $(=m)$, as abscissas, producing the curves shown in Figure 6. Each curve represents a certain value of h/l $(=n)$, the ratio of height to span. The variation in clear span

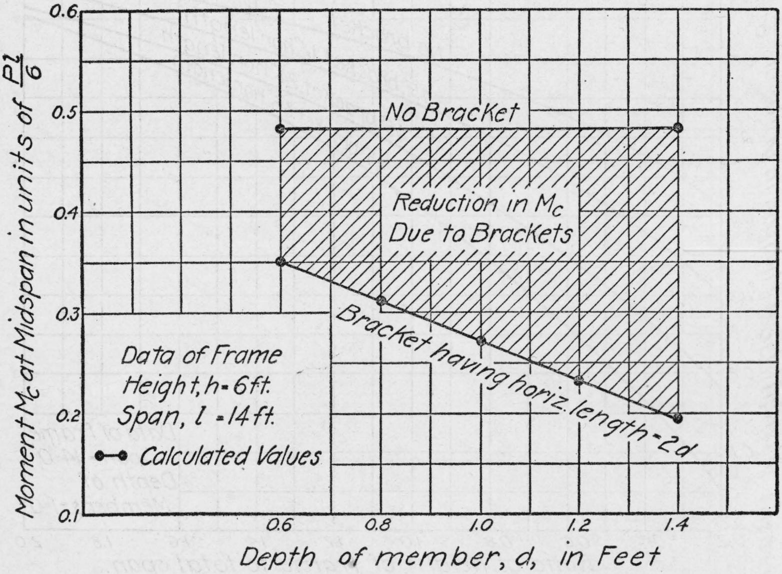

FIG. 5.—*Relation between slenderness of frame and calculated reduction in moment due to bracket*

indicated in this diagram is obtained by varying both the size of the brackets and the slenderness of the members of the frame.

Figure 6 indicates that the moment at mid span varies very nearly as a linear function of the clear span, or distance between bracket edges, for frames of the proportions shown. That is, just as in the case of the curve of Figure 3, here a series of straight lines seem to fit the several groups of calculated points fairly well, the divergence from the linear relation being shown mainly at the extremities of the lines by the dotted curves. The solid straight line represents the range of values ordinarily encountered in the use of brackets; with brackets so small that the ratio s/l becomes 0.9 or more further analysis and supporting test data are needed to determine the exact effect upon the moment distribution.

It is significant that for the ratios of depth to span of members to be found in practice the moment M_c is not greatly affected by a variation in slenderness of members as long as the clear span is not changed. This indicates that the clear span is the variable of major importance. The effect of a variation in slenderness is still less with higher values of n than that shown with values of n equal to 0.25 and 3/7.

It is thought that an equation representing the curves of Figure 6 may be found useful. Such an equation must naturally reduce to

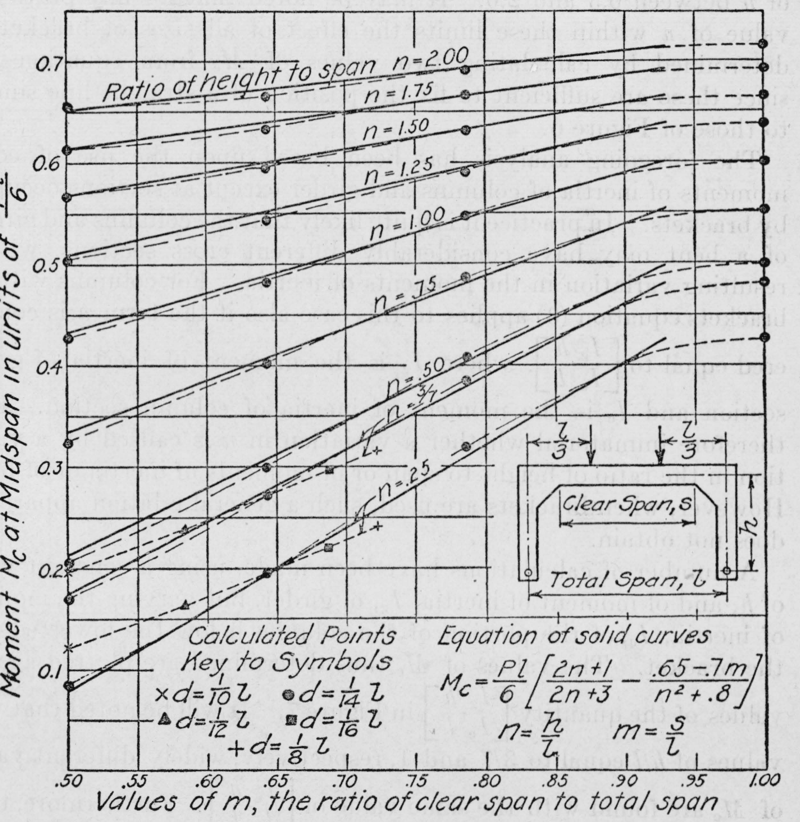

FIG. 6.—*Effect of length of clear span upon moment at mid span*

the ordinary equation for rectangular frames when no brackets are used. If n represents the ratio h/l, the moment at mid span, due to a total load P applied in equal parts at the one-third points of the span of a frame without brackets, is expressed by the equation [3]

$$M_c = \frac{Pl}{6}\left[\frac{2n+1}{2n+3}\right] \tag{3}$$

[3] For demonstration leading to this equation see Bull. 108, Eng. Expt. Sta., University of Illinois, p. 58; 1918.

For frames with brackets [4] it is found that letting $m = s/l$, the ratio of clear span to total span, the straight portions of the curves of Figure 6 may all be expressed by the equation

$$M_c = \frac{Pl}{6}\left[\frac{2n+1}{2n+3} - \frac{0.65-0.7m}{n^2+0.8}\right] \tag{4}$$

This is an empirical equation fitted to the results of semigraphical analyses and hence has a theoretical basis. Its use, however, should be limited to values of m between 0.5 and 0.93 and to values of n between 0.3 and 2.0. It may be noted that for any particular value of n within these limits the effect of all sizes of brackets is determined by calculating two values of M_c from equation (4), since these are sufficient to fix the position of a straight line similar to those of Figure 6.

The foregoing analysis has been based upon the use of equal moments of inertia of columns and girder except at sections occupied by brackets. In practice it is quite likely that the columns and girders of a bent may have considerably different cross sections, with a resulting variation in the moments of inertia. For columns without brackets equation (3) applies to this case also if the term n is considered equal to $\left[\frac{I_g}{I_c} \cdot \frac{h}{l}\right]$, where I_g is the moment of inertia of girder section and I_c is the moment of inertia of column section. It is therefore immaterial whether a variation in n is caused by a variation in the ratio of height to span or of moments of inertia or of both. However, when brackets are used, such a general relation apparently does not obtain.

A number of calculations have been made, using a constant value of h, and of moment of inertia, I_g, of girder, but varying the moment of inertia, I_c, of the portion of the columns below the lower edge of the bracket. The values of M_c thus determined are plotted against values of the quantity $\left[\frac{I_g}{I_c} \cdot \frac{h}{l}\right]$ in Figure 7. It will be noted that with values of h/l equal to 3/7 and 1, respectively, widely different values of M_c are found with the same value of $\left[\frac{I_g}{I_c} \cdot \frac{h}{l}\right]$. Furthermore, these values do not compare at all closely with the moments obtained by assuming $\left[\frac{I_g}{I_c} \cdot \frac{h}{l}\right]$ equal to n in equation (4). It seems unwarranted,

[4] A method of analyzing frames similar to these is given by E. Björnstad in "Die Berechnung von Steifrahmen nebst anderen statisch unbestimmten Systemen." Berlin, 1909. One equation is used for frames both with and without brackets by proper choice of the terms corresponding to n—that is, the members which contain brackets are considered replaced by "equivalent" members of constant cross section throughout. The section of the equivalent member from which n is calculated obviously varies with the size of bracket. This treatment of the subject from a purely theoretical viewpoint neglects the change in shape of the axis of a frame containing brackets. Further, the assumption is made that for a haunch which varies uniformly in depth the moment of inertia also varies as a linear function between the two extremities of the haunch. The results obtained are stated to be approximate.

therefore, to attempt to use equation (4) to investigate frames in which the moments of inertia of the girder is not equal to that of the columns. Figure 7 indicates the general range of values of moments to be obtained in most cases of this sort; frames of other proportions may be analyzed by the application of equation (1).

It has been shown that the moment at mid span decreases with the increase in stiffness at the corners of the frame produced by the use of brackets. If the corners of a rectangular frame could be stiffened without the use of brackets, a decrease in moment at mid span would be accompanied by an equal increase in moment at the corners, since in this case the numerical sum of the maximum positive and

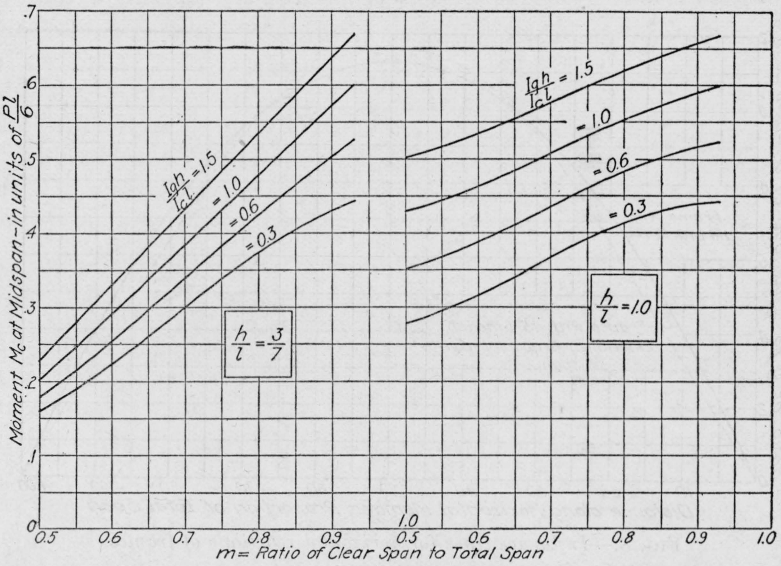

FIG. 7.—*Calculated moments in frames of varying moments of inertia*

negative moments is equal to the maximum moment for a simple beam carrying the same vertical loading. However, where brackets are used the axes of the members do not intersect at right angles, but approach more nearly the pressure line of the resultant force acting on the hinge. The resulting decrease in moment at the corner due to this variation in shape of specimens with brackets approximately offsets the increase in moment at the corner due to the variation in stiffness of the different parts of the frame. Hence, the negative moments in the different types of frame are nearly equal in magnitude for a given load, as will be noted later from the results of tests.

2. EFFECT OF BRACKETS WITH DIFFERENT LOADINGS

Equation (4) has been developed for the special case of third-point loading which was used in the test frames. This form of loading is frequently used in tests because it is easy to apply and produces a moment diagram somewhat similar to that due to a uniformly distributed load. A study of the effect of other loadings shows that equation (4) may be adapted to a form which gives the effect of brackets for such cases.

Influence lines for the horizontal reactions of a two-hinged frame under vertical loads are shown in Figure 8.[5] That is, the ordinate of any point on the influence line represents the relative value of the

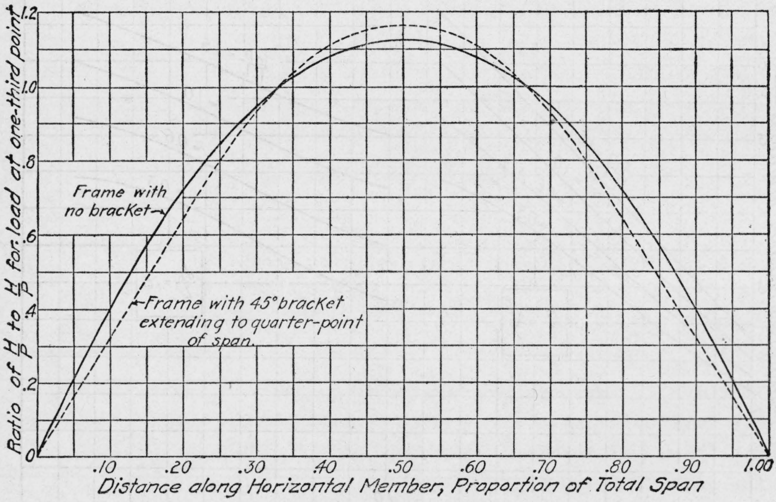

FIG. 8.—*Influence lines for horizontal reactions of frames*

horizontal reaction due to a vertical load at the corresponding point on the span. These relative values are independent of the ratio of height to span of the frame. From these curves it is found that certain common types of loading (with equal total loads) produce the following relative values of the horizontal reaction, H, considering the reaction due to one-third point loading as unity.

[5] The construction of these influence lines is based upon the following theory: With a frame of the type used in this investigation suppose outward thrusts to be applied at the hinges, causing the top member to deflect downward. From Maxwell's theorem of reciprocal displacements it is known that the elastic curve of the top member of this frame is an influence line for the horizontal reactions of a similar frame loaded with vertical loads on the top member and having the hinges at the base held stationary. Figure 8 was, therefore, obtained by computing the shape of the elastic curves of two extreme forms of top member—one having no bracket and one having large 45° brackets extending to the quarter points of the span. It is to be noted that these curves give only relative (not absolute) values of the horizontal reaction, and that the curves can not be used to compare values of the reactions for the two types of frame. However, relative values are sufficient to compare loads at different points on the same frame, which is the purpose of this diagram.

Frame having—	Third point loads	Uniform load	Concentrated center load
No bracket	$H=1.00$	$H=0.750$	$H=1.125$
Large bracket	$H=1.00$	$H=.720$	$H=1.167$

From equations (2) and (4), for the third-point loading,

$$H=\frac{Pl}{6h}\left[\frac{2}{2n+3}+\frac{0.65-0.7m}{n^2+0.8}\right] \tag{5}$$

An expression for the value of H for a frame under uniform load or a concentrated center load may be obtained by multiplying the right-hand member of equation (5) by the corresponding value given in the above table. It is evident that the values 0.750 and 1.125 for the two kinds of loading, respectively, will not be in error more than 4 per cent if used for frames having brackets; using these values in equation (5) and substituting in equation (2) gives

$$M_c=\frac{Pl}{8}\left[\frac{2n+1}{2n+3}-\frac{0.65-0.70m}{n^2+0.8}\right] \tag{6}$$

for a total load P, uniformly distributed, and

$$M_c=\frac{Pl}{4}\left[\frac{2n+1.5}{2n+3}-\frac{0.49-0.53m}{n^2+0.8}\right] \tag{7}$$

for a concentrated load at mid span.

For any other type of loading the relative values of H may be found from Figure 8, and the moment M_c then computed, where P is the total vertical load.

3. EFFECT OF BRACKETS IN VARIOUS TYPES OF FRAMES

While the results of the foregoing analyses may be found useful in the design of two-hinged frames, the question immediately arises as to what quantitative application may be made to other kinds of frames. For example, brackets may well be used in the closed rectangular frame, in the two-legged frame with column bases fixed, in building or viaduct frames and similar structures. In the absence of test data theoretical analyses have been made for two types of frame, and it appears that an application of equation (4) may be made to these cases. In these analyses also the moment of inertia was assumed to vary as $d^{5/2}$.

The frames analyzed are the closed rectangular or quadrangular frame, in which the columns are assumed to be rigidly attached at the bases to a horizontal member of equal section, and the two-legged frame with the columns fixed at the bases. Each of these frames has points of contraflexure in the columns at a distance h_0

equal to two-thirds h or more from the top of the frame. The portion of the frame above these points of contraflexure may be considered as a two-hinged frame in which the bases have been allowed to move apart a small amount. Such an outward movement tends to increase the moment at mid span in a two-hinged frame.

The analysis of these frames has been made by use of the general theory of indeterminate structures used in Section II, 1.[6] The span and depth of members used were the same as those used in the test specimens, while the height and clear span of the frames were varied. Figure 9 shows values of the calculated moment at mid span for

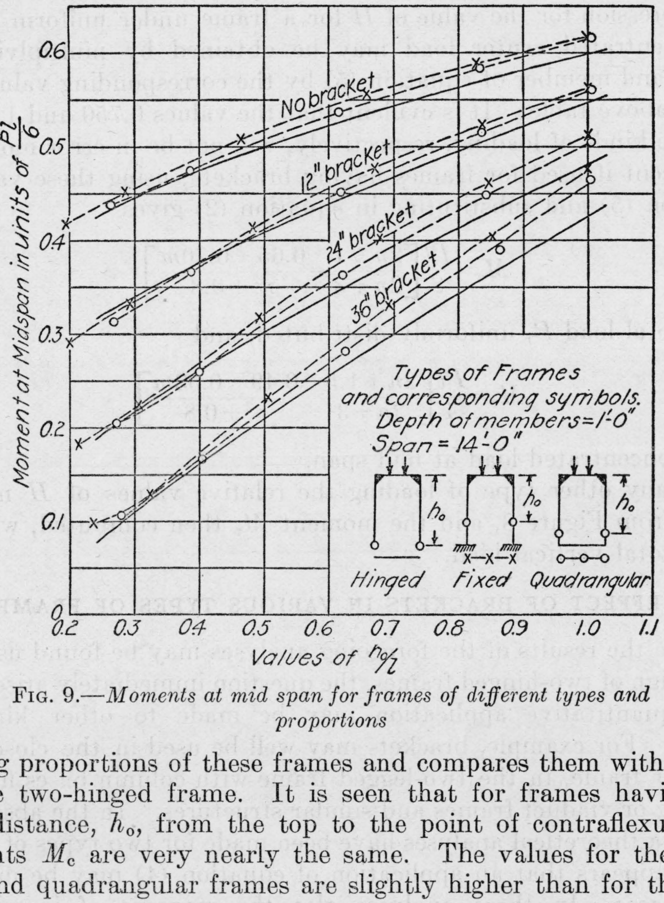

FIG. 9.—*Moments at mid span for frames of different types and proportions*

varying proportions of these frames and compares them with values for the two-hinged frame. It is seen that for frames having the same distance, h_0, from the top to the point of contraflexure, the moments M_c are very nearly the same. The values for the fixed-base and quadrangular frames are slightly higher than for the two-hinged frame because the columns at the point of contraflexure have deflected outward slightly. However, if the value of h_0 has been determined it appears that these frames may all be treated as two-hinged frames by use of equation (4) without appreciable error. For determining h_0 for frames of different proportions, Figure 10 may be used.

[6] See Johnson, Bryan, and Turneaure, Modern Framed Structures, Pt. II, p. 386.

The procedure in determining the effect of certain brackets in a quadrangular or a "fixed-base" frame may be summed up as follows: Knowing the proportions of the frame, the position of the points of contraflexure may be determined from the curves of Figure 10. Placing the value of $\frac{h_0}{l}$ equal to n in equation (4) gives the value of M_c, the moment at mid span of the top member. The moments at all other parts of the frame may be found from the equations of statics. While the value of M_c may be slightly inexact, the error is small.

The curves of Figure 10 giving the position of the points of contraflexure are applicable for any loading on the top member that is symmetrical about the center line of the frame. Hence, if uniform

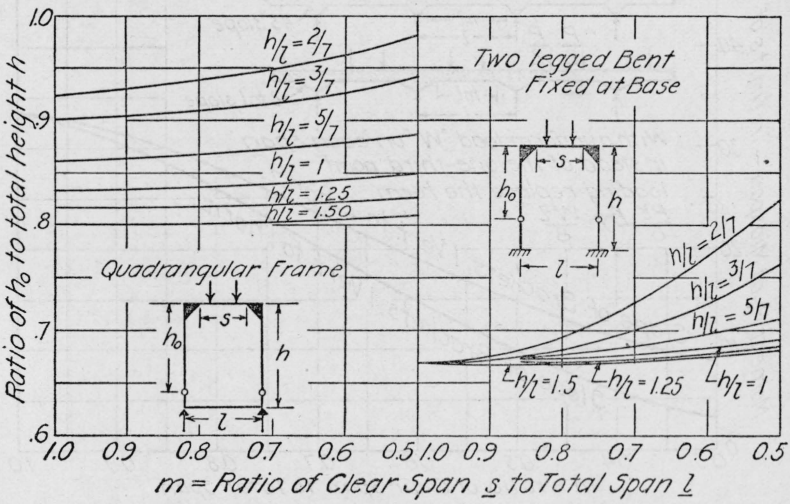

FIG. 10.—*Distance h_o from top of frame to point of contraflexure*

or concentrated loads are used, the procedure is the same except that equations (6) or (7) must be used to determine the value of M_c.

One or two other types of rigidly connected structures may be mentioned. For example, the continuous beam of three spans, with loads on the middle span only, may be treated as a two-legged frame with the columns swung up into the horizontal position. While certain features of the rectangular frame, such as column action, curved beam action, and direct compression due to horizontal thrust, are absent in the continuous beam, brackets used at the supports will occupy the same region of bending moment in the beam as in the frame. Hence, equation (4) will give substantially correct results when used with this special case of a continuous beam.

Another very common form of continuous beam is one having a large number of spans of identical dimensions and with all spans subjected to equal loads. To illustrate the effect of brackets used at the supports in such a beam, two numerical cases have been analyzed by the method explained at the beginning of Section II, 3. In one case brackets with a 45° or 1 to 1 slope were used at both sides of each support; in the other case brackets having a 2 to 1 slope (making an angle of 26° 32′ with the horizontal) were used. The depth of the beam was one-tenth of the span. Figure 11 shows values of the moment at mid span, due to one-third point loading, for various values of the ratio of clear span to total span. Denoting this ratio by m, it is found that approximate formulas for the moment at mid span are:

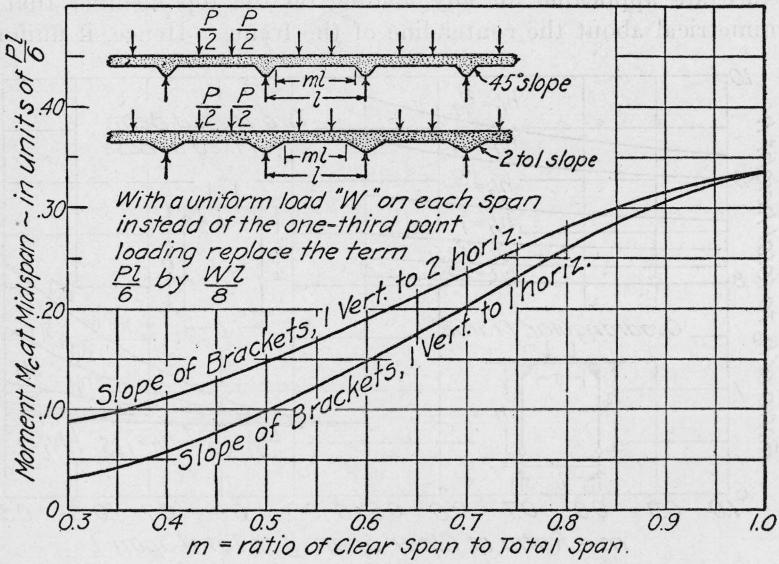

FIG. 11.—*Moments at mid span of continuous beams with brackets*

$$M_c = \frac{Pl}{6}\,\frac{(4m-1)}{9} \tag{8}$$

For brackets with 2 to 1 slope

$$M_c = \frac{Pl}{6}\,\frac{(m)}{3} \tag{9}$$

The moment over the supports in either case is

$$M_s = -\frac{Pl}{6} + M_c \tag{10}$$

Equations (8), (9), and (10) may be applied to the case of a uniformly distributed load W on each span by replacing the quantity $\frac{Pl}{6}$ by $\frac{Wl}{8}$. These equations should not be used for values of m smaller than those shown in Figure 11.

The diagram shows clearly that with very large brackets, corresponding to small values of m, the bracket carries nearly all of the load directly to the support, acting as a very stiff cantilever. The positive moment at mid span accordingly becomes very small.

III. TEST SPECIMENS AND APPARATUS

1. DESCRIPTION OF TEST SPECIMENS

Test specimens of five types were used—two specimens each of types A, B, C, and D, and one of type E. All were two-legged frames with the columns hinged at the bases. The details of these specimens are indicated in Figures 12 to 14. The cross sections of types A, B, C, and D were of T shape, having the following nominal dimensions: Depth, 12 inches; thickness of flange, 3 inches; width of web, 8 inches; width of flange, 30 inches. The flanges were intended to produce the effect of the shell of a ship adjacent to the frame. The cross section of type E was rectangular, being nominally 12 inches deep and 8 inches wide. This type furnished a comparison with type A to show the effect of the difference in section.

All of the test specimens were made 15 feet long and 7 feet high, over all. The span from center to center of hinges was 14 feet, and the height from center of hinges to mid depth of girder was 6 feet. Actual dimensions differed only slightly from these nominal values.

Specimens of types A and E were made with square corners at the intersection of horizontal and vertical members with the exception of a 2-inch fillet at the interior corners. These fillets were used to modify the extremely high stresses which occur at a sharp corner but were not regarded as capable of exerting any appreciable effect as brackets.

Types B, C, and D were made with brackets similar to those actually used in concrete-ship construction. Type B had 45° brackets, 12 inches in horizontal length; the exterior corners were given a 45° chamfer equal in size to the bracket, making the depth of cross section normal to the face of the bracket about 17 inches. Type C was similar to type B, but had a bracket 24 inches in horizontal length, making the depth of cross section at the corner approximately 25 inches. Type D was modified from type C by filling in the angles between the bracket and the main members with two supplementary haunches, so that the inside line of the frame approached the outline of a curved soffit.

The hinge detail at the base of each column was provided by casting in place a steel shoe formed of $\frac{3}{4}$-inch bearing plates and lugs with a 3-inch pinhole at each side of the column connected by a 3-inch pipe sleeve $28\frac{1}{2}$ inches long. A $2\frac{15}{16}$-inch steel pin passing through the pinholes and pipe sleeves engaged similar plates on the test base and formed a simple hinge.

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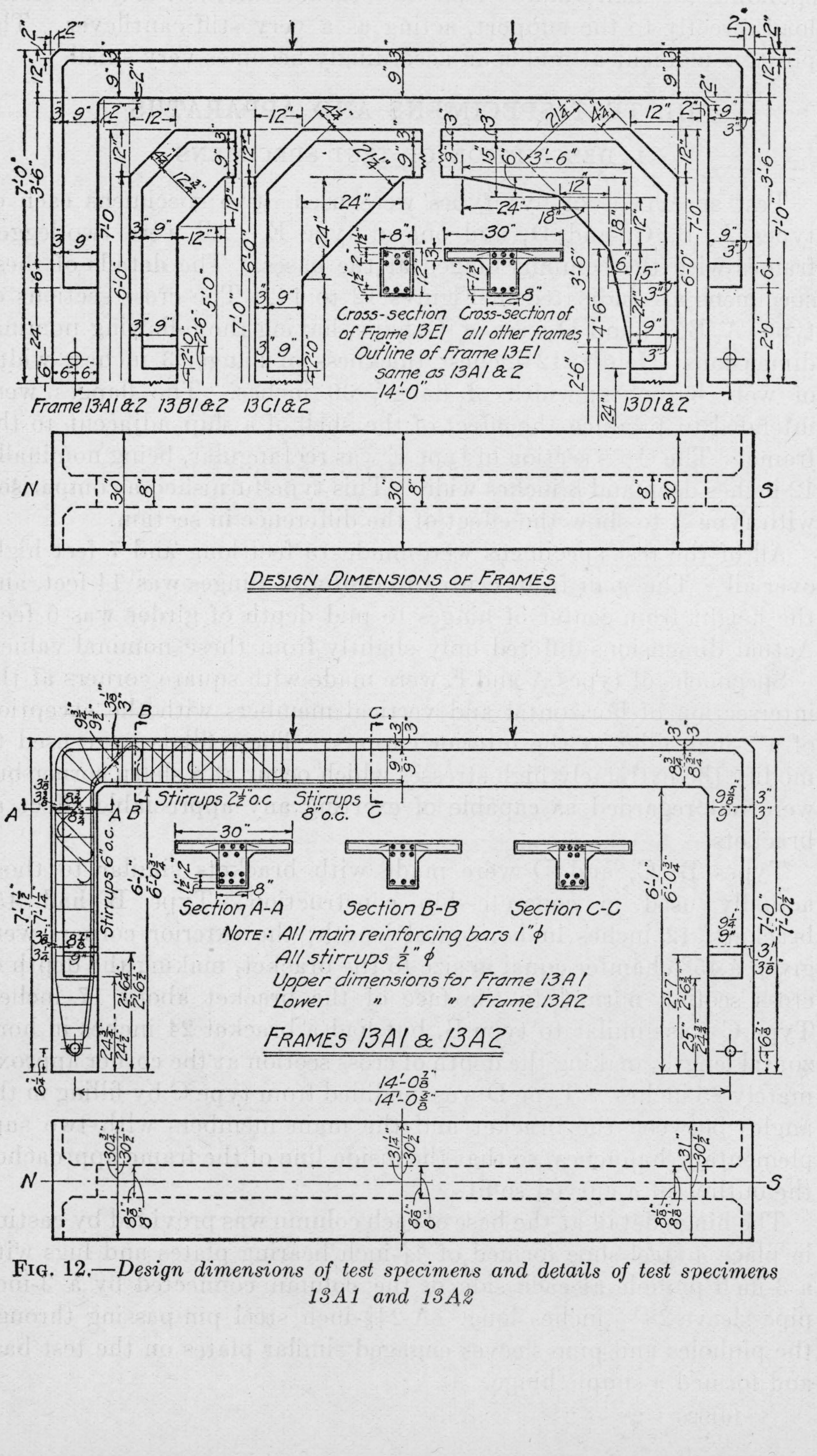

Fig. 12.—*Design dimensions of test specimens and details of test specimens 13A1 and 13A2*

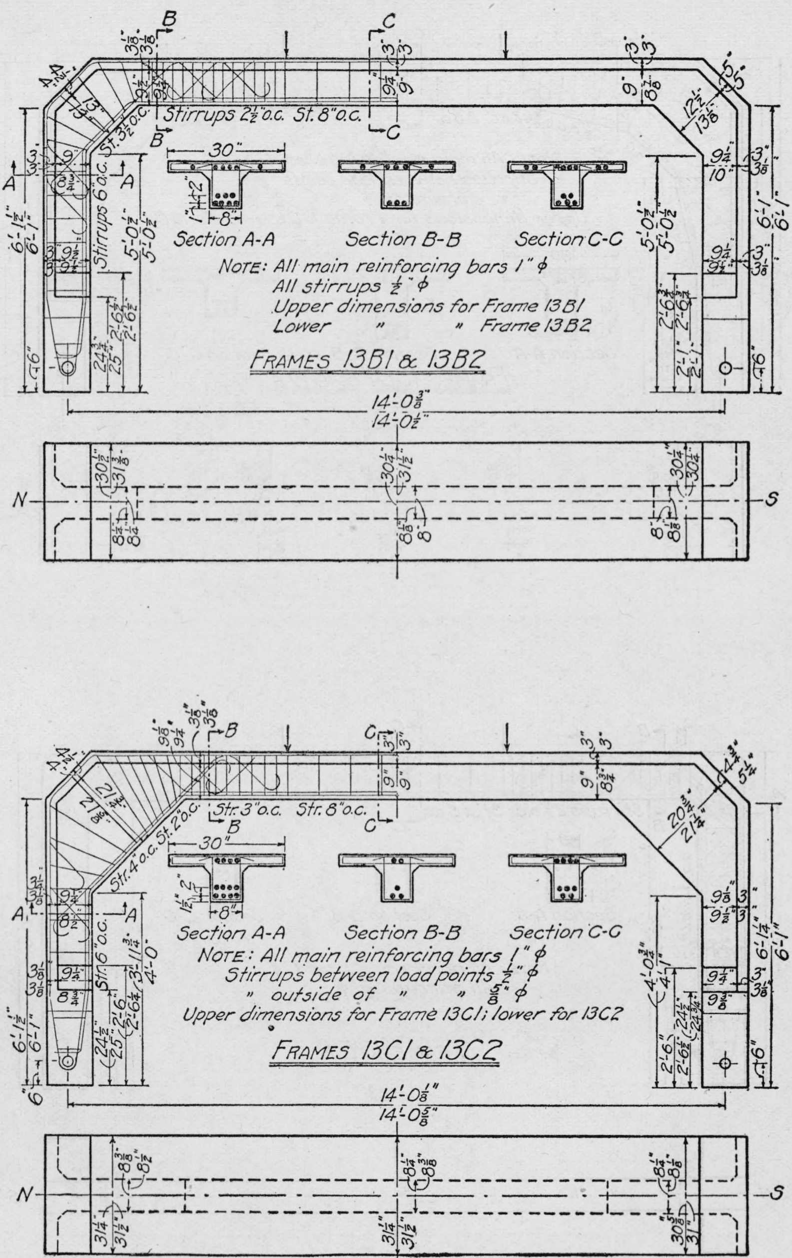

FIG. 13.—*Details of test specimens 13B1, 13B2, 13C1, and 13C2*

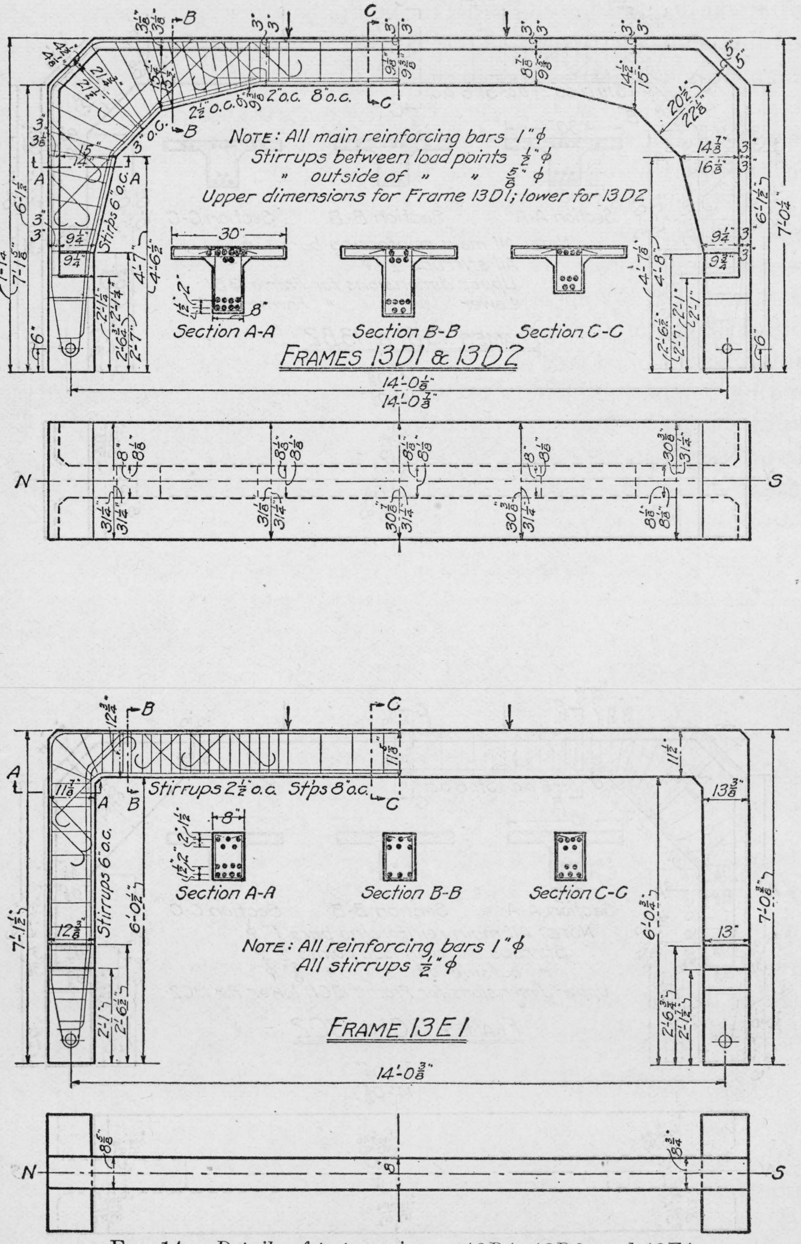

FIG. 14.—*Details of test specimens 13D1, 13D2, and 13E1*

In the design of the reinforcement used in the test specimens the working stresses assumed were 16,000 lbs./in.^{2} for tension in the steel, and 1,500 lbs./in.^{2} for compression in the concrete. The ratio of the modulus of elasticity of steel to that of concrete was taken as 8. The design provided for a reversal of the direction of loading, so that all sections contained a large percentage of compression as well as of tension reinforcement. Details of the reinforcement used in the specimens are shown in Figures 12 to 14.

In designing the members the approximate bending and resisting moments were calculated and the section at which failure by compression would probably occur was determined. From the approximate resisting moment of the compressive stresses at this point the working load for the specimen was calculated, and sufficient tension reinforcement was provided at all points to withstand the external bending moment without exceeding a computed stress of 16,000 lbs./in.^{2} The specimens were heavily reinforced with bent bars and stirrups against diagonal tension failure. Table 1 shows the percentage of longitudinal steel used in all specimens, based on the area of cross section exclusive of flanges.

TABLE 1.—*Percentage of longitudinal reinforcement*

Location of section	Stress in reinforcement (downward loads)	Percentage of reinforcement for specimens Nos.[1]—			
		13A1–2 13E1–	13B1–2	13C1–2	13D1–2
Center of girder	Tension	4.67	4.67	4.67	4.67
	Compression	5.61	7.48	3.74	3.74
Corner of frame [2]	Tension	4.67	3.79	2.05	2.05
	Compression	5.61	4.41	2.46	2.46

[1] This investigation was performed as test series 13; hence, the series number is used as a part of all specimen numbers.
[2] Vertical section for types A and E; section normal to face of bracket in types B, C, and D.

The main reinforcing bars were all 1-inch plain round bars, and the stirrups were either one-half or five-eighths inch plain round bars. The tee flanges were reinforced with cross rods to resist transverse bending in the flanges.

2. MATERIALS AND MAKING OF SPECIMENS

(*a*) CEMENT.—Lehigh Portland cement was used in the making of all the test specimens. It passed the requirements of the United States Government specifications for Portland cement.[7]

(*b*) AGGREGATE.—The sand and gravel were obtained from local deposits at South Bethlehem, Pa. The material was siliceous, clean, and gritty. It was carefully separated by screening into three sizes—(1) Fine sand consisting of grains smaller than one-eighth inch in

[7] Circular of the Bureau of Standards, No. 33; third edition; Jan. 18, 1917.

diameter, (2) coarse sand falling between one-eighth and one-fourth inch in diameter, and (3) gravel exceeding one-fourth inch but less than one-half inch in diameter. The separation of the sand into fine and coarse grades was introduced to avoid the lack of uniformity in the concrete mixture which would result from segregation of sizes in the bin.

(*c*) STEEL.—The reinforcing bars were rolled from rejected shrapnel steel billets of high yield point. The physical properties of this steel are shown in Table 2. Each value in the table is the average of two tests.

TABLE 2.—*Physical properties of reinforcing steel*

Diameter of bar (inches)	Yield point	Ultimate tensile strength	Elongation in 8 inches	Reduction in area
	Lbs./in.²	*Lbs./in.²*	*Per cent*	*Per cent*
½	63, 800	104, 400	16. 6	27. 5
⅝	66, 070	108, 440	18. 0	42. 2
1	55, 800	91, 420	20. 5	38. 2

(*d*) CONCRETE.—The concrete used was mixed in the proportions 1 : 1 : 1 by volume. The unit quantity of sand consisted of 0.8 part fine sand and 0.2 part coarse sand. In preparation for mixing the concrete each kind of aggregate was thoroughly mixed by shoveling, and determinations of the moisture content were made upon samples taken at random. Enough water was added when the concrete was mixed to make the total water 13 per cent of the combined weight of the dry materials, thus producing a rather stiff mixture, considerably drier than is generally used in reinforced concrete construction work.

Six 6 by 12 inch cylinders were made with each test specimen and were stored with the specimen until they were tested. Three cylinders in each lot were tested at the age of 7 days and the remaining three at the age of 40 days, which was the approximate age of the frames when tested. The average compressive strength of the concrete is given in Table 3. The average initial modulus of elasticity of the concrete in the frames was assumed to be 3,750,000 lbs./in.²; this value was found from a large number of compression tests of concrete cylinders having identical proportions and materials but made in connection with other investigations at the laboratory. The stress-strain curves for these cylinder tests were closer to straight lines than is usually expected with concrete.

TABLE 3.—*Compressive strength of concrete*

[Each value represents the average strength of three 6 by 12 inch cylinders]

Made with specimen number	Compressive strength at age of 7 days	Compressive strength at age of 40 days	Made with specimen number	Compressive strength at age of 7 days	Compressive strength at age of 40 days
	Lbs./in.²	*Lbs./in.²*		*Lbs./in.²*	*Lbs./in.²*
13A1	2, 220	3, 110	13D1	2, 870	4, 420
13A2	2, 975	3, 765	13D2	3, 075	5, 860
13B1	2, 880	4, 260	13E1	2, 815	4, 995
13B2	2, 535	3, 665			
13C1	2, 775	4, 505	Average	2, 785	4, 290
13C2	2, 900	4, 040			

(*e*) MAKING OF SPECIMENS.—One wooden form was used for all test specimens, and the inside corners of the form were designed to provide for the variation in shape of the brackets. The inner surfaces of the forms were well oiled. In order to insure plumbing of the specimen and proper alignment of the hinges in the columns of the test specimens, the form was erected in position on the large concrete base used in making the load test, with the steel shoes and hinge pins in position. The reinforcing bars were bent as required, in an Olsen cold-bend testing machine and were wired in place after being set in the form. Concrete was dumped from the mixer into a tight wooden box, carried to the form by a traveling crane, and shoveled into the form. A considerable amount of tamping and rapping was required to get the concrete into place, especially at the corners of the frame. The forms were stripped when the concrete was about 24 hours old, and the specimen was lifted off the test base and transferred to another place in the laboratory. Wet burlap was kept wrapped around the specimens up to the time of testing.

3. TESTING APPARATUS

A heavy concrete test base was made especially for this investigation. A general view of the base with a specimen in position for testing is shown in Figure 16. The base was 22 feet long, 5½ feet high and 2½ feet wide, and was reinforced to withstand a reversal both of vertical loads and horizontal thrusts. A vertical steel link at one end allowed for a practically frictionless horizontal movement at the bottom of one leg of the specimen under test. A 60-ton hydraulic jack acting against this link was arranged to produce a horizontal reaction through the axis of the hinge and either to maintain a fixed distance between the two hinges or to move the hinge in or out any desired amount. Such movement was measured by means of a screw micrometer bar.

Downward loads were applied on the top of all specimens at two points 2 feet 4 inches on each side of mid span. The distance between the load points was one-third of the nominal span from center to center of hinges. The diagram of Figure 15 shows the arrangement for applying and measuring loads and reactions. The vertical loads were produced by two 100-ton hydraulic jacks acting downward on a steel box girder which transmitted the pressure through a heavy knife-edge casting and a roller to the specimen. Steel plates embedded in plaster of Paris were used to distribute the bearing pressure over the concrete. The upward reaction of the jacks was exerted against built-up steel sections connected by six tie rods to other steel sections beneath the test base.

Two strain gauges were used in the tests, one of 4 inch gauge length for measuring deformations of concrete and one of 8 inch gauge

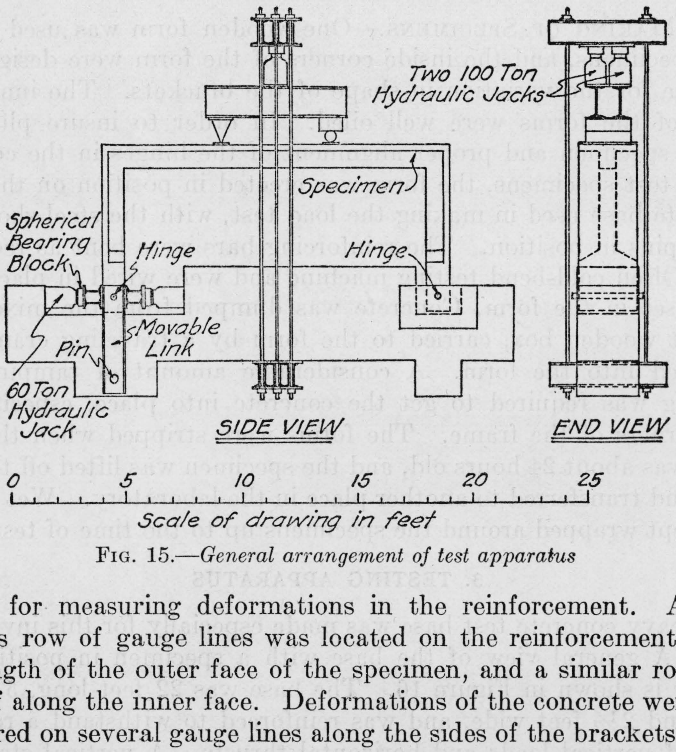

FIG. 15.—*General arrangement of test apparatus*

length for measuring deformations in the reinforcement. A continuous row of gauge lines was located on the reinforcement along the length of the outer face of the specimen, and a similar row was laid off along the inner face. Deformations of the concrete were also measured on several gauge lines along the sides of the brackets.

Deflections were measured at points 1 foot apart on the horizontal member and on the two columns. A black linen thread was stretched at constant tension between points at the two ends, at mid depth of the girder. Similar threads were hung as plumb lines along the sides of the columns. Movement of the specimen with reference to the thread was observed by means of paper scales pasted to small mirrors and attached to the specimen. Readings, which were taken by lining up one edge of the thread with its reflection in the mirror, could be duplicated within 0.01 inch.

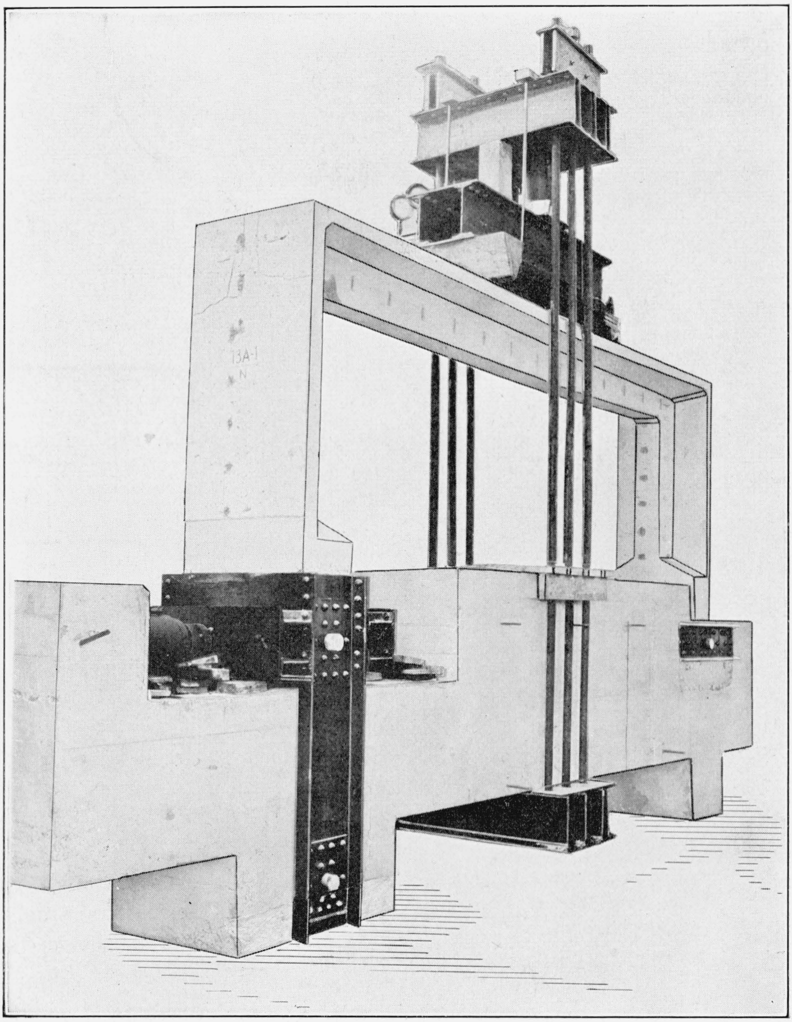

FIG. 16.—*General arrangement of test apparatus*

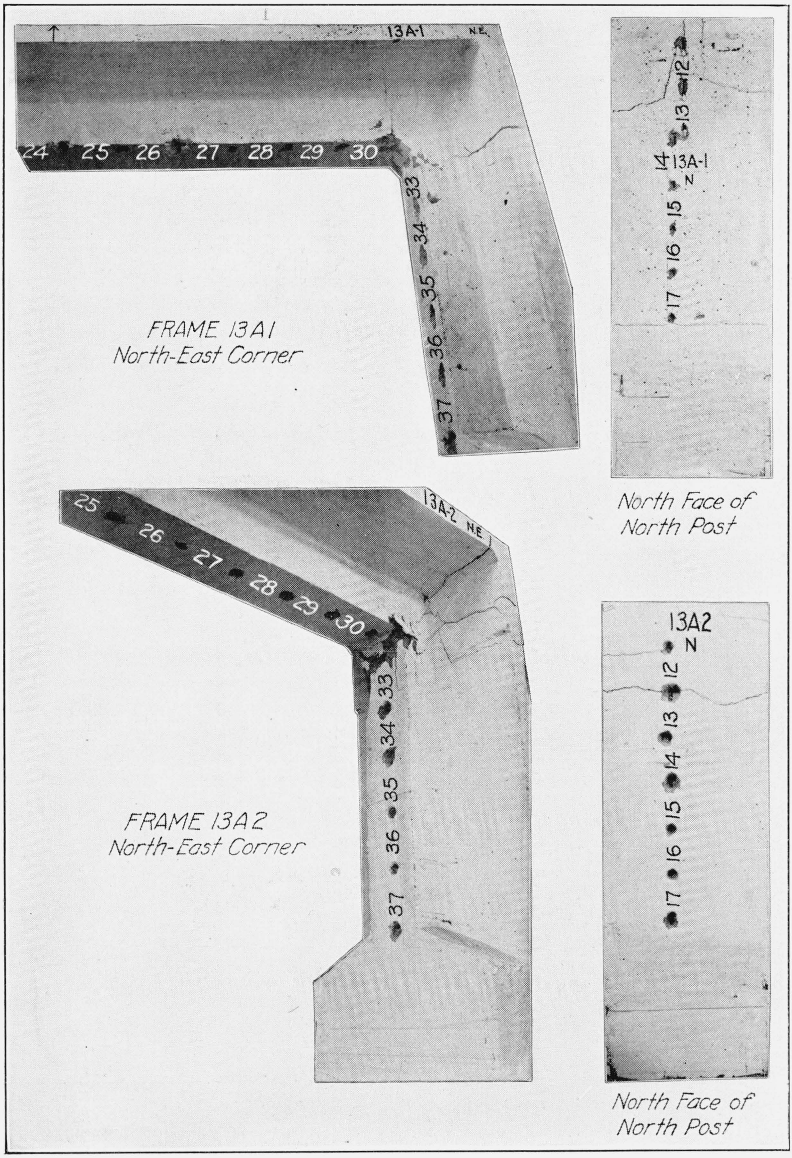

FIG. 17.—*Specimens 13A1 and 13A2 after test*

IV. TEST DATA AND DISCUSSION OF RESULTS

1. PROCEDURE AND PHENOMENA OF TESTS

In general, downward loads were applied to the test specimens in increments of 30,000 pounds, with the hinges at the bases of the columns held in a stationary position. Zero readings were taken with no downward load except that of the loading rig, which weighed about 3,000 pounds, and with just enough horizontal pressure applied at the hinges to tighten up all movable parts of the hinge apparatus.

The effect of a reversal of stress was obtained in the testing of each specimen in the following manner: After readings of deformation and deflection had been taken under a load of 90,000 pounds all downward load was released. The movable hinged end of the specimen was then pushed inward an amount sufficient to produce maximum deformation readings at critical sections as great as those observed under the 90,000-pound vertical load. Following readings under this condition of loading and still without applying any vertical loads, the horizontal jack was swung around to act on the inside of the hinge, and the movable end of the column was thrust outward until stresses were again produced which were comparable to those observed under the 90,000-pound load. The change in distance between hinges was measured in both cases. With the horizontal jack swung back to the outside of the frame and the distance between hinges brought back to its original amount, vertical loading was resumed on the top of the frame. Complete readings of deformation and deflection were taken at a load of 120,000 pounds, and at increments of 30,000 pounds up to the maximum load. Final readings were taken in each case after the maximum load was applied, in order to obtain information on the manner of failure. Figures 17 to 21 show views of the different specimens after failure had taken place. The following paragraphs give a short description of the principal phenomena of the tests.

Specimen 13A1.—Loads were applied as described in Section IV, 1. Numerous cracks were observed under the 60,000 and 90,000 pound loads. After application of end thrusts inward and outward with top load released the movable hinge did not return entirely to its original position but was pushed back into place with little effort. Failure occurred at a maximum load of 120,000 pounds with noticeable crushing and spalling at the north inside corner. This spalling was apparently due largely to slipping of bars at the inside face of the column near the corner. There was also apparent slipping of tension bars near mid span. Tension cracks were numerous in the middle portion of the frame, also across the tee flanges on both vertical and horizontal members near both corners.

Specimen 13A2.—A number of tension cracks appeared near mid span at the 30,000 pound load. Cracks also appeared across the tee flanges about 15 inches from each corner on both horizontal and vertical members. At loads of 60,000 and 90,000 pounds several more cracks opened in the same regions. With inward thrusts several new cracks opened in the top face of the girder within the middle half of the span. With outward thrusts tension cracks were opened at the inside corners of the frame. Failure occurred under vertical load through crushing at the north inside corner and by yielding of the steel in tension at the outside of the same corner. While the tension failure may have occurred first, the yield point was exceeded in the compression reinforcement at the corner, and also in the tension reinforcement under the load points. There were pronounced radial cracks around both corners of the frame. Maximum load, 119,000 pounds.

Specimen 13B1.—Failure began with crushing of the concrete at the north inside corner at the intersection of bracket and girder. After this corner had yielded somewhat a large number of diagonal cracks appeared between the bracket and load point. Cracks were not large at other parts of the frame. Horizontal cracks in the tee flange at the north end of the girder indicated that the flange was shearing loose from the web at failure. Large tension cracks were observed on the top face of the girder at the north end. Some crushing occurred at the top of the south bracket and at the bottom of the north bracket. Maximum load, 152,000 pounds.

Specimen 13B2.—At a load of 30,000 pounds there were a few straight tension cracks on the lower side of the girder. With a load of 60,000 pounds, a number of cracks opened on the outside faces of the columns and on the top face of the girder near the ends. With inward thrusts, a few additional cracks appeared in the top face of the girder, one being between the load points. With outward thrusts, a few cracks opened at the junctions of girder and brackets. At a load of 120,000 pounds pronounced diagonal tension cracks developed in the web of the girder, running outward from the load points at an angle of about 45°. The maximum load was reached at 138,000 pounds. Failure came when the yield point of the steel was reached in the outside face of the south column and at mid span. At about the same time crushing occurred at the junction of the north column and bracket, and the concrete spalled off considerably. There was a slight indication of crushing at the south end, near the junction of the column and bracket.

Specimen 13C1.—Several cracks opened near mid span at a load of 30,000 pounds. At a load of 60,000 pounds cracks opened at about mid height of the outside faces of both columns. With inward thrusts, cracks opened in the outside face of the columns, and one or two opened at each end on the top of the girder. With outward thrusts, cracks opened in the upper part of each bracket and ran down

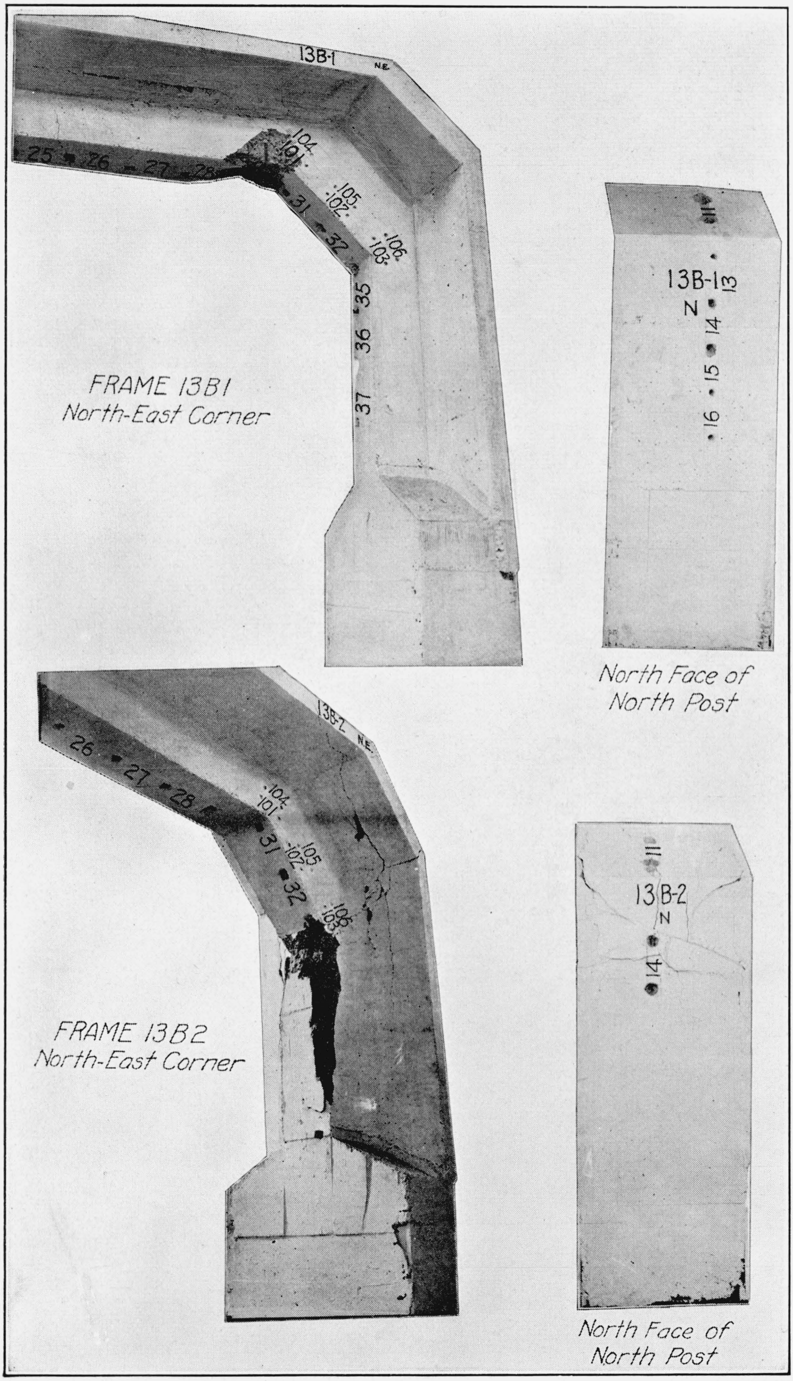

FIG. 18.—*Specimens 13B1 and 13B2 after test*

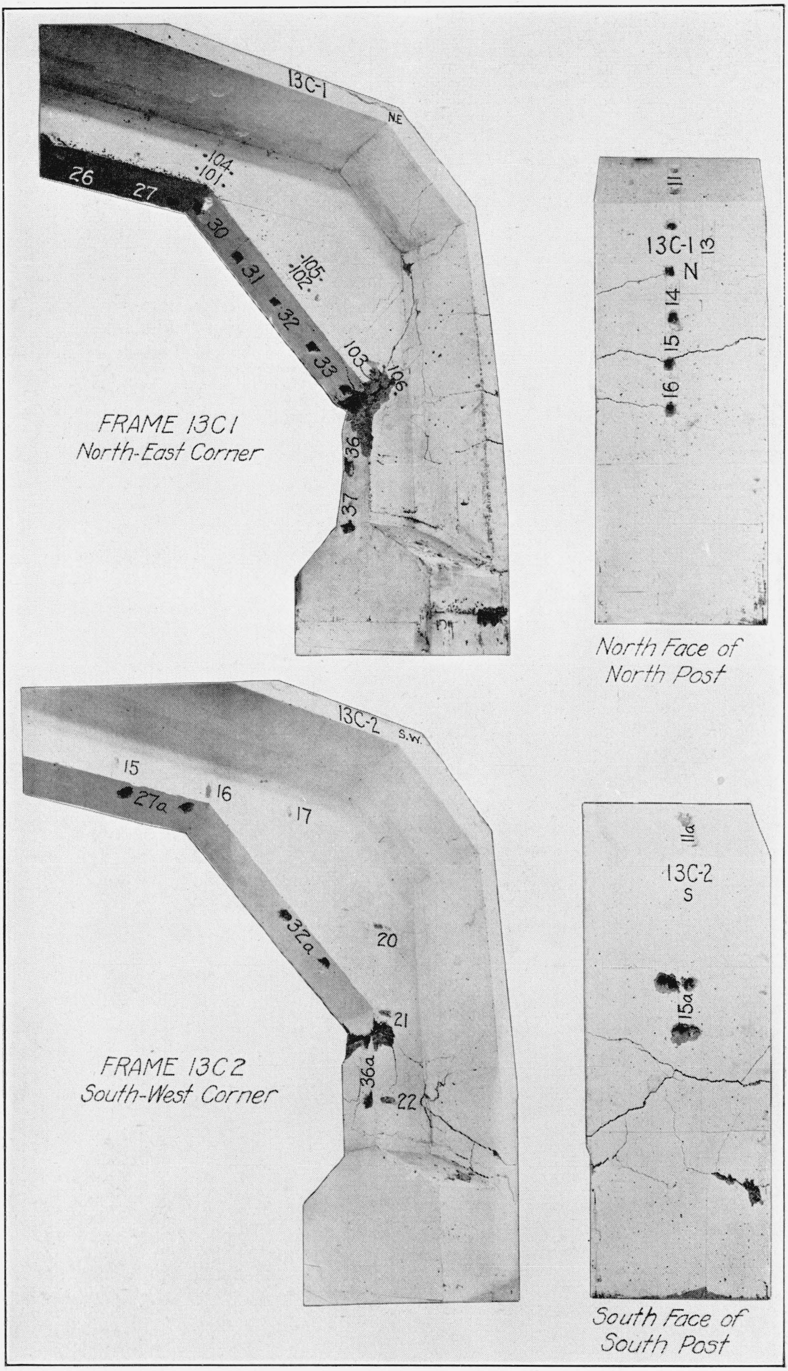

FIG. 19.—*Specimens 13C1 and 13C2 after test*

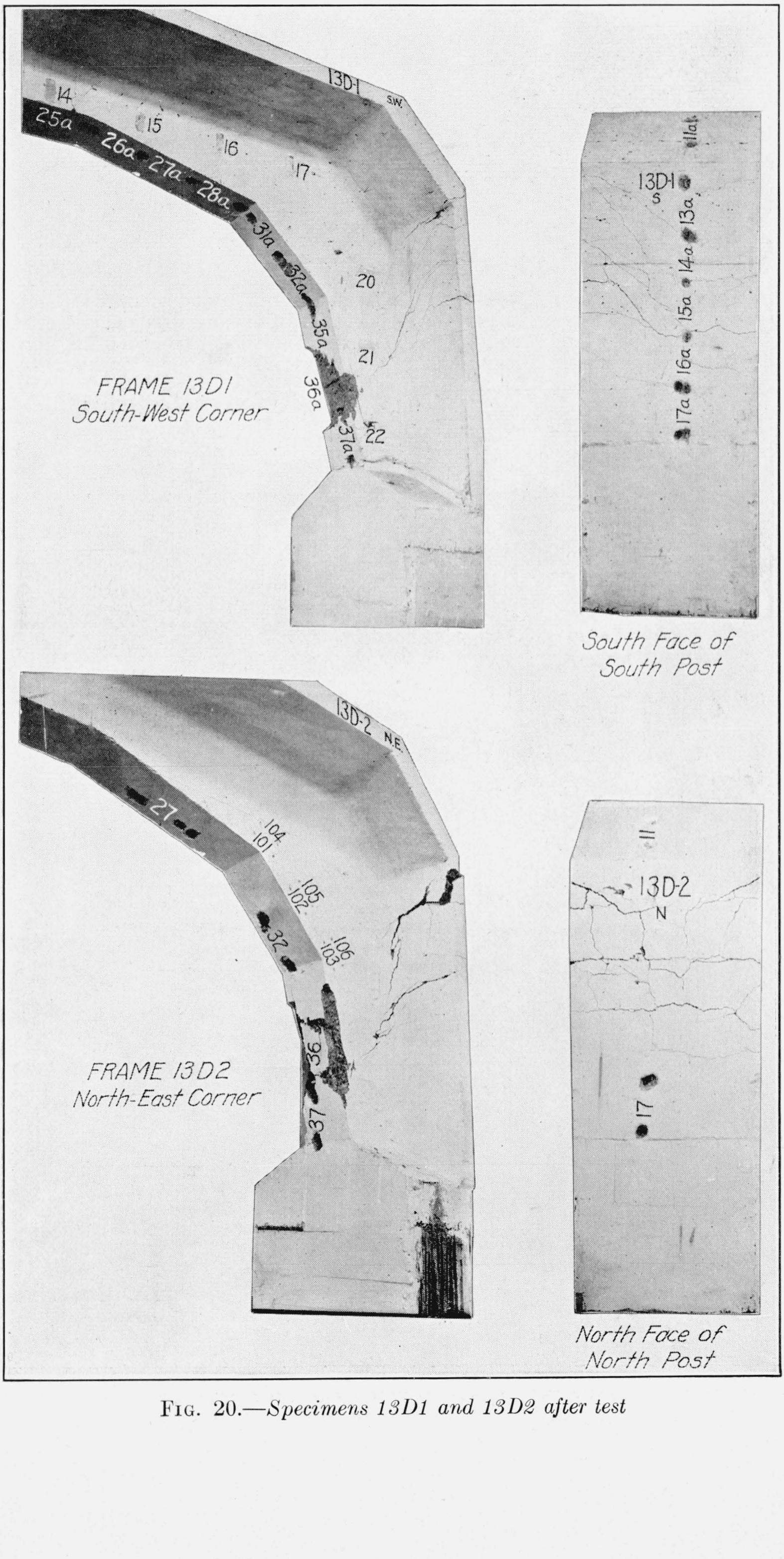

FIG. 20.—*Specimens 13D1 and 13D2 after test*

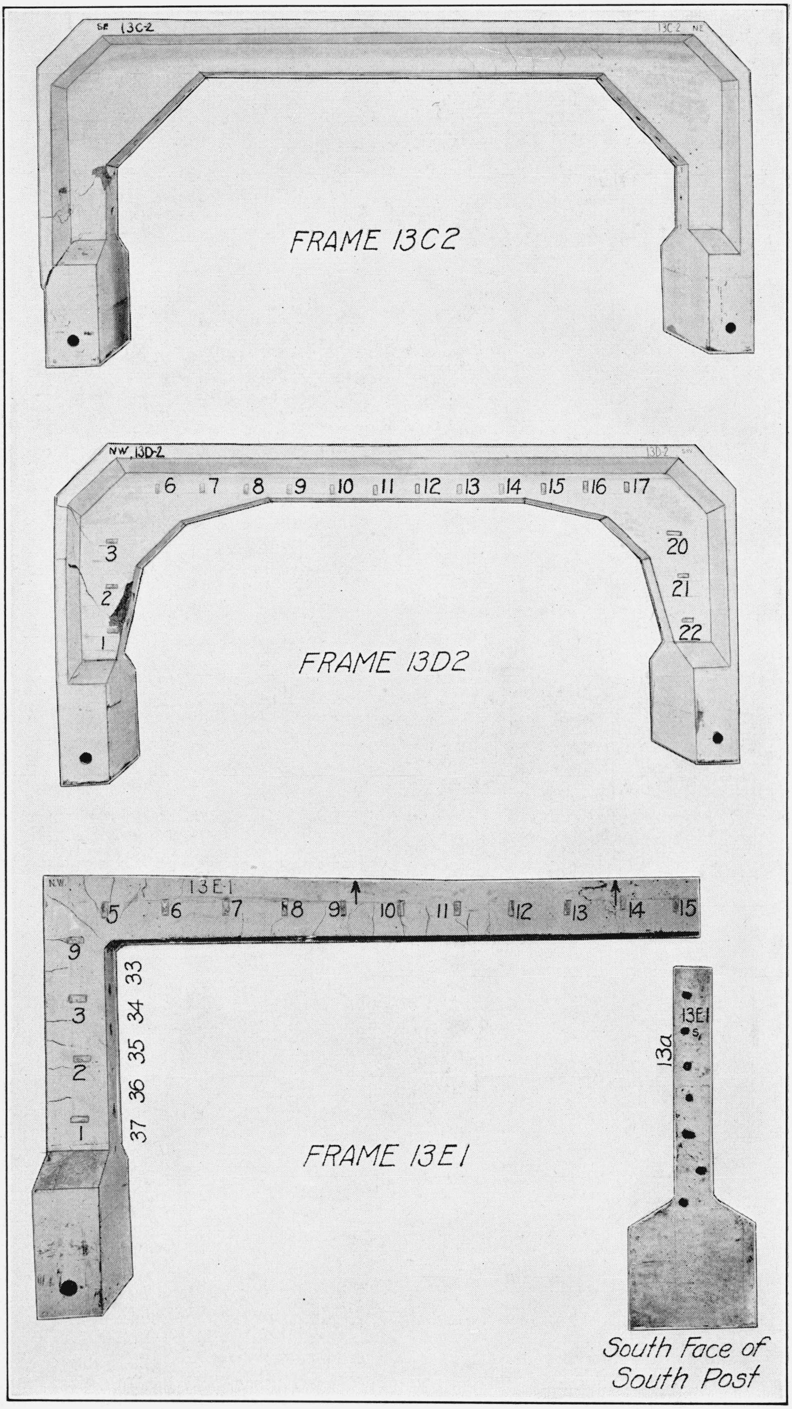

FIG. 21.—*Specimens 13C2, 13D2, and 13E1 after test*

at about 45° with the horizontal, parallel to the face of the bracket. Cracks of this type were produced in all frames with this kind of loading and were at right angles to those produced by inward thrusts. At loads of 120,000 and 150,000 pounds cracks began to run through from the outer face of each column downward diagonally across the webs toward the inside face at the junction of bracket and column. At the maximum load of 182,000 pounds crushing occurred at the bottom of the north bracket. Large cracks opened in the outside face opposite the crushed area, and the yield point of the tension steel was passed here and at mid span.

Specimen 13C2.—A few cracks were observed near mid span at a load of 30,000 pounds, and others opened across the outer face of each column at the loads of 60,000 and 90,000 pounds. Several cracks opened in the upper face of the girder when inward thrusts were applied alone. With outward thrusts the cracks were similar to those found in specimen 13C1. At a load of 120,000 pounds large cracks opened in the outside faces of columns. This specimen was weakened by the accidental omission of two of the four longitudinal reinforcing bars in the outer face of each column. Due to this, failure occurred in the outside face of the south column, opposite the bottom of the bracket where crushing failure rapidly followed. The maximum load was 148,000 pounds.

Specimen 13D1.—A few cracks were observed near mid span at the load of 30,000 pounds, and others opened across the outer faces of both columns at loads of 60,000 and 90,000 pounds. Several cracks opened in the upper face of the girder when inward thrusts were applied and under outward thrusts a large tension crack opened just outside of the south load point. At later loads this crack gave the impression of impending diagonal tension failure; however, failure did not occur in this part of the frame. At a load of 180,000 pounds large cracks appeared at both ends at the top of the vertical faces of the columns. The maximum load was 208,500 pounds. Failure occurred when the yield point of the reinforcement was reached simultaneously in compression on the inside face and in tension on the outside face of the south column at about mid height. The concrete crushed over a considerable area in the locality of the failure.

Specimen 13D2.—A few cracks were observed on the outer faces of the columns near the corners and near mid span at the 60,000-pound load. With inward thrusts, several cracks opened on the top face of the girder. With outward thrusts, cracks on the tension side of the girder were opened. Under the 180,000-pound load larger cracks appeared across the face of the north column, and failure occurred by yielding of the reinforcement of the north column in tension about 3 feet from the top and by crushing on the inside of the column below the junction of the bracket and the lower haunch.

A number of diagonal cracks ran between the sections of tension and crushing failures. Maximum load, 232,000 pounds.

Specimen 13E1.—In this specimen of rectangular cross section the reinforcement was crowded together closely and was probably not as nearly in its designed position as in the other specimens. At a load of 30,000 pounds there were a large number of cracks near mid span, around the corners of the frame, and across the outside faces of the columns. At 60,000-pound load several diagonal cracks had opened between the south load point and the end of the girder. At the maximum load of 77,000 pounds the top of the specimen began to crush just inside the south load point. The crushing extended from the edge of the bearing plate for a distance of several inches. Large tension cracks formed under the south load point and at the south corner. It was noticeable that a large number of tension cracks developed around the corners of the frame, radiating toward the inside corner as a center. These cracks extended to within 3 inches of the inside or compression face.

Values of the ultimate loads and other information regarding the tests are given in Table 4.

TABLE 4.—*Results of tests*

Specimen number	Age	Ultimate load	Manner of failure
	Days	*Pounds*	
13A1	43	120, 000	Bond at outside of north corner, followed by crushing at inside of corner; tension at middle between load points.
13A2	39	119, 000	Compression at inside of north corner; tension at outside of corner and at middle between load points.
Average	41	119, 500	
13B1	42	152, 000	Compression at top of north bracket; some shearing between the flange and web at north corner.
13B2	40	138, 000	Tension at middle of top member and at outside of north corner on column; followed by crushing at bottom of north bracket; shearing between tee and web at corner.
Average	41	145, 000	
13C1	39	182, 000	Compression at bottom of north bracket; tension at outside of north bracket and in top member between load points.
13C2	41	148, 000	Tension at south end in outside face at bottom of bracket, followed by crushing on inside at bottom of bracket; this specimen was weakened by the accidental omission of two of the reinforcing bars in the outside face of each column, where failure occurred.
Average	40	165, 000	
13D1	40	208, 500	Compression at south end at middle of lower haunches and tension in outside face of column at bottom of bracket.
13D2	41	232, 000	Compression at north end at bottom of lower haunch, tension at outside face at bottom of bracket, and in top member between load points.
Average	41	220, 250	
13E1	41	77, 000	Compression just inside of south load point.

2. THRUSTS AND MOMENTS

The ratio of the end thrusts to the downward load for the tests of frames with hinges held stationary was calculated from the gauge readings of the hydraulic jacks. This ratio, in general, seemed about constant for each specimen until near the maximum load, when it decreased considerably. The variation could probably be ascribed to a decrease in the moment of inertia of section at points of high stress. Obviously at stresses near the ultimate strength less elasticity of action would be expected than at lower loads.

Table 5 presents (*a*) ratios of horizontal thrust to total vertical load (average for all but ultimate loads), (*b*) average actual bending moments calculated from the measured vertical loads and horizontal thrusts, and (*c*) bending moments determined by the analytical method described in Section II, 1, on the assumptions that the moment of inertia varied throughout all specimens—first, as the cube of the depth of section and, second, as the 5/2 power of the depth of section. A fair agreement between calculated moments and the average of observed moments is seen, but the variation of individual values from the averages is large.

TABLE 5.—*Comparison of observed and calculated moments*

Specimen number	$\frac{H}{P'}$ average of test	Ratio of bending moment to $\frac{Pl}{6}$					
		From test—		Calculated ($I=Qd^3$)—		Calculated $I=Qd^{5/2}$—	
		At mid span	At corner	At mid span	At corner	At mid span	At corner
13A1	0.168	0.568	0.432				
13A2	.202	.480	.520				
Average	.185	.524	.476	0.481	0.519	0.481	0.519
13B1	.200	.486	.426				
13B2	.236	.392	.520				
Average	.218	.439	.473	.370	.524	.379	.517
13C1	.241	.380	.438				
13C2	.305	.215	.602				
Average	.273	.297	.520	.258	.536	.271	.526
13D1	.312	.198	.574				
13D2	.352	.095	.674				
Average	.332	.146	.624	.186	.590	.203	.577
13E1	.218	.440	.560	.481	.519	.481	.519

Values of the bending moment at mid span, taken from the third and seventh columns of Table 5, are plotted in Figure 22 as ordinates against horizontal lengths of bracket as abscissas. Since the frames of type D had a haunch different in shape from those of the other frames, it was found convenient to reduce it to an equivalent length of 45° bracket. From calculations it was found that the haunch of type D would produce the same moment at mid span as a 45° bracket 32 inches in horizontal length. The points in Figure 22 which represent average bending moments as determined by test are seen to agree fairly well with the calculated bending moments based on equation (1). Considering the variable nature of the materials, the difference in details of design and dimensions, and the various possible sources of error in observations, considerable differences between calculated moments and the moments determined from the measured strain in individual specimens are to be expected.

The variation of the bending moment throughout the frames may be seen in Figure 23, which shows graphically the relative magnitude of the actual bending moments and their distribution in the different types of frame as determined by the tests. This variation is due to two distinct factors—variation in stiffness and variation in the shape of the axis of the frame—as noted in Section II, 1. It is seen that while the moment at mid span varies with the size of the brackets,

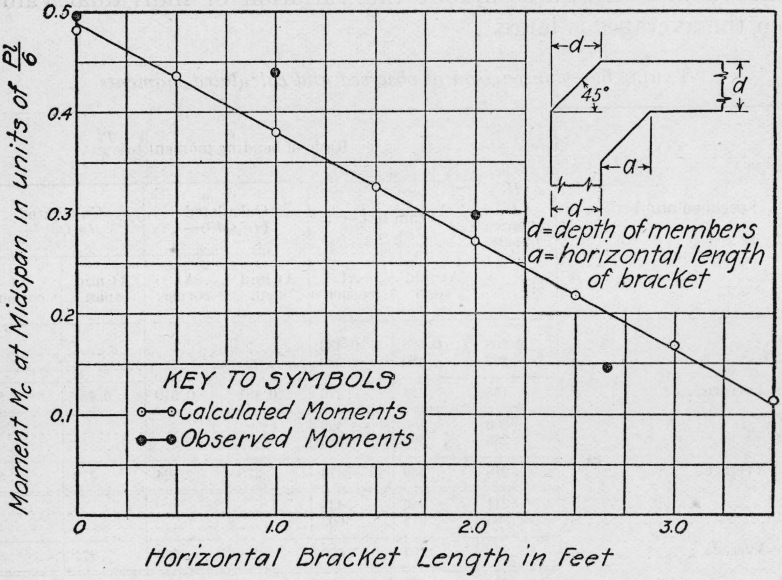

FIG. 22.—*Calculated and observed moments at mid span*

the moment at the corner of the frame is about the same in all types of frame.

3. FLEXURAL STRESSES AND DEFORMATIONS

Tabulated strain gauge data from the tests of all frames are given in Appendix I, which also contains diagrams showing the positions of all gauge lines for measurement of deformations and deflections and shows the positions with respect to these gauge lines of cracks and crushed areas observed at the maximum loads.

Stresses in the steel reinforcing bars as determined from strain gauge measurements are shown graphically in Figures 24 to 26. The stresses were measured on the reinforcing bars which were within the concrete 1½ to 3 inches from the surface. The points where a line lying about 2 inches (to proper scale) within the frame intersects the lines radiating from the axis of the frame indicate the positions of the centers of the gauge lines in which the strains were measured, and the distance from the axis along the radiating line indicates the intensity of the stress. Within straight portions of the frame the solid lines connecting the stress at successive gauge lines give a stress graph which indicates roughly the probable values of the stress at points intermediate between gauge lines. At angles in a frame the stress can not be so interpolated with even approximately correct results. Light broken lines are used to connect the points wherever

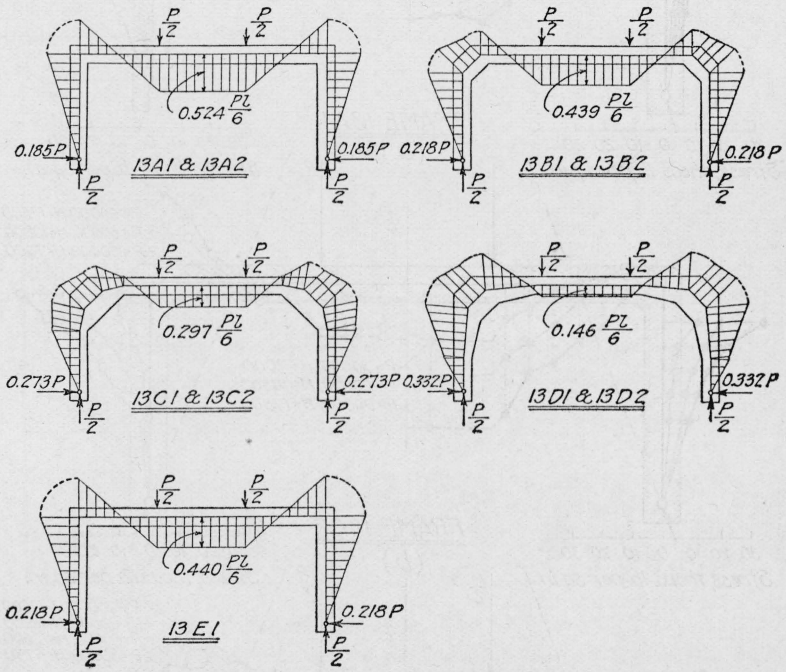

FIG. 23.—*Relative moments in frames*

a direct interpolation of this kind is not permissible. While the stresses shown represent combined flexural and direct stresses, the latter (which were compression for all cases of downward loading) were comparatively small and did not exceed 7 per cent of the total maximum stress in the extreme case of the frames of type D. For the other frames the direct stresses are much less, and, hence, do not have an appreciable influence upon the total stresses measured.

The calculation of stresses in the concrete and steel of these frames is quite laborious, due to the many changes in cross section and reinforcement and the variation in the bending moment. A sufficient number of calculations have been made, however, to show a

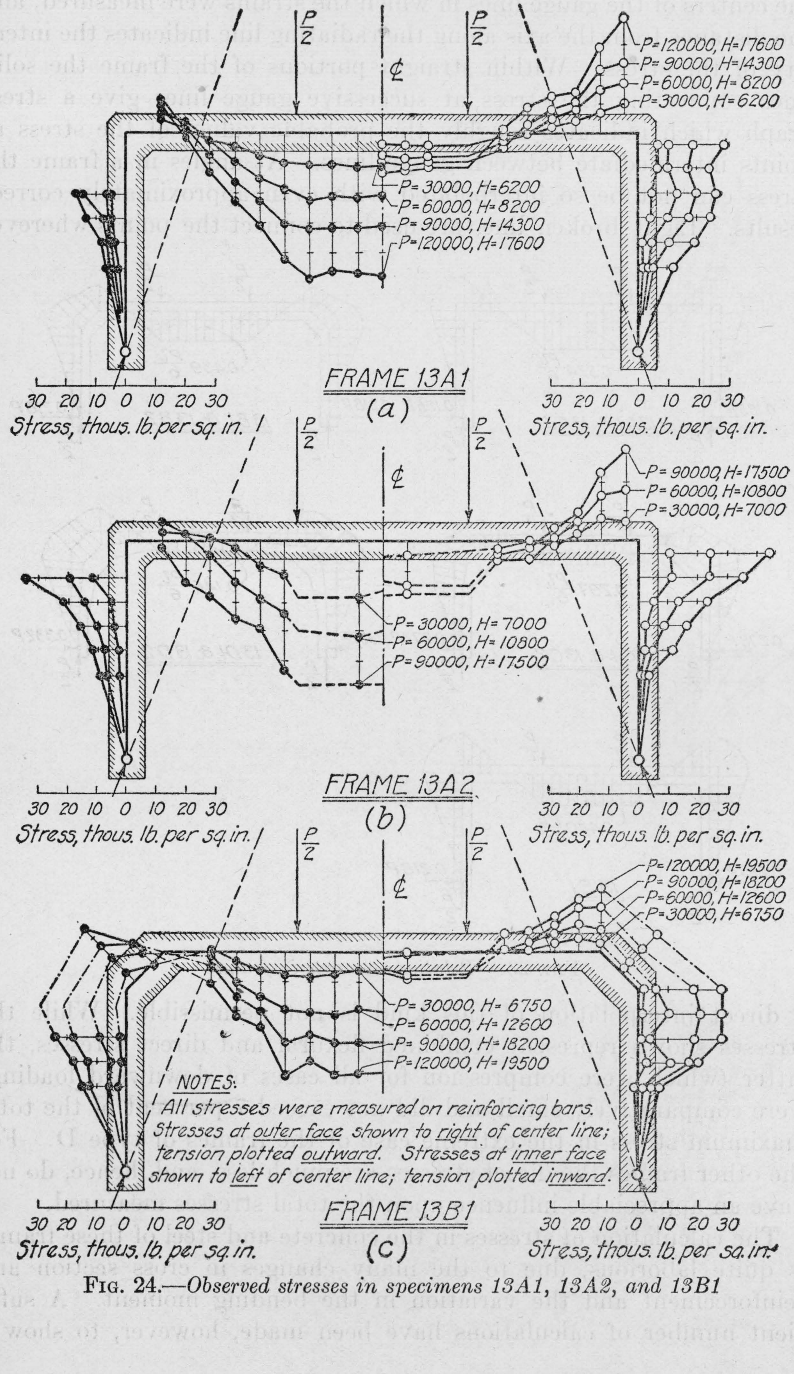

FIG. 24.—*Observed stresses in specimens 13A1, 13A2, and 13B1*

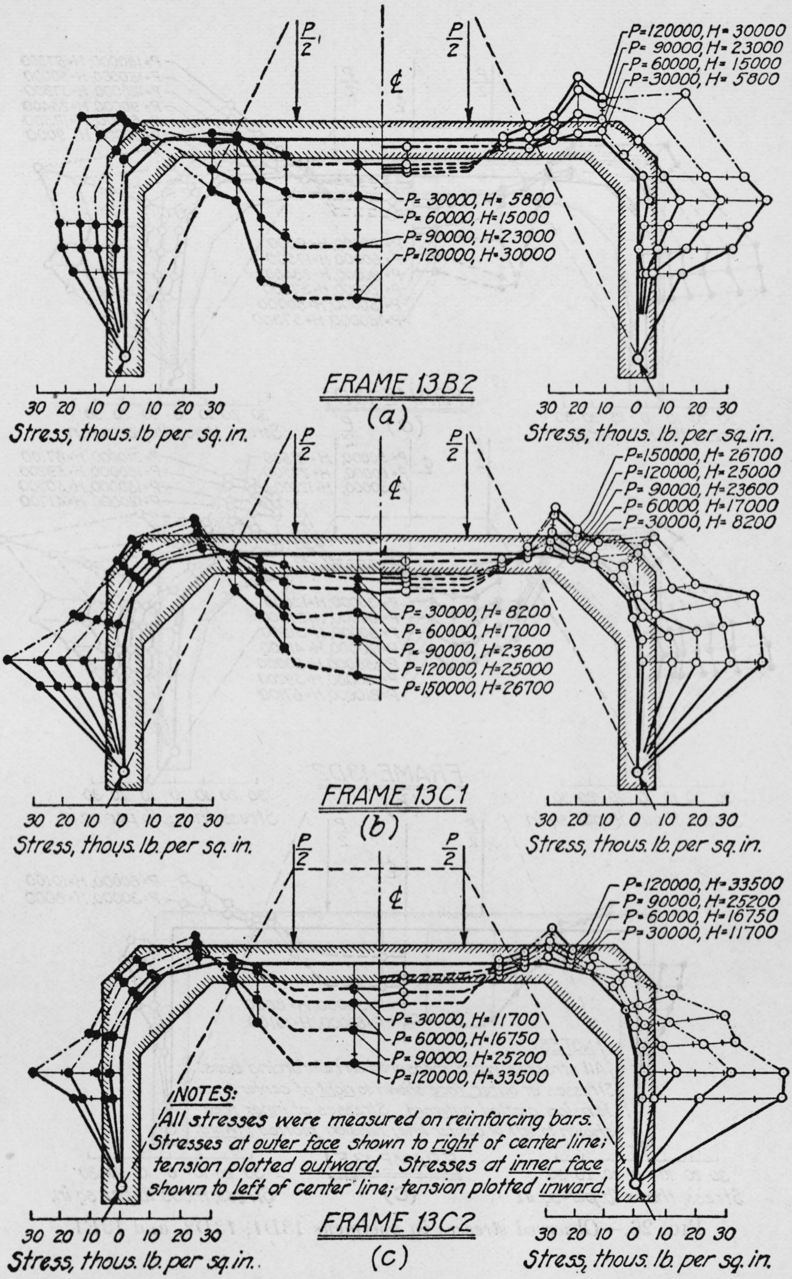

FIG. 25.—*Observed stresses in specimens 13B2, 13C1, and 13C2*

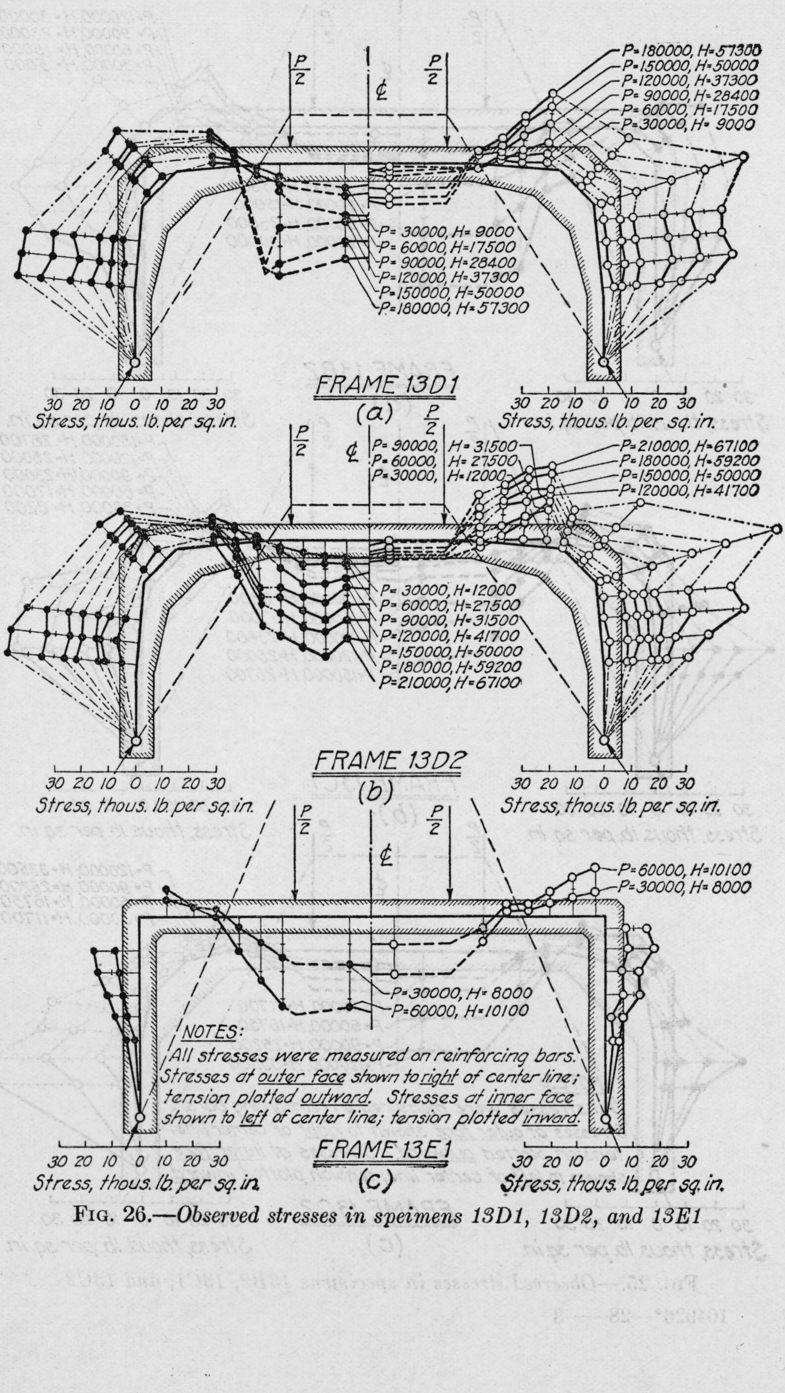

FIG. 26.—*Observed stresses in speimens 13D1, 13D2, and 13E1*

fair agreement with the results of the tests. An example of the comparison between calculated and observed stresses is shown in Table 6, which gives stresses in the tension reinforcement at mid span due to a number of different loads. The observed stresses were determined from the average deformations measured on gauge lines 22 and 22a of each specimen, while the calculated stresses are based on bending moments computed from the known loads and reactions on the frames and include the small compressive direct stresses which existed at the section considered. The calculations were made on the conventional "straight-line theory" of stress distribution and follow the assumption that no tension is carried by the concrete. In most cases the table shows the calculated stress in the reinforcement to be higher than that found from test, a relation between calculated and observed stresses which has often been found to exist.

TABLE 6.—*Comparison of calculated and observed tensile stresses in reinforcement*

Specimen number	Stress	Load on specimen (pounds)				
		30,000	60,000	90,000	120,000	Maximum load
13A1	Calculated	11, 500	32, 000	42, 300	61, 400	61, 400
	Observed	12, 000	21, 800	31, 600	(1)	(1)
13B1	Calculated	10, 000	22, 000	34, 500	55, 900	60, 400
	Observed	7, 100	19, 600	30, 000	40, 100	44, 500
13C1	Calculated	7, 100	13, 000	23, 600	45, 500	46, 500
	Observed	3, 600	13, 700	21, 700	31, 000	(1)
13D1	Calculated	5, 300	11, 700	12, 900	18, 500	50, 000
	Observed	7, 500	11, 000	18, 200	20, 200	58, 200
13E1	Calculated	8, 600	32, 400			42, 900
	Observed	18, 100	34, 900			47, 800

[1] Yield point of steel, 55,000 to 60,000 lbs./in.^{2}

It was anticipated that high stresses would be developed at the corners of the frames, particularly in types A and E, which had only 2-inch fillets at the inside corners, and strain measurements were taken around the corners with the idea of locating the position of the neutral axis. The readings taken did not give very complete information, but it was evident from the information gained that the neutral axis approached the inside corner of the frame very closely, so that high compressive stresses were produced by the usual vertical loading. It is evident that such concentrations of fiber stress should be reduced, and it would seem advisable that a fillet or bracket be used in a sharp corner of this kind. A curved fillet or bracket should produce the best variation of stress around the corner.

4. SHEARING STRESSES

In all cases the frames were highly reinforced against diagonal tension by the use of U stirrups and bent-up longitudinal bars. The ends of stirrups were hooked, being bent out into the flange of the T sections and bent inward in the rectangular sections. The bent-up bars were also anchored by semicircular hooks having a radius of four diameters of bar.

The effectiveness of the web reinforcement was demonstrated by the fact that none of the frames failed by diagonal tension, and that diagonal cracks were, in general, quite small. Knowing this, the shearing unit stresses which were developed seemed quite noteworthy, inasmuch as they are considerably higher than have been

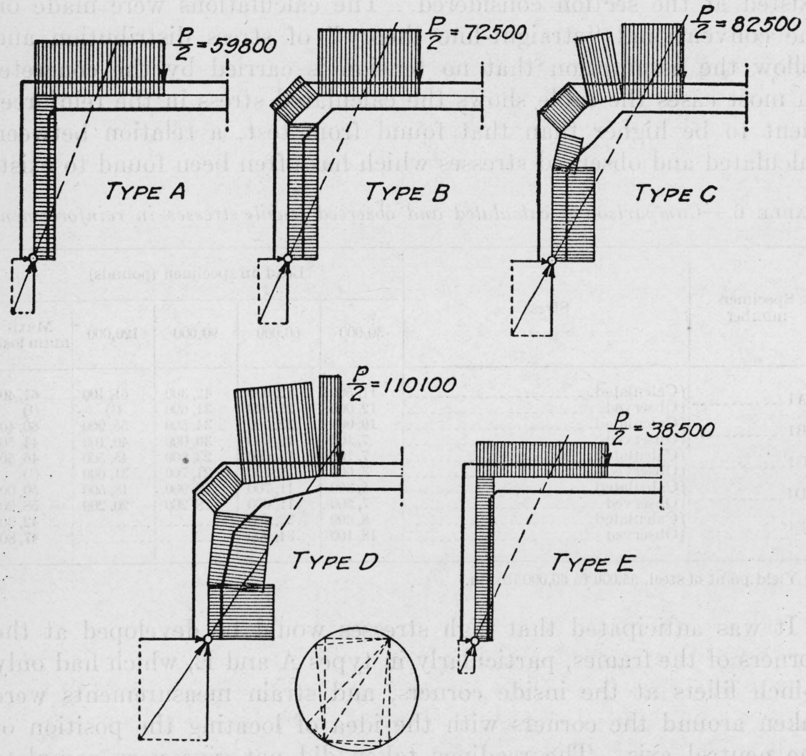

FIG. 27.—*Shear diagrams for frames at maximum load*

found in any tests outside the investigations of the concrete ship section.[8]

Shear diagrams for the different frames at maximum load are shown in Figure 27, and Table 7 gives values of the shearing stresses developed, as calculated at sections of maximum shear just outside the load points. The diagrams show the direction of the axis in each portion of the frame. The total shear at any section in this figure is the component, at right angles to the axis of the frame, of the reaction shown at the lower end of the left end post. The shear-

[8] Slater, Lord, and Zipprodt, Shear Tests of Reinforced Concrete Beams, B. S. Tech. Paper No. 314; 1926.

ing stress, v, equals $\frac{V}{bjd}$, in which V is the total shear, b is the width of the stem of the beam or end post, jd is the distance from the center of the tensile to the center of the compressive bending stresses, and d is the depth of the member. Specimens 13A1 to 13D2, inclusive, were of T section, having a flange 30 inches wide and 3 inches thick, and the value of j used was 0.88. Specimen 13E1 was of rectangular section, and the value of j used was 0.83. A comparison is also made in the table between the shearing stress and the compressive strength of the concrete as determined from tests of 6 by 12 inch cylinders made and tested with the frames.

TABLE 7.—*Maximum observed shearing stresses in frames*

[No failures occurred by diagonal tension]

Specimen number	Maximum vertical shear	Width of section, b	Depth of section, d	$b\,j\,d$	Shearing unit stress v	Cylinder strength f'_c	Ratio $\frac{v}{f'_c}$
	Pounds	*Inches*	*Inches*	*Square inches*	*Lbs./in.²*	*Lbs./in.²*	
13A1	60, 000	8. 12	9. 75	69. 7	860	3, 110	0. 28
13A2	59, 500	8. 00	9. 75	68. 6	865	3, 765	. 23
13B1	76, 000	8. 12	10. 00	71. 5	1, 060	4, 260	. 25
13B2	69, 000	8. 00	9. 75	68. 6	1, 005	3, 665	. 27
13C1	91, 000	8. 25	9. 75	70. 8	1, 285	4, 505	. 29
13C2	74, 000	8. 27	9. 75	71. 8	1, 030	4, 040	. 26
13D1	104, 200	8. 12	9. 87	70. 6	1, 475	4, 420	. 33
13D2	116, 000	8. 12	10. 12	72. 3	1, 605	5, 860	. 27
13E1	38, 500	8. 00	8. 92	59. 2	650	4, 995	. 13

Since there were no failures by diagonal tension, and no strain measurements were taken on the stirrups, there is nothing to show how high a shearing stress could have been developed. The shearing stresses were accompanied by a small, direct compression which balanced a little of the stress on the tension side of the frames and may have reduced somewhat the tendency to diagonal tension failure; still, there were generally fine vertical tension cracks present just outside the load points at very low loads. It does not seem likely that the direct compression produced any considerable increase in the resistance to diagonal tension.

Due to the fact that the positive and negative moments in continuous frames may be equalized by the judicious use of haunches, the magnitude of the moments is usually comparatively small; conversely, in such frames the shearing stresses will be correspondingly large. Hence, it is of considerable value to find that safe shearing strengths may be obtained which are much greater than those commonly allowed in building practice. This is clearly dependent, however, upon the use of a sufficient amount of web reinforcement, properly distributed and anchored, and upon proper anchorage of the longitudinal reinforcement.

5. MOMENT OF INERTIA

In Section II, 1, it was stated that in computing I for the purpose of determining the probable moment distribution in a frame I was assumed to vary as $d^{5/2}$. A study of the variation in I with d and with the increase of load is here made with the purpose of deriving a simple empirical expression for I which, without taking into account small variations in cross section and reinforcement, will indicate the proportional variations in I throughout the frame with sufficient accuracy to justify its use in determining moment distribution in the structure.

The distribution of statically indeterminate bending stresses is governed by the relative stiffness of the various parts of the structure. The stiffness of a member in flexure is usually measured by two quantities; I, the moment of inertia, a function of the size and shape of the cross section, and E, the modulus of elasticity, a physical property of the material. However, in a composite member of steel and concrete, the latter of which is so deficient in tensile strength, the modulus of elasticity, the tensile resisting moment of the concrete, the position of the neutral axis, and, consequently, the moment of inertia about the neutral axis vary with the stress in the member. A large reduction in I occurs when the concrete fails on the tension side of the member, and a further reduction takes place in E and I as the concrete fails to take compressive stress in proportion to its deformation. Throughout this variation in stress distribution for the concrete part of the member the neutral axis changes and this, in turn, slightly affects the moment of inertia of both the concrete and steel areas.

Aside from the question of the effect of stress, the variation in the moment of inertia with the shape of the cross section of a member must be considered. For rectangular areas of width b and depth d, containing equal percentages of reinforcement similarly placed, I varies as bd^3. If such areas contain not equal percentages, but equal areas, A, of steel similarly placed, I varies with powers of d, which lie between 2 and 3. Similarly for a T-beam of constant flange width, constant thickness, and constant area of reinforcement, but varying depth, I varies according to powers of d, which lie between 2 and 3. For a given member, however, the exponent may vary considerably between these limits.

In calculating I for a section of a reinforced concrete beam, especially when combined flexure and direct stress are encountered, a clear distinction must be made between the center of gravity and the axis of zero stress usually termed the neutral axis. One method of calculation (commonly termed the method of transformed sections) is to consider that the effective area of a section consists of the uncracked portion of concrete and an imaginary concrete area which has the same resistance to bending and to direct forces as the reinforcement. Bending moments and moments of inertia of the section are calculated about an axis through the center of gravity of this effective area.

However, the resultant stress at any point in the section will not, in general, be proportional to the distance from an axis through the center of gravity of this section. If there were no direct stresses present, the neutral axis (the axis of zero stress) would pass through the center of gravity of the effective area. The effect of a direct stress is to make the resultant stress at any point greater or less than the stresses due to flexure alone, and thus to shift the axis of zero stress. The resultant stress will then be proportional to the distance from the neutral axis rather than from the gravity axis.

Moments of inertia calculated, by the method described, from the nominal dimensions of the cross section of the test specimens are shown by the broken lines in Figure 28. A section 10.5 inches in effective depth and having 4.75 per cent of steel in both tension and compression was assumed. Using a modulus of rupture of 450 lbs./in.^{2} for this concrete and a value of n equal to 8, the section that may be assumed to be intact at different stages of loading was determined. The discontinuity of the broken line is a recognition of the sudden decrease which should occur in the moment of inertia with the formation of cracks.

As a method of determining the value of the product EI from test data, use was made of the well-known flexure formula $M=\frac{SI}{c}$. This may be written in the form $I=\frac{Mc}{Ee}$, in which e is the unit deformation and c is the distance from the neutral axis to the point where e is measured. Values of M, c, e, and d were obtained from the test data. The sections considered were on members having tension and compression faces parallel, or inclined to each other not more than 14°, as in the case of the tapered haunches of frames 13D, 1 and 2. The effect of this slight amount of taper was neglected and d was measured in all cases on a section normal to the axis of the frame (a line between tension and compression faces). From the measured values of d and the strains on tension and compression gauge lines at a given cross section the position of the neutral axis and the value of c were calculated. For convenience, E has been considered as having a constant value of 3,750,000 lbs./in.^{2}, which was the initial modulus of elasticity for this concrete, and all variation in the quantity EI is included in the single quantity I. Figure 28 shows the variation in I at different values of the compressive stress, f_c, in the concrete as determined from sections approximately 10.5 inches in effective depth.[9] The wide divergence of points shown

[9] A somewhat similar variation in the moment of inertia of a reinforced concrete beam is described by Dr. F. von Emperger in an article "Die Wahre Grosze des Tragheitsmoments im Eisenbetonbalken," Beton und Eisen, June 5, 1916.

may be attributed to errors of observation in M, c, and e to a considerable variation from the nominal depth of 10.5 inches, to the variation from the design dimensions of the section, to the difference in the steel area used in different specimens, to lack of proportionality between stress and distance from the neutral axis, and to the formation of cracks.

Until the concrete in the tension surface begins to crack, I should be expected to remain approximately constant. For Figure 28 the computed value of I with the concrete intact is 3,450 in.^{4}, and this value has been used in the graphs of that figure for all stresses below 500 lbs./in.^{2}

The tension failure of the concrete began generally when the compressive stress was about 500 lbs./in.^{2}, and after this the moment of

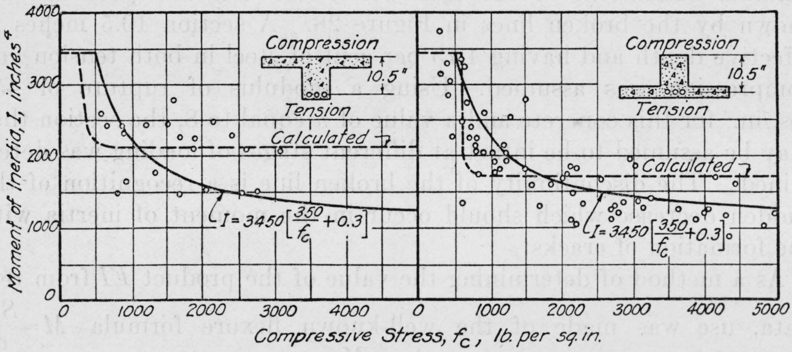

FIG. 28.—*Variation in I with compressive stress*

inertia decreased rapidly. The average values of the experimental data are represented roughly by the hyperbola

$$I = 3{,}450\left(\frac{350}{f_c} + 0.3\right).$$

Further values of I were computed from the test data for members of various depths and for loads giving several different ranges of computed compressive stresses and have been plotted in Figure 29 as ordinates against depths of section as abscissas. From the average curves drawn for each range of compressive stress, I is found to vary approximately as $d^{5/2}$. Hence, each curve represents an equation of the form $I = Qd^{5/2}$. The values of Q are found from Figure 29 to decrease as the values of f_c become larger, in the same general way as was shown by the values of I in Figure 28. This indicates that for a section in which a crack has formed the extent of the crack is a function of the compressive stress, f_c. The following general expression for the relation between moment of inertia and the compressive stress has been found to fit the curves of Figure 29.

$$I = 9.6d^{5/2}\left(\frac{350}{f_c} + 0.3\right) \tag{11}$$

It is assumed that the cracks began to form when the compressive stress was approximately 500 lbs./in.^{2}; before the formation of cracks, the value of f_c in equation (11) may be taken at 500, which modifies equation (11) to

$$I = 9.6d^{5/2} \tag{12}$$

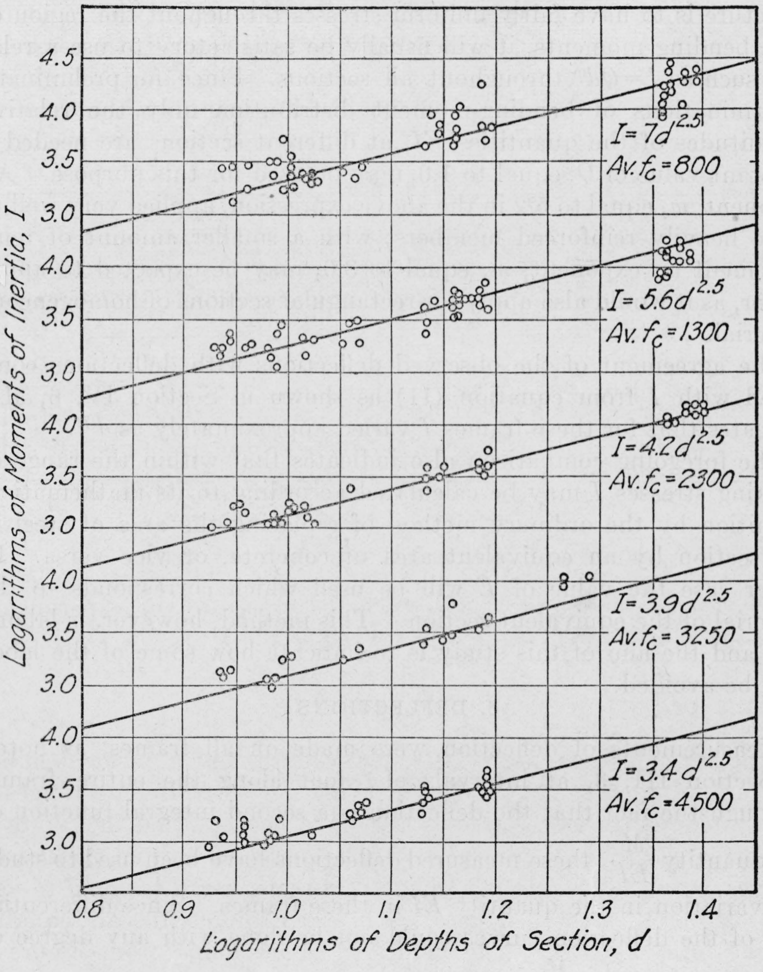

FIG. 29.—*Variation in I with depth of member*

Equation (11) shows that at a compressive stress of 1,500 lbs./in.^{2} in the concrete, the value of I is only about half as great as it was before the tensile strength of the concrete was lost. Hence, in analyzing a structure especially for stresses above ordinary working stresses, the use of the assumption that I varies throughout directly as some power of the depth of members is not exactly logical and will give too high a value of I at points of high stress. This is confirmed by specimens of the types B and C. Equations (11) and (12) can not be expected to apply to members in which the shape of cross section, percentage of reinforcement, or quality of concrete varied greatly from those used in these tests. It is believed, however, that these equations show the general way in which the moment of inertia varies in a reinforced conrete member. For a preliminary determination of the distribution of moment in design, especially if the structure is to have fairly uniform stresses throughout the region of high bending moments, it will usually be satisfactory to use a relation such as $I = Qd^m$ throughout all sections. Since for preliminary determinations of bending-moment distribution only the relative magnitudes of the quantities EI, at different sections are needed a constant value of Q, equal to 1.0, may be used for this purpose. An exponent, m, equal to 5/2 in the above expression, applied very well to these heavily reinforced members; with a smaller amount of reinforcement an exponent, m, equal to 3.0, may be expected to apply better, as it would also apply for rectangular sections of homogeneous material.

The agreement of the observed deflections with deflections computed with I from equation (11) as shown in Section IV, 6, also indicates that for these frames I varied approximately as $d^{5/2}$.

The foregoing comparison also indicates that within the range of working stresses I may be calculated according to its mathematical definition by the ordinary method of replacing the area of steel in the section by an equivalent area of concrete, or vice versa. In either case the value of E will be used which corresponds to the material of the equivalent section. This method, however, is laborious, and the aim of this study is to indicate how some of the labor may be avoided.

6. DEFLECTIONS

Measurements of deflection were made on all frames, as noted in Section III, 3, at intervals of 1 foot along the entire frame. Through the fact that the deflection is a second integral function of the quantity $\frac{M}{EI}$, these measured deflections have been used to study the variation in the quantity EI in these frames. Since differentiation of the deflection curves could not be done with any degree of accuracy, the exact $\frac{M}{EI}$ diagram corresponding to a given elastic curve was not found; however, the reverse operation was performed. Knowing values of M from the observed loads and reactions and assuming certain values of EI, elastic curves were obtained by two successive graphical integrations of the $\frac{M}{EI}$ curves, and these curves were then compared with the experimental curves.

Figure 30 shows values of the observed and computed deflections, as well as the $\frac{M}{EI}$ diagrams upon which the latter were based, for frames of types A, B, C, and D, at a load of 60,000 pounds. In calculating values of EI, the quantity E was assumed constant and equal to 3,750,000 lbs./in.^{2}, and the value of I was computed by use of equation (11). A good agreement is seen between the calculated and observed deflections.

For use with any rectangular frame without brackets, Maney's equation for deflection[10] is readily applicable. For a frame with

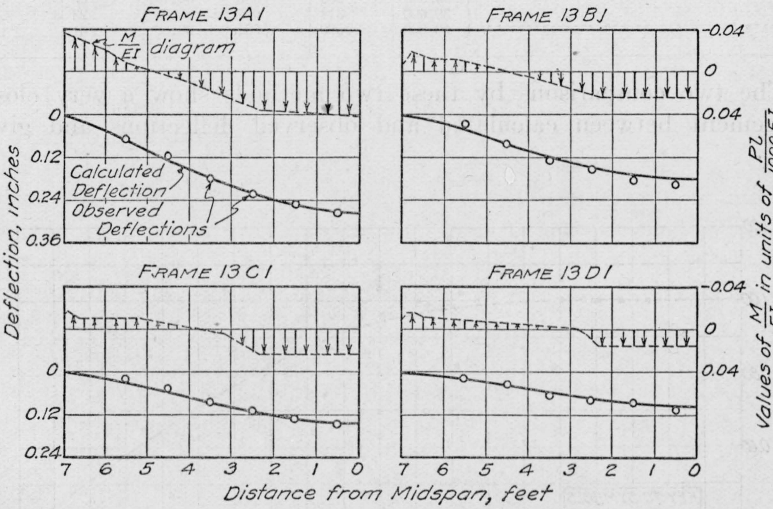

FIG. 30.—*Calculated and observed deflections*

loads at the one-third points in which the maximum moment at the center is $M = k' \frac{Pl}{6}$, the maximum deflection at the center is $\frac{Pl^3}{1{,}296\ EI}(27k' - 4)$. Hence, in Maney's equation, $f = \frac{cl^2}{d}(e_s + e_c)$, the coefficient c becomes $\left(\frac{27k' - 4}{216k'}\right)$. The quantities e_s and e_c must be measured at the point at which the maximum moment is measured. By the aid of Maney's equation, using the measured deformations in the steel and using for d the actual distance center to center of reinforcing bars upon which readings were taken, the computed deflections shown in Table 8 were obtained.

[10] "Relation between deformation and deflection in reinforced concrete beams," by G. A. Maney, Proc. A. S. T. M., Technical Papers, 14, p. 310; 1914.

TABLE 8.—*Calculated and observed deflections at mid span*

Specimen number	Load	k'	c	(e_s+e_c)	Calculated deflection $\left[c\frac{l^2}{d}(e_s+e_c)\right]$	Observed deflection
	Pounds				*Inches*	*Inches*
13A1	30,000	0.466	0.085	0.00052	0.14	0.14
	60,000	.648	.097	.00099	.31	.28
	90,000	.590	.094	.00138	.42	.42
13A2	30,000	.400	.079	.00074	.18	.16
	60,000	.536	.090	.00139	.39	.39
	90,000	.500	.088	.00219	.60	.68
13E1	30,000	.314	.067	.00093	.23	.26
	60,000	.560	.091	.00186	.61	.55

The two comparisons by these two methods show a very close agreement between calculated and observed deflections and give

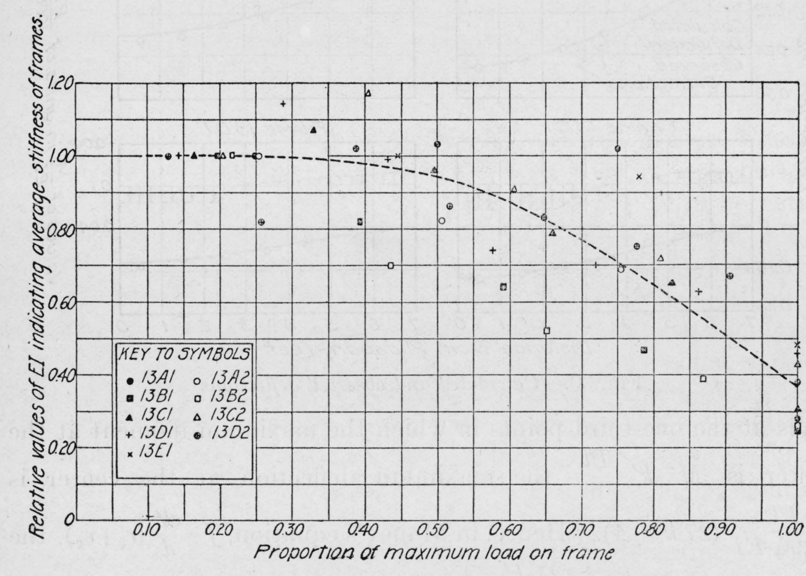

FIG. 31.—*Decrease in stiffness with increasing loads on frames*

further evidence that theoretical relations which were derived for elastic structures also hold true for these frames.

Further use has been made of the measured deflections in studying the variation in stiffness of each frame as a whole during the application of the test loads. In a homogeneous beam within the elastic limit of the material the quantity *EI* is constant and is proportional to the ratio of load to deflection, P/f. In these test specimens the ratio P/f varied, and, hence, the variation in *EI*, which is proportional to P/f, may be calculated from measured values of P/f. On this basis Figure 31 has been constructed, using a relative value of P/f equal to unity for the 30,000-pound load on each frame. The value of f used in each case was the average of measurements on deflection points 10, 11, 12, and 13, near mid span. While the decrease in stiffness with increasing load indicated in Figure 31 is similar to that shown by Figure 28, it must be remembered that in the former the deflections are influenced by the stiffness of all sections of the frame. While the various parts of the frame are subject to widely differing intensities of stress, the sections most highly stressed have the greatest influence upon the deflections at mid span. The decrease in stiffness under increasing load, as shown by both deflection and deformation readings, seems to be a typical phenomenon of reinforced concrete members.

7. CONCLUSIONS

In analyzing the test results it must be remembered that the materials of which the specimens were made were of rather unusual quality. The compressive strength and modulus of elasticity of the concrete were much higher than are usually found in reinforced concrete construction; in a similar way the steel combined a high elastic limit with a fairly high degree of ductility. Materials of this quality were of especial advantage for investigational work but may not be considered as representative of materials generally available for construction work.

The use of a large percentage of longitudinal reinforcement made it possible to utilize much of the compressive strength of the concrete, while the large amount of web reinforcement used permitted the development of exceptionally high shearing stresses without diagonal tension failures.

Certain definite effects have been determined from the use of brackets in the particular test pieces described herein, but more tests are necessary before any broad generalization can be made concerning the effectiveness that a bracket will have in different types and shapes of frames. A number of tentative conclusions, however, may be formulated.

1. From an analysis of the test results it appears that the reinforced concrete frame can be treated with a fair degree of accuracy by analytical methods similar to those used in arch analysis. A study has been made as to the validity of some of the assumptions usually employed in such an analysis.

2. For the purpose of determining the distribution of bending moments, it seems to be sufficiently correct to consider the entire section of a specimen as effective, even at points of sudden change of shape. The effect of such a change in shape upon the stress in the member is a matter which needs further experimental investigation.

3. In the analysis of statically indeterminate frames the modulus of elasticity of the material and the moment of inertia of the cross

section are quantities of primary importance. Within the range of working stresses, the value of the product EI as determined from a large number of test readings agreed closely with the value of EI as computed by the common method of replacing the steel area by an equivalent concrete area and neglecting the tension area of the concrete if cracks have formed. At higher stresses the tests indicate a decrease in the value of EI, resulting in a relative loss of rigidity at points thus stressed. This might produce a slight readjustment of the moment distribution, and some leeway should be allowed in the design of the structure to accommodate such an occurrence. It is to be noted, however, that if the structure can be designed so as to develop nearly uniform stresses throughout, the relative rigidity of different portions will change but little with increasing loads.

4. Fairly consistent quantitative information as to the variation in EI has been obtained from the tests of the different specimens. The value of EI appeared to vary about as the 5/2 power of the depth of the section. After the concrete began to fail in tension the value of EI gradually decreased, in the manner indicated by equation (11).

The assumption that EI can be expressed by a simple equation, $EI=Qd^m$ is evidently not correct, but will usually be satisfactory for preliminary designs. An exponent m equal to 5/2 in the above expression applied very well to these highly reinforced members; with a smaller amount of reinforcement an exponent m equal to 3 may be expected to apply, as it would also for rectangular sections of a homogeneous material. For determining bending moment distribution the relative magnitudes only of the quantities EI at different sections are needed, so that the magnitude of the coefficient Q is immaterial for such calculations.

5. Calculated deflections of the test specimens based on values of moment of inertia from equation (11) agree very well with measured deflections, and also with deflections calculated by use of Maney's equation. This is significant as showing a fairly consistent agreement among the various results of the test.

6. From calculations, the basis of which is confirmed by the tests, it is found that the effect of brackets on the bending moments in a frame may be expressed as a function of the clear span (from edge to edge of brackets), of the ratio of height to span of frame, and of the given loading. The importance of the various factors is indicated in equation (4).

7. The effect of brackets is sometimes considered as a shortening of the span of the loaded member; that is, the bracket is considered a part of the end support, and thus the center of bearing is brought out from the center line of the column. It has been found that this shortening of the span is not constant for a given bracket, but also varies with the ratio of height to span of the frame. For the frames tested, the total span may be considered as reduced by about two-

thirds of the horizontal length of the bracket at each end. While the total moment has been reduced in this way, its distribution between positive and negative sections has varied. The proportional amount of negative moment increased considerably as the size of bracket increased. Hence, while the decrease in total moment was in effect a shortening of the span, this viewpoint does not lead to logical conclusions, since the negative moment actually increased as the span was shortened.

8. The use of 45° brackets in these tests is not intended to imply that this shape is the most effective. For any given frame and loading, the most desirable shape of bracket may be determined. The general rule should be kept in mind that the bending moment diagram for a frame of uniform section is an approximate influence line for the effectiveness of brackets or haunches; that is, an ordinate at any point of this diagram represents the relative usefulness of an increase in section at that point.

9. While brackets will usually have rectilinear outlines, it is evident that the brackets of specimen 13D1 and 13D2, which approached a curved form, had certain advantages. This shape of bracket was very effective in reducing the moment at mid span, and it also produced a fairly uniform distribution of stress throughout. The result was that these frames withstood a much greater load than any frames of the other types.

10. Brackets should not be used for the purpose of reducing stresses, without also determining what effect they will produce upon moment distribution. A large bracket may produce a high moment at a section where it would not occur without the bracket and where the member consequently is not reinforced sufficiently. For example, it is seen from the test of specimen 13D1 that while the bracket was deep enough to provide for the large moment at the corner it caused failure to occur at the weaker section at mid height of the column.

11. The results of these tests confirm the theoretical deduction that brackets can be used to effect a considerable saving of material and of dead weight in a structure. The bracket eliminates the high local stress found at the sharp corners of a frame; it produces a more uniform variation in stress along the frame, thus minimizing the tendency for the formation of cracks; and it reduces bond and shearing stresses at the corners of the frame. Further, by the careful choice of the brackets the bending moments may be varied considerably, and thus a proper balance may be secured between the stresses in regions of positive and negative moments.

The desirable features mentioned apply to all forms of continuous beam and frame construction. In reinforced concrete work the forming of brackets and haunches is comparatively easy, and in important structures the saving of material should considerably overbalance the extra cost of construction.

APPENDIX I

TEST DATA AND SKETCHES OF FRAMES

A summary of deformations, deflections, loads, and reactions observed in the tests is given in Table 9. The table includes all original data of the tests except the stresses shown in Figures 24 to 26.

Following Table 9, detailed sketches showing the position of all strain gauge lines and deflection points, as well as the position of cracks at failure, are given in Figures 32 to 40. It will be noted that strain-gauge points on the steel are marked by solid circles, those on concrete by open circles, and deflection points by open triangles. The gauge lines are numbered to correspond with the data of Table 9. It is felt that the crack drawings furnish considerable information regarding the behavior of the frames under load.

TABLE 9.—*Data of tests*

[Loads are recorded in pounds, deflections or movements in inches, unit stresses in thousands of pounds per square inch, and unit deformations in thousandths of an inch per inch. The + sign indicates tensile stress or deformation and upward or outward deflection and the − sign indicates the opposite]

FRAME NO. 13A1

Observation	Load on frame (pounds)					Base moved outward
	30,000	30,000	60,000	90,000	120,000	
End thrust ____ pounds	6, 200	6, 200	8, 200	14, 300	17, 600	5, 500
End movement ____ inch	−. 01	0	+. 01	−. 04	−. 01	+1. 64
Unit deformation on gauge line—						
101	−. 04	+. 04	−. 04	−. 11	−. 89	________
102	−. 07	−. 06	−. 15	−. 24	−. 02	. 26
103	−. 15	−. 15	−. 22	−. 13	−. 63	. 52
104	+. 07	+. 13	+. 26	+. 69	________	2. 03
105	+. 15	−. 04	−. 02	−. 04	________	+. 11
106	+. 11	−. 04	0	+. 26	+. 94	+1. 26
Deflection on point—						
D1 ____ inch	+. 01	−. 01	+. 02	+. 01	−. 44	+. 47
D2 ____ do	+. 01	0	+. 03	+. 04	−. 22	+. 32
D3 ____ do	+. 01	+. 01	+. 03	+. 03	−. 02	+. 17
D4 ____ do	0	+. 01	+. 01	+. 02	+. 06	+. 04
D5 ____ do	0	0	0	−. 01	−. 05	−. 04
D6 ____ do	−. 03	−. 02	−. 07	−. 10	−. 43	−. 10
D7 ____ do	−. 07	−. 07	−. 13	−. 19	−. 83	−. 14
D8 ____ do	−. 09	−. 09	−. 18	−. 27	−1. 20	−. 18
D9 ____ do	−. 11	−. 12	−. 22	−. 34	−1. 20	−. 20
D10 ____ do	−. 13	−. 14	−. 26	−. 39	−1. 86	−. 22
D11 ____ do	−. 15	−. 15	−. 28	−. 42	−2. 07	−. 24
D12 ____ do	−. 13	−. 14	−. 28	−. 42	−2. 13	−. 23
D13 ____ do	−. 13	−. 14	−. 26	−. 40	−2. 10	−. 23
D14 ____ do	−. 11	−. 12	−. 23	−. 36	−1. 81	−. 20
D15 ____ do	−. 09	−. 10	−. 19	−. 27	−1. 37	−. 17
D16 ____ do	−. 06	−. 07	−. 12	−. 18	−. 93	−. 13
D17 ____ do	−. 05	−. 04	−. 07	−. 10	−. 52	−. 10
D18 ____ do	−. 01	−. 01	−. 02	−. 02	−. 10	−. 04
D19 ____ do	+. 01	0	+. 01	+. 01	+. 08	+. 03
D20 ____ do	+. 01	+. 01	+. 03	+. 04	+. 28	+. 13
D21 ____ do	+. 02	+. 03	+. 04	+. 04	+. 44	+. 25
D22 ____ do	+. 01	+. 01	+. 03	+. 03	+. 57	+. 36

TABLE 9.—*Data of tests*—Continued

FRAME NO. 13A2

Observation		Load on frame (pounds)				Base moved outward
		30,000	60,000	90,000	119,000	
End thrust	pounds	7,000	10,800	17,500	17,700	4,800
End movement	inches	+0.02	+0.01	+0.01	+0.03	+1.66
Unit deformation on gauge line—						
101		−.07	−.17	−.46		+.66
102		−.20	−.53	−.85		+1.22
103		−.30	−.77	−1.66		+.68
104		0	−.09	−.15	−2.52	+3.19
105		+.06	+.46	+1.18	+4.50	+1.16
106		−.20	−.28	−.35		+2.71
Deflection on point—						
D1	inches	0	+.01	−.03	−.84	+.34
D2	do	+.01	0	0	−.47	+.25
D3	do	+.02	+.03	+.04	−.10	+.12
D4	do	+.01	+.02	+.04	+.13	+.03
D5	do	−.01	−.02	−.03		−.03
D6	do	−.04	−.09	−.16	−.51	−.10
D7	do	−.07	−.17	−.30	−.98	−.14
D8	do	−.11	−.25	−.45	−1.47	−.21
D9	do	−.13	−.32	−.58	−1.93	−.25
D10	do	−.15	−.37	−.66	−2.27	−.27
D11	do	−.16	−.39	−.68	−2.47	−.28
D12	do	−.16	−.38	−.68	−2.52	−.28
D13	do	−.14	−.35	−.63	−2.34	−.26
D14	do	−.13	−.31	−.56	−2.00	−.24
D15	do	−.10	−.24	−.44	−1.55	−.20
D16	do	−.06	−.16	−.30	−1.07	−.14
D17	do	−.04	−.09	−.17	−.61	−.10
D18	do	0	−.02	−.03	−.13	−.03
D19	do	+.01	+.02	+.03	+1.0	+.03
D20	do	+.01	+.03	+.06	+.40	+.14
D21	do	+.02	+.04	+.10	+.68	+.29
D22	do	+.02	+.04	+.10	+.96	+.43

TABLE 9.—*Data of tests*—Continued

FRAME NO. 13B1

Observation	Load on frame (pounds)					Base moved outward
	30,000	60,000	90,000	120,000	152,000	
End thrust .... pounds	6,750	12,600	18,200	19,500	26,700	9,100
End movement .... inches	+0.02	+0.02	+0.02	+0.06	-0.01	+1.81
Unit stress on gauge line—						
35	-.4	-6.8	-1.6	----------	-3.2	+8.6
32	-3.4	-10.5	-15.4	-23.3	-28.5	-3.8
28	-1.1	+3.0	+4.5	+10.5	----------	+19.9
22	+9.0	+20.6	+30.0	+40.1	+34.9	+15.4
22a	+5.3	+18.7	+29.9	+40.1	+54.4	+13.9
23a	+7.1	+20.3	+29.6	+40.9	+43.1	+15.4
24a	+8.6	+19.9	+26.6	+36.0	+34.5	+14.6
25a	+6.7	+18.4	+29.8	+40.9	+58.1	+16.1
26a	+6.4	+14.2	+19.1	+24.4	+24.4	+17.6
Unit deformation on gauge line—						
101	-.17	-.39	-.40	-1.47	----------	----------
102	-.13	-.30	-.52	-.99	-1.03	-.24
103	-.18	-.39	-.74	-1.05	-.90	+1.79
104	-.09	-.30	-.13	-.20	----------	+1.25
105	-.18	-.41	-.53	-.81	-.74	-.22
106	-.04	-.28	-.15	-.24	+.40	-.18
Deflection on point—						
D1 .... inches	+.02	+.03	+.03	+.04	-.35	+.46
D2 .... do	+.04	+.03	+.05	+.06	-.21	+.30
D3 .... do	+.01	+.02	+.04	+.06	----------	+.13
D4 .... do	-.01	0	0	+.01	-.04	+.02
D5 .... do	0	-.01	-.01	-.01	+.01	-.02
D6 .... do	-.02	-.04	-.06	-.10	----------	-.18
D7 .... do	-.04	-.03	-.15	-.25	----------	-.23
D8 .... do	-.05	-.13	-.23	-.40	-1.30	-.27
D9 .... do	-.06	-.16	-.32	-.57	-1.67	-.32
D10 .... do	-.08	-.19	-.37	-.66	-1.73	-.35
D11 .... do	-.08	-.20	-.39	-.69	-1.67	-.36
D12 .... do	-.08	-.20	-.39	-.68	-1.56	-.35
D13 .... do	-.08	-.19	-.37	-.66	-1.53	-.34
D14 .... do	-.05	-.15	-.30	-.56	-1.21	-.27
D15 .... do	-.05	-.13	-.24	-.43	-.90	-.25
D16 .... do	-.04	-.08	-.15	-.26	-.56	-.21
D17 .... do	-.02	-.04	-.06	----------	----------	-.14
D18 .... do	0	0	0	-.01	-.01	-.02
D19 .... do	0	0	0	0	+.02	+.01
D20 .... do	+.02	+.03	+.06	+.08	+.20	+.13
D21 .... do	+.02	+.05	+.07	+.10	+.33	+.25
D22 .... do	+.03	+.05	+.07	+.11	+.44	+.34

TABLE 9.—*Data of tests*—Continued

FRAME NO. 13B2

Observation	Load on frame (pounds) 30,000	60,000	90,000	120,000	138,000	Base moved outward
End thrust ... pounds	5,800	15,000	23,000	30,000	31,500	8,300
End movement ... inches	+0.04	+0.04	+0.05	+0.08	+0.07	+1.65
Unit deformation on gauge line—						
101	+.03	−.23	−.40	−.70	−1.40	+6.50
102	−.03	−.40	−.55	−.63	−.65	−.30
103	−.45	−.63	−1.03	−1.03	−2.05	+6.90
104	+.03	+.03	+.03	+.68	−.68	+1.70
105	−.55	−.18	−.30	−.80	−.95	−.45
106	+.70	+.73	+.73	+1.05	+3.60	+2.25
Deflections on point—						
D1 ... inches	+.01	+.01	+.02	+.03	+.35	+.48
D2 ... do	+.01	+.02	+.04	+.07	----	+.28
D3 ... do	+.01	+.03	+.06	+.09	----	+.14
D6 ... do	−.01	−.04	−.07	−.12	−.30	−.13
D7 ... do	−.03	−.09	−.18	−.30	−.65	−.19
D8 ... do	−.05	−.14	−.28	−.49	−1.00	−.24
D9 ... do	−.07	−.20	−.39	−.69	−1.36	−.29
D10 ... do	−.08	−.23	−.45	−.79	−1.58	−.32
D11 ... do	−.08	−.24	−.48	−.84	−1.74	−.33
D12 ... do	−.08	−.24	−.49	−.83	−1.80	−.32
D13 ... do	−.09	−.24	−.47	−.82	−1.73	−.32
D14 ... do	−.07	−.21	−.42	−.74	−1.49	−.30
D15 ... do	−.05	−.16	−.32	−.57	−1.17	−.25
D16 ... do	−.03	−.10	−.19	−.30	−.74	−.18
D17 ... do	−.02	−.05	−.09	−.16	−.37	−.13
D20 ... do	+.01	+.04	+.06	+.11	+.32	+.11
D21 ... do	+.02	+.05	+.08	+.15	+.53	+.21
D22 ... do	+.03	+.05	+.08	+.16	+.72	+.31

FRAME NO. 13C1

Observation	Load on frame (pounds) 30,000	60,000	90,000	120,000	150,000	182,000	Base moved outward
End thrust ... pounds	8,200	17,000	23,600	25,000	26,700	48,000	8,200
End movement ... inches	+0.02	+0.02	+0.02	+0.02	0	−0.03	+1.79
Unit deformation on gauge line—							
101	−.48	−.54	−.81	−.95	−1.27	−1.23	----
102	−.05	+.14	−.09	−.18	−.16	−.53	+.02
103	−.22	−.48	−.54	−.95	−1.03	----	+.46
104	−.03	−.11	−.20	0	−.36	−.36	+5.7
105	−.16	−.01	−.14	−.18	−.11	−.66	+.04
106	−.01	0	0	+.24	+.73	−.64	+.39
Deflection on point—							
D1 ... inches	+.03	+.05	+.08	+.11	+.22	----	+.43
D2 ... do	+.02	+.04	+.07	+.11	+.21	----	+.28
D3 ... do	+.01	+.02	+.04	+.07	+.12	+.31	+.16
D6 ... do	−.03	−.03	−.04	−.07	−.11	−.30	−.15
D7 ... do	−.02	−.05	−.08	−.13	−.22	−.58	−.28
D8 ... do	−.04	−.09	−.14	−.23	−.36	−.89	−.34
D9 ... do	−.04	−.11	−.21	−.32	−.49	−1.18	−.37
D10 ... do	−.08	−.14	−.24	−.38	−.58	−1.40	−.40
D11 ... do	−.08	−.15	−.25	−.40	−.61	−1.51	−.41
D12 ... do	−.08	−.15	−.26	−.41	−.62	−1.55	−.42
D13 ... do	−.07	−.14	−.24	−.39	−.58	−1.47	−.40
D14 ... do	−.05	−.11	−.19	−.31	−.46	−1.26	−.37
D15 ... do	−.05	−.09	−.15	−.24	−.37	−.97	−.35
D16 ... do	−.02	−.05	−.08	−.14	−.22	−.63	−.28
D17 ... do	−.02	−.01	−.05	−.07	−.11	−.33	−.15
D20 ... do	0	+.02	+.04	+.05	+.09	+.30	+.13
D21 ... do	+.02	+.05	+.08	+.12	+.18	+.64	+.28
D22 ... do	+.02	+.05	+.08	+.12	+.21	+.81	+.41

TABLE 9.—*Data of tests*—Continued

FRAME NO. 13C2

Observation	Load on frame (pounds) 30,000	60,000	90,000	120,000	148,000	Base moved outward
End thrust .... pounds	11,700	16,750	25,200	33,500	30,500	13,700
End movement .... inches	+0.02	+0.03	+0.04	+0.03	+0.03	+1.88
Unit deformation on gauge line—						
101	-.35	-.35	-.75	-.67	-.90	----------
102	0	-.32	-.37	-.02	-.05	+.22
103	-.20	-.50	-.75	-.25	-.25	+.63
104	-.10	-.12	-.40	+.52	----------	----------
105	+.07	-.17	-.40	-.12	-.28	-.12
106	+.05	-.15	-.20	+.52	----------	+.38
Deflection on point—						
D1 .... inches	+.01	+.04	+.09	+.13	+.53	+.48
D2 .... do	+.01	+.04	+.08	+.16	+.40	+.32
D3 .... do	+.01	+.02	+.06	+.10	+.24	+.16
D6 .... do	-.01	-.02	-.05	-.08	-.20	-.13
D7 .... do	-.02	-.04	-.09	-.16	-.40	-.27
D8 .... do	-.04	-.08	-.15	-.27	-.61	-.32
D9 .... do	-.07	-.12	-.24	-.40	-.86	-.37
D10 .... do	-.08	-.14	-.27	-.46	-.95	-.40
D11 .... do	- 09	-.15	-.29	-.48	-1.00	-.42
D12 .... do	-.09	-.16	-.29	-.49	-1.02	-.42
D13 .... do	-.08	-.14	-.27	-.46	-.97	-.42
D14 .... do	.07	-.12	-.23	-.41	-.89	-.39
D15 .... do	-.04	-.08	-.16	-.29	-.65	-.36
D16 .... do	-.03	-.06	-.10	-.18	----------	-.32
D17 .... do	-.02	-.03	-.06	-.09	-.24	-.18
D20 .... do	+.01	+.03	+.04	+.08	+.21	+.16

FRAME NO. 13D1

Observation	Load on frame (pounds) 30,000	60,000	90,000	120,000	150,000	180,000	208,500	Base moved outward
End thrust .... pounds	9,000	17,500	28,400	37,300	50,000	57,300	58,500	17,500
End movement .... inches	+0.01	+0.03	+0.04	+0.07	+0.08	+0.05	+0.06	+2.04
Unit deformation on gauge line—								
101	-.02	-.31	-.49	-.81	-1.04	-1.60	-1.78	+2.01
102	-.05	-.13	-.29	-.49	-.49	-.42	+.79	+.16
103	-.14	-.20	-.36	-.18	-.33	-.44	-.51	+.38
104	-.01	-.05	-.20	0	-.14	-.46	-.62	+2.28
105	-.14	-.18	-.35	-.36	-.42	-.53	-.62	+.16
106	-.09	-.16	-.23	-.23	-.22	-.22	-.31	+.12
Deflection on point—								
D1 .... inches	+.02	+.04	+.06	+.09	+.12	+.15	+.34	+.44
D2 .... do	+.01	+.03	+.05	+.08	+.11	+.14	+.28	+.30
D3 .... do	+.01	+.02	+.03	+.05	+.06	+.09	+.16	+.16
D6 .... do	-.01	-.01	-.02	-.04	-.06	-.07	-.15	-.13
D7 .... do	-.02	-.04	-.07	-.11	-.15	-.19	-.33	-.28
D8 .... do	-.04	-.07	-.11	-.18	-.24	-.31	-.53	-.39
D9 .... do	-.05	-.08	-.16	-.25	-.34	-.44	-.73	-.47
D10 .... do	-.06	-.10	-.18	-.30	-.40	-.53	-.86	-.53
D11 .... do	-.06	-.11	-.19	-.32	-.44	-.57	-.93	-.54
D12 .... do	-.07	-.12	-.20	-.34	-.45	-.59	-.97	-.58
D13 .... do	-.06	-.10	-.18	-.33	-.44	-.57	-.94	-.56
D14 .... do	-.05	-.09	-.16	-.30	-.39	-.50	-.81	-.54
D15 .... do	-.03	-.07	-.12	-.22	-.27	-.35	-.59	-.47
D16 .... do	-.03	-.04	-.07	-.12	-.16	-.20	-.36	-.53
D17 .... do	-.01	-.02	-.03	-.06	-.07	-.09	-.17	-.18
D20 .... do	+.01	+.02	+.03	+.06	+.07	+.08	+.17	+.17
D21 .... do	+.01	+.02	+.06	+.09	+.11	+.13	+.30	+.32
D22 .... do	+.03	+.05	+.06	+.10	+.12	+.13	+.23	+.49

TABLE 9.—*Data of tests*—Continued

FRAME NO. 13D2

Observation	Load on frame (pounds)								Base moved outward
	30, 000	60, 000	90, 000	120, 000	150, 000	180, 000	210, 000	232, 000	
End thrust pounds	12, 000	27, 500	31, 500	41, 700	50, 000	59, 200	67, 100	66, 300	16, 800
End movement inches	+0. 02	+0. 02	+0. 02	+0. 05	+0. 06	+0. 06	+0. 07	+0. 08	+1. 74
Unit deformation on gauge line—									
101	—. 25	—. 44	—. 52	—. 61	—. 81	—. 94	—1. 28	—1. 54	—1. 39
102	—. 05	—. 24	—. 31	—. 44	—. 59	—. 66	—. 72	—. 93	+. 04
103	—. 18	—. 35	—. 37	—. 70	—. 81	—1. 18	—1. 18	—1. 18	+. 13
104	—. 12	—. 20	—. 17	—. 40	—. 40	—. 54	—. 63	—. 81	+. 15
105	—. 20	—. 20	—. 26	—. 40	—. 37	—. 31	—. 35	—. 72	+. 04
106	—. 10	—. 15	—. 18	—. 26	—. 24	—. 28	—. 24	—. 40	+. 09
Deflection on point—									
D1 inches	+. 02	+. 04	+. 04	+. 05	+. 08	+. 10	+. 14	--------	+. 38
D2 do	+. 01	+. 03	+. 04	+. 05	+. 08	+. 11	+. 14	+. 45	+. 26
D3 do	+. 01	+. 02	+. 03	+. 04	+. 06	+. 08	+. 10	+. 27	+. 14
D6 do	—. 01	—. 01	—. 02	—. 03	—. 05	—. 06	—. 09	—. 21	—. 12
D7 do	—. 03	—. 05	—. 06	—. 09	—. 12	—. 16	—. 21	—. 46	—. 25
D8 do	—. 03	—. 08	—. 09	—. 13	—. 21	—. 27	—. 36	--------	—. 35
D9 do	—. 05	—. 12	—. 14	—. 23	—. 30	—. 39	—. 53	—1. 05	—. 42
D10 do	—. 07	—. 15	—. 18	—. 28	—. 36	—. 47	—. 62	—1. 22	—. 47
D11 do	—. 06	—. 15	—. 18	—. 29	—. 38	—. 50	—. 66	—1. 32	—. 47
D12 do	—. 07	—. 16	—. 19	—. 30	—. 39	—. 51	—. 68	—1. 36	—. 50
D13 do	—. 05	—. 15	—. 18	—. 29	—. 38	—. 47	—. 65	—1. 33	—. 48
D14 do	—. 05	—. 13	—. 15	—. 25	—. 34	—. 44	—. 58	—1. 18	—. 44
D15 do	—. 03	—. 08	—. 10	—. 17	—. 23	—. 30	—. 41	—. 88	—. 37
D16 do	—. 03	—. 06	—. 07	—. 12	—. 15	—. 19	—. 26	—. 57	—. 29
D17 do	—. 02	—. 04	—. 04	—. 06	—. 07	—. 10	—. 13	—. 30	—. 16
D20 do	+. 01	+. 03	+. 03	+. 05	+. 07	+. 08	+. 11	+. 23	+. 14
D21 do	+. 02	+. 04	+. 06	+. 08	+. 10	+. 13	+. 18	+. 43	+. 27
D22 do	+. 02	+. 05	+. 06	+. 09	+. 12	+. 15	+. 20	+. 57	+. 40

FRAME NO. 13E1

Observation	Load on frame (pounds)		
	30, 000	60, 000	77, 000
End thrust pounds	8, 000	10, 100	14, 500
End movement inches	0. 06	0. 05	0. 05
Unit deformation on gauge line—			
101	—. 22	—. 37	—. 64
102	—. 44	—. 92	—1. 60
103	—. 28	—. 39	—. 53
104	—. 05	—. 13	—. 09
105	—. 02	+. 97	+2. 39
106	—. 09	+. 02	--------
Deflection on point—			
D1 inches	+. 04	+. 09	+. 31
D2 do	+. 05	+. 09	+. 26
D3 do	+. 03	+. 07	+. 18
D4 do	+. 02	+. 04	+. 10
D5 do	—. 01	—. 03	—. 08
D6 do	—. 07	—. 14	—. 31
D7 do	—. 12	—. 25	—. 55
D8 do	—. 17	—. 35	—. 79
D9 do	—. 21	—. 45	—1. 02
D10 do	—. 25	—. 52	—1. 15
D11 do	—. 26	—. 55	—1. 36
D12 do	—. 26	—. 55	—1. 46
D13 do	—. 24	—. 52	—1. 43
D14 do	—. 22	—. 46	—1. 23
D15 do	—. 17	—. 35	—. 88
D16 do	—. 12	—. 25	—. 57
D17 do	—. 06	—. 13	—. 27
D18 do	—. 01	—. 02	—. 01
D19 do	+. 02	+. 02	+. 01
D20 do	+. 03	+. 04	—. 03
D21 do	+. 06	+. 06	—. 07
D22 do	+. 07	+. 06	—. 14

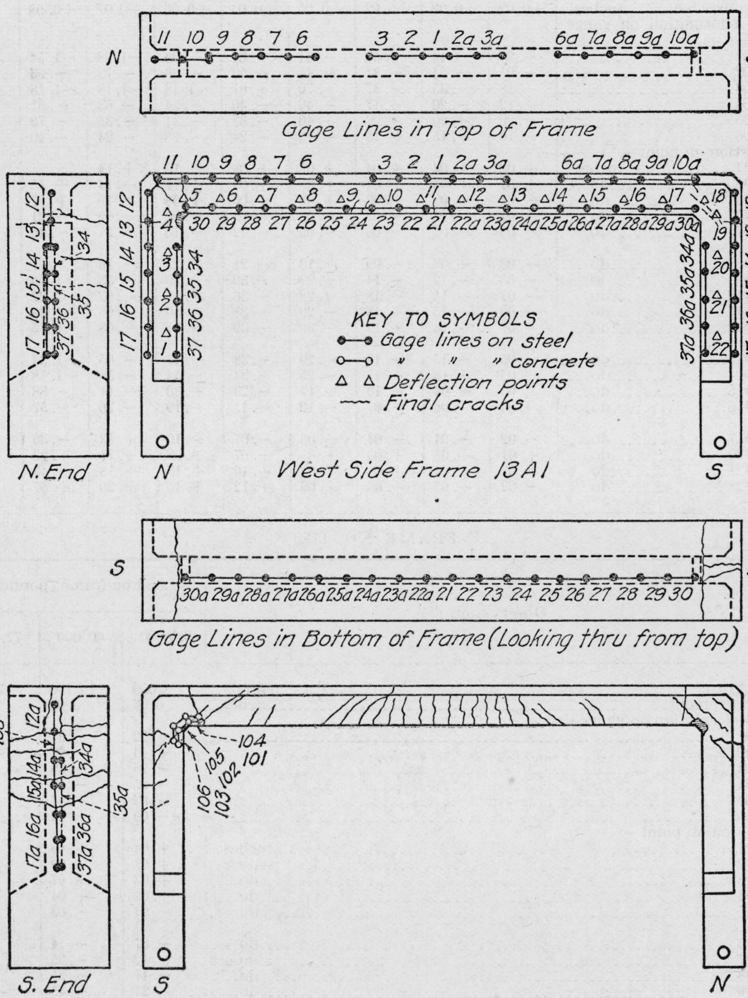

FIG. 32.—*Position of gauge lines and cracks*

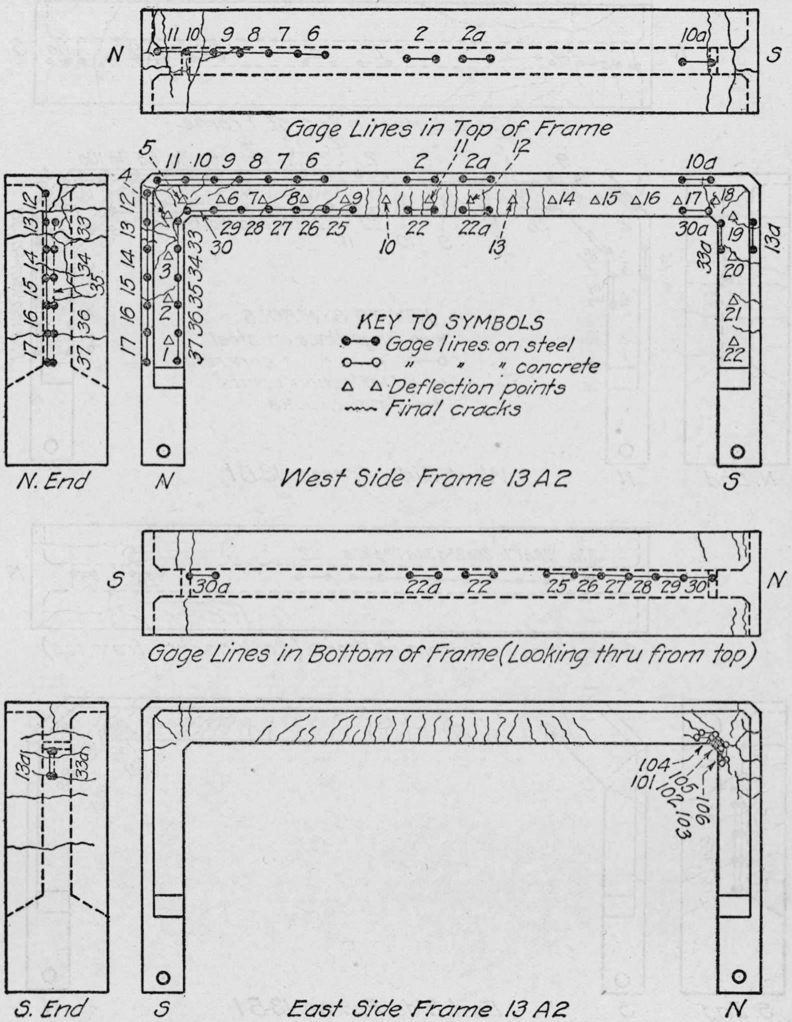

FIG. 33.—*Position of gauge lines and cracks*

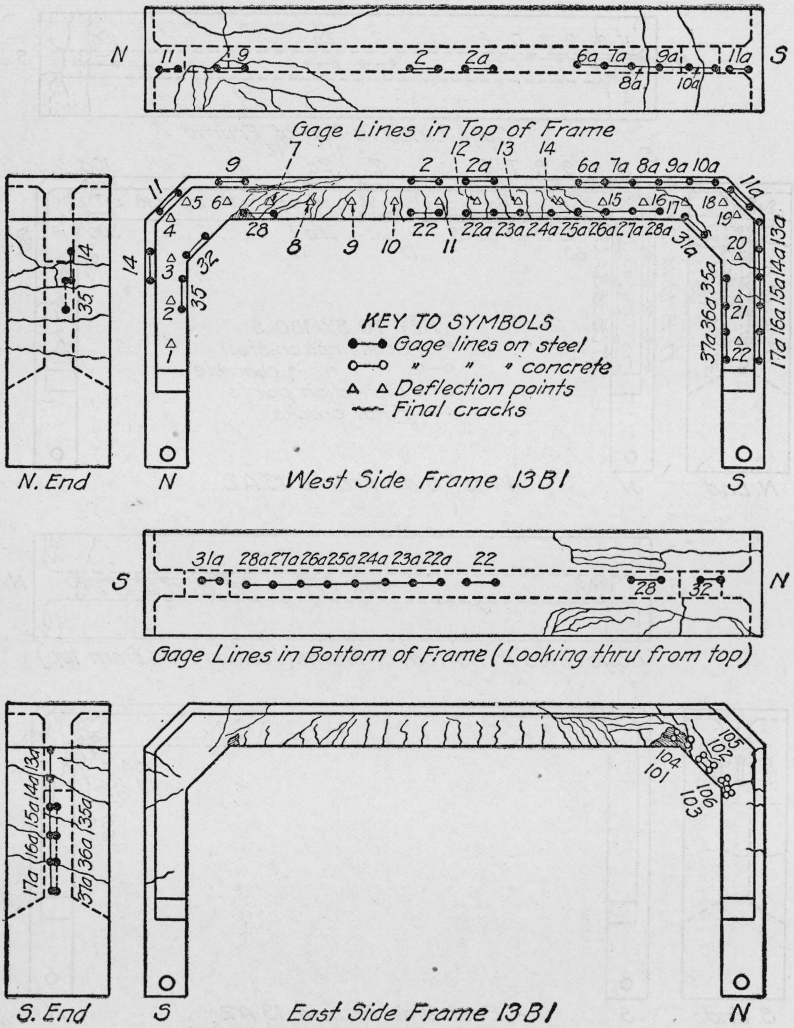

FIG. 34.—*Position of gauge lines and cracks*

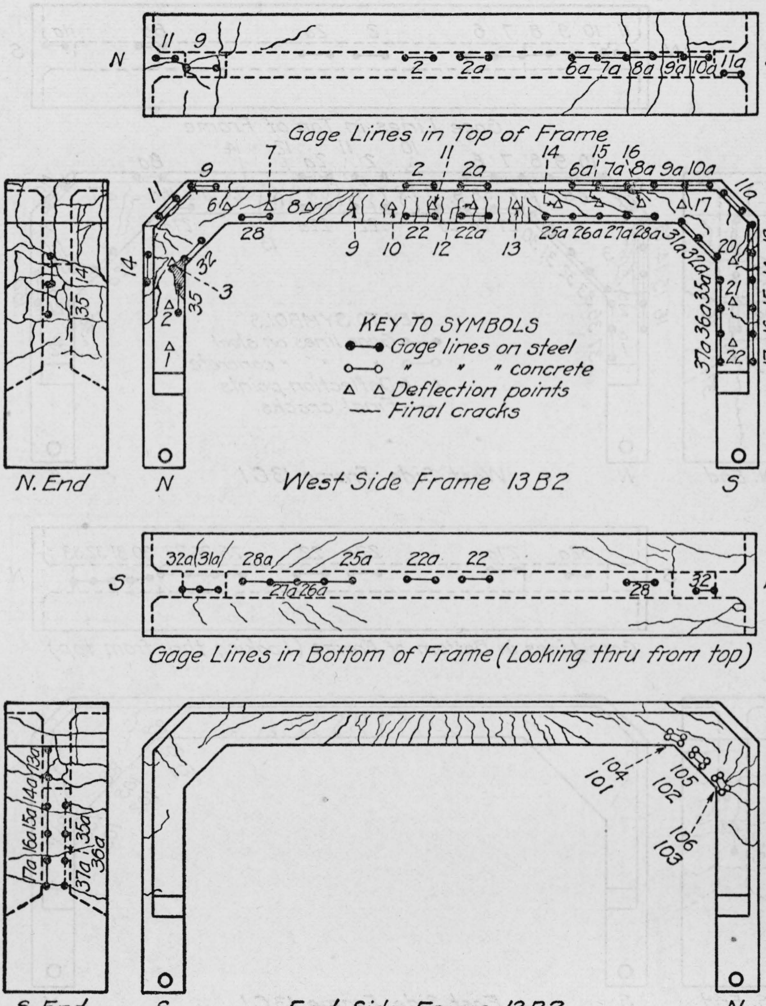

FIG. 35.—*Position of gauge lines and cracks*

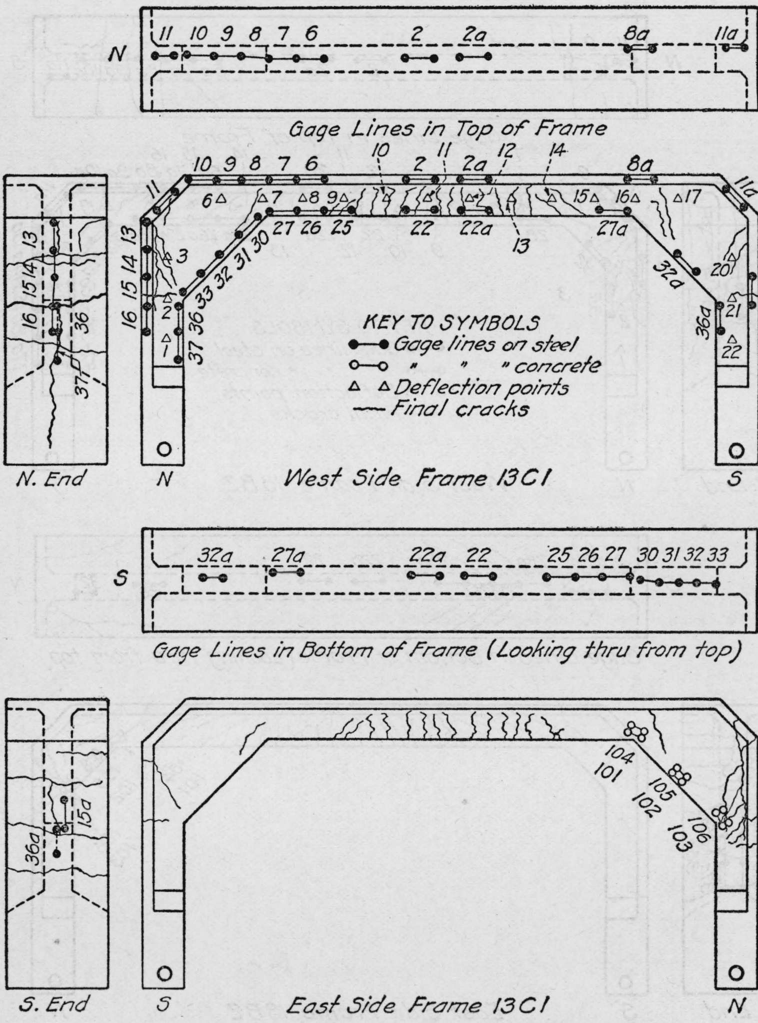

FIG. 36.—*Position of gauge lines and cracks*

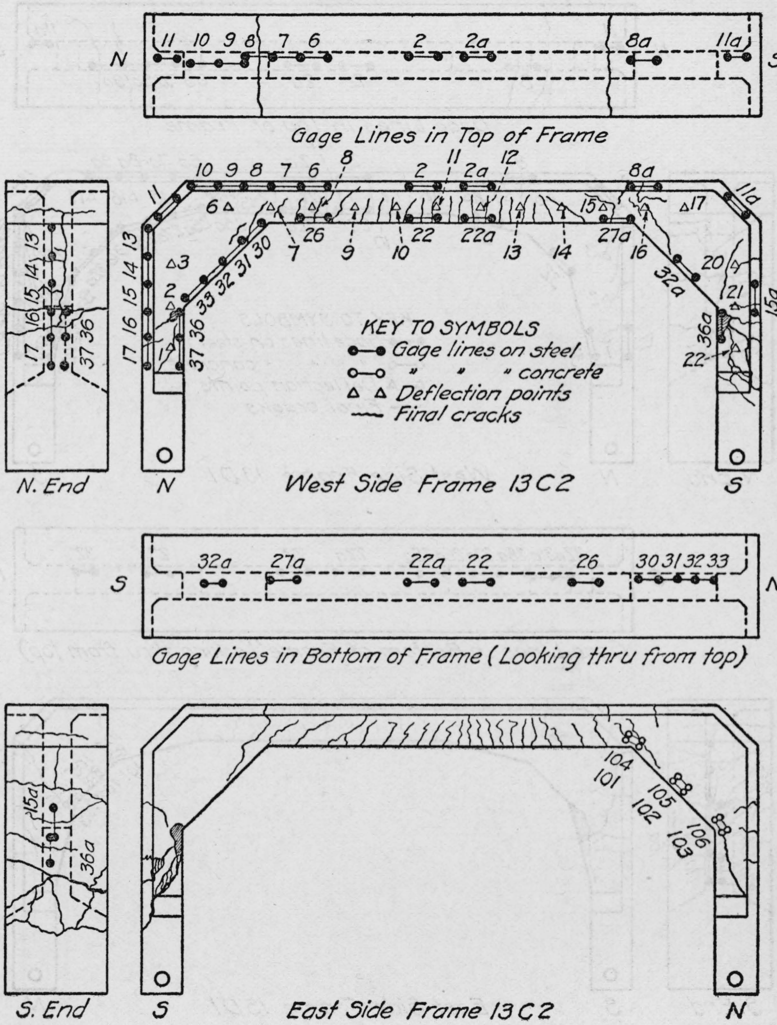

FIG. 37.—*Position of gauge lines and cracks*

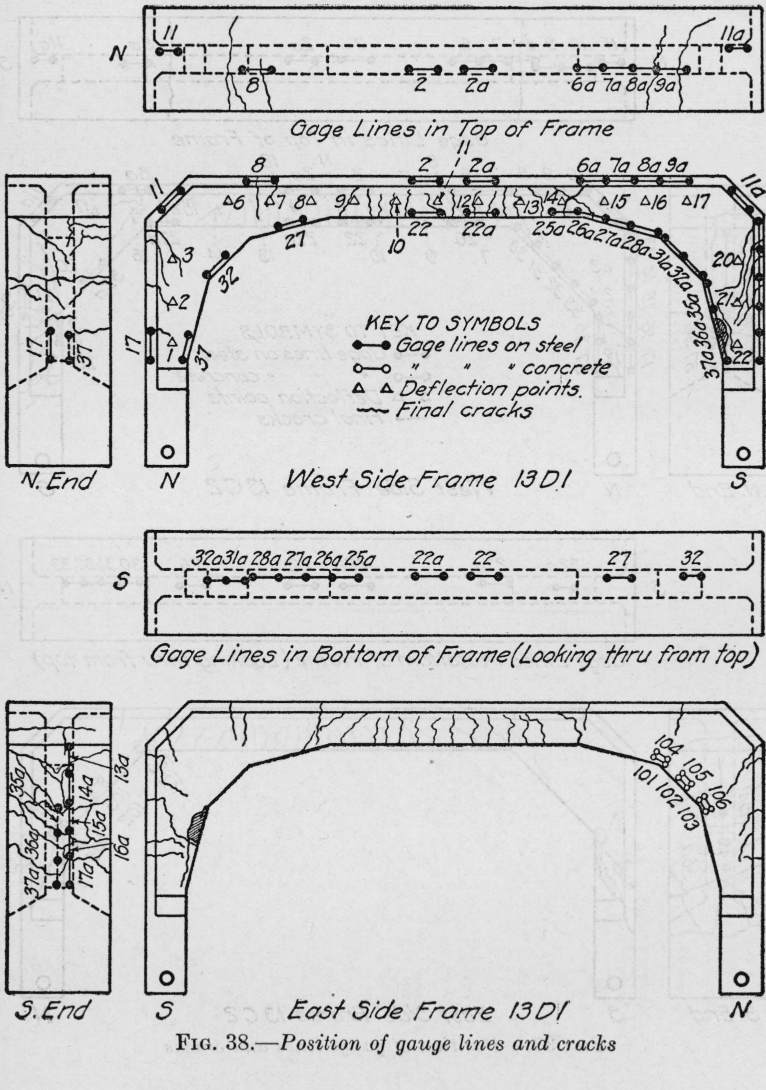

FIG. 38.—*Position of gauge lines and cracks*

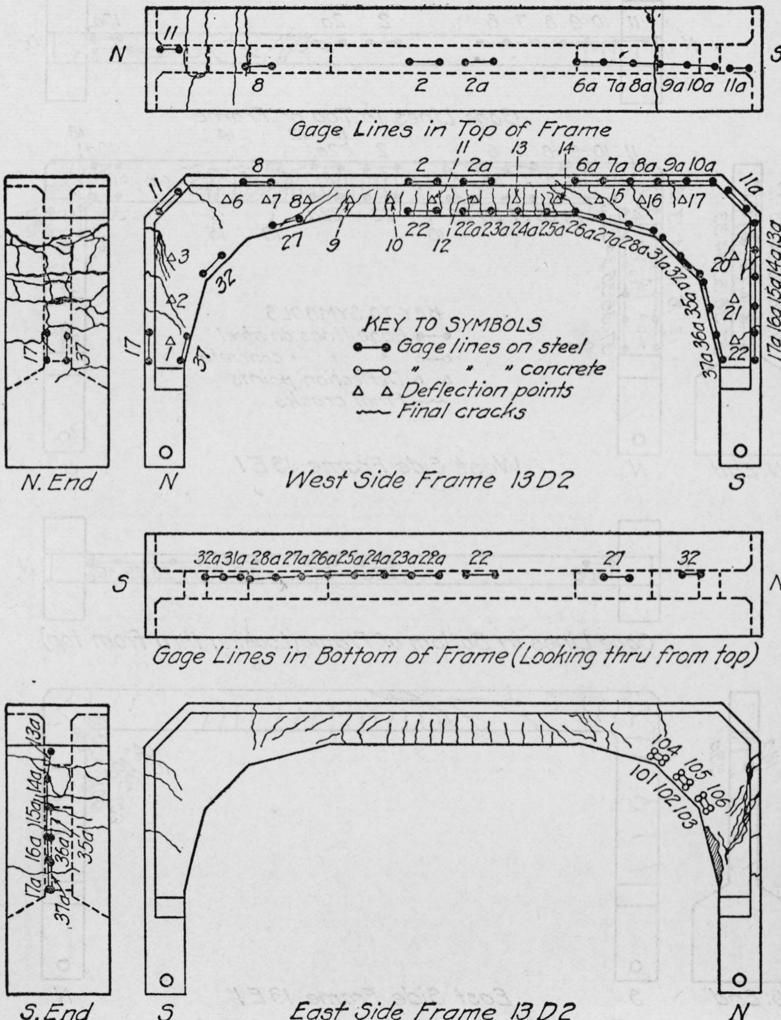

FIG. 39.—*Position of gauge lines and cracks*

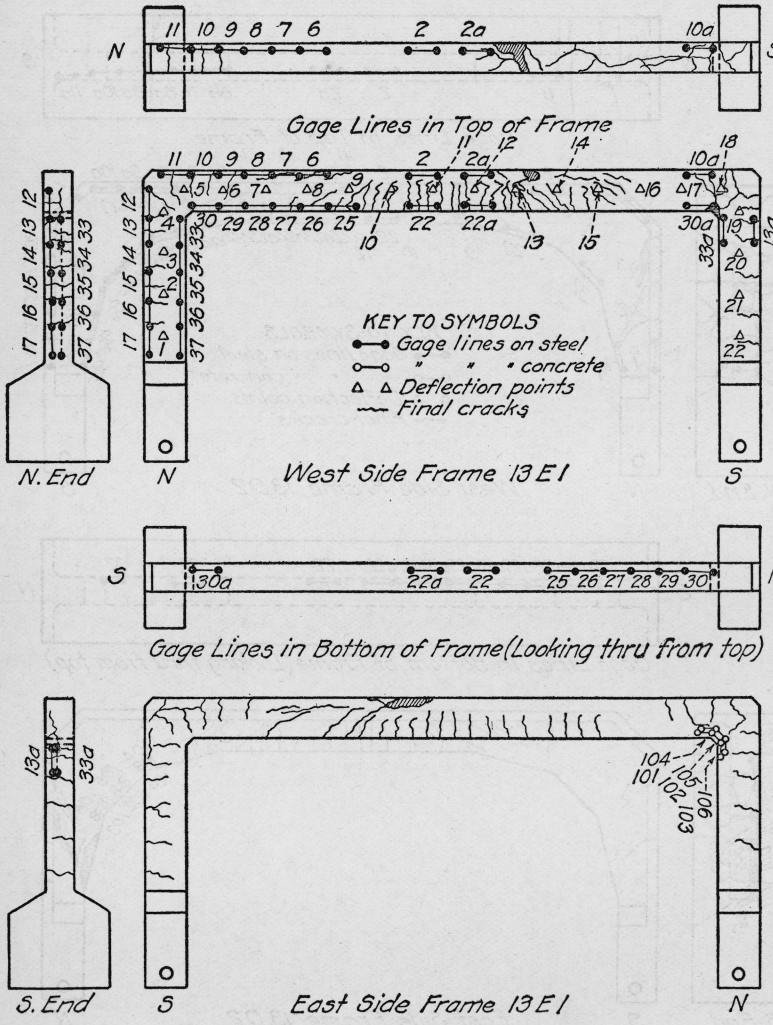

FIG. 40.—*Position of gauge lines and cracks*

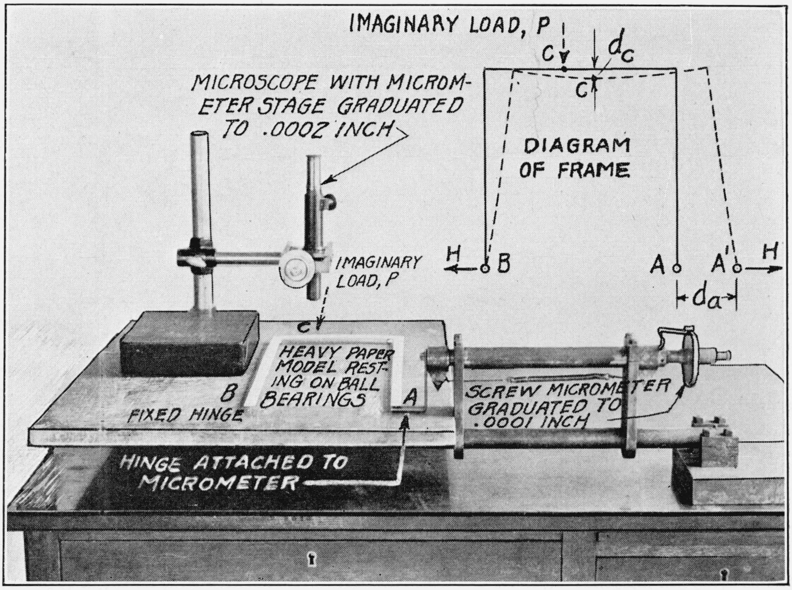

FIG. 41.—*Arrangement of apparatus for testing paper models*

APPENDIX II

SUPPLEMENTARY TESTS OF PAPER MODELS

For comparison with the foregoing analysis and tests a series of tests of paper models has been made by a method [11] devised by Prof. G. E. Beggs, of Princeton University. The tests were made with the assistance of R. L. Brown, of the engineering experiment station, University of Illinois.

By the Beggs method the reactions and moments in indeterminate frames are determined by measuring certain deflections or displacements of a paper or cardboard model of the frame. The theory involved in the tests of models is not new, being based upon Maxwell's well-known "Theorem of reciprocal displacements." The novel feature is the use of small models of paper or other isotropic material and the measurement of deflections by means of microscopes and micrometer gauges. The theory and procedure in the tests may be described with reference to Figure 41, which shows the arrangement of apparatus which was utilized for the purpose. The model of the frame to be tested was placed on a horizontal surface and supported on ball bearings to reduce friction. The hinge B was held stationary by means of a needle, while the hinge A was attached by means of another needle to the screw micrometer D. In forming the hinge between paper and needle, care was taken to make the fit loose enough to obviate high frictional resistance to turning but not loose enough to allow play in the hinge.

To find the horizontal reaction H at A and B, due to a load P acting at the point C, the procedure was to move the hinge A to the right a distance d_a by means of the micrometer D, and with the microscope to read the movement d_c of the point C in the direction of the imaginary load P (which was taken in this case to be at right angles to the direction BA). By the application of Maxwell's theorem the ratio $\frac{d_c}{d_a}=\frac{H}{P}$, or the horizontal reaction $H=P\frac{d_c}{d_a}$. In practice it was found advisable to repeat the operation, moving the hinge A to the left of its initial position and measuring d_c and to use the numerical average of several sets of observations taken in opposite directions in calculating the value of H.

The paper models tested were of types A, B, C, and D, as described in Section III, 1, except that eight different heights of each type of frame were used. To eliminate differences in the quality of the paper of the models, all were made from one sheet of paper, the frame of

[11] An accurate mechanical solution of statically indeterminate structures by use of paper models and special gauges, by G. E. Beggs, Proc. Am. Concrete Inst.; 1922.

type D being first cut out and tested without damaging it, the brackets then cut down to the form of type C and later of types B and A. The paper showed some variability in stiffness, and the values recorded were the average of a number of observations. The values of H/P, the ratio of horizontal reaction to vertical load applied at the one-third points of the top member of the bent, are plotted in Figure 42 for the four types of frame. For comparison, values of H/P have been calculated from equation (5), assuming the haunch of type D to be equivalent to a 45° bracket 4/3 as long as that of type C, and are also plotted in Figure 42. The agreement in the results obtained

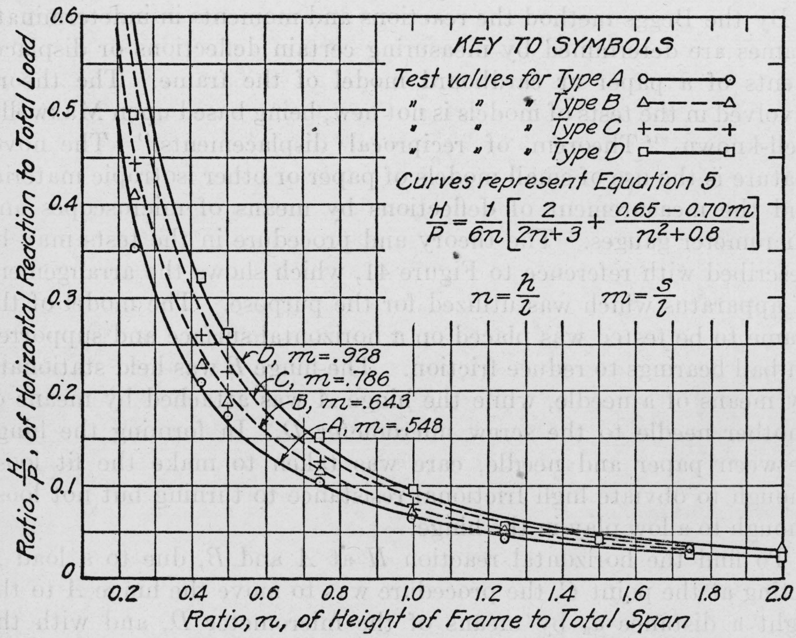

FIG. 42.—*Values of H/P from tests of paper models and from calculations*

by the two methods is very close and may be considered a satisfactory verification of equations (4) and (5).

Another use has been made of the models in determining relative deflections at various points on the top member of the frame. Figure 43 shows relative deflections of the top members of the models of types A, B, and C. These curves are, in effect, influence lines [12] for the horizontal reactions of the frame with a vertical load on the top member. It should be noted, however, that the purpose of these curves, which are similar to those of Figure 8, is to compare values of H/P with the loads at different points on the same frame and not to compare values between different types of frame. It is found that the horizontal reactions with loads at mid span and at the one-third

[12] See footnote 5, p. 200.

points are in the ratios of 1.15, 1.16, and 1.17, for types A, B, and C, respectively; with uniform loads and one-third point loads the reactions are in the ratios 0.75, 0.73, and 0.71 for the three frames, respectively. Comparing these ratios with those calculated in section 4, where for type A the ratio between the reactions for loads at mid span and at one-third points was taken at 1.125, and the ratio for uniform load and one-third point load was taken at 0.75, the agreement is seen to be very close and the correctness of the basis of equations (6) and (7) is thus substantiated.

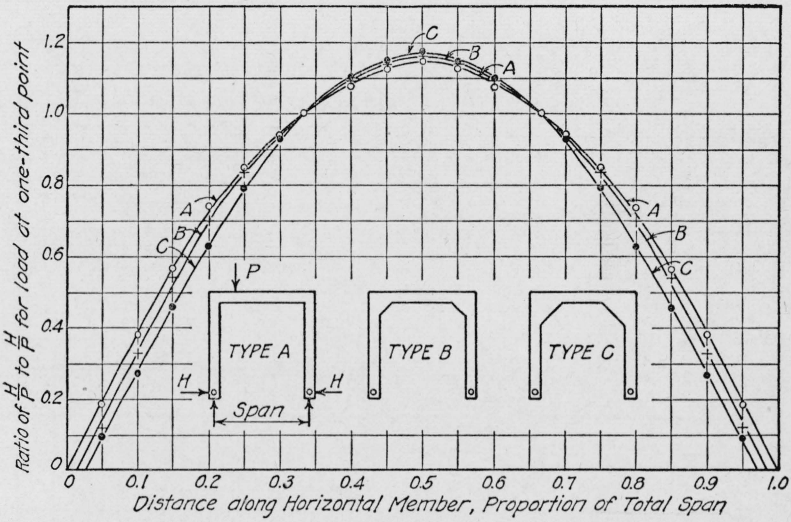

FIG. 43.—*Influence lines for horizontal reactions of frames of Types A, B, and C*

In conclusion it may be said that the tests on paper models gave results which were remarkably close to those found by analysis. However, individual observations varied considerably from the mean value found, and it was necessary to take a number of readings to eliminate errors of observation and manipulation. Further, preliminary tests with these models indicated that some grades of paper are not isotropic or are not uniform in certain properties, so that considerable care must be used in the selection of the material if such tests are to be used for scientific work.

WASHINGTON, April 4, 1928.