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New Results From Previously Reported NBS Fundamental Constant Determinations

B. N. Taylor
National Bureau of Standards, Gaithersburg, MD 20899

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A new treatment of previously reported results of three electric-unit-dependent fundamental constant experiments carried out at NBS over the last decade or so yields accurate, indirect values in SI units for a number of important quantities. These include the fine-structure constant α, the Avogadro constant N_A, the Josephson frequency-voltage ratio $2e/h$, and the quantized Hall resistance $R_H \equiv h/e^2$.

Key words: Avogadro constant; Faraday constant; fine-structure constant; Josephson frequency-voltage ratio; proton gyromagnetic ratio; quantized Hall resistance.

1. Introduction

Over the last decade or so the National Bureau of Standards (NBS) has reported the results of three high accuracy fundamental constant determinations carried out in terms of NBS as-maintained electrical units. The three quantities are the Faraday constant F_{NBS} determined in 1975 using a silver-perchloric acid coulometer [1][1]; the low field gyromagnetic ratio of the proton $(\gamma'_{\mathrm{p,L}})_{\mathrm{NBS}}$ measured in 1978 using the so-called weak or low field method (the prime means the protons are in a spherical sample of pure H_2O at 25 °C) [2]; and the quantized Hall resistance $(R_H)_{\mathrm{NBS}} \equiv (h/e^2)_{\mathrm{NBS}}$ (h is the Planck constant and e the elementary charge) determined in 1983–1984 [3][2] using the quantum Hall effect [6] in GaAs-$\mathrm{Al_xGa_{1-x}As}$ heterostructures. Here and throughout this paper the subscript NBS indicates NBS electrical units; lack of a subscript indicates *Le Système International d'Unités* (SI) units.

About the Author: B. N. Taylor, a physicist, is chief of the Electricity Division in the NBS Center for Basic Standards.

[1]Numbers in brackets indicate literature references.

[2]The result reported in Ref. [3] supplants entirely that given in Ref. [4] since the temperature dependence of R_H, as described in Ref. [5], was not properly taken into account in the earlier work of Ref. [4]. We assume throughout this paper that the quantized Hall resistance is a legitimate measure of h/e^2, i.e., that any corrections are negligibly small.

The NBS as-maintained electrical units in terms of which F_{NBS}, $(\gamma'_{\mathrm{p,L}})_{\mathrm{NBS}}$, and $(R_H)_{\mathrm{NBS}}$ have been measured are $(\mathrm{V_{NBS}}/\Omega_{\mathrm{NBS}}) = \mathrm{A_{NBS}}$ and Ω_{NBS}. Here $\mathrm{V_{NBS}}$ is the NBS volt maintained constant in time since 1972 July 1 using the ac Josephson effect with an uncertainty of a few parts in 10^8, the value of the Josephson frequency-voltage ratio $2e/h$ adopted for this purpose being [7]

$$(2e/h)_{\mathrm{NBS}} = 483593.420\ \mathrm{GHz/V_{NBS}}. \qquad (1)$$

This implies that the ratio $K_V \equiv \mathrm{V_{NBS}}/\mathrm{V}$, where V is the SI volt, is given by

$$K_V \equiv \mathrm{V_{NBS}}/\mathrm{V} \qquad (2a)$$

$$= (2e/h)_{\mathrm{NBS}}/(2e/h) \qquad (2b)$$

and may be assumed to be time invariant within a few parts in 10^8.

The quantity Ω_{NBS} is the NBS ohm defined in terms of the mean resistance of five wire-wound, one-ohm resistors of the Thomas-type. Because Ω_{NBS} is based on artifact standards, one must assume that it is a time-varying unit. This implies that the ratio

$$K_\Omega \equiv \Omega_{NBS}/\Omega \qquad (3)$$

also varies with time.

The NBS ampere A_{NBS} is not separately maintained but is defined in terms of the NBS ohm and volt: $A_{NBS} = V_{NBS}/\Omega_{NBS}$. It thus follows that A_{NBS} also varies with time as does the ratio

$$K_A \equiv A_{NBS}/A \qquad (4a)$$

$$= K_V/K_\Omega \ . \qquad (4b)$$

The actual NBS electrical units in terms of which F_{NBS}, $(\gamma'_{p,L})_{NBS}$, and $(R_H)_{NBS}$ were measured are, respectively, A_{NBS}, A_{NBS}^{-1}, and Ω_{NBS}. Because A_{NBS} and Ω_{NBS} are time varying units as just discussed and the three experiments were carried out over a 10-year period, the three results cannot be readily combined to obtain values for other quantities such as the fine-structure constant α and Avogadro constant N_A. This is unfortunate since, had they been measured at the same time, it would have been a straightforward procedure to derive accurate, indirect values in SI units for these constants as well as for most others of interest. The relevant equations for doing so may be obtained from the known relationships among the constants [8–11]; some of the more important expressions are:

$$\alpha^{-1} = \left[\frac{(\mu'_p/\mu_B)}{2\mu_0 R_\infty} \frac{R_H(t)_{NBS}(2e/h)_{NBS}}{\gamma'_{p,L}(t)_{NBS}}\right]^{1/3} \qquad (5)$$

$$N_A = [R_H(t)_{NBS}(2e/h)_{NBS}F(t)_{NBS}]/2 \qquad (6)$$

$$2e/h = \left[\frac{16^2 R_\infty^2(\mu'_p/\mu_B)(m_p/m_e)^3}{\mu_0^4 c^6 M_p^3}\right.$$

$$\left. \times \frac{R_H(t)_{NBS}^4(2e/h)_{NBS}^4 F(t)_{NBS}^3}{\gamma'_{p,L}(t)_{NBS}}\right]^{1/6} \qquad (7)$$

$$R_H = \mu_0 c \alpha^{-1}/2 = R_H(t)_{NBS} K_\Omega(t)_{NBS} \qquad (8a)$$

$$= \left[\frac{\mu_0^2 c^3(\mu'_p/\mu_B)}{16 R_\infty} \frac{R_H(t)_{NBS}(2e/h)_{NBS}}{\gamma'_{p,L}(t)_{NBS}}\right]^{1/3} \qquad (8b)$$

$$F = \left[\frac{M_p}{(\mu'_p/\mu_B)(m_p/m_e)} \gamma'_{p,L}(t)_{NBS} F(t)_{NBS}\right]^{1/2} \qquad (9)$$

$$K_\Omega(t) = \left[\frac{\mu_0^2 c^3(\mu'_p/\mu_B)}{16 R_\infty} \frac{(2e/h)_{NBS}}{R_H(t)_{NBS}^2 \gamma'_{p,L}(t)_{NBS}}\right]^{1/3}, \qquad (10)$$

where the time dependencies of $(R_H)_{NBS}$, $(\gamma'_{p,L})_{NBS}$, and F_{NBS} have been explicitly indicated [$(2e/h)_{NBS}$ is time independent since V_{NBS} is defined in terms of it through eq (1)]. In these expressions $\mu_0 \equiv 4\pi \times 10^{-7}$ H/m is the permeability of vacuum; $c \equiv 299792458$ m/s is the speed of light in vacuum; μ'_p/μ_B is the magnetic moment of the proton in units of the Bohr magneton (0.012 parts-per-million or ppm current uncertainty[3]); R_∞ is the Rydberg constant for infinite mass (0.0010 ppm current uncertainty); M_p is the molar mass of the proton (0.012 ppm current uncertainty); and m_p/m_e is the ratio of the proton mass to the electron mass (0.043 ppm current uncertainty).

In the past, these and related expressions have been evaluated by assuming that Ω_{NBS} has been constant in time and by allowing an additional uncertainty of about 0.01 ppm per year for its possible drift [1,2,4]. However, recent comparisons of Ω_{NBS} with the resistance units of other national laboratories show that since about 1970 the NBS ohm has likely been decreasing by approximately 0.06 ppm/yr. In the course of reviewing the implications of such a drift on eqs (5–10), it became apparent that the problem of the time variation of Ω_{NBS} could be neatly solved by making use of one other NBS result, namely, the determination in 1973–1974 via the NBS calculable cross capacitor of the ratio $K_\Omega \equiv \Omega_{NBS}/\Omega$ [12]. With the addition of this single measurement, the drift rate of the NBS ohm may be uniquely determined, the values of $(R_H)_{NBS}$, $(\gamma'_{p,L})_{NBS}$, and F_{NBS} converted to the same measurement date, and eqs (5–10) and their numerous extensions readily evaluated. The only critical assumption required is that the drift of the NBS ohm has been linear since the time of the calculable capacitor determination of K_Ω. However, this is supported by the observed linear time dependencies of the measured differences between each of the five resistors which comprise Ω_{NBS} and their mean [13].

We now briefly summarize how the calculation proceeds. The linear drift rate assumption enables one to write

$$K_\Omega(t) \equiv \Omega(t)_{NBS}/\Omega = K_\Omega(t_\Omega)[1 + b(t - t_\Omega)], \qquad (11)$$

where t is the time in years measured from some arbitrary date, t_Ω is the mean date of the NBS calculable capacitor experiment with $K_\Omega(t_\Omega)$ the mean value obtained, and b is the relative drift rate of Ω_{NBS}.[4] It then

[3]Throughout, all uncertainties are one standard deviation estimates.

[4]The time period over which K_Ω was measured was sufficiently short and the random scatter sufficiently large that the effect of the drift of Ω_{NBS} was indiscernible and hence negligible. This is also true of the measurements of F_{NBS}, $(\gamma'_{p,L})_{NBS}$, and $(R_H)_{NBS}$.

follows from this equation and the way Ω_{NBS} enters the determination of F_{NBS}, $(\gamma'_{p,L})_{NBS}$, and $(R_H)_{NBS}$ that

$$F(t)_{NBS}=F(t_F)_{NBS}[1+b(t-t_F)] \tag{12}$$

$$\gamma'_{p,L}(t)_{NBS}=\gamma'_{p,L}(t_\gamma)_{NBS}[1-b(t-t_\gamma)] \tag{13}$$

$$R_H(t)_{NBS}=R_H(t_R)_{NBS}[1-b(t-t_R)], \tag{14}$$

where t_F, t_γ, and t_R are the mean dates of the Faraday constant, proton gyromagnetic ratio, and quantized Hall resistance determinations, respectively, and $F(t_F)_{NBS}$, $\gamma'_{p,L}(t_\gamma)_{NBS}$, and $R_H(t_R)_{NBS}$ are the mean values obtained. The drift rate b may then be calculated by substituting eq (13) for $\gamma'_{p,L}(t)_{NBS}$ into eq (10) and equating the result to eq (11) with $t=t_R$. Once b is in hand, eqs (5–10) may be evaluated with the aid of eqs (12–14). Of course, the law of error propagation must be strictly obeyed in order to obtain correct uncertainties for the calculated quantities.

We have carried out such calculations with the data listed in tables 1 and 2, to which these comments apply:

Table 1. The values quoted for the nonexact constants are based on the results reported in the original references and (with the exception of m_p/m_e) analyses carried out by the author in connection with the 1985 least squares adjustment of the fundamental constants [14]. However, these analyses have generally led to only minor changes in the original results.

Table 2. Only the value of F_{NBS} has been at all changed from that originally reported. The 0.2 ppm net increase arises from a number of positive and negative corrections. New measurements of the Au and Ta impurity content of the silver used in the experiment [20] were specifically undertaken as a result of the author's reanalysis of the experiment for the 1985 adjustment. It should be noted that the 0.031 ppm uncertainty assigned $(2e/h)_{NBS}$ is an estimate of how well the NBS Josephson effect voltage standard apparatus implements the definition of V_{NBS} [see eq (1)]. The uncertainties associated with relating the working voltage standards used in the $(\gamma'_{p,L})_{NBS}$ and F_{NBS} experiments to the voltage standards used to preserve or store V_{NBS} between Josephson effect calibrations are included in the uncertainties assigned F_{NBS} and $(\gamma'_{p,L})_{NBS}$ as given in the table.

The results of the calculations are:

$$\alpha^{-1}=137.035981(12)\ (0.089\ \text{ppm}) \tag{15}$$

$$R_H=25812.8041(23)\ \Omega\ (0.089\ \text{ppm}) \tag{16}$$

$$2e/h=483597.91(32)\ \text{GHz}\cdot\text{V}^{-1}\ (0.67\ \text{ppm}) \tag{17}$$

$$N_A=6.0221438(80)\times10^{23}\ \text{mol}^{-1}\ (1.33\ \text{ppm}) \tag{18}$$

$$F=96485.381(65)\ \text{A}\cdot\text{s}\cdot\text{mol}^{-1}\ (0.67\ \text{ppm}), \tag{19}$$

Table 1. Summary of defined and electric-unit-independent constants used to evaluate eqs (5–10).

Quantity	Numerical value[1]	Uncertainty (ppm)	Refs.
μ_0	$4\pi\times10^{-7}$	defined	
c	299792458	defined	
μ'_p/μ_B	0.001520993127(18)	0.012	[15,14]
R_∞	10973731.529(11)	0.0010	[16,17,14]
M_p	0.001007276470(12)	0.012	[18,14]
m_p/m_e	1836.152470(79)	0.043	[19]

[1]The units for μ_0 are $\text{H}\cdot\text{m}^{-1}$; for c, $\text{m}\cdot\text{s}^{-1}$; for R_∞, m^{-1}; for M_p, $\text{kg}\cdot\text{mol}^{-1}$.

Table 2. Summary of NBS electric-unit-dependent data used to evaluate eqs (5–10).

Quantity	Mean date of measurement	Value[1]	Uncertainty (ppm)	Refs.
$(2e/h)_{NBS}$	Continuous since 1972 July 1	483593.420(15)	0.031	[7]
F_{NBS}	1975 March 15	96486.19(13)	1.33	[1,20,14]
$(\gamma'_{p,L})_{NBS}$	1978 March 22	2.67513229(57)	0.21	[2]
$(R_H)_{NBS}$	1983 November 27	25812.8420(12)	0.047	[3]
$K_\Omega\equiv\Omega_{NBS}/\Omega$	1973 December 2	$1-0.819(27)\times10^{-6}$	0.027	[12]

[1]The units for $(2e/h)_{NBS}$ are $\text{GHz}\cdot\text{V}_{NBS}^{-1}$; for F_{NBS}, $\text{A}_{NBS}\cdot\text{s}\cdot\text{mol}^{-1}$; for $(\gamma'_{p,L})_{NBS}$, $10^8\ \text{s}^{-1}\cdot\text{T}_{NBS}^{-1}$; for $(R_H)_{NBS}$, Ω_{NBS}. Note that in the $(\gamma'_{p,L})_{NBS}$ experiment the NBS tesla $\text{T}_{NBS}\propto\text{A}_{NBS}$.

with $b = (-0.0650 \pm 0.0102)$ ppm/yr.[5] The value for K_Ω on 1985 January 1 is $1-(1.539 \pm 0.107)$ ppm. While it is not the purpose of this paper to make detailed comparisons of these results with others obtained by either direct or indirect means, we do note that in most cases where other values exist the agreement is statistically acceptable. We further point out that the value of $2e/h$ derived here is (9.29 ± 0.67) ppm larger than the value used to define V_{NBS}, implying that V_{NBS} is (9.29 ± 0.67) ppm smaller than the SI volt [see eq (2b)]. This value of $2e/h$ is also (8.09 ± 0.67) ppm larger than the value 483594 GHz/V which is in use in many other national laboratories and which was recommended by the Comité Consultatif d'Electricité in 1972 [23].

In closing we emphasize that the values given here for the various constants, in particular the fine-structure constant, *are the best that can be obtained based solely on existing NBS electrical measurements*.[6] If the three quantities $(\gamma'_{p,L})_{NBS}$, $(R_H)_{NBS}$, and K_Ω had been determined at the same time, then any two would have been sufficient to determine α^{-1}. For example, eq (5) yields a value of α^{-1} from $(\gamma'_{p,L})_{NBS}$ and $(R_H)_{NBS}$. The comparable equations for the two other pairs of measurements are[7]

$$\alpha^{-1} = \left[\frac{c(\mu'_p/\mu_B)}{4R_\infty} \frac{(2e/h)_{NBS}}{K_\Omega(\gamma'_{p,L})_{NBS}}\right]^{1/2} \tag{20}$$

$$\alpha^{-1} = 2(R_H)_{NBS} K_\Omega/\mu_0 c. \tag{21}$$

By treating the drift rate of the NBS ohm as a variable we remove the redundancy or overdetermination inherent in the three measurements and obtain a unique value for α^{-1} and all other quantities. Since this approach eliminates the distinction between the so-called Josephson effect value of α^{-1} traditionally derived from eq (20) and the quantum Hall effect value derived from eq (21), perhaps the value given here should simply be referred to as the NBS condensed matter value.

[5]This value is in agreement with $b = -(0.0562 \pm 0.0048)$ ppm/yr which the author derived from a linear least squares fit to the results of direct and indirect comparisons of Ω_{NBS} with the unit of resistance Ω_{NML} maintained constant in time by Australia's National Measurement Laboratory (NML) with the NML calculable cross capacitor [21]. We also note that the Bureau International des Poids et Mesures unit of resistance Ω_{BIPM} is based on the mean of six resistors, two of which are of the same type used to define Ω_{NBS}. Comparisons of Ω_{BIPM} with Ω_{NML} dating back to 1964 show that Ω_{BIPM} varies quite linearly with time. Since the time dependencies of the measured differences between Ω_{BIPM} and each of the two NBS resistors which partially define Ω_{BIPM} are also observed to be linear [22], it may be concluded that the linear drift rate assumption for Ω_{NBS} is justified. If in fact one assumes the existence of a quadratic component, any reasonable estimate of its magnitude is sufficiently small that its effect is inconsequential.

[6]"Best" or "recommended" values in the traditional sense would, of course, require taking into account the relevant data available from all sources.

[7]Note that eliminating K_Ω from eqs (20) and (21) yields eq (5) while equating the two yields eq (10).

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