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An Intercomparison of Pressure Standards Between the Istituto di Metrologia "G. Colonnetti" and the National Bureau of Standards

J. C. Houck, G. F. Molinar, and R. Maghenzani
National Bureau of Standards, Washington, DC 20234

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Intercomparisons were performed between a primary standard gas piston gauge of the Istituto di Metrologia "G. Colonnetti" (IMGC) and two gauges at the National Bureau of Standards. The agreement between the average pressure generated by the IMGC primary standard and the NBS transfer gauge was within 7 ppm (over the range 0.75 to 5.0 MPa) and the agreement between the IMGC primary standard and the NBS primary standard was within 6 ppm (over the range 0.5 to 1.5 MPa). The agreement is well within the estimated uncertainties of the gauges: 24 ppm for the IMGC primary gauge, 30 ppm for the NBS transfer gauge, and 28 ppm for the NBS primary gauge.

Key words: effective area; intercomparison; piston gauge; pressure; primary standard; transfer standard.

Intercomparisons of pressures generated by a primary standard gas piston gauge (IMGC5) of the Istituto di Metrologia "G. Colonnetti" were made with pressures generated by a transfer gas piston gauge (PG23) of NBS and a primary standard gas piston gauge (PG24) of NBS.

A rather comprehensive treatment of piston gauges is given by Heydemann and Welch [1].[1] With the permission of the authors, selected portions of this treatment are modified and presented as background for readers not familiar with piston gauges. Pressure is defined as force per unit area and its value can be determined by a direct force-per-unit area measurement with a piston gauge. A *primary* standard piston gauge is one for which the area is determined by direct dimensional measurements. A *transfer* standard piston gauge is one for which the area is determined by intercomparison with another piston gauge or or monometer.

About the Authors: J. C. Houck is with the Temperature and Pressure Measurements and Standards Division, NBS, and NBS guest workers G. F. Molinar and R. Maghenzani have returned to IMGC, Strada delle Cacce 73, 10135, Torino, Italy.

[1] Figures in brackets indicate the literature references at the end of this paper.

The pressure is applied to the end face of a piston of known cross sectional area and the resulting force is measured by loading the piston with weights. Figure 1 is a schematic cross section of a piston gauge showing a vertical piston, which in operation is supported by the applied pressure balancing the force due to gravity acting on the piston, the weight carrier, and the weights.

The pressure p generated by a piston gauge at its reference level is given by eq (1)

$$p=\frac{\sum_{i=1}^{n} M_i\, g(1-\rho_{\mathrm{air}}/\rho_{Mi})+\gamma C+T_w}{A_0\,[1+(\alpha_c+\alpha_p)(T-T_{\mathrm{ref}})][1+bp]} \tag{1}$$

where the symbols have the following meanings:

M_i, ρ_{Mi} mass and density of weight i,

ρ_{air} density of air at the temperature, barometric pressure, and humidity prevailing in the laboratory,

γ	surface tension of pressure-transmitting fluid,
C	circumference of the piston where it emerges from the fluid,
T_w	tare weight or error,
A_0	effective area of the assembly at zero pressure,
α_c,α_p	thermal expansivities of cylinder and piston,
T	temperature of the assembly,
T_{ref}	temperature to which A_0 is referred,
b	pressure coefficient of the effective area.

The term $(1-\rho_{air}/\rho_{Mi})$ is the air buoyancy correction for weight i. γC is the force exerted on the piston by the surface tension of the fluid. The term $[1+(\alpha_c+\alpha_p)(T-T_{ref})]$ corrects the area for thermal expansion. The term $[1+bp]$ describes the change of the effective area with pressure. This important correction term will be the one discussed in this article. Several effects contribute to the change of effective area with pressure: the distortion of the piston under the combination of the longitudinal stress due to the applied load and the hydrostatic pressure over part of its length; and the distortion of the cylinder due to the internal pressure over part of its length, and external pressure and end-loading where applicable.

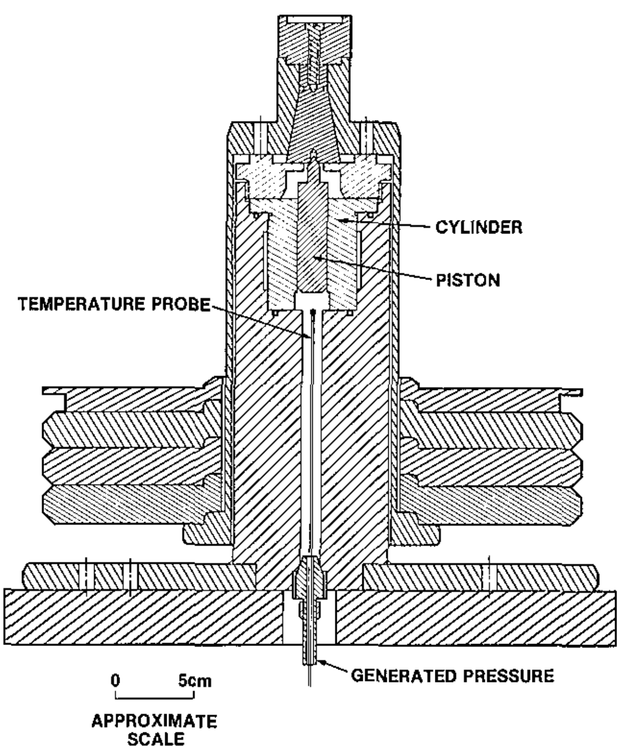

Figure 1-IMGC5 simple piston and cylinder dead weight piston gauge. The temperature probe is a thermocouple referenced to 0 °C. For this figure as well as figures 2 and 3, the piston and cylinder materials are given in table 1; the shadings in the figures do not indicate the materials of the components.

Several authors have made significant contributions to the calculation of these distortions. See reference [1] for more details and additional references. The change in radius r_p of a solid piston subjected to end pressure p, surrounded by the pressure p_c in the clearance is given according to Johnson and Newhall [2] by

$$\frac{r_p(p)-r_p(0)}{r_p(0)} = \frac{\mu p}{E} + \frac{p_c}{E}(\mu-1) \tag{2}$$

where μ is Poisson's ratio and E is the modulus of elasticity. The change in the radius r_c of a hollow cylinder of outside radius R_c is given by

$$\frac{r_c(p)-r_c(0)}{r_c(0)} = \frac{p_c}{E}\left[\frac{(1+\mu)R_c^2+(1-\mu)r_c^2}{R_c^2-r_c^2}\right] - \frac{p_0}{E}\left[\frac{2R_c^2}{R_c^2-r_c^2}\right] + \frac{\mu p_e}{E} \tag{3}$$

where p_0 is the pressure on the outside of the cylinder, and p_e is the pressure on the end faces.

In eqs (2) and (3), p_c varies along the piston from a maximum at the lower end to zero at the top of the cylinder, and the exact correction for piston and cylinder distortion depends on the geometry of the clearance. In practice the geometry of the clearance may change with pressure.

The effective area is the arithmetic mean between the areas of the piston and of the cylinder

$$A_{eff}=1/2(A_p+A_c) \tag{4}$$

Equations (2) and (3) can now be introduced into eq (4), and, assuming for simplicity that $p_c=0.5\ p$, we have for the pressure coefficient b of eq (1)

$$\begin{aligned}
b &= -(1-3\mu)/2E_p \quad \text{(piston distortion)}\\
&+ \{(1+\mu_c)R_c^2+(1-\mu_c)r_c^2\}/\{2E_c(R_c^2-r_c^2)\}\\
&\quad \text{(cylinder distortion due to internal pressure)}\\
&- (p_0/pE_c)\{(2R_c^2/(R_c^2-r_c^2)\}\\
&\quad \text{(cylinder distortion due to external pressure)}\\
&+ (p_e/p)(\mu_c/E_c)\\
&\quad \text{(cylinder distortion due to end loading)}
\end{aligned} \tag{5}$$

Let us get an estimate for the size of these corrections to the area. For steel $E=2\times10^{11}$ N/m^2 and $\mu=0.28$; further let $R_c/r_c=3$ and we have for the pressure coefficient

$$b_{steel}=-4\times10^{-13}+38\times10^{-13}-112\times10^{-13}(p_0/p) + 14\times10^{-13}(p_e/p) \tag{6}$$

Table 1. Main characteristics of the three piston gauges used in the intercomparison.

Piston gauge designation	IMGC5	NBS-PG23	NBS-PG24
Piston-cylinder type	Simple	Partially re-entrant	Controlled-clearance
Piston material, cylinder material	Tungsten-carbide, tool steel	Tungsten-carbide, tungsten carbide	Tungsten-carbide, tool steel
Fluid	Dry nitrogen	Dry nitrogen	Dry nitrogen
Range (MPa)	0.5–5.0	0.7–5.0*	0.5–1.5
Reference temperature (°C)	23	23	23
Effective area A_0 at atm. pressure and at 23 °C(m^2)	2.000662×10^{-4}	8.390170×10^{-6}	5.067132×10^{-4}
Pressure coefficient (MPa^{-1})	1.0×10^{-6}	0.0*	-5.1×10^{-7}
Thermal coefficient of expansion α_p for piston ($°C^{-1}$)	5.5×10^{-6}	4.55×10^{-6}	4.9×10^{-6}
Thermal coefficient of expansion α_c for cylinder ($°C^{-1}$)	12.0×10^{-6}	4.55×10^{-6}	12.0×10^{-6}
Total estimated relative uncertainty of the pressure† ($\Delta p/p$) (ppm)	24.0	30.0	28.0

* NBS-PG23 has a range 0.7 to 17.2 MPa but the zero pressure coefficient is applicable only in the range 0.7 to 5.0 MPa.

† Total estimated uncertainty includes uncertainties of effective area at atm. pressure and 23 °C, pressure coefficient, thermal expansion coefficients of piston and cylinder, gravity, and masses.

all in m^2/N.

For cemented tungsten carbide with $E=6\times10^{11}$ N/m^2 and $\mu=0.2$ the pressure coefficient is

$$b_{TC}=-3.4\times10^{-13}+12\times10^{-13}-38\times10^{-13}(p_0/p)$$
$$+3\times10^{-13}(p_e/p) \qquad (7)$$

all in m^2/N. It is immediately obvious that, depending on the design of the gauge, the pressure coefficient can be positive, negative, or even zero. Different ways of dealing with the pressure coefficient b have led to the design of various types of piston gauges—three different ones were used in this intercomparison. Table 1 gives the relevant characteristics of each of the gauges.

Figure 1 is a schematic cross section of IMGC5, a simple piston and cylinder in which the effective area at atmospheric pressure (determined from the average of the areas of the piston and of the cylinder) was obtained from direct dimensional measurements of the diameters. The pressure coefficient of the effective area ($b=1.0\times10^{-6}$ MPa^{-1}) used in these intercomparisons was determined by IMGC by comparison with another IMGC simple piston and cylinder gas piston gauge of 2 MPa range whose pressure coefficient had been calculated theoretically.

Figure 2 is a schematic cross section of NBS-PG23 which has a partially re-entrant piston and cylinder with the effective area at atmospheric pressure and the pressure coefficient determined by comparison with a primary standard. PG23 has a pressure range of 0.7 to 17.2 MPa and was calibrated against the primary standard PG24, but only over the range of 0.7 to 1.9 MPa. From this calibration we obtained a value for A_0 and concluded that $b\cong0.0$. On a theoretical basis the pressure coefficient would be expected to be between -2.4×10^{-6} MPa^{-1} (the calculated value for the lower re-entrant half of the piston and cylinder) and $+0.7\times10^{-6}$ MPa^{-1} (calculated for the upper half of the piston and cylinder which behaves like a simple piston and cylinder). Future comparisons with NBS gauges

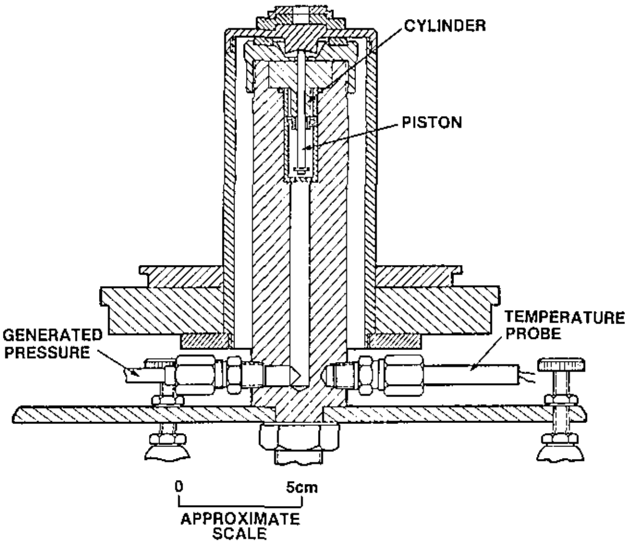

Figure 2–NBS-PG23 partially re-entrant piston and cylinder dead weight piston gauge. The temperature probe is a 100-ohm Platinum Resistance Thermometer (PRT).

operating at higher pressures will give a value of the pressure coefficient for PG23 to use over a greater range than was used for this intercomparison.

Figure 3 is a schematic cross section of NBS-PG24 which is a primary standard controlled-clearance piston gauge with the effective area derived from dimensional measurements of the piston only, with an empirically determined correction based on extrapolation of jacket pressure required to close the cylinder on the piston, and a theoretical pressure coefficient (-5.1×10^{-7} MPa^{-1}) applied to the deformation of the piston only.

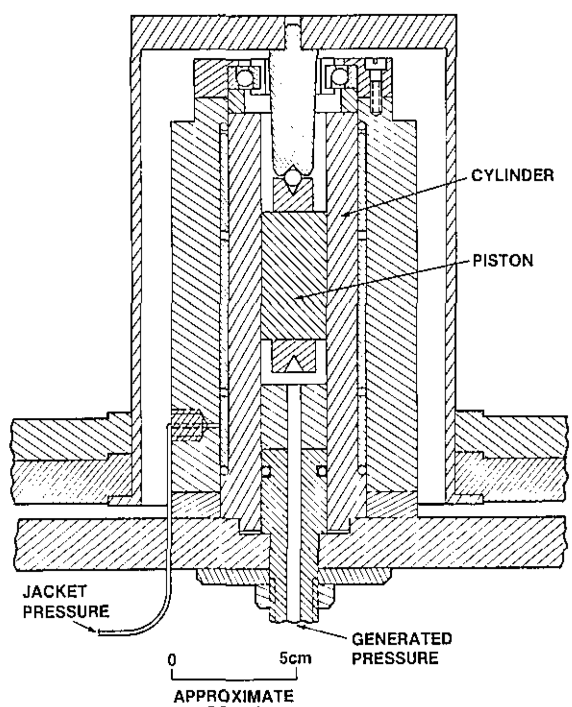

Figure 3–NBS-PG24 controlled-clearance dead weight piston gauge. A 100-ohm platinum resistance thermometer for temperature measurement was mounted on the base plate at a position not shown in the figure.

The calculations for pressure measurements by controlled-clearance piston gauges, and by other types of piston gauges, as well as considerations of direct comparison, are also given by Heydemann and Welch [1]. Dry nitrogen was used as the pressure fluid. A pressure head correction for nitrogen was applied for the difference in level between the bottom of pistons of the gauges at their operating levels. The gauges were operated at temperatures near 23 °C. Using the thermal expansion coefficients given in table 1, the effective areas were corrected for the operating temperature. The pistons were manually rotated in the CW direction at a frequency between 0.5 to 2.0 Hz.

Electronic position indicators were used to monitor the vertical position of each gauge while determining the pressure balance to ensure that each gauge was used at its correct operating height. The rate of change of position was used to determine that the rate of fall was appropriate for indicating that the pressures were balanced during the intercomparison.

A total of 21 comparisons at 10 different pressures was made between IMGC5 and NBS-PG23. One method of evaluating the data was to regard PG23 as the standard and IMGC5 as a test gauge to be calibrated using the NBS computer program for calibrating test gauges. This program determines the effective area and the pressure coefficient of the test gauge in terms of those of the standard.

In eq (8), p is the pressure generated at the reference level of the test instrument by the standard gauge.

$$p=\frac{F^{T}}{A_0^{\mathrm{T}}(1+b_1^{\mathrm{T}}p+b_2^{\mathrm{T}}p^2)} \tag{8}$$

where

$$F^{T}=\frac{Mg[1-(\rho_{\mathrm{air}}/\rho M)]+\gamma C+T^{\mathrm{T}}}{1+(\alpha_{\mathrm{p}}+\alpha_{\mathrm{c}})(T-T_{\mathrm{r}})} \tag{9}$$

is the force exerted on the test gauge piston, A_0^{T} is the effective area of the test gauge, b_1^{T} is the fractional change of effective area with pressure of the test gauge, and b_2^{T} is the fractional change of effective area of the test gauge with the square of the pressure.

Note that for simplicity the temperature correction of the area has been lumped with the force, F^{T}.

The RHS of eq (8) represents the pressure generated by the test gauge at its reference level. By adjusting F^T this pressure is made equal to p. The effective area A_0^T and the coefficients b_1^T and b_2^T can then be obtained by fitting

$$F^T = A_0^T p(1 + b_1^T p + b_2^T p^2) - T^T \tag{10}$$

to the (F^T, p) data obtained from the comparisons.

If eqs (11–18) are fitted to the data at very low pressures, the terms involving the pressure coefficients b_1 and b_2 are usually insignificant and either eq (11) or (12) is used to characterize the gauge. Note that T may be either a tare error or a coefficient necessary to characterize the behavior of the gauge properly. At higher pressures the coefficient b_1 and occasionally also b_2 become significant and must be included in the function fitted to the data.

$$F^T = pA_0^T \tag{11}$$

$$F^T = pA_0^T - T \tag{12}$$

$$F^T = pA_0^T(1 + b_1^T p) \tag{13}$$

$$F^T = pA_0^T(1 + b_1^T p) - T \tag{14}$$

$$F^T = pA_0^T(1 + b_1^T p + b_2^T p^2) \tag{15}$$

$$F^T = pA_0^T(1 + b_1^T p + b_2^T p^2) - T \tag{16}$$

$$F^T = pA_0^T(1 + b_2^T p^2) \tag{17}$$

$$F^T = pA_0^T(1 + b_2^T p^2) - T \tag{18}$$

A high-speed digital computer will perform these computations in a few seconds and, apart from the coefficients, will determine the standard deviations of the coefficients, the residual standard deviations, and the residuals. A plot of the residuals as functions of pressure will show at a glance whether any gross errors have been made in recording and entering the data.

The proper fit is finally selected by comparing the residual standard deviations of the various fits and the standard deviations of the coefficients. The standard deviation of the residuals is reduced as more coefficients are used to characterize the gauge. However, since the number of degrees of freedom is reduced simultaneously, the uncertainty of the coefficients increases. Selected, therefore, is the fit with the least number of coefficients, which gives low residual standard deviation and for which each coefficient is greater than three times the standard deviation of the coefficient. The lowest order eq (11) ($F=pA$) was selected and gives an effective area of the IMGC gauge of 2.000661×10^{-4} m^2 at 23 °C with a standard deviation of the area of 0.6 ppm. The area of the IMGC gauge given by IMGC is 2.000662×10^{-4} m^2 at 23 °C. The areas are seen to differ by 0.5 ppm which is less than one standard deviation.

Another method of evaluating the intercomparison was to calculate the pressure generated by each piston gauge according to the method used by the respective laboratories. The results of the 21 direct comparisons (the same points used in the first method) are shown in table 2. The pressures were referenced to the base of the IMGC piston to account for head corrections. The average of the pressures calculated by IMGC minus the pressures calculated by NBS is −12.7 Pa with a standard deviation of the mean of 2.0 Pa. The average of the pressures calculated by IMGC minus the pressures calculated by NBS divided by the NBS

Table 2. Comparison of pressures measured by IMGC5 and by NBS-PG23 piston gauges in chronological order.

Pressure defined by IMGC5 (MPa)	Pressure defined by NBS-PG23 (MPa)	Pressure difference IMGC5 NBS-PG23 (Pa)	Pressure difference / Pressure: (IMGC5-NBS-PG23) / NBS PG23 (ppm)
.750184	.750193	− 9	−12.0
2.998337	2.998362	−25	− 8.3
4.996813	4.996836	−23	− 4.6
4.996824	4.996845	−21	− 4.2
3.997585	3.997591	− 6	− 1.5
2.998342	2.998356	−14	− 4.7
1.999101	1.999100	1	.5
.999861	.999865	− 4	− 4.0
.750184	.750200	−16	−21.3
1.499478	1.499485	− 7	− 4.7
2.498710	2.498724	−14	− 5.6
3.497947	3.497975	−28	− 8.0
4.497188	4.497202	−14	− 3.1
4.497188	4.497217	−29	− 6.5
3.997576	3.997591	−15	− 3.7
3.497947	3.497969	−22	− 6.3
2.498711	2.498713	− 2	− .8
1.999101	1.999111	−10	− 5.0
1.499478	1.499483	− 5	− 3.3
.999862	.999861	1	1.0
.750183	.750188	− 5	− 6.7
Mean value		−12.7	− 5.4
Standard deviation of the mean		2.0	1.0

pressure is –5.4 ppm with a standard deviation of the mean of 1.0 ppm.

Sixteen comparisons were made between IMGC5 and NBS-PG24 at five different pressures. The same lowest order eq (11) ($F=pA$) was selected for this pair of gauges. It gives an effective area of the IMGC gauge of 2.000649×10^{-4} m^2 at 23 °C with a standard deviation of the area of 0.6 ppm. The difference in area of the IMGC gauge determined by this NBS standard (NBS-PG24) in this comparison with that given by IMGC is –6.5 ppm. The second method of evaluating the intercomparisons was also applied to this pair of gauges. The results of the 16 direct comparisons are shown in table 3. The average of the pressures calculated by IMGC minus the pressures calculated by NBS is –6.8 Pa with a standard deviation of the mean of 1.1 Pa. The average of the pressures calculated by IMGC minus the pressures calculated by NBS divided by the NBS pressures is –6.5 ppm with a standard deviation of the mean of 0.6 ppm.

Figure 4 is a plot of the differences in pressure calculated from the characteristics of the gauges versus pressure, and figure 5 is a plot of the *relative* difference in pressure calculated from the characteristics of the gauges versus pressure. While a systematic difference in pressure is evident in the data, it is small compared to the estimated systematic uncertainties.

Table 3. Comparison of pressures measured by IMGC5 and by NBS-PG24 piston gauges in chronological order.

Pressure measured by IMGC5 (MPa)	Pressure measured by NBS-PG24 (MPa)	Pressure difference IMGC5 NBS-PG24 (Pa)	Pressure difference / Pressure: (IMGC5-NBS-PG24) / NBS PG24 (ppm)
.750240	.750244	- 4	- 5.3
.999918	.999927	- 9	- 9.0
1.249856	1.249868	-12	- 9.6
1.499533	1.499539	- 6	- 4.0
1.499534	1.499547	-13	- 8.7
1.249855	1.249861	- 6	- 4.8
.999917	.999925	- 8	- 8.0
.750238	.750242	- 4	- 5.3
.750238	.750242	- 4	- 5.3
.999917	.999924	- 7	- 7.0
1.249854	1.249866	-12	- 9.6
1.499532	1.499547	-15	-10.0
.500319	.500321	- 2	- 4.0
.500319	.500322	- 3	- 6.0
.500319	.500321	- 2	- 4.0
.500318	.500320	- 2	- 4.0
Mean value		- 6.8	- 6.5
Standard deviation of the mean		1.1	0.6

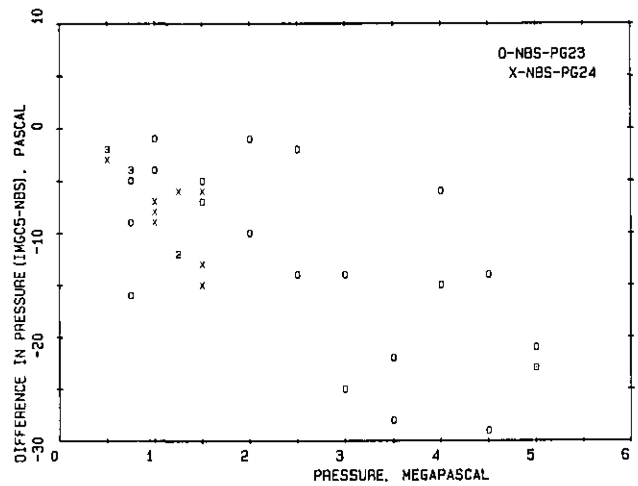

Figure 4–Difference in pressure (Pa) versus pressure (MPa) for IMGC5 against NBS-PG23 and IMGC5 against NBS-PG24. The numbers "2" and "3" represent the number of replicate data for IMGC5 against NBS-PG24 plotted at the same point.

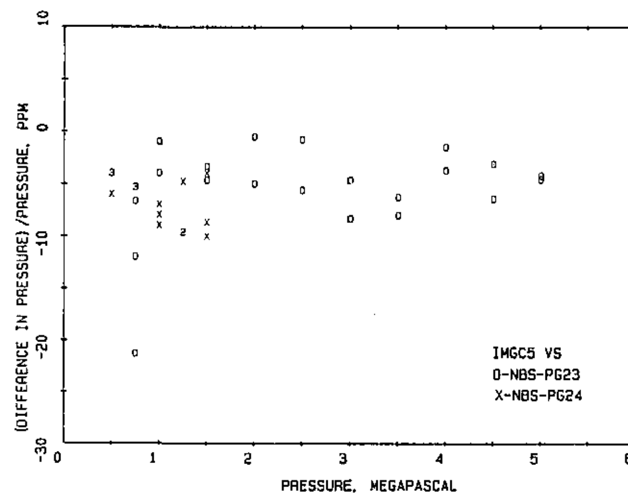

Figure 5–Difference in pressure divided by pressure (ppm) versus pressure (MPa) for IMGC5 against NBS-PG23 and IMGC5 against NBS-PG24. The numbers "2" and "3" represent the number of replicate data for IMGC5 against NBS-PG24 plotted at the same point.

Both methods of expressing the results of the intercomparisons show significantly better agreement between the gauges (1 to 7 ppm) than the estimated systematic uncertainty of each of the gauges (IMGC5, 24 ppm; NBS-PG23, 30 ppm; and NBS-PG24, 28 ppm).

The differences observed between IMGC5 and NBS-PG24 (the primary standard), 6.8 ppm by area comparison and 6.5 ppm by pressure comparison, indicate that the two different methods of calculating effective areas are well verified at this pressure range. The differences observed, between IMGC5 and NBS-PG23, the transfer standard, 0.5 ppm by area comparison and 5.4 ppm by pressure comparison, indicate that the latter gauge serves very well as a transfer standard in the given pressure range.

References

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