Efficiency of 4π-Crystal-Scintillation Counting: 2. Dead-Time and Coincidence Corrections

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The dead-time and coincidence corrections to be applied in 4π-crystal-scintillation counting have been both approximately and rigorously derived. It has been shown that under special conditions the non-randomness due to "true" counts appearing in both channels requires additional correction terms.

1. Introduction

Two methods of using the 4π-crystal-sandwich-scintillation counter, initiated by one of us [1] [1] for the standardization of radioactive sources, have been described, namely, the "addition" and "coincidence" methods. The former may be used where the counts due to noise in the multiplier phototubes are low compared with the counts due to the radioactive source, whereas the latter method is applied where the "true" counts due to the source are low compared with the noise counts and it is desired to reduce the effect of these random noise counts.

In the addition method the dead-time correction is completely orthodox, it being necessary only to allow for the true counts lost due to the total dead time arising from a given number of counts in the one channel, each count having associated with it a gate or dead time τ_g.

2. Approximate Derivation

In the coincidence method, however, a non-random element is introduced that makes the calculation of the dead-time and coincidence losses somewhat more difficult. But even so, provided that the counting-rate losses due to both noise and source, or true, counts are low, an approximate approach to the problem may be used.

Let us therefore assume, in the first place, that in the coincidence method, the counting rates due to noise in the two channels (N'_n and N''_n per second) and that due to the radioactive source (N_t per second) are such that there is no appreciable interaction. If, further, N_{tc} is the number of true counts per second traversing each channel and being recorded as coincidences, while N(I) and N(II) are the total number of counts observed in each channel respectively, then

$$N(\text{I})=N'_n+N_{tc} \qquad (1)$$

and

$$N(\text{II})=N''_n+N_{tc}. \qquad (2)$$

If N_{tc} were extremely small, then the dead times in each channel for a gate dead time of τ_g would be $N'_n\tau_g$ and $N''_n\tau_g$, and the probabilities of detecting a true count in channels I and II would be, respectively,

$$P_{\text{I}}=(1-N'_n\tau_g) \qquad (3)$$

and

$$P_{\text{II}}=(1-N''_n\tau_g). \qquad (4)$$

As these noise counts are completely random, the probability of a true count being observed in both channel I *and* channel II, that is to say as a recorded coincidence, is given by the multiplication together of the probabilities given in eq (3) and (4). That is,

$$P^n_c=(1-N'_n\tau_g)(1-N''_n\tau_g)$$
$$=1-(N'_n+N''_n)\tau_g+N'_nN''_n\tau_g^2, \qquad (5)$$

the last term on the right-hand side representing the overcorrection on account of overlapping of noise counts in the two channels.

If, on the other hand, there were negligible noise counts but a number of true counts N_{tc} per second, then the dead-time correction would be represented by a probability P^t_c, where

$$P^t_c=1-N_{tc}\tau_g. \qquad (6)$$

When the quantities $N'_n\tau_g$ and $N''_n\tau_g$ are small, as in the conditions of the present experiments, it can be assumed that there is no appreciable interaction between noise and true counts, and that the total probability of observing a true count as a coincidence is given by the product of the probabilities of eq (3), (4), and (6),

$$P_c\sim 1-(N'_n+N''_n+N_{tc})\tau_g+N'_nN''_n\tau_g^2, \qquad (7)$$

neglecting $N_{tc}N'_n\tau_g^2$ and $N_{tc}N''_n\tau_g^2$, which are usually an order of magnitude less than $N'_nN''_n\tau_g^2$. $N'_nN''_n\tau_g{}^2$ itself involves a correction of at most 0.5 percent, even at high noise counting rates.

[1] Figures in brackets indicate the literature references at the end of this paper.

Substituting for N'_n and N''_n from eq (1) and (2), we have

$$P_c \sim 1-(N(\text{I})+N(\text{II})-N_{tc})\tau_g + N'_n N''_n \tau_g^2 \quad (8)$$

and

$$N_{tc} = N_t\{1-(N(\text{I})+N(\text{II})-N_{tc})\tau_g + N'_n N''_n \tau_g^2\}. \quad (9)$$

Equation (8) neglects an overcorrection of the form $(\tau_c/2)(N'_n+N''_n)$, where τ_c is the resolving time of the coincidence analyzer. However, as $\tau_c \ll \tau_g$ and we have already assumed that $N'_n\tau_g$ and $N''_n\tau_g$ are small, the approximation should still be a good one.

The coincidence count will also include, however, accidental coincidences arising from the random noise counts. These accidental coincidences are equal to $2\tau_c N'_n N''_n$ per second, where τ_c is the resolving time of the coincidence analyzer. Therefore, the observed coincidence-counting rate of N_c per second is related to the true-count coincidence rate by the expression,

$$N_c = N_{tc} + 2\tau_c N'_n N''_n. \quad (10)$$

From eq (9)

$$N_t = \frac{N_{tc}}{1-(N(\text{I})+N(\text{II})-N_{tc})\tau_g + N'_n N''_n \tau_g^2}. \quad (11)$$

N_{tc} is not directly observable because the observed coincidence-counting rate, N_c, is the sum of N_{tc} and the accidental rate due to noise. Substituting for N_{tc} from eq (10) into eq (11) we have that

$$N_t = \frac{N_c - 2\tau_c N'_n N''_n}{1-\{N(\text{I})+N(\text{II})-(N_c-2\tau_c N'_n N''_n\}\tau_g + N'_n N''_n \tau_g^2}. \quad (12)$$

In a blank experiment (*without* a source in the crystal-sandwich scintillator)

$$(N_c)_b = (N_{tc})_b + 2\tau_c (N'_n)_b (N''_n)_b. \quad (13)$$

In this expression, $(N_{tc})_b$ will arise from cosmic radiation, possible contamination of the crystal faces, and fluorescence in the glass of the multiplier phototubes due to potassium–40. However, in the individual channels, $(N_{tc})_b \ll N(\text{I})_b$, $N(\text{II})_b$, so that within a few tenths of a percent,

$$N'_n = (N'_n)_b = (N(\text{I}))_b - (N_c)_b + 2\tau_c(N(\text{I})_b N(\text{II})_b) \quad (14)$$

and

$$N''_n = (N''_n)_b = (N(\text{II}))_b - (N_c)_b + 2\tau_c(N(\text{I})_b N(\text{II})_b). \quad (15)$$

Substitution of these values for N'_n and N''_n in eq (12), and subtraction of $(N_{tc})_b$, as given by eq (13), from N_t, as calculated from eq (12), yields the true disintegration rate of the source to within a few tenths of a percent.

This approximate derivation of the dead-time losses is, however, based on certain premises that may not always be strictly applicable in practice. In the first place, if the noise counts are negligible, the dead-time losses would be given exactly by eq (6); but in practice the noise counts can never be neglected. If, on the other hand, the true counts are very low, the correction for dead-time losses would be given by eq (5); but then the method would be impractical, as long periods of counting would be required to give good statistics. The third possibility that has been assumed in the derivation, namely, that of reasonable numbers of noise and true counts but limited interaction between the two due to low counting rates, reflects the conditions of the present experiments.

It was felt that several of the assumptions made in the above approximate derivation are in a measure intuitive, and that the experimental results obtained would be placed on a more firm footing if a rigorous derivation were developed to establish the relative orders of magnitude of the various interactions. In addition, once one enters into the realm of very high counting rates with large counting-rate losses, there will be an interaction between noise and true counts, and it will be impossible to multiply the probabilities P_c^n and P_c^t to give the combined probability of observing a true count coincidence, P_c, of eq (7). Moreover, because a great many true counts in channel I are linked to true counts in channel II, it will not be permissible to derive the probability by multiplying together two individual probabilities for the separate channels as was done to derive P_c^n (eq (5)) from P_I and P_II (eq (3) and (4)), although the individual probabilities P_I and P_II do refer to processes in channel I and in channel II that are completely random.

3. Rigorous Derivation

In determining the dead-time loss and accidental-coincidence corrections that must be applied in the method of 4π-crystal-scintillation counting, it is necessary to apply a rigorous analysis of the events occurring in the two multiplier-phototube channels that lead to the coincidence analyzer. This is necessitated by the fact that a number of the events occurring in one channel are linked to those occurring in the other, and it may be therefore no longer valid to apply the kind of considerations that would be possible if all the events were completely randomly distributed.

Let us assume that N_t counts per second, the so-called true counts, arise in the composite crystal scintillator due to the radioactive source and the background. Then, of these counts, a number N_{tc} per second will be recorded by the coincidence analyzer. Further, let N'_n and N''_n be the number of noise counts per second passing along channel I and channel II, respectively, and let N_c be the total number of coincidences recorded per second by the coincidence analyzer.

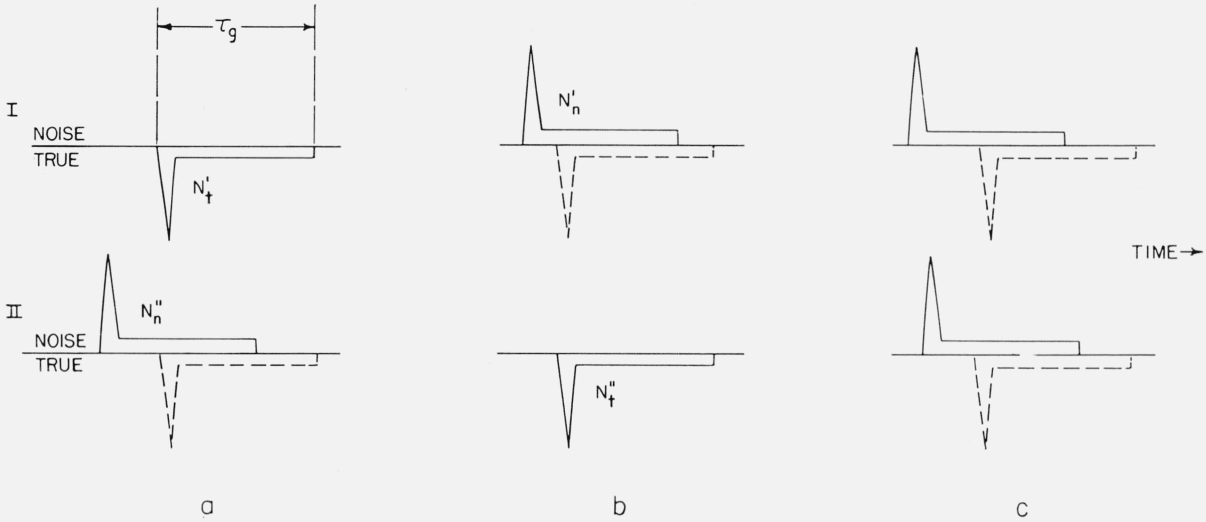

FIGURE 1. *Diagrammatic representation of different kinds of counting losses.*

It is, however, also necessary to consider, in each channel, what may be conveniently designated as "divorced true counts" which we will assume to be respectively N'_t and N''_t per second. These arise in the following manner. A true count, if it can pass through both channels unimpeded, arrives at the coincidence analyzer as a "perfect" coincidence with no delay whatsoever and, as an "N_{tc}" event, will be recorded. If, however, the pulse due to a true count happens to coincide with the gate of a preceding noise pulse in one channel, the true count in that channel will be lost and the pulse in the other channel due to the true count will proceed to the coincidence analyzer linked to the noise pulse that has "consumed" its partner. A divorced true count is thus always linked temporally to a noise count in the other channel, and such divorced true counts are therefore no longer randomly distributed with respect to the other counts.

The diagrams in figure 1 will help to illustrate the relationships of the various kinds of counts one to another. In these diagrams the pulses of noise counts are shown above the time axis of each channel, whereas the pulses due to true counts are shown below the axes, although in reality they are experimentally indistinguishable. Figure 1(a) illustrates the circumstances in which a divorced true count (N'_t per second) can arise in channel I, figure 1(b) the circumstances giving rise to a divorced true count (N''_t per second) in channel II, and figure 1(c) those in which the true counts can be lost in both channels I and II. In all cases the length of the pulse is chosen to signify the time τ_g in which the electronic gates in either channels I or II will be closed to the transmission of further pulses.

What then are the numbers of divorced true counts, N'_t and N''_t per second, in channels I and II, respectively?

These correspond to the numbers of overlaps between true-count pulses and *preceding* noise-count pulses in the *other* channel, for times greater than τ_c, the resolving time of the coincidence analyzer.

If a pair of true counts is impressed on the circuit within a time interval less than τ_c after a single noise pulse or a coincident pair of noise pulses, the coincidence analyzer will accept these as a true coincidence. The effective noise-pulse dead time for the creation of divorced counts in the other circuit is thus $\tau_g-\tau_c$.

In channel II, the noise counts N''_n per second give rise, therefore, to an effective total dead time of $N''_n(\tau_g-\tau_c)$ per second for the creation of divorced true counts (N'_t per second) in channel I. During this time N_t true counts will be submitted to channels I and II. Therefore, if there were no interaction between channels I and II,

$$\text{divorced true counts in channel I} = (\tau_g-\tau_c)N''_nN_t \text{ per second.} \quad (16)$$

Similarly,

$$\text{divorced true counts in channel II} = (\tau_g-\tau_c)N'_nN_t \text{ per second.} \quad (17)$$

These expressions as they stand give, however, somewhat higher values than N'_t and N''_t, because they also include cases of overlap, as in figure 1(c), in which both true-count pulses are rejected by two *preceding* noise counts.

The numbers of overlaps between a pair of true-counts and a preceding noise count in channels I and II are, respectively, $(\tau_g-\tau_c)N'_nN_t$ and $(\tau_g-\tau_c)N''_nN_t$ per second.

The time in which a second noise count in channel II (fig. 2) must occur to eliminate N''_t varies therefore from 0 (fig. 2(a)) to $(\tau_g-\tau_c)$ (fig. 2(b)), giving an average time per overlap of $\frac{1}{2}(\tau_g-\tau_c)$.

Thus, considering figure 2, the number of such divorced true counts passing along channel II is $(\tau_g-\tau_c)N'_nN_t$ per second (eq 17), and the average dead time for each of these events for the acceptance of a *preceding* noise count in channel II which will eliminate the "N''_t" pulse is $\frac{1}{2}(\tau_g-\tau_c)$. This corre-

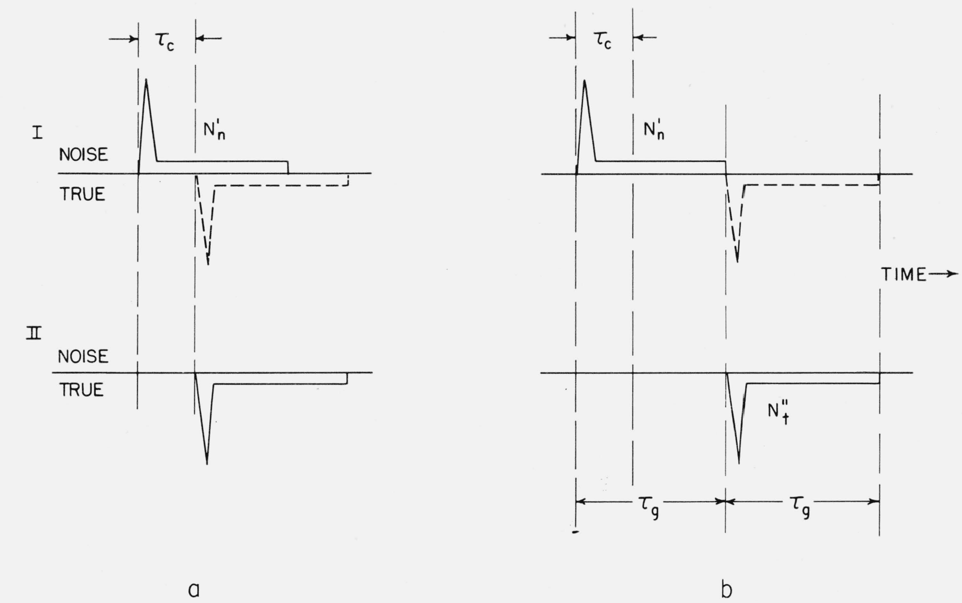

FIGURE 2. *Overlap between true-count pulse and preceding noise-count pulse.*

sponds for such events to a dead time per second of $\frac{1}{2}(\tau_g-\tau_c)^2N_n'N_t$. With N_n'' noise counts per second in channel II, the number of triple events causing the loss of both of a pair of true counts in channel I and channel II is $\frac{1}{2}(\tau_g-\tau_c)^2N_n'N_n''N_t$ per second. It is seen that the same result will be obtained by proceeding via "N_t''" events in channel I (eq 16) and the N_n' counts in channel I.

We thus derive the following corrected values for the divorced true counts in channels I and II:

$$N_t'=(\tau_g-\tau_c)N_tN_n''\{1-\tfrac{1}{2}(\tau_g-\tau_c)N_n'\}\text{ per second,} \tag{18}$$

and

$$N_t''=(\tau_g-\tau_c)N_tN_n'\{1-\tfrac{1}{2}(\tau_g-\tau_c)N_n''\}\text{ per second.} \tag{19}$$

It is now possible to proceed to the computation of the true counts lost as a result of the events that are summarized in figure 1. In the first place, every "N_t'" and "N_t''" event corresponds to a lost true-count pair. In addition, true counts can be lost in the dead times initiated in each channel by the divorced true counts, N_t' and N_t'' per second. The dead times involved in "N_t'" or "N_t''" events vary from τ_g to $\tau_g-\tau_c$, as illustrated for the "N_t''" events in figure 3.

The dead time is only that which *succeeds* an "N_t'" or "N_t''" pulse. In no case does the *preceding* noise count contribute to the dead time, for, from the very nature of the divorced true counts, an earlier true-count pair following a noise count merely creates an earlier divorced true count.

It is a curious fact that, as shown in figure 3(c), an intermediate time interval, varying from 0 to τ_c, becomes available for acceptance of true counts immediately following the noise-count dead time in channel I. This is effected when the channel-I pulse of the true-count pair registers as a coincidence with the "N_t''" pulse at the coincidence analyzer. The fraction of the "N_t'" or "N_t''" events in which this window becomes available is τ_c/τ_g, and its width is on the average equal to $\frac{1}{2}\tau_c$. Thus, the true counts that can be accepted in this window for "N_t'" and "N_t''" events are, respectively, $\frac{1}{2}\tau_c^2/\tau_gN_t'N_t$ and $\frac{1}{2}\tau_c^2/\tau_gN_t''N_t$ per second.[2]

It therefore follows that the total true counts lost in channels I and II due to the dead times succeeding "N_t'" and "N_t''" events are respectively $N_t'N_t(\tau_g-\frac{1}{2}\tau_c^2/\tau_g)$ and $N_t''N_t(\tau_g-\frac{1}{2}\tau_c^2/\tau_g)$ per second.

In addition to losses of true counts N_t', N_t'', $N_t'N_t(\tau_g-\frac{1}{2}\tau_c^2/\tau_g)$, and $N_t''N_t(\tau_g-\frac{1}{2}\tau_c^2/\tau_g)$, true counts can also be lost in the triple events illustrated by figure 1(c). These number $\frac{1}{2}(\tau_g-\tau_c)^2N_n'N_n''N_t$ per second. These triple events in which a pair of true counts is completely eliminated by *two* noise counts, one in each channel, are illustrated in greater detail in figure 4.

As can be seen from inspection of figure 4, the dead time due to the second noise count varies from τ_g (fig. 4(a)) to the average of τ_c and τ_g, or $\frac{1}{2}(\tau_g+\tau_c)$ (fig. 4(b)). This is equivalent to a total average dead time of $\frac{1}{4}(3\tau_g+\tau_c)$. The true counts lost in triple events such as that of figure 1(c) are therefore $\frac{1}{8}(\tau_g-\tau_c)^2\ (3\tau_g+\tau_c)N_n'N_n''N_t^2$ per second. Therefore, the total true counts lost per second as a result of

[2] In the actual experiment the resolving time of the amplifier was greater than τ_c, so that coincidences such as are indicated in figure 3(c) do not occur. Figure 3(c) is valid only for very sharp pulses whose duration is much less than τ_c.

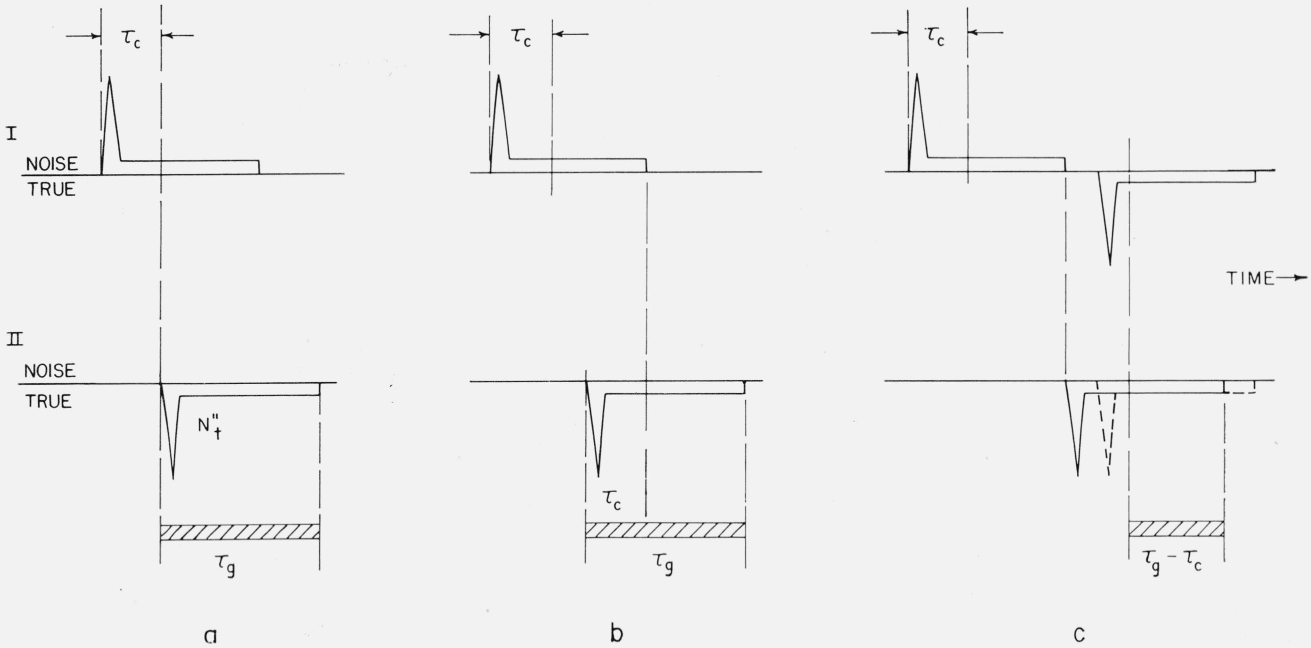

FIGURE 3. *Dead times of divorced true counts.*

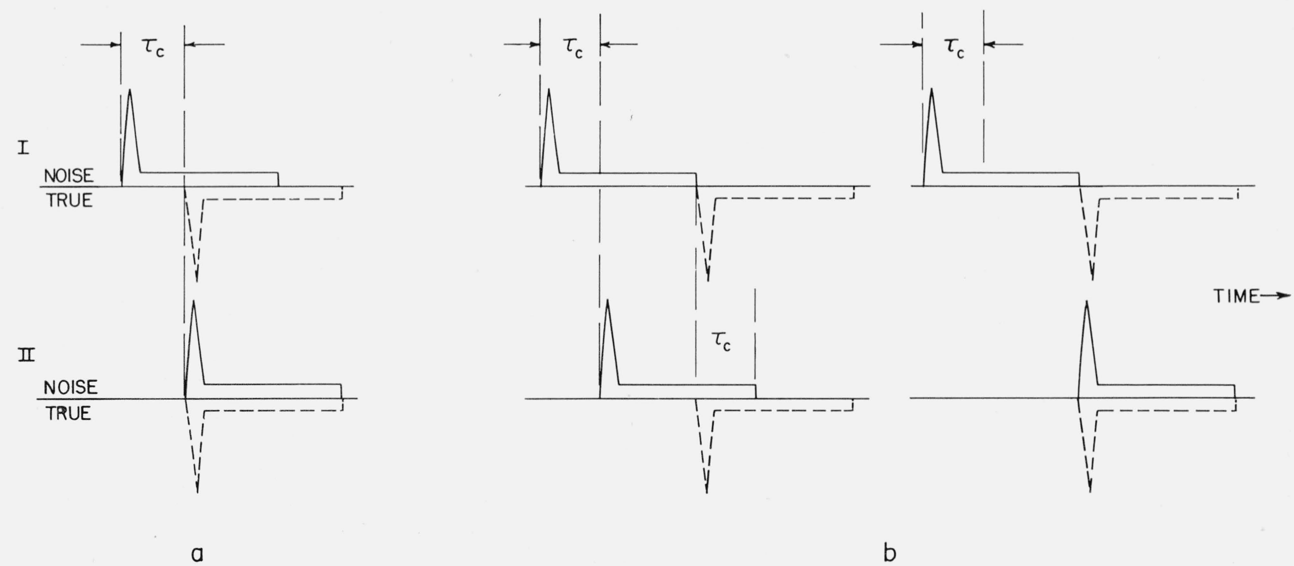

FIGURE 4. *Dead times of "triple events."*

losses in both channels I and II due to overlapping true-count and noise-count pulses are

$$\Sigma N_{t1}=N'_t+N''_t+N_t\left(\tau_g-\tfrac{1}{2}\frac{\tau_c^2}{\tau_g}\right)(N'_t+N''_t)$$

$$+\tfrac{1}{8}(\tau_g-\tau_c)^2(3\tau_g+\tau_c)N'_nN''_nN_t^2.$$

$$\therefore \Sigma N_{t1}=(N'_t+N''_t)\left\{1+N_t\left(\tau_g-\tfrac{1}{2}\frac{\tau_c^2}{\tau_g}\right)\right\}$$

$$+\tfrac{1}{8}(\tau_g-\tau_c)^2(3\tau_g+\tau_c)N'_nN''_nN_t^2. \quad (20)$$

$$\sum\frac{N_{t1}}{N_t}=(\tau_g-\tau_c)\left\{N'_n(1-\tfrac{1}{2}(\tau_g-\tau_c)N''_n)\right.$$

$$\left.+N''_n\{(1-\tfrac{1}{2}(\tau_g-\tau_c)N'_n)\right\}\left\{1+N_t\left(\tau_g-\tfrac{1}{2}\frac{\tau_c^2}{\tau_g}\right)\right\}$$

$$+\tfrac{1}{8}(\tau_g-\tau_c)^2(3\tau_g+\tau_c)N'_nN''_nN_t. \quad (21)$$

N_{tc} is the number of true-count coincidences per second that are registered by the coincidence analyzer and therefore pass through channels I and II unimpeded. Each of these counts imposes a gate

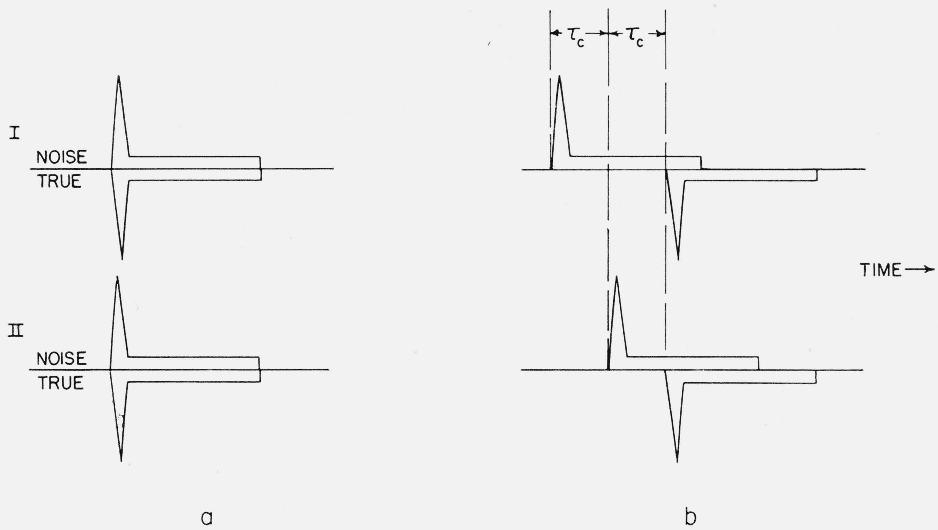

FIGURE 5. *Coincidences between accidental noise coincidences and true counts.*

dead time of τ_g, and gives rise therefore to $N_{tc}\tau_g N_t$ losses per second. Therefore, the total of all true counts lost from any cause whatsoever is equal to $N_{tc}N_t\tau_g+\sum N_{t1}$.

The true counts per second, N_t, are therefore given by

$$N_t=N_{tc}+N_{tc}N_t\tau_g+\sum N_{t1}$$

$$N_t=\frac{N_{tc}}{1-N_{tc}\tau_g-\frac{\sum N_{t1}}{N_t}}, \qquad (22)$$

N_{tc} being the number of true-count pairs that pass through channels I and II unimpeded. N_c is, however, the *total* number of coincidence counts recorded per second. Thus the difference between N_{tc} and N_c will be essentially due to accidental noise coincidences that are of the order $2\tau_c(N_n'-N_t'')(N_n''-N_t')$ per second. Thus,

$$N_{tc}\sim N_c-2\tau_c(N_n'-N_t'')(N_n''-N_t'). \qquad (23)$$

The reason for subtracting N_t'' from N_n' and N_t' from N_n'' in the accidental noise-coincidence term is that any noise count associated with an "N_t'" or "N_t''" event can *never* contribute to an accidental coincidence. This accidental coincidence correction in eq (23) is, however, an overcorrection in that true-count pulses can arrive during the time the accidental noise coincidences are being accepted by the coincidence analyzer. The time during which such true counts can be accepted as coincidences is seen by inspection of figure 5 to vary from 0 to $2\tau_c$, giving an average of τ_c.

The number of coincidences between accidental noise coincidences and true-count pulses is thus equal to $2\tau_c(N_n'-N_t'')(N_n''-N_t')\tau_cN_t$ per second. In other words, the accidental noise-coincidence term must be multiplied by a factor equal to $(1-\tau_cN_t)$ to allow for true counts coinciding with accidental noise coincidences. Thus, as a closer approximation,

$$N_{tc}\sim N_c-2\tau_c(N_n'-N_t'')(N_n''-N_t')(1-\tau_cN_t). \qquad (24)$$

There is, however, still one other possible source of coincidence counts that cannot be overlooked. If we consider figure 1(a) in detail it will be seen, as shown in figure 6, that a window also exists for a very small number of coincidences of "N_t'" and "N_t''" events with noise counts, just as a window was previously noted to exist (fig. 3(c)) for the acceptance of true counts.

The numbers of "N_t'" and "N_t''" events that can be available for such coincidences are equal respectively to $[\tau_c/(\tau_g-\tau_c)]N_t'$ and $[\tau_c/(\tau_g-\tau_c)]N_t''$ for $2\tau_c<\tau_g$ or N_t' and N_t'' for $2\tau_c>\tau_g$. The average time that the window is open for the acceptance of a noise count in coincidence is $\frac{1}{2}\tau_c$, provided that $2\tau_c<\tau_g$, or $\frac{1}{2}(3\tau_c-\tau_g)$ if $2\tau_c>\tau_g$. The numbers of such coincidences for "N_t'" and "N_t''" events, respectively, are therefore $\frac{1}{2}[\tau_c^2/(\tau_g-\tau_c)]N_t'N_n''$ and $\frac{1}{2}[\tau_c^2/(\tau_g-\tau_c)]N_t''N_n'$ per second as, in general, $2\tau_c<\tau_g$. As these coincidences can also be in coincidence with true-count pulses, they too represent an overcorrection and must be multiplied by the same correction factor $(1-\tau_cN_t)$ used to correct the accidental noise-coincidence counts of eq (24).[3]

We can therefore now write exactly that

$$N_{tc}=N_c-\left\{2\tau_c(N_n'-N_t'')(N_n''-N_t') +\frac{1}{2}\frac{\tau_c^2}{\tau_g-\tau_c}(N_t'N_n''+N_t''N_n')\right\}(1-\tau_cN_t). \qquad (25)$$

[3] Here again, however, because the resolving time of the amplifier was greater than τ_c, these coincidences could not, in practice, occur.

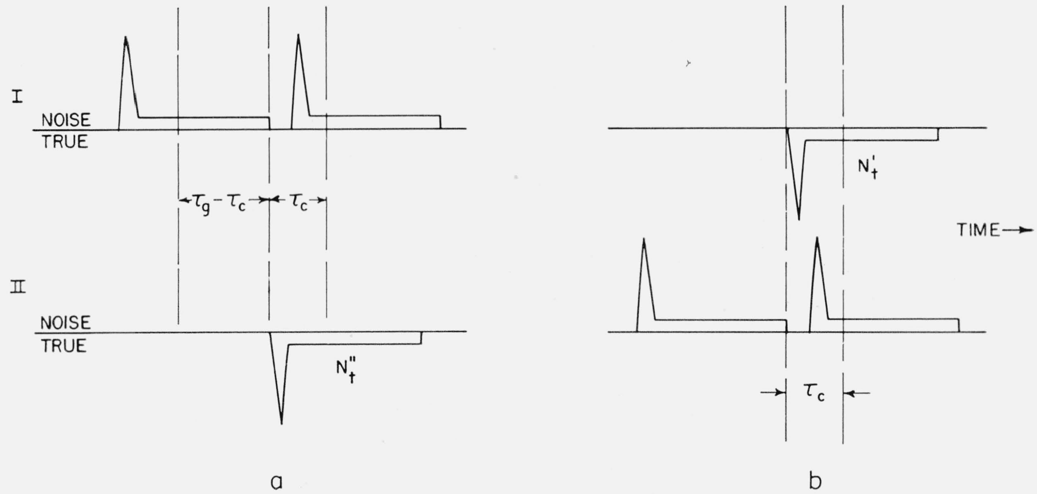

FIGURE 6. *"Window" for coincidences between noise counts and divorced true counts.*

The true counts per second, N_t, are now given by

$$N_t = \frac{N_{tc}}{1 - N_{tc}\tau_g - \dfrac{\Sigma N_{t1}}{N_t}}, \tag{22}$$

where N_{tc} is given by eq (25) and $\dfrac{\Sigma N_{t1}}{N_t}$ by eq (21).

The expression for N_t can therefore be written down exactly for those who desire to experience such an exercise or whose experimental conditions demand it. In the interest of economy of space, however, let us at this point make an approximation that is justified by the physical quantities involved under the experimental conditions [1], namely, to neglect all terms involving higher powers than τ_g^2 ($\tau_g \simeq 10$ μsec) and τ_c (about ½ μsec) in eq (21). Then, by substitution from eq (21) into eq (22),

$$N_t = \frac{N_{tc}}{1 - N_{tc}\tau_g - (\tau_g - \tau_c)\{N'_n + N''_n - (\tau_g - \tau_c)N'_n N''_n\}\{1 + N_t\tau_g\}}. \tag{26}$$

It is seen from eq (18) and (19) that substitution for N'_t and N''_t in eq (25) for N_{tc} will introduce terms involving $\tau_c(\tau_g - \tau_c)$ and $\tau_c(\tau_g - \tau_c)^2$, which can be neglected by comparison with $2\tau_c N'_n N''_n$. Equation (26) can therefore be further simplified to give

$$N_t = \frac{N_c - 2\tau_c N'_n N''_n}{1 - \tau_g(N_c - 2\tau_c N'_n N''_n) - (\tau_g - \tau_c)\{N'_n + N''_n - (\tau_g - \tau_c)N'_n N''_n\}}. \tag{27}$$

As a further simplification, because

$$N'_n + N''_n \simeq N(\mathrm{I}) + N(\mathrm{II}) - 2(N_c - 2\tau_c N'_n N''_n),$$

if all products of τ's containing τ_c are neglected,

$$N_t \simeq \frac{N_c - 2\tau_c N'_n N''_n}{1 - \tau_g N_c - (\tau_g - \tau_c)\{N(\mathrm{I}) + N(\mathrm{II}) - 2N_c\} + \tau_g^2 N'_n N''_n}. \tag{28}$$

In order to determine N'_n, and N''_n a blank experiment can be carried out with the crystal removed and with an opaque obstruction between the two multiplier photo tubes, to prevent phosphorescence in one photocathode causing a simultaneous signal in both. Then N_t is zero and the noise counts will be registered directly by the scalers in each of channels I and II. By inserting the crystal and carrying out experiments with and without a source, the values for N_t, for both the source plus background and background, respectively, can then be determined in terms of the number of coincidences and the known quantities τ_c and τ_g and the noise counts.

It is interesting to note from an inspection of eq (18) and (19) that if $\tau_g = \tau_c$, the numbers of divorced counts N'_t and N''_t become zero. For such a condition, ΣN_{t1} (from eq (21)) also becomes zero and eq (22) gives without any approximation

$$N_t = \frac{N_c - 2\tau_c N'_n N''_n (1 - \tau_c N_t)}{1 - N_c\tau_c + 2\tau_c N'_n N''_n (1 - \tau_c N_t)}. \tag{29}$$

In considering the case for $\tau_g = \tau_c$, it is also necessary to use the form of eq (25), for N_{tc}, where $2\tau_c > \tau_g$, namely,

$$N_{tc} = N_c - \left\{2\tau_c\,(N'_n - N''_n)(N''_n - N'_t) + \frac{1}{2}(3\tau_c - \tau_g)(N'_t N''_n + N''_t N'_n)\right\}\left(1 - \tau_c N_t\right), \tag{30}$$

in which, however, the second term in the first bracket becomes zero when $\tau_g = \tau_c$ by virtue of both N'_t and N''_t becoming zero. However, this is never experimentally feasible, for (a) if τ_c is *increased* to equal τ_g, the accidental rate will be too high, and (b) if τ_g is *decreased* to equal τ_c, the non-randomness tends to disappear and the rigorous derivation becomes unnecessary.

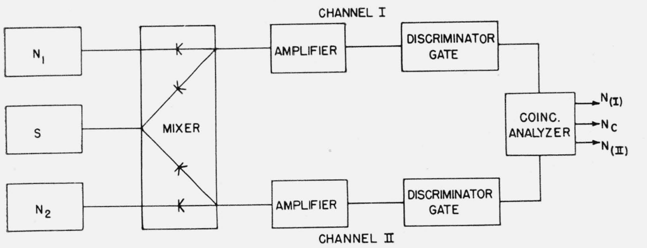

FIGURE 7. *Experimental arrangement for testing validity of rigorous and approximate derivations of the dead-time and coincidence losses.*

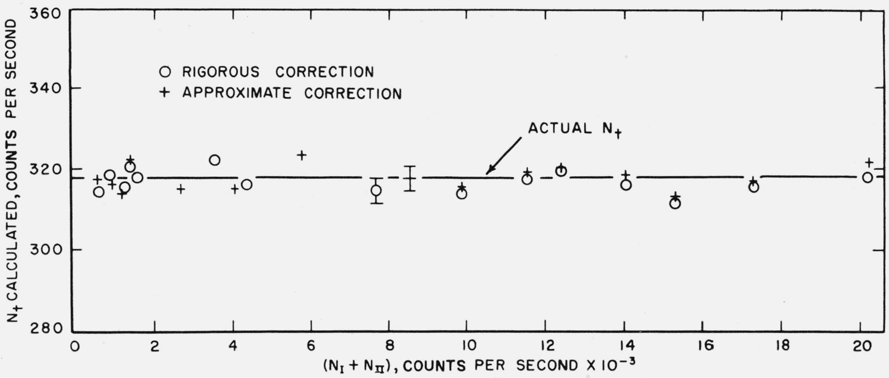

FIGURE 8. *"True" counts after correction by both the rigorous and approximate methods.*
Limits of error are indicated on only two points.

4. Comparison of Rigorous and Approximate Derivations

It is of interest to compare experimentally the validity of the corrections required by the approximate derivation, eq (12), and the rigorous derivation, eq (28). The arrangement shown schematically in figure 7 was designed to test directly both derivations over the range of phototube noise encountered in the experiments.

A sandwich source of Tl^{204}, covered by a hemispherical aluminum reflector, was counted by phototube S. These counts, *and the noise counts* from phototube S, were fed into channels I and II *simultaneously*, thus providing a source of "true" pulses. Separate phototubes, N_1 and N_2, were then used as independent sources of random noise, and pulses from N_1 and N_2 were fed at the same time into channels I and II, respectively. The counting rate from S was maintained constant.

The voltages across the dynodes of N_1 and N_2 were varied stepwise so as to increase the random noise from 0 to 11,000 counts per second, covering the most extreme noise measurements encountered in the actual experiments. The noise pulses from N_1 and the "true" pulses from S passed through channel I, and similarly the noise pulses from N_2 and the same "true" pulses from S passed through channel II. N(I), N(II), and N_c, were then analogous to the values obtained in the actual experiment, except in this case N_t was known exactly (by counting S alone) and N'_n and N''_n were known exactly (by counting N_1 and N_2 separately). In figure 8 are shown the values of N_t (approximate) and N_t (rigorous) obtained by means of eq (12) and (28), respectively, as functions of $N(I)+N(II)$, the sum of the single-channel counting rates. As can be seen, both methods of correction are applicable over the range of phototube noise encountered in the experiment.

The foregoing derivations will be applicable to any system where coincidence between two counters is used to select particular events from a high-background environment.

We gratefully acknowledge the assistance of Carter C. Smith in carrying out the experimental comparison of the rigorous and approximate derivations.

5. Reference

[1] C. C. Smith, H. H. Seliger, and J. Steyn, J. Research NBS **57**, 251 (1956) RP2716.

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