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Axial Performance of Spectacle Lenses[1]

Francis E. Washer

The measured values of the axial meridional powers for 311 spectacle lenses are reported. The spherical refractive powers range from +7.00 to −20.00 diopters with cylindrical powers of 0.00, 1.00, and 2.00 diopters. The departures of the measured from the nominal values are shown. The probable errors of measurement are discussed. A set of tolerance values is suggested and the degree of compliance with these suggested tolerances is shown. Theoretical and experimental justifications for the proposed tolerance range are given.

1. Introduction

For a number of years, there has been evidence of a need for a performance standard for spectacle lenses. Various organizations have from time to time attempted to formulate standards relating to the quality and performance of spectacle lenses. Many of these standards are quite good, but they frequently differ in the magnitude of the suggested tolerances, which tends to confuse the user.

Because of the lack of agreement and incompleteness that prevailed among existing standards, the National Bureau of Standards was asked by the Veterans' Administration to assist in the preparation of specifications for use in the purchase of spectacle lenses. In the course of the work leading to a specification based on performance, the axial and marginal meridional powers of 311 spectacle lenses were measured [1, 2].[2]

On account of the evident interest in spectacle-lens performance and particularly in the proper magnitude of the tolerances in axial power that may be set in future standards, it seems worthwhile to report the measured values of the axial powers of these lenses. Analysis of these results shows that lenses are already being made to quite close tolerances. In addition, the values obtained for these lenses can be used to determine the suitability of any set of tolerances of axial power that may be specified.

2. Selection of Lenses for Measurement

The selection of a series of spectacle lenses for use in preparing a set of tolerances is not a simple matter. There are so many varieties, each designed for a specific use. These include single-vision, bifocal, and trifocal lenses; also glasses that give best over-all performance when used in viewing distant objects and those that give best performance when used for near objects. In addition, for a single type there are a great number of steps required to cover the entire range of powers. For example, for lenses having spherical power only, there are 41 possible lenses to be considered, if one proceeds from a power of +20.00 to −20.00 diopters in 1-diopter steps. For each halving of the steps, the number to be considered is doubled, so for ¼-diopter steps, the total number is 161.

If lenses for astigmatic correction are also included, the number of different lenses to be considered are increased enormously. Because of the impracticability of measuring the performance of a representative number of all available lenses, it was early decided to limit the scope of the investigation to cover lenses used for distant-vision only. The number was further reduced by deciding to measure only the type called corrected ophthalmic lenses. A corrected ophthalmic lens is an ophthalmic lens having the total spherical and cylindrical powers divided between the front and rear surfaces in a manner that minimizes as far as practicable, the differences between powers measured at points in the peripheral region and the optic axis [3]. Lenses of this nature are known to the trade as members of a "corrected curve" series.

Under this limitation, the range for study was accepted as being from +7.00 to −20.00 diopters. In order to reduce the number to be studied still further to some practicable figure, percentage-of-use tables prepared by the Veterans' Administration were studied [4]. A recent version of these percentage-of-use tables is shown in table 1. On the

TABLE 1. *Expected percentage of use for single-vision lenses having spherical powers ranging from +20.00 to −20.00 diopters and cylindrical powers ranging from 0.00 to +6.00 diopters*

Based on information contained in Veterans' Administration specification of 1955.

Range of spherical power	Percentage having cylindrical power in the range—				
	0.00 to 0.00	0.25 to 2.00	2.25 to 3.00	3.25 to 4.00	4.25 to 6.00
Diopters					
16.25 to 20.00	0.03	0.05	0.005	0.005	0.02
12.25 to 16.00	.35	.40	.05	.05	.005
9.25 to 12.00	1.30	.40	.05	.02	.02
7.25 to 9.00	0.20	.10	.05	.02	.005
6.25 to 7.00	.20	.05	.005	.005	.005
4.25 to 6.00	1.40	6.20	0.005	0.05	0.005
2.25 to 4.00	12.20	6.30	.15	.05	.02
0.00 to 2.00	23.50	14.00	.25	.10	.10
.00 to 0.00	0	4.70	.15	.10	.05
.00 to −2.00	5.00	14.20	.45	.15	.08
−2.25 to −4.00	1.20	2.80	0.45	0.20	0.07
−4.25 to −6.00	0.40	1.00	.10	.10	.10
−6.25 to −7.00	.10	0.15	.05	.02	.005
−7.25 to −9.00	.10	.10	.02	.01	.01
−9.25 to −20.00	.05	.05	.02	.02	.02
Total	46.03	50.50	1.94	0.945	0.515

[1] This work was performed in connection with a research project sponsored by the Veterans' Administration.

[2] Figures in brackets indicate the literature references at the end of this paper.

basis of these studies, the major number of lenses were selected having spherical power ranging from +7.00 to −6.00 diopters and cylindrical power ranging from 0.00 to +2.00. The number selected outside this range is small but is perhaps sufficient to indicate probable performance.

It also seemed desirable to make measurements on more than one lens of a given power. To achieve this end, lenses of identical powers were purchased from five different manufacturers. Some power combinations were not readily available from each of the five, but generally, at least three makers were able to do so. The final total of lenses that were measured was 311, comprised of 3 to 5 lenses in each of 68 power combinations.

3. Nomenclature

In the early days of spectacle making, spherical surfaces only were used. After the existence of astigmatism in the eyes of certain people suffering from poor vision was established, it was found that the combination of spherical and cylindrical surfaces on spectacle lenses resulted in improved vision particularly in the axial region. Because only spherical and cylindrical surfaces were used, the custom arose of describing or prescribing such lenses in terms of spheres and cylinders.

With the advent of toric surfaces for the correction of astigmatism over a wide vista, the practice of prescribing spectacles in terms of spheres and cylinders might well have been discontinued. However, the practice still persists and frequently leads to misunderstanding because a given prescription can usually be written in two ways. For example, a given prescription can be written prescribing a positive sphere and with a positive cylinder or as a larger positive sphere with a negative cylinder. The process of changing the lens prescription from one form to another equivalent form is called transposition.

When the study that finally led to the preparation of a performance specification for the Veterans' Administration was initiated, it soon became evident that it was much simpler to describe the performance in terms of meridional powers. There are two principal meridional powers for an astigmatic lens, and the equivalent cylindrical power is given by the simple difference of the two principal meridional powers. Actually, when using this concept, the power of a lens is completely specified by giving the values of the two principal meridional powers. The advantage of this method is still more evident when one is comparing measured performance in the peripheral region with that in the axial region.

In the course of this study of spectacle performance, measurement, analyses, and reporting of results have been done in terms of the meridional powers so as to avoid any possible ambiguity. However, as the terms *spherical* and *cylindrical power* are in general use in the prescribing of lenses, a summary of the relations connecting meridional powers and the usual prescribed powers is given below.

V_P = One of the principal meridional powers. It is given by the sum of the prescribed spherical and prescribed cylindrical power. It is the maximum meridional power when the prescribed powers are positive.

H_P = The second principal meridional power. It is usually the same as the spherical power. It is the minimum meridional power when the prescribed powers are positive.

C_P = Prescribed cylindrical power. It is the difference between the principal meridional powers and is given by the relation $C_P = V_P - H_P$.

V_O = Measured value of one of the meridional powers at the optical center of the lens. It is the measured power for vertical lines.

H_O = Measured value of the second meridional power at the optical center of the lens. It is the measured power for horizonal lines. (In the absence of cylindrical power, $V_O = H_O$.)

C_O = The measured cylindrical power at the optical center of the lens. It is obtained from the relation, $C_O = V_O - H_O$.

3.1. Additional Relations

The following formulas are used in computing the departures of the measured values from the specified values for the axial region. The values V_P, H_P, and C_P are derived from the nominal prescribed powers.

$$\Delta V_O = V_O - V_P$$
$$\Delta H_O = H_O - H_P$$
$$\Delta C_O = C_O - C_P.$$

Figure 1 is a schematic drawing showing the two principal meridians in which measurements are made. For convenience, the lens is always so oriented that $V_O - H_O \geq 0$, which is equivalent to limiting the study to lenses having zero or positive cylindrical power.

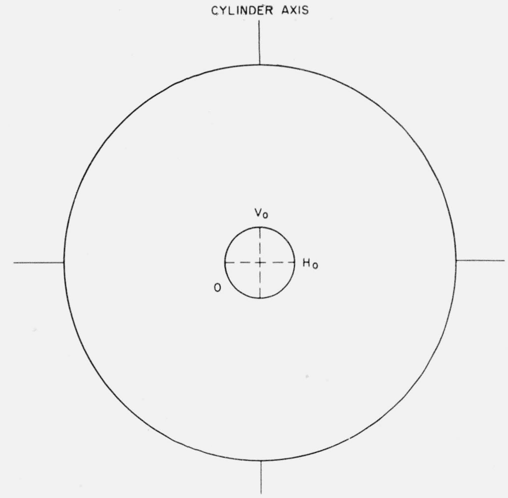

FIGURE 1. *Schematic drawing showing the two principal meridians in which measurements are made.*

For convenience, the lens is always so oriented that $V_O - H_O \geq 0$, which is equivalent to limiting the study to lenses having zero or positive cylindrical power.

4. Method of Measurement

The powers of all lenses were measured on standard vertex power-measuring instruments. Instruments of this type are actually calibrated to read meridional power directly, which provides an additional reason for recording and reporting the results of measurement in terms of meridional power. Before beginning the actual measurement of the various sample lenses, the instruments were calibrated with the aid of a series of standard lenses whose powers had been carefully determined on a visual optical bench. These standard lenses were used from time to time during the course of the investigation to make certain that no inaccuracy of the vertex power-measuring instruments had developed. In addition, an extended study of the sources and magnitude of possible errors was made before proceeding with the main part of the work. This study is presented in some detail in the following section.

4.1. Variation in Axial Power for One Observer

When making a measurement of dioptric power of a spectacle lens with vertometer, lensometer, or similar measuring instrument, it is usual to first set the zero in accordance with the manufacturer's instructions. The lens to be measured is then properly positioned in the instrument and a setting for best focus made. The measured power is then read directly from the calibrated drum. Several sources of variation in the reported power are at once apparent. First, the range through which the target reticle can be moved without marked deterioration of the observed imagery is appreciable, so that the observer must attempt setting the reticle in a mean position and trust that he has gone neither too far nor not far enough. For best results, it appears desirable to pass completely through the region of good focus, reverse and pass back through it, then approach again, stopping at what is believed to be the optimum position. To reduce systematic errors by a single observer, it is preferable that the drum always be rotated in the same direction in the approach to the final setting.

In order to give an idea of the magnitude of the variation possible from this depth of focus effect, measurements were made to determine the range of setting possible with tolerable focus. The values obtained are shown in table 2.

TABLE 2. *Variation of readings possible when setting the drum for satisfactory definition, but stopping at the beginning of the range*

All values are given in diopters

Spherical power	Setting made with drum moving—		Focal range
	Clockwise	Counterclockwise	
+4.00	+4.13	+4.03	+0.10
2.00	2.13	2.01	.12
0.00	0.10	−0.01	.11
−1.00	−.88	−.99	.11
−2.00	−1.78	−1.89	.11
−4.00	−3.84	−3.98	.14
−6.00	−5.94	−6.06	.12

In making the observations, shown in table 2, several lenses were selected at random to cover the central region of the instruments range. It is clear that if the observer maintains the same criterion and same direction of drum motion, he will get approximately the same value for either manner of setting. For example, the measured power for the lens with nominal power four is given by either

$$4.13 - 0.10 = 4.03 \text{ diopters}$$

or

$$4.03 - (0.01) = 4.04 \text{ diopters.}$$

However, if reversed conditions are used, then the measured power may range from 3.93 to 4.14 diopters.

A second source of variation is the size of the interval separating individual settings. For example, when the smaller scale division is 0.12 diopter, estimating the reading to the nearest hundredth is rather difficult. Perhaps the best that can be expected for such an interval is an estimate between one-sixth and one-eighth of a scale division or approximately ± 0.02 diopter. For other regions, the size of a scale division is greater, and the error in the estimate is proportionately greater.

4.2. Variations in Axial Power Measurements for Three Observers

Because it was realized that the differences between marked and measured powers observed in the course of this work might serve as a basis in establishing tolerance for purchase specifications of spectacle lenses, all possible precautions were taken to insure accuracy. Too much reliance, therefore, was not placed on the findings of a single observer. There exist always possibilities of differences in observational criteria among observers that may lead to systematic differences in results. There are also possibilities of gross errors or mistakes when considering the results of a single observer, which are usually detected when the measurements are repeated by a second observer because it is quite unlikely that both will make the same error for the same lens.

In order to determine the probable variations among observers, the same lenses were measured on the same instrument by three different observers. None of the observers was permitted to see the values obtained by the others during the experiment. One observer completed his work on the entire series before the next one started.

Table 3 shows the values obtained by each of three observers, identified as D, G, and B, for a representative group of six lenses. The fourth line in each case is the average value obtained and is the one accepted as correct. The symbols V_O, H_O, and C_O signify, respectively, the measured maximum and minimum meridional powers and the measured cylindrical power following the procedure set forth in section 3. It is clear that the departure from the average is relatively small for each observer.

TABLE 3. *Values of the axial powers for 6 lenses obtained by each of 3 observers (D, G, and B)*

The average values for the three observers are accepted as correct. All values are expressed in diopters.

Obs.	Sph.	Cyl.	V_o	H_o	C_o	ΔV_o	ΔH_o	ΔC_o
D			8.90	6.97	1.93	−0.10	−0.03	−0.07
G	7	2	8.97	6.97	2.00	−.03	−.03	.00
B			8.96	6.93	2.03	−.04	−.07	.03
Average			8.94	6.96	1.99	−.06	−.04	−.01
D			6.13	4.02	2.11	+0.13	+0.02	+0.11
G	4	2	6.12	4.00	2.12	.12	.00	.12
B			6.12	4.03	2.09	.12	.03	.09
Average			6.12	4.02	2.11	.12	.02	.11
D			3.06	1.01	2.05	0.06	0.01	0.05
G	1	2	3.07	1.05	2.02	.07	.05	.02
B			3.05	1.00	2.05	.05	.00	.05
Average			3.06	1.02	2.04	.06	.02	.04
D			2.09	0.02	2.07	0.09	0.02	0.07
G	0	2	2.01	.01	2.00	.01	.01	.00
B			2.00	.00	2.00	.00	.00	.00
Average			2.03	.01	2.02	.03	.01	.02
D			1.01	−0.93	1.94	0.01	0.07	−0.06
G	−1	2	1.01	−.98	1.99	.01	.02	−.01
B			1.03	−.99	2.02	.03	.01	.02
Average			1.02	−.97	1.98	.02	.03	−.02
D			−2.01	−3.99	1.98	−0.01	0.01	−0.02
G	−4	2	−2.00	−3.98	1.98	+.00	.02	−.02
B			−2.04	−4.00	1.96	−.04	.00	−.04
Average			−2.02	−3.99	1.97	−.02	.01	−.03
D			−3.89	−5.89	2.00	+0.11	0.11	+0.00
G	−6	2	−3.91	−5.96	2.05	.09	.04	.05
B			−3.97	−5.95	1.98	.03	.05	−.02
Average			−3.92	−5.93	2.01	.08	.07	.01

Considering both of the above factors and ignoring, for the present, possible systematic errors arising from initial improper adjustment of the instrument and from possible errors of calibration, it seems probable that a single observer should be able to repeat a given setting within one-fourth of a scale division. Two settings are, however, necessary to make a single power determination so the probable error is increased by a factor of 1.4. The probable error of a single determination by a single observer may accordingly be estimated as one-third scale division. For small dioptric powers, this means that the probable error of a single determination is ± 0.04 diopter. For higher dioptric powers, where the size of a scale division is 0.25 diopter, this error may reach ± 0.07 diopter. In making the measurements on the lenses reported herein, the observers customarily make five complete determinations. The spread of the recorded values tend to corroborate the above conclusions. The error in the reported values of dioptric powers should, of course, be appreciably lower, as the error of an average of five determinations is less than half that of a single.

An analysis was made of the results of measurements on 36 lenses by 3 observers to determine possible observer bias or systematic error. The results of this analysis are shown in table 4. Departures from the averages were determined for each observer for each of the 36 lenses. Ideally the sums of the departures from averages should total zero, on the assumption that although different lenses are involved, the nature of the measurements is essentially the same. Actually this sum is not zero, so a small observer bias is indicated. The average magnitude for each observer is as shown in table 4. It is clear that for none of the observers does the magnitude of the systematic error exceed ± 0.01 diopter, which is practically negligible, being of the order of magnitude of one-tenth scale division. In addition, the probable error of a single determination was computed for each observer and found not to exceed ± 0.02 diopter. Considering the small magnitude of the observer bias and the probable error of a single determination for each observer, it can safely be said that the error of a determination of V_O, H_O, or C_O will generally be less than ± 0.03 diopter.

TABLE 4. *Systematic errors in* V_O, H_O, *and* C_O *for 3 observers, D, G, and B, based on measurements from 36 lenses*

Observer	Average departure from the average of—		
	V_o	H_o	C_o
	Diopters	*Diopters*	*Diopters*
D	+0.011	+0.013	−0.002
G	−.003	−.006	+.003
B	−.006	−.009	−.001

To give a clearer view of the agreement among the three observers, table 5 shows a frequency distribution of the errors. For example, in the determination of V_O, the value obtained by observer D is identical with $\overline{V_O}$ (the average for the three observers) for 14 percent of the lenses. All of the observers are within ± 0.03 diopter of $\overline{V_O}$ for 80 percent of the lenses. It is clear from these tables, that the determinations of each of the three observers are within ± 0.03 diopter of the accepted average values of $\overline{V_O}$, $\overline{H_O}$, and $\overline{C_O}$ for 90 percent of the lenses and are within ± 0.01 diopter for 50 percent of the lenses. The findings shown here in table 5 tend to corroborate the belief that a probable error of a complete determination for a single observer will generally not exceed ± 0.03 diopter.

TABLE 5. *Frequency distribution of departures from the average for each of 3 observers for determinations of $\overline{V_O}$, $\overline{H_O}$, and $\overline{C_O}$ on a group of 36 lenses*

A

$\delta\overline{V_O}$	Percentage of determinations that do not depart from $\overline{V_O}$ by more than $\delta\overline{V_O}$ for observer—		
	D	G	B
Diopters			
0.00	14	25	11
±0.01	39	67	56
.02	61	89	78
.03	83	97	89
.04	92	97	92
.05	95	100	100
.06	100	100	100

B

$\delta\overline{H_O}$	Percentage of determinations that do not depart from $\overline{H_O}$ by more than $\delta\overline{H_O}$ for observer—		
	D	G	B
Diopters			
0.00	17	25	17
±0.01	56	58	58
.02	70	89	84
.03	78	98	92
.04	92	100	98
.05	95	100	100
.06	100	100	100

C

$\delta\overline{C_O}$	Percentage of determinations that do not depart from $\overline{C_O}$ by more than $\delta\overline{C_O}$ for observer—		
	D	G	B
Diopters			
0.00	28	22	28
±0.01	58	55	58
.02	70	84	75
.03	86	89	84
.04	92	100	98
.05	98	100	98
.06	100	100	100

5. Results of Measurement on 311 Spectacle Lenses

The two principal meridional powers of each lens were measured by each of three observers. Each observer made five determinations of each quantity for each lens. The results of measurement for all observers were averaged and the results are listed in tables 6 to 12. For convenience, each table lists results for the entire range of spherical powers for a single value of the cylindrical powers. The two principal meridional powers, V_O and H_O, are the only quantities directly measured. The value of the equivalent cylindrical power is given by the difference between the corresponding values of V_O and

TABLE 6. *Measured values of the axial meridional powers, V_O and H_O, for 72 spectacle lenses having zero cylindrical power and with spherical powers ranging from +7.00 to −18.00 diopters*

The nominal prescribed spherical powers are listed under "Sph". The values of the deviations from the nominal powers in the principal meridians are listed under the headings ΔV_O, ΔH_O, and $\Delta C_O=(\Delta V_O-\Delta H_O)$.

All values are expressed in diopters.

Sph.	V_O	H_O	C_O	ΔV_O	ΔH_O	ΔC_O
7.00	6.98	6.98	0.00	−0.02	−0.02	0.00
	7.01	7.00	.01	+.01	+.00	.01
	7.01	7.01	.00	.01	.01	.00
6.00	6.08	6.07	0.01	0.08	0.07	0.01
	6.07	6.07	.00	.07	.07	.00
	5.98	5.96	.02	−.02	−.04	.02
	6.02	6.05	−.03	+.02	+.05	−.03
	6.00	5.98	+.02	.00	−.02	+.02
5.00	5.05	5.05	0.00	0.05	+0.05	0.00
	4.99	4.99	.00	−.01	−.01	.00
	5.04	5.05	−.01	+.04	+.05	−.01
	4.97	4.96	+.01	−.03	−.04	+.01
	5.04	5.02	.02	+.04	+.02	.02
4.00	3.99	3.99	0.00	−0.01	−0.01	0.00
	3.99	4.00	−.01	−.01	+.00	−.01
	4.02	4.01	+.01	+.02	.01	+.00
	4.02	4.00	.02	.02	.00	.02
	4.01	4.02	−.01	.01	.02	−.01
3.00	3.01	3.02	−0.01	0.01	0.02	−0.01
	2.98	2.99	−.01	−.02	−.01	−.01
	3.01	3.01	+.00	+.01	+.01	+.00
	3.09	3.09	.00	.09	.09	.00
	3.01	3.01	.00	.01	.01	.00
2.00	2.02	2.01	0.01	0.02	0.01	0.01
	2.03	2.04	−.01	.03	.04	−.01
	1.99	1.99	+.00	−.01	−.01	+.00
	1.97	1.97	.00	−.03	−.03	.00
	2.02	1.99	.03	+.02	−.01	.03
1.00	0.99	0.99	0.00	−0.01	−0.01	0.00
	1.01	1.01	.00	+.01	+.01	.00
	1.01	1.00	.01	.01	.00	.01
	1.00	1.02	−.02	.00	.02	−.02
	0.99	1.00	−.01	−.01	.00	−.01
0.00	0.01	0.01	0.00	+0.01	0.01	0.00
	−.01	−.01	+.00	−.01	−.01	.00
	+.01	+.00	.01	+.01	+.00	+.01
	.01	.00	.01	.01	.00	.01
	.00	.00	.00	.00	.00	.00
−1.00	−0.99	−0.99	0.00	0.01	0.01	0.00
	−.96	−.96	.00	.04	.04	.00
	−1.01	−1.02	.01	−.01	−.02	.01
	−1.00	−1.00	.00	+.00	+.00	.00
	−1.00	−1.00	.00	.00	.00	.00
−2.00	−2.01	−2.02	0.01	−0.01	−0.02	0.01
	−1.96	−1.95	−.01	+.04	+.05	−.01
	−1.96	−1.97	+.01	.04	.03	+.01
	−2.01	−2.01	.00	−.01	−.01	.00
	−1.92	−1.96	.04	+.08	+.04	.04
−3.00	−2.98	−2.97	−.01	0.02	0.03	−0.01
	−2.97	−2.95	−.02	.03	.05	−.02
	−2.99	−3.00	+.01	.01	.00	+.01
	−2.97	−2.96	−.01	.03	.04	−.01
	−3.02	−2.98	−.04	−.02	.02	−.04
−4.00	−4.01	−4.01	+0.00	−0.01	−0.01	+0.00
	−4.00	−4.00	.00	+.00	+.00	.00
	−4.00	−4.00	.00	.00	.00	.00
	−4.00	−4.00	.00	.00	.00	.00
	−3.90	−3.87	−.03	.10	.13	−.03
−6.00	−5.89	−5.90	+0.01	0.11	0.10	+0.01
	−6.02	−6.00	−.02	−.02	.00	−.02
	−6.00	−6.01	+.01	+.00	−.01	+.01
	−6.01	−6.01	.00	−.01	−.01	.00
	−6.00	−6.00	.00	+.00	+.00	.00
−10.00	−10.06	−10.06	0.00	−0.06	−0.06	0.00
	−9.89	−9.89	.00	+.11	+.11	.00
	−9.96	−9.96	.00	.04	.04	.00
−14.00	−13.92	−13.92	0.00	0.08	0.08	0.00
	−14.07	−14.07	.00	−.07	−.07	.00
	−13.98	−13.98	.00	+.02	+.02	.00
−18.00	−17.95	−17.95	0.00	0.05	0.05	.00
	−17.90	−17.90	.00	.10	.10	.00
	−17.97	−17.97	.00	.03	.03	.00

TABLE 7. *Measured values of the axial meridional powers, V_O and H_O, for 76 spectacle lenses having a cylindrical power of 1.00 diopter and with spherical powers ranging from +6.00 to −19.00 diopters*

The nominal prescribed spherical powers are listed under "Sph". The values of the deviations from the nominal powers in the principal meridians are listed under the headings ΔV_O, ΔH_O, and $\Delta C_O = (\Delta V_O - \Delta H_O)$.

All values are expressed in diopters.

Sph.	V_O	H_O	C_O	ΔV_O	ΔH_O	ΔC_O
7.00	8.04	7.00	1.04	0.04	0.00	0.04
	8.00	6.98	1.02	.00	−.02	.02
	8.00	7.02	0.98	.00	+.02	−.02
6.00	6.99	5.92	1.07	−0.01	−0.08	+0.07
	7.16	6.06	1.10	+.16	+.06	.10
	7.09	6.03	1.06	.09	.03	.06
	7.01	5.94	1.07	.01	−.06	.07
	7.00	6.02	0.98	.00	+.02	−.02
5.00	6.09	5.10	0.99	0.09	0.10	−0.01
	6.06	5.08	.98	.06	.08	−.02
	5.98	4.97	1.01	−.02	−.03	+.01
	6.01	5.01	1.00	+.01	+.01	.00
	6.00	4.93	1.07	.00	−.07	.07
4.00	5.00	3.97	1.03	0.00	−0.03	0.03
	5.00	3.98	1.02	.00	−.02	.02
	5.00	4.03	0.97	.00	+.03	−.03
	5.07	4.01	1.06	.07	.01	+.06
	5.03	4.17	0.86	.03	.17	−.14
3.00	3.97	2.96	1.01	−0.03	−0.04	+0.01
	4.01	3.05	0.96	+.01	+.05	−.04
	3.98	2.98	1.00	−.02	−.02	.00
	3.95	3.01	0.94	−.05	+.01	−.06
	4.08	3.05	1.03	+.08	.05	+.03
2.00	3.01	2.04	0.97	0.01	0.04	−0.03
	3.09	1.96	1.13	.09	−.04	+.13
	2.99	2.00	0.99	−.01	+.00	−.01
	3.01	2.00	1.01	+.01	.00	+.01
	3.01	2.03	.98	.01	.03	−.02
1.00	2.01	1.00	1.01	0.01	0.00	+0.01
	1.99	1.00	0.99	−.01	.00	−.01
	2.02	0.99	1.03	+.02	−.01	+.03
	2.01	1.00	1.01	.01	+.00	.01
	2.03	1.01	1.02	.03	.01	.02
0.00	0.98	0.01	0.97	−0.02	0.01	−0.03
	.98	−.01	.99	−.02	−.01	−.01
	.99	−.02	1.01	−.01	−.02	+.01
	1.00	+.00	1.00	+.00	+.00	.00
	1.00	−.01	1.01	.00	−.01	.01
−1.00	0.02	−0.99	1.01	0.02	0.01	0.01
	−.03	−1.01	0.98	−.03	−.01	−.02
	+.00	−0.97	.97	+.00	+.03	−.03
	−.04	−.99	.95	−.04	.01	−.05
	−.02	−1.00	.98	−.02	.00	−.02
−2.00	−0.94	−1.95	1.01	+0.06	0.05	0.01
	−.98	−2.00	1.02	.02	.00	.02
	−.97	−1.97	1.00	.03	.03	.00
	−1.00	−1.99	0.99	.00	.01	−.01
	−1.00	−1.98	.98	.00	.02	−.02
−3.00	−1.97	−3.01	1.04	0.03	−0.01	+0.04
	−1.97	−2.97	1.00	.03	+.03	.00
	−1.95	−2.96	1.01	.05	.04	.01
	−2.03	−3.01	0.98	−.03	−.01	−.02
	−2.02	−3.00	.98	−.02	+.00	−.02
−4.00	−2.98	−3.94	0.96	+0.02	0.06	−0.04
	−2.92	−4.01	1.09	.08	−.01	+.09
	−2.92	−4.00	1.08	.08	+.00	.08
	−2.96	−3.98	1.02	.04	.02	.03
	−3.00	−4.03	1.03	.00	−.03	.03
−6.00	−4.93	−5.96	1.03	0.07	+0.04	0.03
	−4.93	−5.93	1.00	.07	.07	.00
	−4.99	−5.95	0.96	.01	.05	−.04
	−5.02	−6.02	1.00	−.02	−.02	+.00
	−4.91	−5.97	1.06	+.09	+.03	.06
−10.00	−9.01	−10.01	1.00	−0.01	−0.01	0.00
	−9.07	−10.07	1.00	−.07	−.07	.00
	−8.98	−9.97	0.99	+.02	+.03	−.01
	−8.83	−9.87	1.04	.17	.13	+.04
−11.00	−9.96	−10.96	1.00	0.04	0.04	0.00
	−9.83	−10.87	1.04	.17	.13	.04
	−9.97	−11.02	1.05	.03	−.02	.05
−15.00	−13.88	−14.86	0.98	0.12	0.14	0.02
	−14.07	−15.00	.93	.00	−.07	.07
	−13.99	−14.97	.98	.01	.03	.02
−19.00	−17.78	−18.83	1.05	0.22	0.17	0.05
	−17.98	−18.92	0.94	.02	.08	−.02
	−17.87	−18.75	.88	.13	.25	−.12

TABLE 8. *Measured values of the axial meridional powers, V_O and H_O, for 71 spectacle lenses having a cylindrical power of +2.00 diopters and with spherical powers ranging from +7.00 to −20.00 diopters*

The nominal prescribed spherical powers are listed under "Sph." The values of the deviations from the nominal powers in the principal meridian are listed under the headings ΔV_O, ΔH_O, and $\Delta C_O = (\Delta V_O - \Delta H_O)$.

All values are expressed in diopters.

Sph.	V_O	H_O	C_O	ΔV_O	ΔH_O	ΔC_O
7.00	8.94	6.96	1.98	−0.06	−0.04	−0.02
	8.96	7.00	1.96	−.04	+.00	−.04
6.00	8.02	5.99	2.03	+0.02	−0.01	+0.03
	7.99	6.02	1.97	−.01	+.02	−.03
	8.05	6.05	2.00	+.05	.05	+.00
	8.11	6.01	2.10	.11	.01	.10
5.00	6.99	4.95	2.04	−0.01	−0.05	0.04
	7.03	5.13	1.90	+.03	+.13	−.10
	7.11	5.06	2.05	.11	.06	+.05
	6.99	5.01	1.98	−.01	.01	−.02
4.00	6.12	4.02	2.10	+0.12	0.02	0.10
	6.03	3.98	2.05	.03	−.02	.05
	6.01	4.03	1.98	.01	+.03	−.02
	6.05	4.06	1.99	.05	.06	−.01
3.00	5.10	3.16	1.94	0.10	0.16	−0.06
	4.98	2.95	2.03	−.02	−.05	+.03
	5.04	3.04	2.00	+.04	+.04	.00
	4.97	3.00	1.97	−.03	−.00	−.03
	5.00	3.04	1.96	.00	+.04	−.04
2.00	3.98	2.01	1.97	−0.02	0.01	−0.03
	4.04	2.00	2.04	+.04	.00	+.04
	4.00	2.04	1.96	.00	.04	−.04
	4.02	2.04	1.98	.02	.04	−.02
	4.03	2.01	2.02	.03	.01	+.02
1.00	3.02	1.03	1.99	0.02	0.03	−0.01
	3.04	1.02	2.02	.04	.02	+.02
	2.95	0.99	1.96	−.05	−.01	−.04
	3.01	.99	2.02	+.01	−.01	+.02
	3.06	1.02	2.04	.06	+.02	.04
0.00	2.05	0.02	2.03	0.05	0.02	0.03
	2.03	.01	2.02	.03	.01	.02
	2.01	−.03	2.04	.01	−.03	.04
	2.01	−.01	2.02	.01	−.01	.02
	2.01	−.01	2.02	.01	−.01	.02
−1.00	1.03	−1.00	2.03	0.03	0.00	0.03
	1.00	−0.99	1.99	.00	+.01	−.01
	0.96	−.99	1.95	−.04	.01	−.05
	1.02	−.97	1.99	+.02	.03	−.01
	1.00	−.98	1.98	.00	.02	−.02
−2.00	−0.03	−1.97	1.94	−0.03	0.03	−0.06
	−.03	−1.96	1.93	−.03	.04	−.07
	+.00	−1.99	1.99	.00	.01	−.01
	.02	−1.99	2.01	+.02	.01	+.01
	.03	−1.96	1.99	.03	.04	−.01
−3.00	−0.99	−2.98	1.99	0.01	0.02	−0.01
	−.97	−2.99	2.02	.03	.01	+.02
	−.95	−2.95	2.00	.05	.05	.00
	−.94	−2.93	1.99	.06	.07	−.01
	−1.01	−2.99	1.98	−.01	.01	−.02
−4.00	−1.97	−4.02	2.05	+0.03	−0.02	+0.05
	−1.95	−4.01	2.06	.05	−.01	.06
	−2.02	−3.99	1.97	−.02	+.01	−.03
	−2.00	−3.94	1.94	+.00	.06	−.06
	−1.98	−4.03	2.05	.02	−.03	+.05
−6.00	−3.92	−5.98	2.06	0.08	0.02	0.06
	−4.02	−6.07	2.05	−.02	−.07	.05
	−4.02	−6.03	2.01	−.02	−.03	.01
	−3.92	−5.93	2.01	+.08	+.07	.01
−10.00	−7.94	−10.09	2.15	0.06	−0.09	0.15
	−7.93	−9.93	2.00	.07	+.07	.00
	−7.87	−10.02	2.15	.13	−.02	.15
	−7.93	−9.93	2.00	.07	+.07	.00
−12.00	−9.97	−11.99	2.02	0.03	0.01	0.02
	−10.05	−11.99	1.94	−.05	.01	−.06
	−9.89	−11.96	2.07	+.11	.04	+.07
−16.00	−14.07	−16.05	1.98	−0.07	−0.05	−0.02
	−13.95	−15.95	2.00	+.05	+.05	+.00
	−13.86	−15.86	2.00	.14	.14	.00
−20.00	−17.82	−19.62	1.80	0.18	0.38	−0.20
	−17.87	−19.73	1.86	.13	.27	−.14
	−17.61	−19.46	1.85	.39	.54	−.15

Table 9. *Measured values of the axial meridional powers, V_O and H_O, for 43 spectacle lenses having a cylindrical power of +3.00 diopters and with spherical powers ranging from +6.00 to −10.00 diopters*

The nominal prescribed spherical powers are listed under "Sph". The values of the deviations from the nominal powers in the principal meridian are listed under the headings ΔV_O, ΔH_O, and $\Delta C_O=(\Delta V_O-\Delta H_O)$.

All values are expressed in diopters.

Sph.	V_O	H_O	C_O	ΔV_O	ΔH_O	ΔC_O
6.00	9.03	6.00	3.03	0.03	0.00	0.03
	9.01	6.02	2.99	.01	.02	−.01
	8.97	5.96	3.01	−.03	−.04	+.01
	9.00	5.94	3.06	+.00	−.06	.06
5.00	8.06	5.07	2.99	0.06	+0.07	−0.01
	8.05	4.99	3.06	.05	−.01	+.06
	8.04	5.16	2.88	.04	+.16	−.12
	8.06	4.97	3.09	.06	−.03	+.09
4.00	6.99	3.98	3.01	−0.01	−0.02	0.01
	7.20	4.09	3.11	+.20	+.09	.11
	7.09	4.02	3.07	.09	.02	.07
	7.00	4.06	2.94	.00	.06	−.06
3.00	6.12	3.08	3.04	0.12	0.08	+0.04
	6.02	2.98	3.04	.02	−.02	.04
	5.99	2.97	3.02	−.01	−.03	.02
	6.02	3.00	3.02	+.02	.00	.02
−1.00	1.99	−1.05	3.04	−0.01	−0.05	0.04
	2.04	−0.99	3.03	+.04	+.01	.03
	2.00	−.97	2.97	.00	.03	−.03
	2.04	−.92	2.96	.04	.08	−.04
	2.03	−.95	2.98	.03	.05	−.02
−2.00	1.05	−2.01	3.06	0.05	−0.01	+0.06
	0.99	−1.98	2.97	−.01	+.02	−.03
	1.00	−2.03	3.03	+.00	−.03	+.03
	0.97	−1.99	2.96	−.03	+.01	−.04
	1.10	−1.88	2.98	.10	.12	−.02
−3.00	−0.03	−3.00	2.97	−0.03	0.00	−0.03
	+.02	−2.96	2.98	+.02	.04	−.02
	−.02	−3.03	3.01	−.02	−.03	+.01
	+.08	−2.82	2.90	+.08	+.18	−.10
	.07	−2.91	2.98	.07	.09	−.02
−4.00	−0.97	−3.99	3.02	0.03	0.01	+0.02
	−1.04	−4.03	2.99	−.04	−.03	−.01
	−0.99	−3.98	2.99	+.01	+.02	−.01
	−.93	−3.92	2.99	.07	.08	−.01
−6.00	−3.06	−6.04	2.98	−0.06	−0.04	−0.02
	−2.98	−5.99	3.01	+.02	+.01	+.01
	−2.99	−5.98	2.99	.01	.02	−.01
	−3.04	−6.07	3.03	−.04	−.07	+.03
−10.00	−6.94	−9.97	3.03	+0.06	0.03	0.03
	−6.91	−10.05	3.14	.09	−.05	.14
	−6.94	−9.85	2.91	.06	+.15	−.09
	−6.85	−9.86	3.01	.15	.14	+.01

Table 11. *Measured values of the axial meridional powers, V_O and H_O, for eight spectacle lenses having a cylindrical power of +5.00 diopters and with spherical powers of −1.00 and −2.00 diopters*

All powers are expressed in diopters.

Sph.	V_O	H_O	C_O	ΔV_O	ΔH_O	ΔC_O
−1.00	3.98	−0.99	4.97	−0.02	0.01	−0.03
	3.99	−.88	4.87	−.01	.12	−.13
	3.97	−1.00	4.97	−.03	.00	−.03
	3.89	−1.02	4.91	−.11	−.02	−.09
−2.00	2.95	−1.98	4.93	−0.05	+0.02	−0.07
	3.02	−1.91	4.93	+.02	.09	−.07
	3.00	−2.00	5.00	.00	.00	+.00
	2.90	−2.08	4.98	−.10	−.08	−.02

Table 10. *Measured values of the axial meridional powers, V_O and H_O, for 33 spectacle lenses having a cylindrical power of +4.00 diopters and with spherical powers ranging from +5.00 to −10.00 diopters*

The nominal prescribed spherical powers are listed under "Sph". The values of the deviations from the nominal powers in the principal meridians are listed under the headings ΔV_O, ΔH_O, and $\Delta C_O=(\Delta V_O-\Delta H_O)$.

All values are expressed in diopters.

Sph.	V_O	H_O	C_O	ΔV_O	ΔH_O	ΔC_O
5.00	9.04	5.09	3.95	0.04	0.09	−0.05
	9.07	5.01	4.06	.07	.01	+.06
	9.01	4.98	4.03	.01	−.02	.03
	9.20	5.03	4.17	.20	+.03	.17
4.00	7.98	4.03	3.95	−0.02	0.03	−0.05
	7.99	3.97	4.02	−.01	−.03	+.02
	8.21	4.10	4.11	+.21	+.10	.11
	8.07	4.09	3.98	.07	.09	−.02
−1.00	3.05	−0.95	4.00	0.05	0.05	0.00
	3.04	−.97	4.01	.04	.03	+.01
	2.96	−1.00	3.96	−.04	.00	−.04
	3.00	−0.95	3.95	+.00	.05	−.05
−2.00	2.03	−1.94	3.97	0.03	0.06	−0.03
	2.05	−1.98	4.03	.05	.02	+.03
	1.93	−1.98	3.91	−.07	.02	−.09
	2.03	−1.99	4.02	+.03	.01	+.02
−3.00	1.06	−2.95	4.01	0.06	0.05	0.01
	0.97	−3.01	3.98	−.03	−.01	−.02
	1.09	−2.98	4.07	+.09	+.02	+.07
	1.14	−2.85	3.99	.14	.15	−.01
−4.00	0.09	−3.92	4.01	0.09	0.08	+0.01
	−.04	−4.00	3.96	−.04	.00	−.04
	.01	−3.97	3.98	+.01	.03	−.02
	−.04	−4.06	4.02	−.04	−.06	+.02
	.01	−3.96	3.97	+.01	+.04	−.03
−6.00	−2.00	−6.06	4.06	0.00	−0.06	+0.06
	−1.98	−5.98	4.00	.02	+.02	.00
	−1.84	−5.83	3.99	.16	.17	−.01
	−1.99	−6.01	4.02	.01	−.01	+.02
−10.00	−5.95	−10.00	4.05	0.05	0.00	0.05
	−5.94	−9.90	3.96	.06	+.10	−.04
	−5.97	−9.99	4.02	.03	.01	+.02
	−5.92	−9.90	3.98	.08	.10	−.02

Table 12. *Measured values of the axial meridional powers V_O and H_O, for 8 spectacle lenses having a cylindrical power of +6.00 diopters and with spherical powers of −1.00 and −2.00 diopters*

All powers are expressed in diopters.

Sph.	V_O	H_O	C_O	ΔV_O	ΔH_O	ΔC_O
−1.00	4.97	−0.98	5.95	−0.03	+0.02	−0.05
	5.12	−.91	6.03	+.12	.09	+.03
	5.02	−.85	5.87	.02	.15	−.13
	5.02	−1.00	6.02	.02	.00	+.02
−2.00	3.89	−2.03	5.92	−0.11	−0.03	−0.08
	4.03	−1.92	5.95	+.03	+.08	−.05
	4.00	−1.83	5.83	.00	.17	−.17
	4.06	−1.95	6.01	.06	.05	+.01

H_O. The deviations from the nominal values of V_O, H_O, and C_O (derived from the prescribed values of sphere and cylinder) are listed under the headings ΔV_O, ΔH_O, and ΔC_O.

Cursory examination of the measured values of V_O, H_O, and C_O listed in tables 6 through 12 shows the departure from the nominal values to be quite small.

6. Establishment of Tolerances

In the establishment of tolerances for axial power for spectacle lenses, two factors should be kept constantly in mind. First, the tolerances should be sufficiently large that lenses now made in accordance with recognized good practice should not show too high a percentage of rejection. Second, the tolerances should be as small as practicable to insure uniformity of performance. In other words, the tolerance should be such that if a user is supplied with the nominally correct lens or a lens at either extreme of the proper tolerance, he is not likely to perceive any difference in the quality of vision.

6.1. Proposed Tolerances for Axial Powers

Following a careful analysis of the magnitudes of the axial deviations ΔV_O, ΔH_O, and ΔC_O, a set of tolerances were established for use in a purchase specification [5]. The tolerances decided upon are specified as follows:

"The refractive power measured through the optical center shall agree with the prescribed power to within the following tolerances:

Powers 0 through____	6. 00	±0.06 diopter.
Powers 6.25 through_	12. 00	±1 percent of power.
Powers above________	12. 00	±0.12 dioper.

"Where cylindrical power is present, the above tolerances shall apply separately to each principal meridian of the lens and to the cylinder alone."

The total range of powers covered by this specification is +9.00 to −20.00 diopters. It is probable that it can be extended to cover the range +20.00 to −20.00 diopters.

6.2. Degree of Compliance With Proposed Tolerances

The results of measurement made on the 311 lenses that are contained in tables 6 to 12 have been considered with respect to these tolerances. For clarity, the results of these considerations are shown graphically in figures 2 to 6. In the graphs showing deviations in meridional power, the abscissas represent values of meridional power, V_O, while the ordinates give the magnitude of the deviation, ΔV_O or ΔH_O, for a given value of V_O. The solid lines bounding the horizontal zone in the central region of the graphs are the tolerance lines. Points that lie outside this zone indicate departures from the specified values in excess of tolerance. To avoid plotting a multiplicity of points, only those points are plotted where one or both of a pair fall outside tolerance. For example, if for a given lens ΔV_O is greater than the specified tolerance, the value of ΔV_O is plotted on the graph and in addition the value of ΔH_O is plotted, whether it exceeds the specified tolerance or not. In order to show the number of lenses complying, a bar graph is shown at the bottom of each frame. The total number of boxes at a given value of V_O corresponds to the total

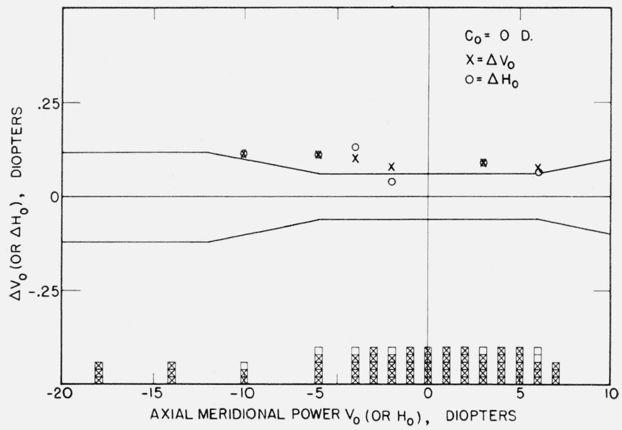

Figure 2. *Degree of compliance with suggested tolerances for axial power for 72 lenses having zero cylindrical power.*

Only those points are plotted for which at least one member of a pair falls outside the tolerances indicated by the solid lines. The bar graph at the bottom shows the number of lenses in the sample; boxes containing an X indicate number of lenses complying; and vacant boxes indicate number of lenses not complying in at least one meridian.

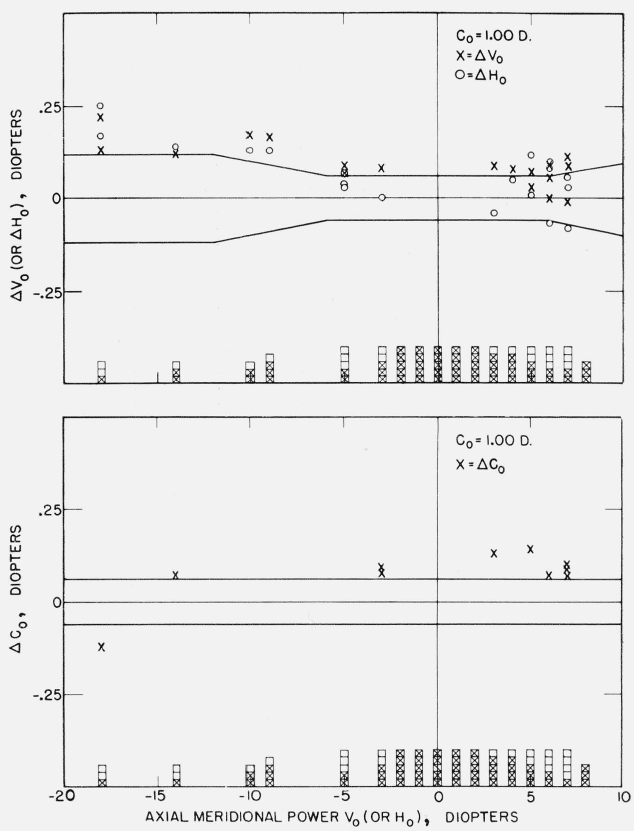

Figure 3. *Degree of compliance with suggested tolerances for axial power for 76 lenses having cylindrical power of 1 diopter.*

The upper frame shows degree of compliance for the meridional powers and the lower frame shows the degree of compliance for cylindrical power. Boxes containing an X in the bar graph in the lower frame indicate the number of lenses that comply fully with the requirements for both meridional and cylindrical power.

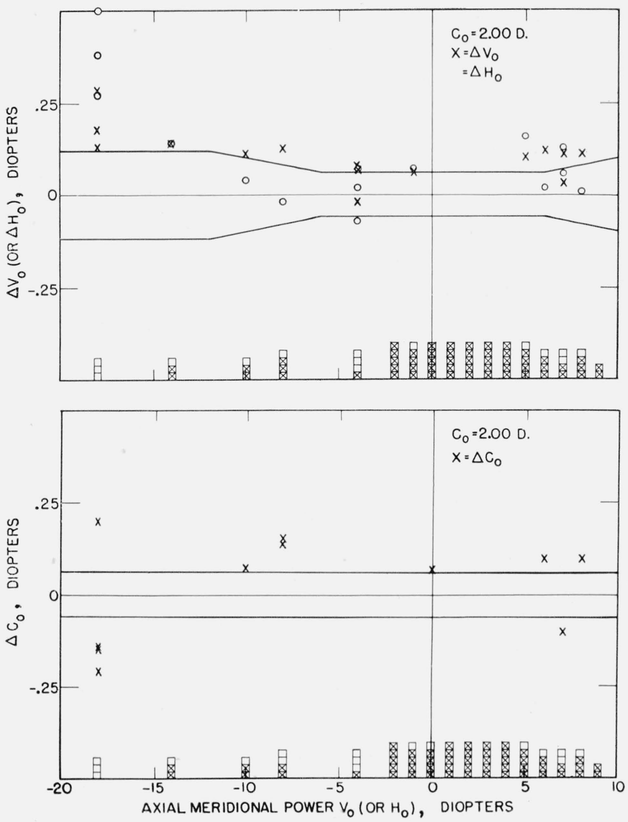

FIGURE 4. *Degree of compliance with suggested tolerances for axial power for 71 lenses having cylindrical power of 2 diopters.*

The upper frame shows degree of compliance for the meridional powers and the lower frame shows the degree of compliance for cylindrical power. Boxes containing an X in the bar graph in the lower frame indicate the number of lenses that comply fully with the requirements for both meridional and cylindrical power.

number of lenses in the sample of the given power. The number of boxes containing an X is the number of lenses of the given power that comply; the number of empty boxes give the number of lenses having values of ΔV_O or ΔH_O in excess of the proposed tolerance. For example in figure 2, which shows the degree of compliance for the principal meridional powers on axis for 72 lenses having zero cylindrical power, there are 5 boxes at $V_O=6.00$ diopters, indicating 5 lenses measured, 3 of which comply with the tolerances for V_O and H_O. None of this group of 72 lenses showed cylindrical power in excess of the tolerances, so no graph is given for ΔC_O.

The upper frames of figures 3 to 6 show similar results for lenses having cylindrical powers of 1.00, 2.00, 3.00, and 4.00 diopters. The lower frames show the magnitudes of the deviations in cylindrical power ΔC_O for the same lenses. Here, too, only those values are plotted that are in excess of the tolerance ± 0.06 diopter for cylindrical power which is shown by the tolerance lines on the graph. The bar graph at the bottom of the lower frames combine the results for both meridional and cylindrical power. A box containing an X in this graph indicates that the given lens complies in full with the tolerances for meridional and cylindrical power.

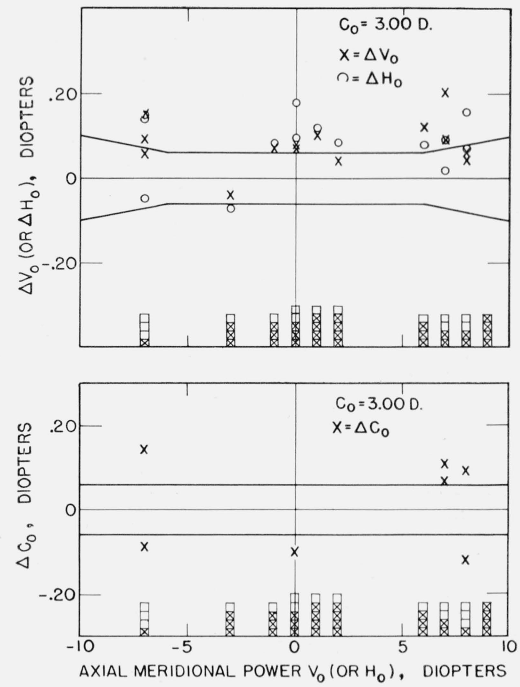

FIGURE 5. *Degree of compliance with suggested tolerances for axial power for 43 lenses having cylindrical power of 3 diopters.*

The upper frame shows degree of compliance for the meridional powers and the lower frame shows the degree of compliance for cylindrical power. Boxes containing an X in the bar graph in the lower frame indicate the number of lenses that comply fully with the requirements for both meridional and cylindrical power.

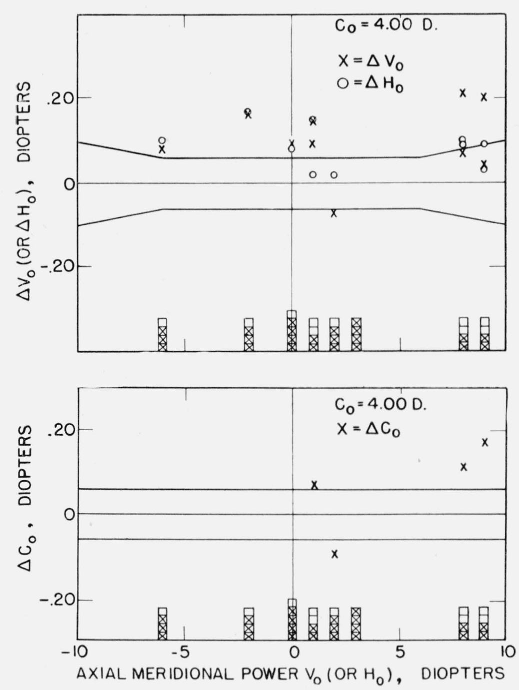

FIGURE 6. *Degree of compliance with suggested tolerances for axial power for 33 lenses having cylindrical power of 4 diopters.*

The upper frame shows degree of compliance for the meridional powers and the lower frame shows the degree of compliance for cylindrical power. Boxes containing an X in the bar graph in the lower frame indicate the number of lenses that comply fully with the requirements for both meridional and cylindrical power.

It is interesting to note in these figures, that there is only one instance, (fig. 4, $V_O = -18$ D), where no single lens of a given group complies in full with the tolerances. In all other cases, at least one, and generally two or more comply in full which indicates that these tolerances are being satisfied and can be satisfied.

The degree of compliance with the suggested tolerances is shown in tabular form for the 311 lenses in tables 13 to 18. In these tables, the total number of lenses of a given power in each sample is given, together with information on the number complying in each phase of the specification. At the bottom of each table, the percentage of lenses complying in V_O, H_O, and C_O separately is given and finally the percentage complying fully in V_O, H_O, and C_O.

TABLE 13. *Degree of compliance with suggested requirements for axial power for 72 lenses having meridional powers ranging from +7.00 to −18.00 diopters and zero cylindrical power*

Meridional power ($V_O=H_O$)	Number of lenses in sample	Number of lenses in sample complying on axis in—			
		V_O	H_O	C_O	V_O, H_O, and C_O
7.00	3	3	3	3	3
6.00	5	3	3	5	3
5.00	5	5	5	5	5
4.00	5	5	5	5	5
3.00	5	4	4	5	4
2.00	5	5	5	5	5
1.00	5	5	5	5	5
0.00	5	5	5	5	5
−1.00	5	5	5	5	5
−2.00	5	4	5	5	4
−3.00	5	5	5	5	5
−4.00	5	4	4	5	4
−6.00	5	4	4	5	4
−10.00	3	2	2	3	2
−14.00	3	3	3	3	3
−18.00	3	3	3	3	3
Total	72	65	66	72	65
Percentage	------	90	92	100	90

TABLE 14. *Degree of compliance with suggested requirements for axial power for 76 lenses having meridional powers ranging from +8.00 to −18.00 diopters and a cylindrical power of +1.00 diopter*

Meridional power ($V_O=H_O$+1.00)	Number of lenses in sample	Number of lenses in sample complying on axis in—			
		V_O	H_O	C_O	V_O, H_O, and C_O
8.00	3	3	3	3	3
7.00	5	3	4	2	1
6.00	5	4	2	4	2
5.00	5	4	4	4	3
4.00	5	4	5	5	4
3.00	5	4	5	4	4
2.00	5	5	5	5	5
1.00	5	5	5	5	5
0.00	5	5	5	5	5
−1.00	5	5	5	5	5
−2.00	5	5	5	5	5
−3.00	5	3	5	3	3
−5.00	5	2	4	5	2
−9.00	4	3	3	4	3
−10.00	3	2	2	3	2
−14.00	3	3	2	2	1
−18.00	3	1	1	2	1
Total	76	61	65	66	54
Percentage	------	80	86	87	71

TABLE 15. *Degree of compliance with suggested requirements for axial power for 71 lenses having meridional powers ranging from +9.00 to −18.00 diopters and a cylindrical power of +2.00 diopters*

Meridional power ($V_O=H_O$+2.00)	Number of lenses in sample	Number of lenses in sample complying on axis in—			
		V_O	H_O	C_O	V_O, H_O, and C_O
9.00	2	2	2	2	2
8.00	4	3	4	3	3
7.00	4	3	3	3	2
6.00	4	3	4	3	3
5.00	5	4	4	5	4
4.00	5	5	5	5	5
3.00	5	5	5	5	5
2.00	5	5	5	5	5
1.00	5	5	5	5	5
0.00	5	5	5	4	4
−1.00	5	5	4	5	4
−2.00	5	5	5	5	5
−4.00	4	2	2	4	1
−8.00	4	3	4	2	2
−10.00	3	2	3	2	2
−14.00	3	2	2	3	2
−18.00	3	0	0	0	0
Total	71	59	62	61	54
Percentage	------	83	87	85	76

TABLE 16. *Degree of compliance with suggested requirements for axial power for 43 lenses having meridional powers ranging from +9.00 to −7.00 diopters and a cylindrical power of +3.00 diopters*

Meridional power ($V_O=H_O$+3.00)	Number of lenses in sample	Number of lenses in sample complying on axis in—			
		V_O	H_O	C_O	V_O, H_O, and C_O
9.00	4	4	4	4	4
8.00	4	4	2	2	1
7.00	4	2	3	2	2
6.00	4	3	3	4	3
2.00	5	5	4	5	4
1.00	5	5	4	5	4
0.00	5	3	3	3	3
−1.00	4	3	3	4	3
−3.00	4	4	3	4	3
−7.00	4	2	2	2	1
Total	43	35	31	35	28
Percentage	------	81	72	81	65

TABLE 17. *Degree of compliance with suggested requirements for axial power for 33 lenses having meridional powers ranging from +9.00 to −6.00 diopters and a cylindrical power of +4.00 diopters*

Meridional power ($V_O=H_O$+4.00)	Number of lenses in sample	Number of lenses in sample complying on axis in—			
		V_O	H_O	C_O	V_O, H_O, and C_O
9.00	4	3	3	3	2
8.00	4	3	2	3	2
3.00	4	4	4	4	4
2.00	4	3	4	3	3
1.00	4	2	3	3	2
0.00	5	4	4	5	4
−2.00	4	3	3	4	3
−6.00	4	3	4	4	3
Total	33	25	27	29	23
Percentage	------	76	82	88	70

TABLE 18. *Degree of compliance with suggested requirements for axial power for 16 lenses having meridional powers −1.00 and −2.00 diopters, and cylindrical powers of +5.00 and +6.00 diopters*

The summary in the lower part of the table shows the degree of compliance for the entire group of 311 lenses.

Meridional power ($Vo=Ho+5.00$)	Number of lenses in sample	Number of lenses in sample complying on axis in—			
		Vo	*Ho*	*Co*	*Vo*, *Ho*, and *Co*
Diopters					
−1.00	4	3	3	2	2
−2.00	4	3	2	2	1
Total	8	6	5	4	3
Percentage	------	75	62	50	38
($Vo=Ho+6$):					
−1.00	4	3	2	3	2
−2.00	4	3	2	2	1
Total	8	6	4	5	3
Percentage	------	75	50	62	38
Summary of all lenses	311	257	260	272	230
Percentage	------	83	84	87	74

At first glance, the percentage of lenses complying seems so low that one might think the tolerances were too severe. There are, however, a number of factors to be considered that tend to show that the tolerances are eminently fair. For example, these are not results for lenses from a single source but from a variety of sources. In addition, these are not lenses specially made for this study but are representative of lenses that are in routine manufacture. Consequently, if only a single lens of a given sample of 3 to 5 lenses complies with the suggested tolerances, it signifies that a lens of that power can be and is being made that will satisfy the suggested tolerances. Moreover, from the graphs in figures 2 through 6, it is evident that only minor adjustments need be made in the manufacturing process to bring many of the lenses now outside of tolerance within the bounds of the suggested tolerances. In connection with this view, it is noteworthy that of all the 68 combinations used in this study, only one failed to have a single member of the sample comply. This is the sample, where $V_O=-18.00$ and $C_O=2.00$ diopters. It is probable that lenses of this combination can be made that will comply with the tolerances. Inasmuch as at least one lens in 67 out of 68 samples complied with suggested requirements, one can say that with present-day routine manufacturing techniques, there is potentially a 98-percent degree of compliance.

The degree of compliance ought also to be considered with respect to the expected percentage of use, which is shown in table 1. For lenses having zero cylindrical power, the expected percentage of use table indicates that 91 percent of these lenses will have powers ranging from +4.00 to −4.00 diopters. For lenses of these powers, table 13 shows that 42 out of 45, or 93 percent, comply. For lenses having cylindrical power 0.25 to 2.00, the expected percentage of use in conjunction with spherical power from +4.00 to −4.00 diopters is 83 percent. For this range, table 14 shows 41 out of 45, or 91 percent, complying and table 15 shows 40 out of 44, or 91 percent, complying. It is accordingly clear that in the ranges of powers commonly used, approximately 90 percent of the lenses as presently made should comply easily with these tolerances.

6.3. Depth of Focus and the Proposed Tolerances

In preparing a tolerance for axial power, it is proper to consider factors other than degree of compliance of existing lenses. Some thought should be given as to the probable effect on the vision of the user, if it is assumed that the lens has been correctly prescribed, properly positioned, and is affected only by disparities arising from deviations of the measured from the prescribed power. With this in mind, a brief study was made to determine magnitude of the variation in power of the viewing lens that would still permit a fixed eye to resolve discrete objects separated by 1 minute of arc in the object space. This figure of 1 minute of arc is the angular width of the lines in the letters of vision-testing charts, which are just legible to a person of normal (or 20/20) vision when viewed under standard conditions.

The computation of the magnitude of the permissible power variation was done in a manner closely paralleling that used in determining depth of focus for a lens, with the exception that the variation is expressed as change in dioptric power of the viewing lens rather than as displacement of the focal plane in the image space of the viewing lens. The final results are shown as curve 1 in figure 7. In the figure, both the positive and negative values are plotted so that the space separating the two branches is a measure of the total range of variation of dioptric power of the viewing lens for which the limit of resolution is 1 minute of arc in the object space. It is clear that range of variation or depth of focus is least for the high positive powers and increases steadily as one moves from the region of high positive power to the region of high negative power.

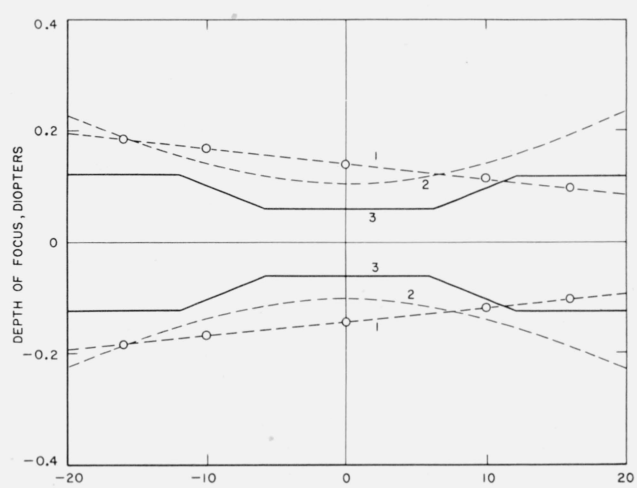

FIGURE 7. *Curves showing depth of focus in dioptric power variation of the viewing lens for a limit of resolution of 1 minute of arc in the object space.*

The zone bounded by the curves marked 1 shows the theoretical depth of focus; the zone bounded by the curves marked 2 shows the value found experimentally; and the zone bounded by the curves marked 3 shows the suggested tolerances.

The depth of focus was also determined experimentally by photographing test charts through the spectacle tester at a series of settings through the region of focus. The results of these measurements are shown in the curves marked 2 in figure 7. Although there is a pronounced difference between curves 1 and 2, it is not particularly alarming so far as purposes of the present discussion are concerned. It must be remembered that curve 1 shows the depth of focus for an ideal lens, whereas curve 2 shows the depth of focus for an actual lens that is not free from aberrations. It is probable that further study would locate the causes of the discrepancies. The prime thing to remember is that both of these approaches indicate the existence of upper limits on lens tolerances or depth of focus, if one wishes to maintain a given level of vision.

The suggested tolerances, discussed in section 6.1, are also shown as curve 3. It is clear that these tolerances are at all times within the range covered for curve 2, (the results of experiment) and extend beyond the range covered by curve 1, (the results of theoretical computation) in the case of high positive powers. It must be borne in mind that curves 1 and 2 delineate the extreme ranges for which 1-minute resolution is possible, and that it is probable a reasonable level of contrast for the distinguishing of close objects will be maintained over about 0.7 of the total range indicated by curves 1 and 2. It is, therefore, clear that the suggested tolerance range agrees closely with the range derived from both theoretical and experimental considerations. It is also evident that any marked increase in the suggested tolerances may result in reduction of the image quality for a user who happens to obtain a spectacle lens for which the variation of the actual from the prescribed power is near the limit of the extended tolerance range.

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7. References

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WASHINGTON, August 25, 1956.