# Temperature of a Copper Arc 

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#### Abstract

The tempersture of a copper aro in sir has been messured by comparing 31 sets of transition probabilities for spectra of 20 elements with speatral-line intensities from those elements separately added to the copper. The infersitiea sre taken from the recently published NBA Tables of Speotral-line Intensities bad the transition probabilities from the literature. The individual determinations are disousaed. The set of determinations is shown to follow a Gausaian distribution about a mean of $5100^{\circ} \mathrm{K}$. The average deviation of the determinations is $470^{\circ} \mathrm{K}$ sind the standerd deviation of the mean if $110^{\circ} \mathrm{K}$. This temperature mey be ured In the evaluation of thousands of atomic transition probabilittees from the fintensities in the new tableg. The effect of the uneertainty in the temperature on derived tramition probebilities if evaluated as a function of excitation potential.


## 1. Introduction

Physicists have a natural interest in measuring quantities of extreordinary magnitude. This is especially true when the quantity measured permits new or more accurate evaluations of physical constants.

Such a quantity is the temperature of the electric arc in air between metallic electrodes. Well above the melting points of all materials and the boiling points of nearly all, the arc provides one of the hottest locations generally available in the physical laboratory. In such an arc it is possible to study the properties of atoms whose behavior is described by Boltzmann's lav.

One particular arc, the temperaturo of which is eapecially interesting is the are between copper electrodes, in which the spectra reported in the NBS Tables of Spectral-line Intensities were excited [1]. ${ }^{\text {. }}$ It happens that the temperature of this arc is susceptible of mcasurement by means of these very spectral-line intensity date. In itsolf, this would tempt us to carry out the determination; the fact that an accurate value of the temperature may enable us to evaluate transition probabilities for the 25,000 classified lines in the NBS Tables makes the temptation irresistible.

The arc has been described in the NBS Tables [1]. It was struck between two copper pellets $1 / 4$ inch in diameter, $1 /$ inch long, and weighing 1.5 grams each. The pelleta were fomed in a hydraulic press at 5000 psi from fine copper powder. Two pellets were mounted in water cooled clamps, cathode nbove the anode, with an arc gap of 3 mm . A direct curront of 10 amperes, controlled by resistive ballast, was drawn from a 220 -volt line. Exposures with which the spectra were taken varied in duration from 1 sec to 5 min dopending on spectregraphic efficiency and photographic plate sensitivity in different spectral regions. Separate sets of spectrograms were made for each element, using for each exposure fresh pellets to which an element had been added in the proportion of 1 atom of the element for every 1000 gatoms of copper. The element was thoroughly mixed with the copper powder before pressing.

[^0]A lens at the slit of the spectrograph formed an image of the are which fell entirely within the aperture of the grating, consequently the spectra represented radiation from all parts of the arc.
The method of temperature measurement adopted here is based on comparisons of published experimental transition probabilities of atomic lines with the intensities of the same lines measured in the copper arc. This method yields a temperature which is precisely that appropriate to the calculation of transition proba bilities from our intensities.

If $N_{n}$ atoms in the arc column are in an excited state $n$, the number of transitions per second to a lower state $m$ will be $N_{n} A_{n w}$ where $A_{\mathrm{nm}}$ is Einstein's probability coefficient of spontaneous emission [2]. The power from the transition is

$$
\begin{equation*}
I=h_{v} N_{n} A_{\mathrm{nm}} \tag{1}
\end{equation*}
$$

where $I$ is the intensity of the omitted line, $h$ is Planck's constant, $p$ is the frequency of the line and $h v$ is the energy of a single photon. The temperature enters the comparison through its role of populating the upper level in accordance with Boltzmann's law

$$
\begin{equation*}
N_{\#}=N_{0} \frac{g_{z}}{g_{0}} e^{-\frac{B}{k T}} \tag{2}
\end{equation*}
$$

where $N_{0}$ is the population of the ground state and $g_{0}$ is its statistical weight $(2 J+1), g_{4}$ is the statistical weight of the upper level, $E$ is the energy of the upper level, $k$ is Boltzmann's constant and $T$ is the absolute temperature.

Subetituting (2) into (1) we have

$$
\begin{equation*}
I=\frac{N_{0}}{g_{0}} \lambda \nu g_{\mathrm{N}} A_{n \mathrm{~m}} e^{-\frac{E}{k T}} \tag{3}
\end{equation*}
$$

Substituting $\nu=c / \lambda$, where $c$ is the velocity of light and $\lambda$ the wavelength of the line, and changing in to logaritlumic form (base 10), (3) becomes

$$
\begin{equation*}
\log \frac{I \lambda}{g_{\pi} A_{4 m}}=\log \frac{N_{0} h c}{g_{0}}-\frac{0,434 E}{k T} \tag{4}
\end{equation*}
$$

The first term on the right of (4) is a constant for
any particular element in this arc. We set it equal to $O$ and solve for $T$. Introducing a value for $t$,

$$
\begin{equation*}
T=-\frac{625 E}{\log \frac{7 \lambda}{g A}-C} \tag{5}
\end{equation*}
$$

where $T$ is in ${ }^{\circ} \mathrm{K}, g A=q_{n} \mathcal{A}_{n m}$ and $E$ is in units of $10^{5} \mathrm{~cm}^{-1}$ which we will cail kilokaysers ( $1 \mathrm{kK}=$ $1000 \mathrm{~cm}^{-1}$ ) in accordance with a suggestion of Meggers [3]. The value of $T$ is most conveniently obtained from a plot of $\log \Omega / g A$ versus $E$ for each spectrum in which relative values of $g A$ are known. The constant $C$ does not affect the slope of the line from which $T$ is determined.

The ordinats is equivelent to $\log N_{n} / g_{n}$ so the plot is in effect a display of the population of excited levels in the atom as a function of the level value. Plots for eight sets of transition probsbilities taken from the literature and compared with our intensities are given in figures 1 to 8 . The straightness of the plots clearly ehows that the observations are in accord with the Boltzmann distribution. The population of levels of equal statistical weight declines exponentially with increasing energy values.

If the $f$-value is known instead of $A$, we may use Ladenburg's [4] equation

$$
\begin{equation*}
g_{n} A_{n+1}=\frac{8 \pi^{2} e^{2}}{m c} \frac{\xi_{m n} f_{m q}}{\lambda^{2}} \tag{6}
\end{equation*}
$$

for converting eq (5) to the form


Figure 1. Lop ratio of intentatigy $\times \lambda^{3}$ to gf-talte for mudtiplets of Ca 1 in the copper are plotted ner gus upper Ievel.
din ara temperature of $4800^{\circ} \mathrm{K}$ is derived from tbe alope of the line of beat ft.

$$
\begin{equation*}
T=\frac{-625 E}{\log \frac{I \lambda^{3}}{g f}-C^{r}} \tag{7}
\end{equation*}
$$

where $g f=g_{m} f_{m n}\left(=g_{\pi} f_{n \pi}\right)$, and $C^{\prime}$ is a constant.
We assume that the temperature of the are was constent during the exposuro of each of the hundreds of spectrograms in which our intensity observations were made. Great care was taken in the preparation of the electrodes and in the operation of the are to insure uniformity in the experimental conditions from exposure to exposure. The NBS intensity data used in the 31 temperature detarminations reported in section 4 of this paper are taken from about 50 separate exposures. It will bo shown in the discussion that the variation in the determinations arises, at least in part, from systematic errors amongst the sets of tramsition probabilities.

## 2. Fscitation in the Arc in Air at Atmospheric Pressure

The question arises whether we should be able to assign a valid temperature which can be used to describe the intensity of spectral lines emitted in accordance with eq (3). If there are several processes of excitation a.t work, each operating at a different effective temperature, then eq (3) is not valid. In a classical investigation, Ornstoin and Brinkman [5] have shown that excitation in the column of the ordinary arc in air is controlled by collisions between atoms and molerules in a strictly thermal fashion. The same process is responsible for ionization and dissociation. Thay found that the excitation was well described by Boltzmann's law and that ionization followed Snan's equation, which also demands \& thermal aggregation of particles. Experiments by Elenbaas described by Cobine [6] show that in arcs at pressurcs above 20 mm Hg , the electron and gas


Figure 2. Log ratio of intenaity $\times \lambda^{2}$ to of-ualue for murdipleto of Co C in the copper and plolled versus wpper level.
An are temperature of $3490{ }^{\circ} \mathrm{K}$ is derived from the slape of the Itne of beet fit


Figure 3. Lay ratio of intersity $\times h^{3}$ to gf-talue for matieplets of Cr 1 in the copper are ploted teessus wpper tevel.



Figure 4. Log ratio of intensity $\times \lambda^{2}$ to gf-value for mudiplets of Fe 1 in the copper arc ploded versur upper level.
An are tamperatury of $8600^{\circ} \mathrm{E}$ is cietived frum the stope of the Ithe of best fit.


Figure 5. Log ratio of interaity $\times \mathrm{A}$ to ga for Iines of Tinin the empper are plotied verius wpper level.
An sec tempersture of $5: 00^{\circ} \mathrm{K}$ t derived trom the slope of the line of best fit.


Figure 6. Log ralio of intemaity $\times h^{2}$ to gf-value for mailipleta of V 1 in the copper arc plotted versus upper leved.
An tre temperatire of t3T0 ${ }^{\circ} \mathrm{K}$ is derived fram the stope of the ane of best at.


France 7. Iog ratio of intensity $\times h^{3}$ bo fif-calue for multiplete of V I in the capper arc plothed tersus upper tevel.
An arc temperature of $\mathbf{d i c} 00^{\circ} \mathrm{K}$ \& derived from the alope of the line or best 4 t.
temperatures are equal. Suits [7] has measured the gas temperature and positivo ion temperature in arcs at atmospheric pressure and found them equal within an experimental error of $300^{\circ} \mathrm{K}$.

The investigations quoted above are of interest in considering excitation in ares. However, in this investigation we need not depend on them. The remainder of this paper is depoted to a discussion of graphical cyaluations of temperature from eq (5) or (0), using 31 sets of transition probabilities taken from the literature. Examples are shown in figures 1 through 8. The linearity of these plots establishes beyond doubt that the excitation in our arc is described by Boltamann's law. The fact that the excitation does follow Boltzmann's Iaw enables us to calculate transition probabilitios from intensitios and conversely. The question of the existence of complete thermodyoamical equilibrium per se becomes irrelevant for our purpose.

## 3. Choice of Transition Probabilities

The values of relative transition probabilities available for comparison with our intensities may be divided into four categories: (1) Experimental valtues based on independent temperature determinations; (2) experimental values based on temperature determinations obtained with data from category (1); (3) values calculated by quantum-mechanical methods (e.g., Hartree-Fock); and (4) values calculated with the approximate theory of Bates and Damgaard. Several trials with data from categories (3) and (4) resulted in plots of eq (7) exhibiting such a large scatter of points that no reasonably reliable temperature could be derived. Allen [8], in a similar investigation, states that use of Batess and Damgaard


Figure 8. Log ratio of intensity $\times \lambda$ to ga for tines of ZnI in the copper are plotted wersu* upper tevel.

values leads to impossibly high are temperatures, particularly in complex spectra. Category (2) is obviously inadrissible for our purposes, since such a set of date would merely duplicate the information from cntegory (1) with which it was calibrated.

We are left with the experimental data of eategory (1) for our temperature detarmination. These data are reviewed up to 1954 by R. B. King [9] who noted about 2000 known raluea. The number has aince increased to about 2200 .

## 4. Results

A rather complete search of the published literature on experimental transition probabilities disclosed 31 sets of data suitablo for comparison with the intensities in the NBS Tables. All independent sets of data with more than 4 lines in common with those of our tables (and from which temperatures could be derived) were included, with one exception discussed in section 5. The results are summarized in table 1. Column 1 gives the spectrum investiguted by the authors in col. 2. The dita came from 5 laboratories, indicated in col. 3, and listed below.
Physical Institute of the State University at Leningrad11 sets
Physical Laboratory of the University ofUtrecht10 sets
Norman Bridge Jaboratory of the Cali-formie Institute of Technology -....... 7 setsPhysical Institute, Academy of SciencesUSSR, Moscow2 sets
Zeeman Laboratory at the University of Amsterdam ..... 1 set

Table 1．Individual determinations of topper are temperature

| 8pactrum | Author | Lelipratory | Dets | Ret， | Metbod | Na ot ］lues | Prange of $\mathbf{E}$ | $\Delta E$ | T | Dev． |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | K | ＊K | ${ }^{*} \mathbf{K}$ | ${ }^{\square} \boldsymbol{K}$ |
| AEI | Terpsits tt $\mathrm{s}_{3}$ ， | Ctrecht | 1906 | ［10］ | Are | 8 | 48－54 | 11 | 1800\％ | 500 |
| Bis | Kruithof | Strecltt | 1943 | 11. | Arc | 10 | 20－36 | 9 | 1280 | 850 |
| 磁1 | Ostrovery et al． | Lenlngesd | 1880 | 12 | Hook | 5 | 14－366 | 23 | 5000 | 100 |
| Cg 1 | Seltiltteragr et al． | Utrout | 1943 | 13 | Are | 12 | 20－43 | 11 | 1580 | 220 |
| 宥 1 | Olsen 6t al． | Cullimma | 1950 | （1） | Fum．ubs． | 6 | 18－4 | 34 | 4900 | 900 |
| C11 | Hentitum et 51 | Utracht | 1056 | ［15］ | ATE A | 15 | 61－65 | 11 | 5080 | 25 |
| CdI |  | Lenilngrad | 1900 | （16） | Hinok and am， | 13 | 31－＊6 | 14 | 4750 | 350 |
| Del 1 | King et al， | Callumis | 1055 | 17 | Fura，abes． | 181 | 24－46 | 23 | 6306 | 280 |
| Qol | Orgtorstcy at al， | Lemingrad | 1898 | 18 | Hoak | 30 | 24－37 | 11 | 6450 | 380 |
| CrI | Hill et el． | Cbilforits | 1001 | 10 | Furb，部㐌， | 127 | 24－50 | 边 | 1280 | E20 |
| $\mathrm{F}_{6} \mathrm{I}$ |  | Caliprnim | 1988 | ［20］ | Furs． Furd．日m， | 180 | 18－62． | 83 | 4830 | 206 |
| $\mathrm{F}_{6}$ ： | Eobrlev（table 3） | Mosocr | 1943 | 182 | ATC | 15 | 38－15 | 13 | 4850 | 250 |
| Pet | Sabotev（table 5） | M0800\％ | 1943 | 20 | dre | 32 | 27－40 | 18 | 1020 | 200 |
| Pet | A ATts ek al． | Amplerdsm | 1964 | 2 2 | Forn．em． | 59 | 20－38 | 18 | 5500 | 400 |
| Qsi | Ontrovily et $\mathrm{a}_{\text {a }}$ ． | Inamind | 1968 | 24， | Eopl | $\sqrt{5}$ | $28-36$ | 10 | 5750 | 610 |
| 且 1 | Sthruten et al． | Utracht | 1943 | 120］ | Are | 8 | 82－71 | 0 | \＄200 | 1100 |
| In I | Ostropsky etys， | Lemingtad | 1958 | ［24］ | Hook | $\delta$ | 21－33 | 9 | 4510 | 580 |
| F T | Fan dar Elald et al． | Utrecht | 19060 | 26） | Fland | 8 | 13－2\％ | 14 | 3100 | 0 |
| Mf： | Finrston et al | Utrecht | 1841 | 近 | Art | IE | 11－58 | 13 | 5160 | 686 |
| MnI | Optutrsky et al． | Lenngrad | 1957 | （28） | Hool | 9 | 1R－88 | 18 | 8000 | 1200 |
| N1I | Kiny | Colibrinio | 1948 | ［29］ | Furn，fibst． | 80 | 98－37 | 9 | 1880 | 440 |
| 81 | Qutionsly et al． | Lentngrad | 1057 |  | Hools | 33 | 16－3＊ | 17 | 51.00 | 0 |
| Bri | Schuttaymer ef al， | Ttrock | 1093 | ［13） | Arc | 8 | 15－40 | 25 | 410 | 800 |
| TV1 | King ot ab． | Cuhborna | 1020 | \％20 | Furn blus | 204 | 19－40 | 21 | 17060 | 10 |
| T1 | vap Btekelenlrury et al． | Ukraht | 1818 | 801 | Are | 88 | 19－82 | 13 | 5 | 180 |
| TY 1 | Ostrimsty el Bh． | Edinlugrad | 108 | ［32］ | Hook | $\sqrt{61}$ | 19－34 | 15 | 5400 | 800 |
| T4t | Flippor el el． | Tamingrad | 1083 | ［3］ | Hook | 5 | 25－42 | 17 | \＄400 | 1200 |
| V1 | King | Obldiornks | 197 | 919 | Furn，abe， | 224 | 18．82 | 2 | 8110 | 210 |
| VI | Oftrovsky | Lemingrag | 1008 | 18． | Hook | 78 | 21－33 | 12 | 5100 | 1000 |
| En | gehmitersar et al． | Utrecht | 1913 | 36 | dre | 12 | $31-68$ | 28 | 560 | 500 |
| ズ1 | Poralricets］． | Leminspad | 1980 | （18） | Hooksand em． | 12 | 54－68 | 14 | blth | 60 |

The date of publication is given in col． 4 and the reference numiber，as listed at the end of this paper in col 5．The method used in the paper is indicated in col．6．In general，the several laboratories use different methods．At Leningrad the anomalous dispersion of the vapor in the neighborbood of the absorption lines from a furnnce is employed according to the＂hook＂method of Rogestwensky［36］some－ times with varistions．The temperature of the tube furnace is measured with an optical pyrometer．At Utrecht measurements are made usually with the arc method originated by Ormstein and his co－work－ ers［5］and carried on by Professor Smit［37］．In that procedure，the temperature is usually measured from the intensity distribution in the CN bands in the spectrum．The California experiments generally follow the method of total absorption as developed by King and King［38］．The furnace temperature is mensured with an optical pyrometer．

The number of spectral lines utilized in each deter－ mination is given in col．7．The largest numbers are from California．The minimum and maximum values of upper excitation potential and the differ－ ences between them are given in cols． 8 ard 9 ．These determine the range of the abscissae of the plots and thus affect to some extent the precision of the slope determination．The temperature determined from the slope of the plot appears in col． 10 and its devia－ tion from the mean value in col．11．The average value of the 31 individual determinations is $5100^{\circ} \mathrm{K}$ and the average deviation is $470{ }^{\circ} \mathrm{K}(9 \%)$ ．The standard deviation of an individual determination， which is a measure of the scatter of the determina－ tions，is given by

$$
\sigma=\sqrt{\frac{\overline{\Sigma \operatorname{dev}^{2}}}{n-1}}
$$

and is equal to $600{ }^{\circ} \mathrm{K}$（12\％）．The standard de－ viation of the mean，which measures the precision of the result，is given by

$$
\sigma_{m}=\sqrt{\frac{\Sigma \text { dev. }}{n(n-1)}}
$$

and is equal to $110^{\circ} \mathrm{K}(2 \%)$ ．The median value is also $5100^{\circ} \mathrm{K}$ ．
It is of interest to compare the reaults from the three mafor contributors of transition probability data．This is done in table 2．The three mear values agree amongst themselves within their un－ certainties．Another test of the data was made by selecting 12 sets of data which，on the basis of smath scatter and uniform distribution of the points，and adequate range of $E$ ，seemed to give promise of better determinations．The average of these 12 is $5000^{\circ} \mathrm{K}$ with an average deviation of $500^{\circ} \mathrm{K}$ ． These values are not significantly different from the general averages．The above mentioned tests pro－ vide some confidence in the adoption of an un－ weighted mean as the best value for the temperature．

Eight of the plots from which the temperatures were derived are reproduced in the figures．In each figure is given the spectrum，the reference from which the transition probabilities were taken，the tempera－ ture derived from the plot and a note whether each point on the plot represents a siagle spectral line or the average value for a multiplet．

Table 2. Comparison of laboratories

| Spectuan | Temp. | Dey. |
| :---: | :---: | :---: |
| Caluarmia |  |  |
| Car | 1000 | 630 |
| OI | 4881 | 546 |
| Nt | 480 | 240 |
| Fel | 1500 | 0 |
| Tij | 00 co | 2 z 0 |
| 7 | 5810 | 400 |
| Col | 5380 | 580 |
| ATE | 4820 | 390 |
| $\pi=470$ |  |  |
| Lentngrad |  |  |
| MnI | 3000 | 1330 |
|  | 1510 | 720 |
| CdI | 1750 | 180 |
| Bal | 8000 | 280 |
| Set | 5100 | 180 |
| 201 | 5160 | 70 |
| T1t | 8400 | 170 |
| Col | 6490 | 250 |
| Giat | 5780 | 520 |
| V11 | 8190 6400 | 880 |
| Avg | 5230 | 510 |
|  |  |  |
| \% $=780$ |  |  |
| Utrecht |  |  |
|  |  |  |
| Bat | 42.50 | 870 |
| 8 Cl | 4110 | 410 |
| Agt | 4600 | 540 |
| Cat | 4980 | 240 |
| 施 1 | 8080 | 40 |
| K I | 6104 | 0 |
| T11 | 52000 | 140 |
| 7ni | 80000 | 680 |
| Mg : | 5780 | 6ftb |
| H:\% | 62.50 | 1130 |
| A FES | 6120 | 480 |
| $\underline{=1930}$ |  |  |

## 5. Discussion of Resulta

It might not be smiss to discuss briefly the individusl determinations. Ag gave a good plot, but with only a small range of $E$ ' and a small number of lines. The Leningrad Ba data are excellent, except for low palues of ${ }^{\circ}$ for 4132 and 3889 A which were probably caused by the proximity of strong lines at 4130 and 3891 A . The Utrecht Ba plot is good. Both plots of Cd are good and they agree-well. Both Ca plots are excellent; the Califormia one covers an unusually long range of $E$ (see fig. 1). The plot of the Co data from California is long and straight but shows some scatter: the Leningrad one is shorter but with less scatter (see fig- 2). The low temperature which is obtained with the California Cr data has been noticed by sevcral other people and is discussed by Goldberg, Muller, and Aller [39]. Otherwise a good plot is obtained (fig. 3), A set of Cr data from Loningrad shows such a large scatter and small range of $E$ that no temperature could be derived [28].

In the California Fe plot, all the Iines below 3200 A stand above the line. These were ignored in deriving the slope. The Moscow Fe data show a rather large scatter but yield satisfactory temperatures. The same paper [22] gives a set of data for Bi, which, altbough apparently in good order, yields a temperature of $8700^{\circ} \mathrm{K}$, which differs from the general mean by six times the standard deviation and it has bean rejected, as noted ahove. The Arnsterdam Fe data form an excellent plot with a good range of $E$ and a small scatter (see fig. 4).

The Ga and In data are deficient in number and involve only resonance lines which are generally subject to various experimental difficulties, but they give a satisfactory plot. The plot for Hg covers too short a range of $E$ because the NBS intensity of the resonance line was affected by self-absorption and could not be used. $K$ represents the only data obtained from flames, and, although necessarily deficient in number of lines, yields a satisfactory plot. It involves the resonance lines, which showed no self-abeorption in the NBS spectra. The Mg data show a good distribution and a very emall scatter. The Mg resonance line was omitted because of self-absorption in the NBS line. The Mn resonence lines fall high on the Mn plot, indicating too low a value of of from the hook method, and they were not used in finding the slope. Ni gives a good plot but with a small range of $E$. As in the case of Mn , the resonance lines of Sc are high on the plot and were ignored in determining the slope. The hook method appareutly tends to give low ralues for the strongest lines. The Utrecht Sr data make a nice plot over a good range of $E$. Another set of Sr data obtained by Eberbagen at Gättingen [40] by using the Kiel water-arc forms a set of points with too large a scatter to permit a detcrmination of temperature.

Data for Ti are available from three laboratories. The California set has a large number of points well distributed over a long range of $E$ and a very small scatter. A preliminary plot of these data is shown in ref. [1]. The Utrecht data are fewer and have a smaller range, but shown almost no scatter. The maximum vertical deviation of a point from the best line is $0.09 \log$ units which represents a maximum deviation of 23 percent (see fig. 5). The Leningrad data have more pointe but a considerable scatter. It may be mentioned at this point that Miss Rountree [41] has calculated a temperature for our are from our Ti data and King's. Her value is slightly high because she omitted the $\lambda^{3}$ factor of eq (7). The Tl data from Leningrad have been frequently cited in the Russian literature. There are only 5 lines in common with ours. The fact that the temperature derived from these data shows the maximum deviation in the group of 31 values may arise from the location of 4 of the 5 lines in the short wavelength regions below 2800 A where the accuracy of the intensities declines. The V data from Caliiornia have the largest number of lines in the group and the plot shows a moderate scatter of points well distributed over a long range of $E$, permitting an accurate determination of 7 . It shows no suggestion of curvature, indicating strict adherence to Boltz-
mann's law (see fig. 6). The Leningrad data cover a shorter range of $E$ (see fig. 7). The Utrecht data for Zn cover the longest range of $E$ of the whole group and give a very straight plot (see fig. 8). The Leniugrad data ornit the intersystem line at 3075 A and so bave a more restricted range of $E$ :

Taken as a whole, the results exhibit a nearly normal (Gaussian) distribution of values (see table 3). Consideration of the results in table 1 suggests that the effective temperature of the copper arc is substantially constant and that the variations arise from syatematic errors in the verious sets of transition probabilities. For example, the two sets of $a f$-values for $y$ I exbibited in figures 6 and 7 are compared with the same intensity numbers in each case. The temperature difference of $780^{\circ} \mathrm{K}$ between the values from each plot car arise only from systematic errors inherent in the two sets of of-values. Similarly, the transition probabilities of van Hengstum and Smit [15] for Cd i are systematically smaller relative to those of Penkin and Redko [16] at the larger values of $E$, so although the identical NBS intensity numbers are used, the two plots yield temperatures which differ by $330^{\circ} \mathrm{K}$. The diversity of methode represented and the independent nature of the laboratories leads us to suppose that these systematic errors have offiset one another to some extent in contributing to the general average value.

Various correlation plots have been made to determine other systematic infuences on the derived temperatures. There appears to be no correlation with the number of lines used, with atomic number, with range of $E$ (abscissae of the plots), with magnitude of $E$, nor with ionization potential.

Table 3. Distribution of individual temperatare determinations aboud the mean, in intertals of is.

| Intrypl | $\begin{aligned} & 3300- \\ & 3800 \end{aligned}$ | $\begin{aligned} & 3200- \\ & 4500 \end{aligned}$ | $\begin{gathered} 5000- \\ 5100 \end{gathered}$ | $\begin{gathered} 5100- \\ 5700 \end{gathered}$ | $\begin{aligned} & 8200- \\ & 6500 \end{aligned}$ | $\begin{aligned} & 6800- \\ & 6800 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Inditidus) } \\ & \text { detterminations } \end{aligned}$ | 8800 | $\begin{aligned} & 4070 \\ & 4250 \\ & 4250 \\ & 4410 \end{aligned}$ | 4510 4820 4800 4750 4820 4800 4850 6000 7080 5060 6100 | 5100 <br> 3160 <br> 6200 <br> 5010 <br> 5380 <br> 5100 <br> 3469 <br> 5600 | $\begin{aligned} & 6750 \\ & 3790 \\ & 67 \times 0 \\ & 4260 \end{aligned}$ | 6400 |
| Observad irbetion. Expected (Crupstant)...- | $\begin{gathered} 0.03 \\ .02 \end{gathered}$ | $\begin{array}{r} 0.13 \\ .14 \end{array}$ | $\begin{array}{r} 0.36 \\ .34 \end{array}$ | 0.38 .34 | 0.13 .14 | 0.08 .02 |

## 6. Effect of Uncertainty in Temperature on Uncertainty in Derived Transition Probability

The purpose of determining the best value of the temperature of the copper are is to enable us to calculate values of $g A$ or $o f$ from the intensities of the classified lines in the NBS Tables of Spectral-line Intensities. We must find out how the uncertainty of the value of $T$ will influence the uncertainty of the $g A$ or $g f$.

If we differentiate eq (3) with respect to $T$ and divide the result by eq (3) we have

$$
\frac{d(g A)}{g A}=-\frac{E}{k T} \frac{d T}{T} .
$$

Introducing numerical values we find

$$
\frac{d(g A)}{g A}=-\frac{E}{3.54} \frac{d T}{T}
$$

Now if $\frac{d T}{T}$ is 2 percent, then

$$
\frac{d(g A)}{g A}=-0.00565 E
$$

Símilarly

$$
\frac{d(g f)}{g f}=-0.00585 E
$$

The magnitude of the error for values of the upper level from 10 to 100 kilokaysers is given in table 4, assuming a temperature of $5100{ }^{\circ} \mathrm{K}$ and an uncertainty in tho temperature of 2 percent. Most of the lines in the NBS tables have upper levels leas than 50 kK , so most of the errore introduced by the temperature error will be less than 30 percent. This is about the same as the errors in the intensity numbers themselves.

Table 4. Erfor in of of A a correapanding to a 2 percent error in T at $5100^{\circ} \mathrm{K}$

| $E \ln 15$ | W. Crom im sfor 94 |
| :---: | :---: |
| 10 | 0 |
| 20 | 11 |
| 31 | 17 |
| 40 | 20 |
| 50 | 28 |
| 60 | 34 |
| 70 | 40 |
| 900 | 45 |
| 500 | 61 68 |

## 7. Conclusion

A temperature for the copper arc in which the intensities published in the NBS Tables of Spectralline Intensities [1] wore measured has been determined. Comparison of NBS intensities with 1650 published transition probabilities for 20 different spectra yields the value

$$
5100 \pm 110^{\circ} \mathrm{K}
$$

With this parametcr, relative traasition probabilities for the 25,000 classified lines in the NBS Tables can be computed.

Further studiea are in progress to determine the state of ionization of each element in the arc. This will allow the relative values for each spectrum to be adjusted to the same scale. It may then be possible to calibrate the scale on an absolute basis as Allen
has done [42]. All the pertinent data for the 25,000 lines are on punched cards and the computation can be performed very quickly, onee the proper parameters and factors have been determined.

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## 8. References

[1] W. F. Meggers, C. H. Corlies, sid B. F. Scribner, Tables of Spectral-line Intensities, NBS Monogreph 32 (1961), (0.S. Covernment Printing Office, Wrshington, D.C.)
[2] A. Einstein, Phy日, 2. 18, 121 (1917).
(3) Transactions of the Joint Cornmission for Epectroscopy. J. Opt. Soc. Am. 43, 410 (1953).
[4] R. Ladenburg, Z. Pbysik 4, 451 (i921).
(5) I. S. Ormetein and H. Brinkman, Physiea 1,797 (1934).
[6] J. D. Cobine, Gaseous Conductora, p. 290, Dover Pubscatione, New York (1958).
(7) C. G. Euits, Physice ©, 190 and 315 (1936).
[s] C. W. Allen and A. S. Agasd, Mon. Not. Roy. Astronom. soc. 115, 571 (1965).
[9] R. B. King, Proc. of Cont. on Steilar Atmospheres, Indiana Univ. (1954).
[10] J. Terpatra and J. A. Smit, Fhysics 24, 037 (1058).
[11] A. M. Kruithof, Physics 10, 403 (1943).
[12] Y. I. Oetroveky and N. P. Penkin, Optics and Spectroscapy 9,371 (1860).
[13] J. W. Sohuttevaer M. J. de Bont, and T. H. van den Briek, Physica 10, 544 (1943).
[14] K. H. Oisen P M. Routly, and R. B. King, Astrophys. J. 130,688 (1959).
[15] J. P. A. van Hengstum and J, A. Smit, Physien 28, 86 (1956).
[16] N. P. Penkin and T. P. Red'ko, Optics and Spectroseopy - 360 (1960)
[17] f. B. King, B. Parneg, M. Dayfes, and K. H. Olsen, J. Opt. Goc. Am. 45, 350 (1965).
[18] Y. I. Ostrovsky and N. P. Penk[u, Optika i Spoktroskopiya 5, 345 (1958).
[19] A. J. Hili and R. B. Kingt J. Opt. Soe. Am. 41, 315 (1951).
[20] R. B, King sund A, B. King, Astrophys, J. 82, 24 (1938),
${ }^{21]}$ W. W. Carter, Phys. Rev.76, 962 (1949).
$[22]$ N. N. Sobolev, J. Exp. Theoret. Phys. 17, 13I (1943),
23] J. Aarts, D. Harting, and C. J. Bakler, Physica 20, 1250 (1054).
[24] Y. I. Ostrovsky and N. P. Penkin, Optika i Spektroskopiys 4, 719 (1058).
[25] J. W. Schouten and J. A. Smit, Physica 10, 061 (1943).
[26] E. F. M. van der Held and J. H. Heierman, Physiea 3, 81 (1936)
[27] J. A. H. Karsten and L. S. Ornateln, Physiea E, 1124 (1941).
[28] Y. I. Ostrovsky and N. P. Penkin, Optika i Spektroskoplys 3, 193 (1957).
[29] R. B. Kligg, Astrophys. J. 188, 87 (1948).
[20] Y. I. Ostravaky and N. P. Penkir, Optika i Speltrioskopiys 3, 391 (2957).
[31] L. H. M. van Stekelenburg and J. A. Bmit, Physica 14, 185 (1048).
[32] X. J. Ostrovaky, G. F. Parcheveky, and N. P. Penkin, Optika i Spektraskepiga 1, 821 (1956).
33] A. Filippov and W. Prosofiey, $Z$. Phymik 85, 647 (1933).
34] A. B, King, Astrophys, J. 105,376 (1947).
35 J. W. Schuttevaer and J. A. Smit, Physica 1\%, 502 (1943).
(30] D. S. Rogestwensky and N. P. Penkin, J. Phys. U.S.S.R. 5. 319 (1941).
[37] J. A. Slmit, Phyaies 18, 683 (194B).
(38) R. B. King and A. S. King, Astrophys. J. B2, 377 (1935).
[39) L. Goldberg, E. A. Murller, and L. H. Aller, Aatrophys. J. Supplement No. $455_{1} 1$ (1960).
[40] A. Eberhagen, Z. Physik 143, 302 (1255).
[41] J. C. Rountree, Anuales d'Astrophyaique 2F, 638 (1960).
[42] C. W. Allen and A. B. Asasd, Mon. Not. Roy. Astrox. Soc. 117, 36 (1957); C. W. Allen, ibid. 117, 622 (1957); C. W. Alten, ibid, 121, 299 (1960).
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