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The Force-Elongation Curve of a Thin Fibrous Network

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Specimens from low-density weblike handsheets were tested in a tensile tester. In a test the direction of extension was frequently reversed and the specimen reextended to obtain a series of force-elongation curves. For Kraft woodpulp specimens the force-elongation behavior was well represented by an exponential equation involving three parameters: a modulus of elasticity C_2, a length parameter x_c related to average segment length between network bonds, and an elongation value x_0 at which the curve starts. The unstrained length of a specimen l increases, and the parameters x_c and C_2 tend to decrease with each successive reextension curve. For a series of specimens of increasing area density representative values of x_c/l tend to decrease and C_2 to increase. For a series of specimens made from pulps beaten increasing amounts representative values of x_c/l tend to decrease and C_2 to increase. Some features of the tensile behavior can be modeled by a system of parallel filaments of equal length to which longer parallel filaments with an exponential length distribution have been added. Upon extension the filaments assume load successively, thus simulating the force-elongation behavior of a paper network. By thinking in terms of this model it is possible to anticipate intuitively much of the behavior of a paper network.

Key words: nonwoven fabrics, tensile behavior; paper, tensile behavior; paper fibers, adhesion; paper fibers, bonding; paper pulps, characterization.

1. Introduction

In the making of paper, a mat of fibers usually obtained from woodpulp is deposited from a slurry onto a screen. The fibers in the mat bond together to form paper, the quality of which depends upon the kind of pulp and additives used, the beating treatment to which the pulp is subjected, and the subsequent processing of the mat after it has been formed. Such important paper properties as strength and durability are dependent upon the strength and permanence of the interfiber bonds. Therefore, a knowledge of pulp characteristics associated with good bonding could be very useful in making paper from recycled pulps, for example, or in producing papers with improved durability.

In order to estimate bond strength, it is necessary to determine the behavior of a paper specimen subjected to mechanical strain and to interpret this behavior in terms of a suitable theory. To this end Dodson [1][1] has reviewed literature on the nature of bonds in paper and the behavior of paper under mechanical strain. The experimental and theoretical behavior of nonwoven fabrics subjected to tensile stress has been studied by Backer and Petterson [2,3] and by Hearle et al. [4–8]. Dent [9] has reviewed existing theories for calculating Young's modulus of filament webs.

The "two-dimensional" fiber network discussed by Kallmes and Corte [10] is especially suitable for studies of bond strength. In such a network the fibers lie in a single plane except at the fiber-fiber contacts; most of the contacts involve only two fibers and the total area of the crossings is a small fraction of the sheet area.

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[1] Figures in brackets indicate literature references at the end of this paper.

Such a network can be approximated, for instance, by a handsheet of 2.5 g/m^2 mass per unit area made from spruce sulfite pulp. The structure of this network can be subjected to theoretical analysis, and handsheets approximating this network are sufficiently durable for testing.

If a specimen cut from such a handsheet is stretched to break in a sensitive tensile tester, a jagged force-elongation curve is obtained in which the force rises and dips through one or more peaks, and eventually diminishes to zero. Such a curve is exemplified by the recorder trace, figure 1, obtained by testing a specimen from a 2.5 g/m^2 handsheet made from Southern Kraft pulp that had been beaten 5,000 revolutions in a laboratory beater. The specimen, 2 cm long and 1 cm wide, was extended at a rate of 0.2 cm/min; 1 cm of chart travel corresponds to an extension of 0.01 cm, and full-scale force for the recorder chart is 49 mN (5g).

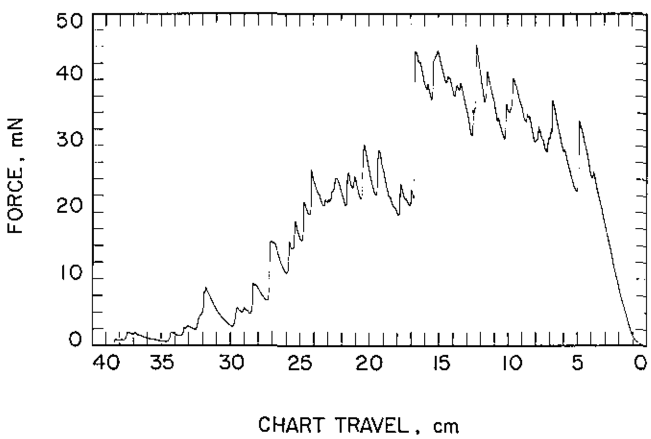

Figure 1–Force-elongation curve for a handsheet specimen of beaten Southern Kraft woodpulp, 2.5 g/m^2 mass per unit area.

Each jag in the curve signifies the breaking of a bond between fibers. (It is assumed that the fibers are much stronger than the bonds between them and that the fibers themselves do not break.) When a bond break occurs, energy previously stored in the network is dissipated, and Dodson [11] has suggested that the average strain energy dissipated per bond break could be used as an energy parameter to characterize bonding. Smith [12–15] has evaluated this and other parameters from tests on a number of different handsheets, but the resulting values were of low precision. These values were obtained by a data analysis partly dependent upon subjective judgment.

It is believed that much of this subjectivity could be eliminated were it possible to characterize the force-elongation behavior of a test specimen at various stages during a tensile test. A semiempirical method for accomplishing this was discovered, but the method must be checked by additional experiments before its applicability can be established. This cannot be done at the present time inasmuch as this research has been discontinued. The method, however, seems promising and is presented herewith.

2. Characterization of the Force-Elongation Curve

During a tensile test the following behavior sequence is observed in the fibrous network: the fibers tend to align themselves in the direction of the extension with the straightest and most perfectly aligned fiber segments bearing most of the load. As the specimen is elongated it contracts laterally forming a fluted structure in the portions not restrained by the end clamps. Bond breaks often occur at random points, causing only local damage and leaving the surrounding network essentially intact. However, a breakage of some bonds, usually in the vicinity of network flaws or irregularities, results in more extensive damage. Often a succession of breaks occurs in the same region because of stress concentrations that develop. In the regions of more extensive damage, parts of the network lose their strain and sag away. As the extension continues, further deterioration occurs. Portions of the network midway between the ends of the specimen tend to collapse into threadlike structures in which the longitudinal forces are concentrated. These forces in turn are diffused through the uncollapsed portions of the network at either end. In the final stages of the breaking process, often only one threadlike structure remains. The specimen then sustains relatively large elongations before breaking.

Consider the recorder trace, figure 2, obtained by testing a specimen from a 2.5 g/m^2 handsheet made from Northern Kraft woodpulp that had been beaten 5,000 revolutions in a laboratory beater. The specimen, 2 cm long by 1 cm wide, was extended at a rate of 0.2 cm/min. Chart travel of 1 cm corresponds to a speciman extension of 0.01 cm, and full-scale force for the specimen is 196 mN (20 g). At intervals the test was stopped, the extension was reversed until the tensile force in the specimen was zero, and the specimen then reextended to obtain a new force-elongation curve. Recording traces during the reversal of the crosshead travel are not shown.

The general condition of the specimen at various stages is noted in figure 2. For the first eight reextensions the specimen did not develop any large holes or tears, and the state of the specimen at the

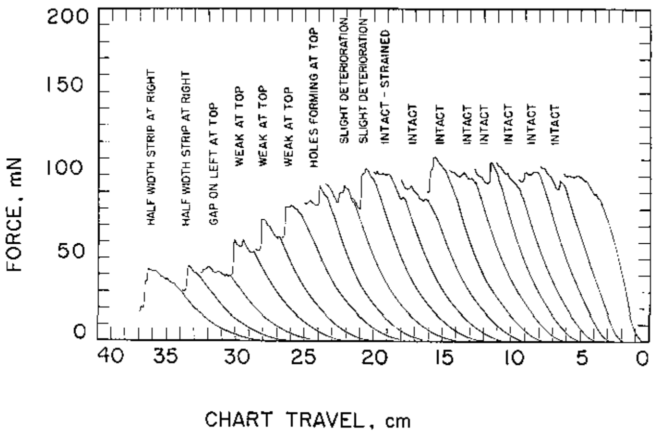

Figure 2–Cycled force-elongation curves for a handsheet specimen of Northern Kraft woodpulp, 2.5 g/m² mass per unit area.

beginning of the reextension is described as "intact." At later stages of the test the specimen developed holes and tore at the edges as it deteriorated.

In figure 3 are shown plots of F', or force per unit elongation, versus force F for the first 11 reextended force-elongation curves of figure 2. The origin of the F' scale has been shifted vertically for each of these plots, and arbitrary units have been selected to display the plots to best advantage. The curve corresponding to each plot is identified by its region of chart travel. The plots are seen to be linear and to have positive intercepts on the F' axis. Plots similar to these have resulted from most of the tests on specimens formed from woodpulps.

It will be noticed that the early points corresponding to small values of F usually fall below the straight line. At these beginning force values the slope of the force-elongation curve increases rapidly as the network fibers start to orient. It is only after the initial orientation that the network behaves so that F' increases linearly with F. Also it has been found in general that F, F' data from the initial force-elongation curve in a test do not fit well to a straight line. In this instance F' usually rises rapidly, becomes constant until F is moderately large, and then increases linearly with F. Data from the first two reextension curves often tend to plot in a similar fashion. This behavior is thought to arise from an initial maldistribution of forces in a network. It is only after the network has been "broken in" that F' has a good linear correspondence with F. In the plot of figure 3, data from the initial force-elongation curve therefore have been omitted.

Sometimes when a test specimen is unloaded the recorder trace does not return to the original zero force level. This behavior is usually an effect of

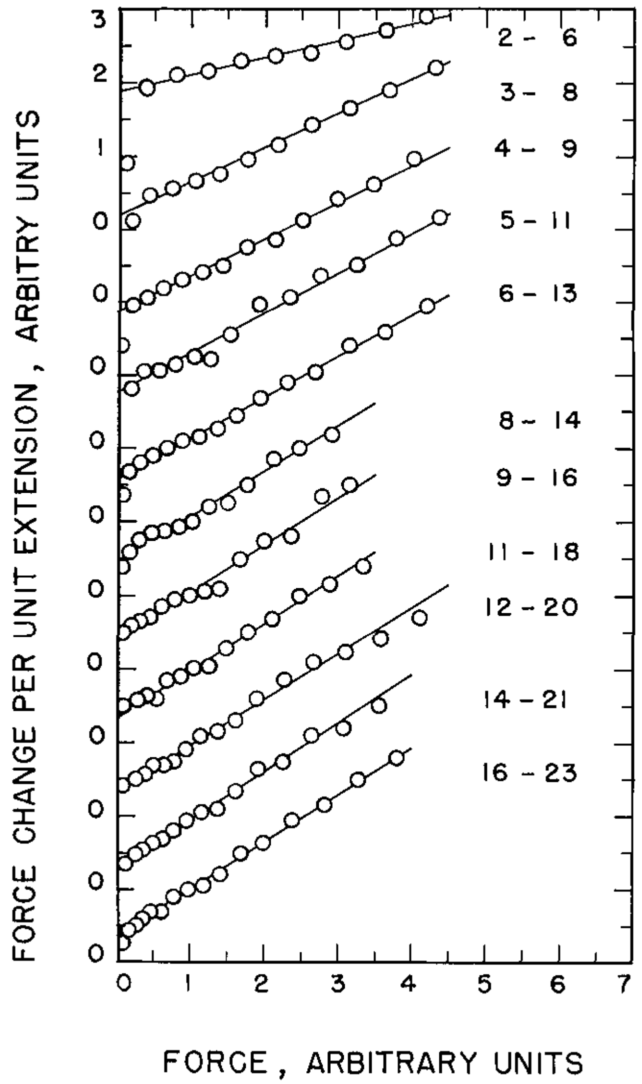

Figure 3–Plots of force change per unit extension versus force for curves of figure 2. Numbers at right give the region of chart travel in cm for each associated force-elongation curve.

hysteresis. Its consequences can be minimized by waiting a few minutes before reextending the specimen, but it is more practical to reextend immediately and to correct data from the new curve for the change in baseline. Sometimes a shift of baseline is due to a change in the recorder system, a circumstance especially likely because of the high sensitivities required by these tests. Thus a correction for baseline shift should be made in any event.

Force-elongation curves of these paper network specimens exhibit viscoelastic, or hysteretic behavior. Thus the slopes of the force-elongation curves will depend upon the rate of extension. The rate of extension therefore must be maintained at some standard value in all of the tests in order to obtain comparable results. This precaution should minimize the influence of viscoelasticity, but the possibility of other hysteretic effects should be kept in mind in the interpretation of data.

Let the equation of a straight line faired through a plot such as those shown in figure 3 be

$$\frac{dF}{dx} = \frac{1}{x_c}(C_2+F) \tag{1}$$

where x_c is the reciprocal of the slope of the straight line, and C_2/x_c is the intercept on the F' axis. This differential equation can be integrated to obtain

$$F = C_1 e^{x/x_c} - C_2 \tag{2}$$

In eqs (1) and (2) F is the tensile force sustained by the specimen, and x is a distance variable measured in the direction of the elongation. C_1, C_2 and x_c are constants characterizing the force-elongation curve.

Ordinarily the values of C_1, C_2 and x_c would be obtained by fitting eq (2) to a curve, using the least squares technique. This method, however, involves solving a complicated transcendental equation, so a procedure providing a good approximate solution is used instead. The x, F data from the curve to be fitted is differentiated to obtain values of F', and eq (1) is fitted to the F, F' data to evaluate C_2 and x_c. Equation (2) with these known values of C_2 and x_c is then fitted to the curve to evaluate C_1.

According to eq (2), F becomes zero when

$$x = x_0 = x_c \ln\ (C_2/C_1) \tag{3}$$

Thus eq (2) can be put in the form

$$F = C_2[e^{(x-x_0)/x_c} - 1] \tag{4}$$

x_0 is the value of the distance variable at which the force just increases from zero, so x–x_0 is the elongation of the network. As the specimen fibers become better aligned, and as the specimen deteriorates, the unstrained length of the specimen increases. Thus for each successive reextension curve of figure 2, successively increasing values of x_0 should be expected.

Equation (2) or (4) usually fits test data on woodpulp specimens very well. For instance, values of C_1, C_2, x_c and x_0 were found for each of the force-elongation curves of figure 2, and corresponding curves, eq (4), were calculated. Plots of these calculated curves superposed the original data for practical purposes. The only noticeable deviations occurred at small values of F. The calculated curves started to rise at slightly larger values of x, and had larger initial slopes.

It should be emphasized that the force-elongation curves characterized here were obtained from tests on specimens formed from Kraft woodpulps. In these pulps strong bounds are formed only between adjacent fibers that are compatible in shape and are in intimate contact. Bondability is enhanced by a beating treatment that causes the fibers to become pliable and to assume a ribbonlike shape. In some other types of pulps beating causes fibrillation, and the bonds formed consist of many microbonds between the fibrils and adjacent fibers. Similar strong bonds extending over a large area are formed when the beaten pulp contains appreciable amounts of fines which act as a glue.

Test data obtained on network specimens formed from these latter pulps differ from data obtained on woodpulp specimens. In the plots of F' versus F, F' rises slowly at first or stays at a constant value, and only increases linearly with F after F has become moderately large. A straight line through this late linear portion usually has a negative intercept on the F' axis. Force-elongation data for these pulps cannot be fitted by eq (4). A more complicated equation involving an additional linear or exponential term is required.

3. A Mechanism for Force-Elongation Behavior

When a test specimen from a fibrous network is stretched, segments of the fibers composing the network tend to orient themselves in the direction of the extension and to resist the extension through the combined action of forces along their lengths. As the extension proceeds, more and more of these segments between bonds become oriented and bear load. Other fiber segments have a curved configuration between bond points, and cannot bear load until the network is stretched sufficiently to straighten them out. As a result of these processes the force-elongation curve has a slope that increases with increasing elongation. This reaction of the network to extension can be simulated by a system of parallel filaments of unequal lengths, each filament adding to the resistive force of the system as the extension is increased. The flexural stiffness of the fibers and the restraining action of the clamps at the ends of the specimen also contribute to the network's reaction to stretching, but these and other effects are considered secondary in importance and can be disregarded.

Consider as a model a system of parallel filaments of unequal length. Let the unstrained length of the system be l, and let there be ν filaments of this length. Let the length of any other filament be $l+\eta$, and let the number of filaments having length increments between η and $\eta+d\eta$ be $\nu f(\eta)d\eta$. Let each of the filaments be linearly elastic with spring constant k, neglecting the decrease in k with increase in unstrained length. If this system of parallel filaments is extended from the length l to a length $l+\xi$, the sum S of the tensile forces in the filaments is

$$S=\nu k\xi+\nu k\int_0^\xi f(\eta)(\xi-\eta)d\eta \qquad (5)$$

To find the proper value of $f(\eta)$, S can be set equal to the force F, as defined by eq (4). Thus if it is assumed that

$$f(\eta)=\frac{1}{x_c}e^{\eta/x_c} \qquad (6)$$

eq (5) becomes

$$F=\nu k\left\{\xi+\frac{1}{x_c}\int_0^\xi e^{\eta/x_c}(\xi-\eta)d\eta\right\}$$

$$=\nu k\left\{\xi+x_c\left[e^{\xi/x_c}-\frac{\xi}{x_c}-1\right]\right\}$$

$$=\nu k x_c\left[e^{(x-x_0)/x_c}-1\right] \qquad (7)$$

upon substituting the elongation $x-x_0$ for ξ. Thus $f(\eta)$ is properly defined by eq (6), and comparison of eqs (7) and (4) yields the relationship

$$C_2=\nu k x_c \qquad (8)$$

The model is thus seen to consist of ν filaments of unstrained length l in parallel with additional filaments of length $l+\eta$ distributed such that the number of filaments with length increments between η and $\eta+d\eta$ is

$$\frac{\nu}{x_c}e^{\eta/x_c}d\eta.$$

If such a model is elongated to a length $l+\xi$, the number of filaments should be equal to $\nu e^{\xi/x_c}$. It would be unrealistic to expect that this model would apply in situations were ξ becomes very large because too many stretched filaments would be required. In fact it is often observed that at large elongations the curvature of the force-elongation curve does not increase as rapidly as eq (1) would require. At these elongations, however, bond breaks occur; thus the exponential distribution just derived for filaments having lengths greater then l is adequate for practical purposes.

The response of a fibrous network to tensile strain may now be understood more clearly. Suppose that a typical test specimen (1 cm wide by 2 cm long) is gathered together at each end, and extended to form a threadlike structure. In an ideal network in which there are no fluctuations in density and the fibers have no flexural stiffness, no force would be required during this part of the extension. The threadlike structure would consist of a number of interconnected fiber pathways of perfectly oriented fiber segments. Most of the fiber segments interlinking the pathways would hang in slack loops, and would not sustain force until the structure is extended or until bonds maintaining the shorter pathways are broken. A test specimen with the original width maintained at the clamped ends would approximate the response just described for this idealized threadlike structure.

The tensile response of the threadlike structure when elongated would be similar to that of the parallel filament model, but the mechanism of the response is different. Instead of independent filaments there are interconnected pathways, and instead of additional filaments that assume load, interconnections between pathways become oriented and assume load as the elongation progresses. The number ν of initially strained filaments and the distribution function $f(\eta)$ for the additional filaments no longer have precise meanings, but are still useful for interpretations of behavior in terms of the parallel filament model.

From an inspection of eq (4) it is seen that x_c has dimensions of length. If the magnitude of x_c is small, the force-elongation curve will rise rapidly as $x-x_0$ increases. This is the kind of behavior that might be expected to result from testing a network in which the fiber segments between bonds are small. This small segment size insures that additional parallel pathways that sustain force are rapidly formed during extension of the network. This suggests that x_c is a characteristic length having a value that increases or decreases with average segment size of the network. For a given network the value of x_c found from a test also depends upon the length l of the test specimen. In order that force-strain curves be comparable for specimens of different length taken from the same handsheet, it is necessary that the characteristic strain $\epsilon_c=x_c/l$ have a constant value characteristic of the network. Thus the value of x_c should vary proportionally with the specimen length.

It is seen from eq (4) that C_2 is a parameter having dimensions of force; therefore its value should not depend upon specimen length. From eq (8) the value of C_2 for the parallel filament model is expressed as the product of ν initially strained filaments and the quantity kx_c. But k, the spring constant of the filament, can be set equal to m/l, where m is the tensile modulus of elasticity of a filament. Thus eq (8) becomes

$$C_2=\nu\frac{m}{l}x_c=\nu m\epsilon_c \qquad (9)$$

Although the value of C_2 may be independent of specimen length, it is expected to have a value that increases as the width increases. For long specimens the value of C_2 should be directly proportional to the width.

4. Effect of Deterioration

During a test the unstrained length of the specimen just before each reextension is given by

$$l=l_0+x_0-x_s \qquad (10)$$

where l_0 is the original unstrained length, x_0 is calculated for each reextension curve using eq (3), and x_s is the value of x at which the initial force-elongation curve just increases from zero. For the curves of figure 2 the test specimen had an initial unstrained length l_0 of 20.0 mm. The value of x_s, 5.0 mm on the chart, was 0.05 mm after multiplying by a scale factor of 0.01. In eq (10) the values of l_0 and x_s are constant for each of the successive reextension curves obtained in a test, but the value of x_0 increases with each successive curve because of progressive deterioration. Thus l, the unstrained length of the specimen just before each reextension, also increases with progressive deterioration, and the fractional increase in unstrained length l/l_0-1 might be taken as a measure of deterioration. The values of the parameters C_2 and x_c characterizing the successive reextension curves also change with progressive deterioration. These changes of C_2 and x_c as functions of deterioration, measured by l/l_0-1 are studied here.

Table 1 gives the values of constants characterizing the force-elongation curves of figure 2. According to this table the characteristic strain x_c/l for the specimen decreases in value as the specimen deteriorates. This behavior is demonstrated in figure 4 where values of x_c/l for the 2nd through the 11th reextension curves are plotted versus the corresponding values of l/l_0-1. The value of x_c/l for the first reextension curve was exceptionally high and was not plotted. A straight line approximating the dependence of x_c/l upon l/l_0-1 is passed through the data points determined from the third through the eighth reextension curves. Evidently the characteristic strain x_c/l decreases rapidly at first relative to deterioration as measured by l/l_0-1, but as the test progresses x_c/l decreases more slowly and in a more linear fashion. Similar behavior is almost always observed in tests on specimens formed from woodpulp.

According to table 1 the constant C_2 decreases very rapidly as the specimen deteriorates during a test. From eq (9) part of this decrease in value is attributable to decrease in value of x_c/l and the remainder to decrease in value of $\nu m=lC_2/x_c$. The decrease in value of lC_2/x_c is still very rapid but may

Table 1. Values of constants C_2, x_c and x_0 for curves of figure 2.

Curve	x_0 (mm)	l (mm)	l/l_0-1 (%)	x_c (mm)	C_2 (mN)	x_c/l (%)	$\log_e lC_2/x_c$ ([1])
2–6	0.21840	20.168	0.842	0.84484	153.66	4.1889	8.2075
3–8	0.31207	20.262	1.310	0.42984	49.456	2.1214	7.7542
4–9	0.41506	20.365	1.825	0.38783	31.634	1.9044	7.4152
5–11	0.53146	20.482	2.407	0.35375	24.750	1.7272	7.2675
6–13	0.65520	20.605	3.026	0.35872	21.343	1.7409	7.1115
8–14	0.79018	20.740	3.701	0.33553	16.005	1.6178	6.8970
9–16	0.93808	20.888	4.440	0.33543	14.312	1.6058	6.7926
11–18	1.0905	21.041	5.203	0.32164	11.572	1.5287	6.6293
12–20	1.2613	21.211	6.057	0.33259	11.854	1.5680	6.6280
14–21	1.4338	21.384	6.919	0.31462	9.500	1.4713	6.4703
16–23	1.6796	21.630	8.148	0.30249	10.861	1.3985	6.6549

[1] C_2 is expressed in mN units, x_c/l as a fraction.

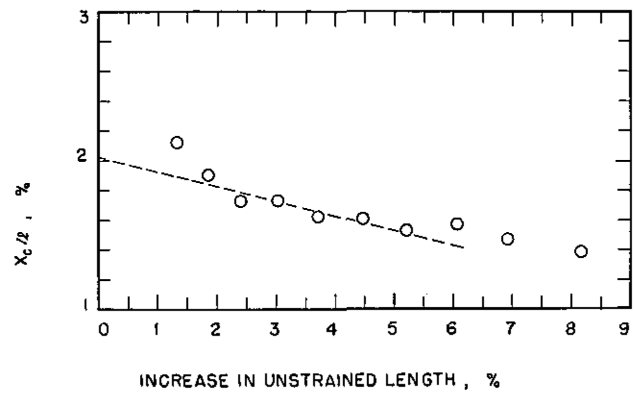

Figure 4–Plot of x_c/l versus percent increase in specimen length, data of table 1.

be mitigated for comparison purposes by plotting the natural logarithm of this quantity versus the fractional increase in length l/l_0-1, as is done in figure 5. A straight line approximating this dependence is passed through data points determined from the third through the eighth reextension curves. The trend of this plot is similar to that observed for test data from other specimens formed from woodpulp.

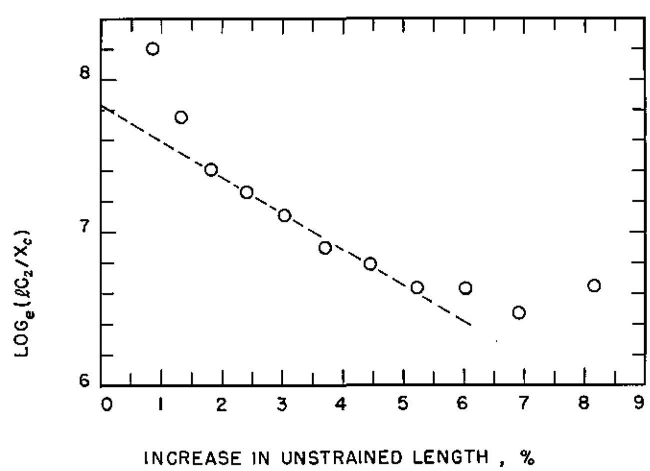

Figure 5–Plot of $\log_e(lC_2/x_c)$ versus percent increase in specimen length, data of table 1. C_2 is expressed in mN units, x_c/l as a fraction.

A mechanism accounting for the decrease of x_c/l and $lC_2/x_c=\nu m$ as the specimen deteriorates is not intuitively evident. However one might speculate that the following process occurs: assume that the specimen network is formed of fiber segments of size distribution such that longer segments occur less frequently than shorter segments. Assume also that the longest segments are the most likely to sustain forces and thereby dominate the force-elongation behavior of the network. Thus for force-elongation curves obtained early in a test the fiber pathways sustaining force are channeled through the long segments. In this instance the network might react to an elongation as though it were a coarse network composed of a small number of fiber threads with long segment lengths between bonds. The measured value of $lC_2/x_c=\nu m$ would be proportional to the modulus of one of these threads, and thus be large. The measured value of x_c/l would be proportional to the segment length, and thus be large also. Bonds associated with the long segments are subjected to large forces and tend to break preferentially, so that as the test proceeds the fiber pathways sustaining force are channeled through the more numerous fiber segments of shorter length. Thus the values of x_c/l and $\nu m=lC_2/x_c$ tend to decrease progressively as the test continues.

The effects of the mechanism just described are observed at that stage of the test when the specimen is "broken in" but still intact. For the initial and first one or two reextension curves extremely high values of x_c/l and lC_2/x_c are observed, and these values decrease rapidly as the test proceeds. Much of this initial behavior may attributed to fluctuations in density of fiber distribution in the specimen, which causes the forces in the fiber segments to be distributed nonuniformly. The bonds subjected to the high force concentrations are rapidly broken and the force distribution becomes more uniform.

In many specimens data points obtained from successive reextension curves tend to plot in a scattered fashion so that it is not always possible to fit a smooth curve to them with good confidence. This behavior may also be attributed to density distributions which cause successive bond breaks to be localized during a test, the locality of the breaks shifting as the test progresses.

In the later stages of a test, holes form in the specimen and deterioration is apparent visually. Data points for x_c/l and lC_2/x_c obtained when the specimen is in this condition tend to fluctuate and to plot above the trend line fitted to "intact" data. The last three data points in figures 4 and 5 demonstrate this effect. Conceivably at this stage of a test the force distribution in a specimen has become sufficiently uniform that values of x_c/l and lC_2/x_c do not change much until gross deterioration occurs.

5. Materials Studied and Experimental Procedures

Low-density open-web handsheets were prepared from Northern and Southern Kraft woodpulps. Fibers from the Northern woodpulp were thin and easily assumed a ribbonlike configuration after a moderate amount of beating. The Southern woodpulp fibers were coarser and needed more beating to become pliable and attain a ribbonlike texture. For handsheets of the same mass per unit area made from the two pulps the Southern pulp handsheets had fewer fibers per unit area and fewer bonds per unit area than did the handsheets of Northern pulp. Methods used to prepare the handsheets are described elsewhere [12,13].

Force-elongation curves were obtained for a series of Northern Kraft handsheets having densities varying from 1.5 to 3.5 g/m^2, and for a series of Southern Kraft handsheets having densities varying between 1.75 and 3.5 g/m^2. Force-elongation data were also obtained for handsheets prepared from Northern and Southern Kraft woodpulps that had been subjected to various amounts of beating. In addition, data were obtained on specimens of different dimensions taken from the same handsheet in order to study the effects of specimen size and shape.

Tensile tests were conducted on specimens 2 cm long and 1 cm wide except for several specimens of 5 cm length. Crosshead speed of the tensile tester was 0.2 cm/min for the specimens of 2 cm length and 0.5 cm/min for the specimens of 5 cm length. Chart speed was 20 cm/min. During the tests the direction of extension was frequently reversed to unload the specimens and then resumed to obtain a series of force-elongation curves such as those shown in figure 2. The constants C_2, x_c, x_0 and l associated with each of these curves were determined by the methods described previously.

Tests were performed in a laboratory maintained at 20 °C, 50 percent R.H. on specimens subjected to this environment for at least 12 hours before testing. The handsheets from which the specimens were obtained were not always stored under these controlled conditions, and may have changed somewhat during the time interval between preparation and testing.

In the experimental verifications to be discussed it is expedient to characterize the tensile behavior of each test specimen by representative values of x_c/l and lC_2/x_c. These values were obtained by plotting x_c/l and $\log_e(lC_2/x_c)$ versus the corresponding values l/l_0-1, and extrapolating to the origin where $l/l_0-1=0$, as was done in figures 4 and 5. The extrapolated values thus obtained are quite imprecise and the extrapolation process involved some degree of subjective judgment. The precision could have been improved by additional testing especially devoted to this purpose, but the improvement was not deemed necessary for these demonstrations where the intent is merely to show a qualitative trend of the values of x_c/l and lC_2/x_c that is consistent with expected behavior.

6. Effect of Specimen Size and Shape

A short series of tests was made to see if the values of C_2 and x_c/l depend upon the size and shape of the specimen. For these tests specimens of width × length 1×2, 1×5 and 2×5 cm were cut from the same handsheet. The handsheet was of 2.5 g/m^2 density formed from Northern Kraft woodpulp beaten 5,000 revolutions in a laboratory beater. Extrapolated values of x_c/l and $\log_e(lC_2/x_c)$ derived from these tests are given in table 2. Values of C_2 calculated from these derived values are also given.

Table 2. Extrapolated values of x_c/l and $\log_e(lC_2/x_c)$ for different size specimens from the same handsheet

Specimen (w×l cm)	x_c/l [1] (%)	$\log_e(lC_2/x_c)$ [2]	C_2 (mN)
1×2	2.16±0.52 [3]	8.24±0.30 [3]	82
1×5	2.04±0.34	7.81±0.22	50
2×5	1.85±0.13	8.70±0.22	111

[1] Average of extrapolations from five specimens.
[2] C_2 is expressed in mN units, x_c/l as a fraction.
[3] Standard deviation.

Values of x_c/l for the three specimens are approximately the same, indicating that the value of x_c/l is not dependent upon the width of the specimen. This observation is consistent also with the idea that x_c/l is a strain parameter independent of specimen length and characteristic of the handsheet network. It is interesting to note that the standard deviations of the values of x_c/l decrease markedly as the areas of the specimens increase. This suggests that as the area of the specimen increases the effects of textural inhomogeneities are smoothed out.

The value of C_2 for the 2×5 cm specimen is approximately twice as large as that for the 1×5 cm specimen, in agreement with the expectation that for long specimens C_2 has a value directly proportional to specimen width. One might expect the value of C_2 for the 1×2 cm specimen to be closer to that for the 1×5 cm specimen; however the length of this specimen is quite short relative to the width, so that specimen shape can also influence the value of C_2 in this case.

The tests just described indicate that specimen dimensions of 2×5 or 1×5 cm yield the most precise results. But the 1×5 cm specimen might be preferable for certain tests, because with it the force drops that occur during a test are more clearly delineated. In addition, the long narrow configuration tends to minimize edge effects at the clamped ends, which prevent proper orientation of the network fibers and an even sharing of the tensile force throughout the width of the specimen.

7. Effect of Handsheet Density

According to the previous discussion it is expected that lC_2/x_c should increase as the density of the test specimen is increased, because the higher density specimens have more fibers per unit area and hence more pathways to sustain a tensile force. It is also expected that x_c/l should decrease as the density of the test specimen is increased, because the higher density specimens have smaller segement lengths between bonds. In order to verify these expectations, extrapolated values of $\log_e(lC_2/x_c)$ and x_c/l were obtained for a series of Northern and a series of Southern Kraft woodpulp handsheets of various densities. These values and their standard deviation limits are plotted versus handsheet density in figures 6 through 9.

The extrapolated values of $\log_e(lC_2/x_c)$ for the Northern and Southern pulps tend to increase and the values of x_c/l tend to decrease as the handsheet density increases, in agreement with expectations. Linear regression lines have been drawn through the data points to make this more obvious. Although not demonstrated directly, values of C_2 for the Northern and Southern pulps also increase with handsheet density. As the values of lC_2/x_c increase exponentially and the values of x_c/l decrease only linearly, the product of these quantities, C_2, should therefore

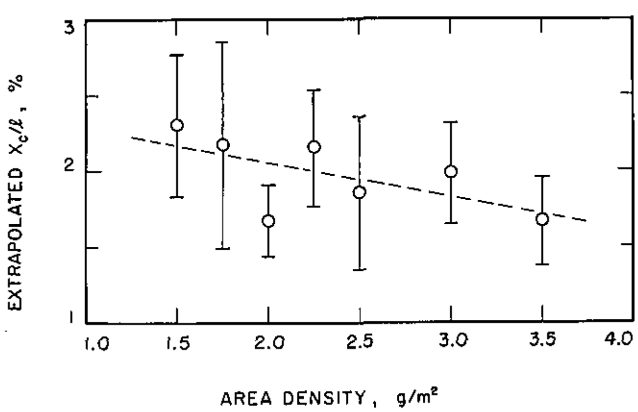

Figure 6–Plot of extrapolated x_c/l values versus area density for handsheets formed from Northern Kraft woodpulp.

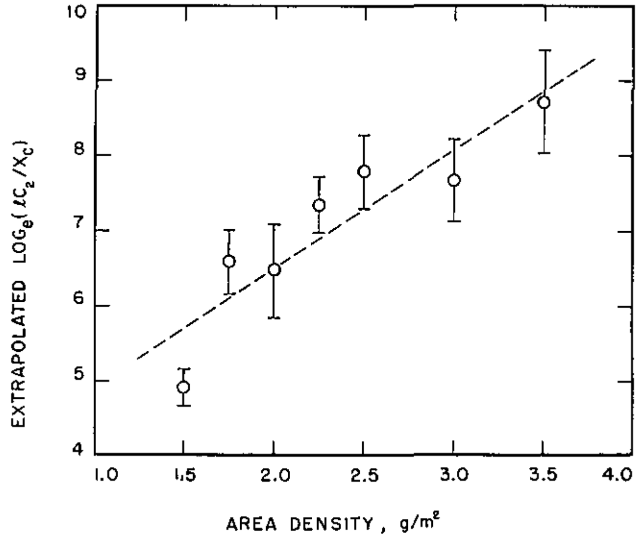

Figure 7–Plot of extrapolated $\log_e(lC_2/x_c)$ values versus area density for handsheets formed from Northern Kraft woodpulp. C_2 is expressed in mN units, x_c/l as a fraction.

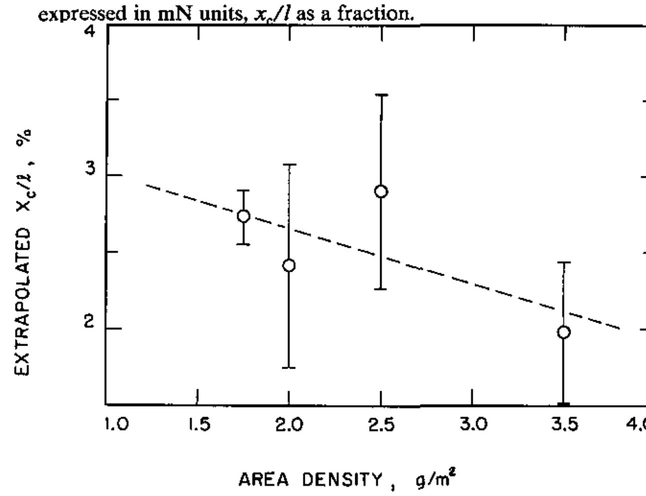

Figure 8–Plot of extrapolated x_c/l values versus area density for handsheets formed from Southern Kraft woodpulp.

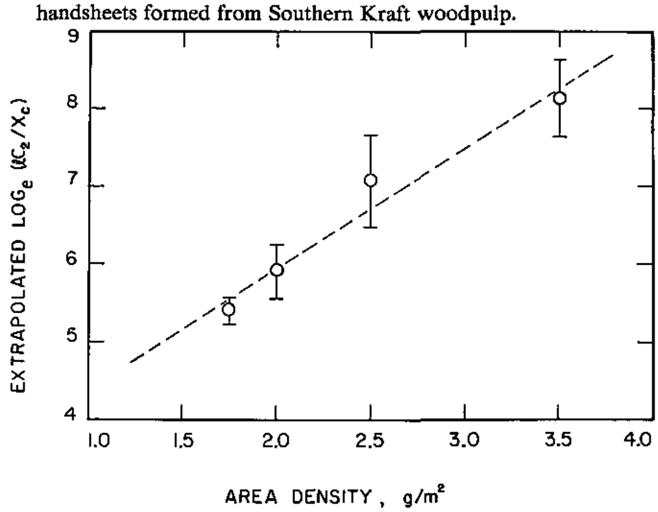

Figure 9.–Plot of extrapolated $\log_e(lC_2/x_c)$ values versus area density for handsheets formed from Southern Kraft woodpulp. C_2 is expressed in mN units, x_c/l as a fraction.

increase. This increase of C_2 with handsheet density has been confirmed by direct calculation.

The fibers in the Southern pulp furnish are coarser than the fibers in the Northern pulp furnish. Thus a handsheet formed from the Southern pulp would have fewer fibers per unit area than would a Northern pulp handsheet of the same area density. The segment lengths and hence x_c/l for the Southern pulp specimen thus would be larger. The data of figures 6 and 8 confirm these expectations. According to figure 6 the value of x_c/l read from the regression line is 0.0194 for a Northern pulp handsheet specimen of density 2.5 g/m^2. Similarly, from figure 8, the value of x_c/l is 0.0248 for a Southern pulp handsheet specimen. This larger value for the Southern pulp handsheet is in agreement with expectations.

A similar intuitive estimate of the relative values of C_2 for Northern and Southern pulp handsheets does not seem possible. However values of $\log_e(lC_2/x_c)$ for 2.5 g/m^2 handsheets can be read from figures 7 and 9. The corresponding values of lC_2/x_c are 1480 mN for the Northern pulp handsheet and 829 mN for the Southern pulp handsheet. Multiplying by the values of x_c/l found from figures 6 and 8 yields C_2 values of 28.7 mN for the Northern pulp handsheet and 20.5 mN for the Southern pulp handsheet.

8. Effect of Beating

Extrapolated values of x_c/l and $\log_e(lC_2/x_c)$ were determined for a series of handsheets of the same density prepared from Northern Kraft woodpulp beaten various amounts in a laboratory beater. These values were also determined for a similar series of handsheets prepared from Southern Kraft woodpulp. The results are plotted in figures 10–13. The effect of beating should be to increase the number of bonds per unit area of the handsheet, so that the number of pathways sustaining tensile force should increase, and the average segment length between bonds should decrease with the degree of beating. Thus the values of x_c/l should decrease and those for $\log_e(lC_2/x_c)$ should increase with the degree of beating. Inspection of figures 10 and 11 shows that these expectations are borne out for the Northern Kraft pulp handsheets. The dependence of x_c/l and $\log_e(lC_2/x_c)$ upon beating is not so pronounced for the Southern pulp. The linear regression lines plotted in figures 12 and 13, however, show the expected trend.

9. Summary

It has been suggested that the strength and durability of bonds between fibers in paper can be studied by preparing low-density "two-dimensional" handsheets from paper pulps and subjecting specimens cut from these handsheets to tensile tests. If such a specimen is stretched to break, a jagged force-elongation curve is obtained which rises and dips through one or more peaks, and eventually diminishes to zero. This behavior is interpreted to mean that the specimen deteriorates slowly during a test as the interfiber bonds break successively. Thus if during a test the extension were frequently reversed until the tensile force in the specimen were zero, and the specimen then reextended, a series of force-elongation curves would be obtained. By studying the changes in these curves, deterioration in the specimen could be evaluated and some means of compensating for this

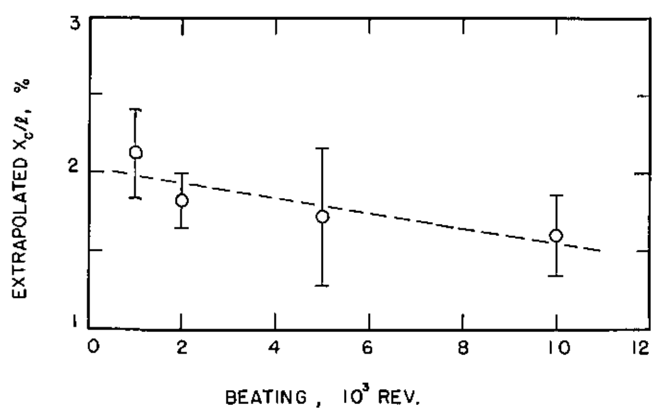

Figure 10.–Plot of extrapolated x_c/l values versus degree of beating for handsheets formed from Northern Kraft woodpulp.

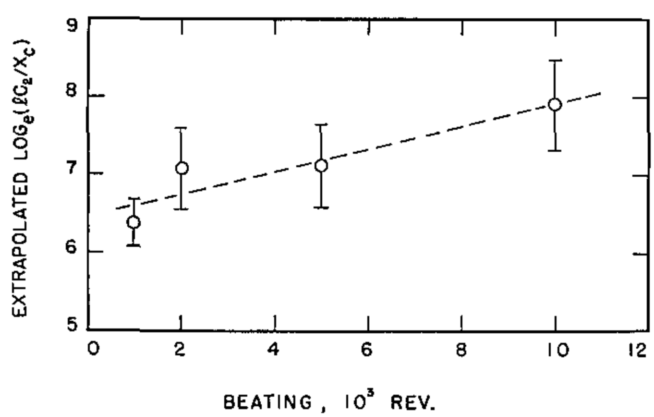

Figure 11.–Plot of extrapolated $\log_e(lC_2/x_c)$ values versus degree of beating for handsheets formed from Northern Kraft woodpulp. C_2 is expressed in mN units, x_c/l as a fraction.

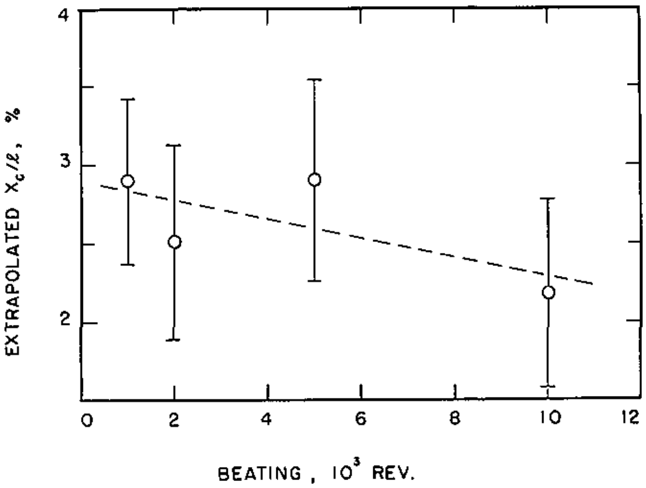

Figure 12.-Plot of extrapolated x_c/l values versus degree of beating for handsheets formed from Southern Kraft woodpulp.

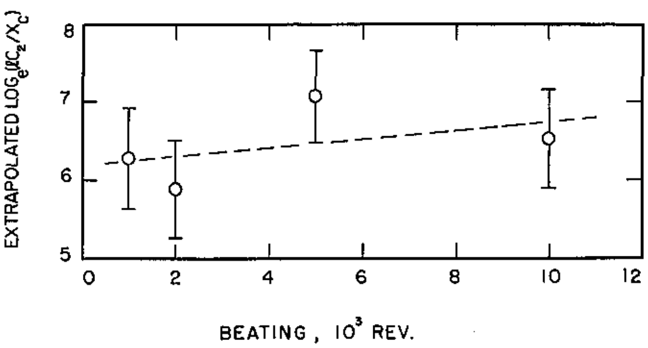

Figure 13.-Plot of extrapolated $\log_e(lC_2/x_c)$ values versus degree of beating for handsheets formed from Southern Kraft woodpulp. C_2 is expressed in mN units, x_c/l as a fraction.

deterioration devised. It might then be possible to reduce data taken at various stages of a test to a comparable basis.

The tensile behavior of low-density specimens formed from Northern and Southern Kraft woodpulps was studied. The force-elongation curves were found to be well represented by an equation of the type $F=C_2[\exp(x-x_0)/x_c-1]$, where F is the force and C_2 is a parameter having the dimensions of force, x is an elongation variable, x_0 is the elongation at which the force just rises from zero, and x_c is a parameter having the dimensions of length. The unstrained length of a specimen at the start of a test is l_0, but the unstrained lengths l corresponding to successive force-elongation curves have increasing values. Thus the value of l/l_0 increases as the test progresses, and can be used as a measure of deterioration. It is found that the value of x_c/l tends to decrease gradually, and the value of C_2 decreases rapidly as the specimen deteriorates. If representative values of x_c/l and C_2 are determined for handsheets of various densities, it is found that x_c/l decreases and C_2 increases as the density of the handsheet increases. Also x_c/l decreases and C_2 increases for handsheets made from pulps subjected to increasing amounts of beating.

Some features of the tensile behavior just described can be modeled by a system of parallel filaments of equal length to which longer parallel filaments with an exponential length distribution have been added. As this system is extended the filaments assume load successively, and the force-elongation behavior is governed by the same mathematical relationship as that found for the "two-dimensional" handsheet specimens. By thinking in terms of this model it is possible to anticipate intuitively much of the behavior just described for the handsheet specimens.

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