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Investigation of an Alternating-Current Bridge for the Measurement of Core Losses in Ferromagnetic Materials at High Flux Densities

Irvin L. Cooter and William P. Harris

The use of bridge methods for measuring core losses in ferromagnetic materials has generally been restricted to measurements at low flux density. However, accurate values can be obtained at higher flux densities if they are corrected by the application of a term derived from the harmonic components of the exciting current. This correction term is obtained by considering the ferromagnetic material to absorb energy from current and voltage at the fundamental frequency and to return energy to the circuit at harmonic frequencies.

1. Introduction

A knowledge of the total core loss and its components is of vital importance to design engineers who use ferromagnetic materials that are subject to changes in direction or magnitude of magnetic flux. Generally this information is obtained from measurements made on strip samples in an Epstein frame, using a wattmeter. In many cases, however, there are certain advantages, such as increased sensitivity and greater frequency range, in making these loss measurements by a bridge method. For measurements at low inductions (flux densities) and at high frequencies, the bridge methods have been used successfully for several years [1, 2]. [1] However, at inductions above 10 to 12 kilogausses in nonoriented silicon steels or 16 to 18 kilogausses in oriented silicon steels, the distortion in the wave form of the exciting current is considerable, and it has been found that the results obtained with the bridge methods hitherto used differ from those obtained with the wattmeter. This difference increases rapidly with increase in induction, the bridge method always indicating larger power losses.

The discrepancies between results obtained at high flux density by the two methods have been attributed to harmonic components in the exciting current, which result from the nonlinear element (iron-cored coil) in the circuit. Several attempts [3, 4, 5, 6, 7] have been made to resolve the differences and to obtain a method for correcting the bridge results in order to determine the actual power loss in the iron. These investigations have been only partially successful. The present paper describes a method for obtaining a "harmonic" correction term for the bridge that results in excellent agreement in core-loss measurements between the bridge and wattmeter even when the flux density is high.

[1] Figures in brackets indicate the literature references at the end of this paper.

1.1. Wattmeter Method

The generally accepted way of measuring total core loss in sheet material at power frequencies is by the Epstein method as specified by the American Society for Testing Materials [1]. The 25-cm Epstein test requires flat-rolled material cut into strips 3 cm wide and at least 28 cm long. The strips are assembled in the four arms of the Epstein test frame, with double-lap joints at the corners. The diagram of connections for the core-loss test is shown in figure 1.

The current coil of a wattmeter, W, is connected in series with the primary winding of the test frame. The voltage coil is connected to the secondary winding of the test frame. The average-indicating voltmeter is used (1) for determining the maximum induction in the specimen in accordance with Camilli's method [8], and (2) in conjunction with the root-mean-square-indicating voltmeter for calculating the form factor of the secondary voltage.

The wattmeter method is used principally for measurements at flux densities above 1,000 gausses. The chief limitations of the method are its lack of sensitivity for small specimens, and the errors of the wattmeter at the higher frequencies.

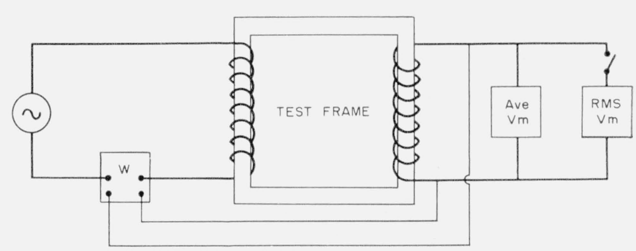

FIGURE 1. *Connections for core-loss test, using wattmeter.*

1.2. Bridge Method

Several different types of bridges [1, 2] have been used for measuring core loss. For the present investigation, any bridge capable of measuring the impedance of an iron-cored coil could have been used. However, for convenience, attention was confined to the Maxwell-Wien circuit illustrated in figure 2, in which

R_a, R_c=product arms of the bridge,
R_{dt}, L_d=measured values of resistance and inductance of the d (unknown) arm,
R_b, C_b=balancing resistance and capacitance,
R_s=resistance of the source,
R_L=resistance of the leads, contacts, etc.,
D=a detector, tuned to the fundamental frequency.

The quantities R_{dt} and L_d are determined from the balance equations

$$\text{(a)} \quad R_{dt}=\frac{R_aR_c}{R_b} \quad \text{and} \quad \text{(b)} \quad L_d=R_aR_cC_b. \quad (1)$$

In order to use this bridge circuit for the measurement of power loss, the primary of an Epstein test frame is connected in the bridge, as shown in figure 3.

The maximum flux density, $B_{\max}$, is calculated from the equation

$$E_{\text{avg}}=4fB_{\max}NA\times10^{-5},$$

where

E_{avg}=average absolute value of the secondary voltage,
f=frequency in cycles per second,
$B_{\max}$=maximum flux density in kilogausses,
N=number of turns in the secondary winding of the Epstein frame (700),
A=cross-sectional area of the ferromagnetic specimen in square centimeters.

The test frame may be represented by a series combination of inductance and resistance, as shown in the equivalent circuit diagram, figure 4, where

R_w=ohmic resistance of the primary winding of the frame,
R_d=increase (relative to an air-cored coil) in the resistance of the primary winding,[2]
L_d=measured inductance of the primary winding.

The bridge arms are selected so that

$$R_a \text{ and } R_b \gg R_c+(R_d+R_w).$$

The current, I', in R_a and R_b is therefore negligible as compared to the current, I, in R_c and R_d. The exciting current, I, depends upon the values of R_w, R_d, L_d, R_c, R_L, R_s, and E_s. If R_{dt} is the total resistance of the test frame, then

$$R_d=R_{dt}-R_w=\frac{R_aR_c}{R_b}-R_w. \quad (2)$$

[2] A more complete consideration of the nature of R_d is given in section 2.

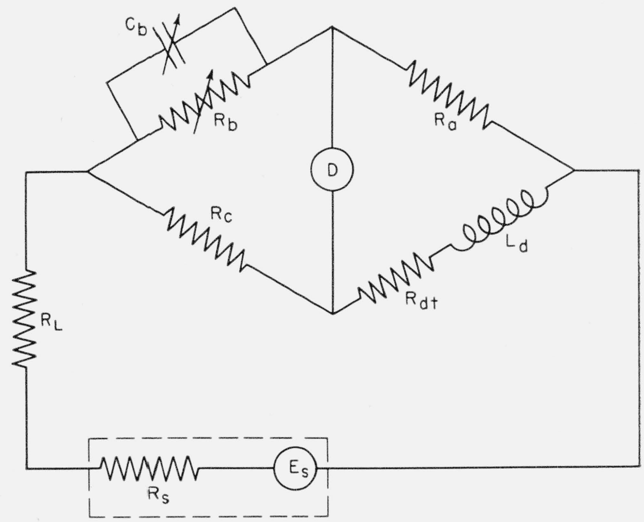

FIGURE 2. *Maxwell-Wien bridge circuit.*

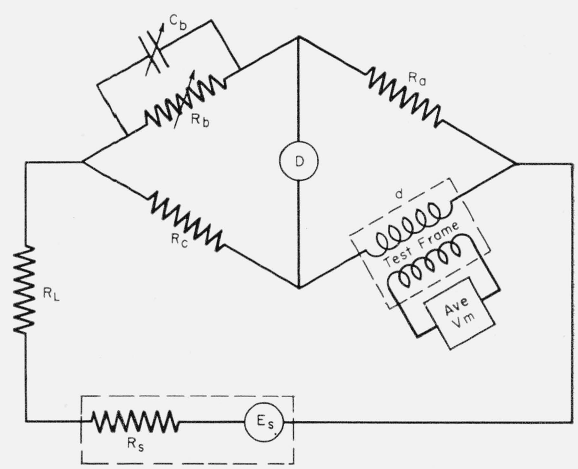

FIGURE 3. *Maxwell-Wien bridge circuit for measuring core loss, using Epstein test frame.*

Balance at fundamental frequency is obtained by adjusting R_b and C_b so that the tuned detector indicates a null. If the induction in the ferromagnetic specimen is low, and if the output voltage of the power source has a negligible amount of distortion, the power dissipated in the core is I^2R_d. However, if there are harmonics present in the power source, or if the induction is sufficiently high in the ferromagnetic material, the total core loss in the material may differ considerably from that calculated from the increase in apparent resistance, R_d, and the current, I.

2. Theory of the Harmonic Correction

In order to derive the relationships involved in a bridge circuit containing nonlinear elements, let us again consider figures 3 and 4. The nonlinear impedance of the Epstein test frame causes the

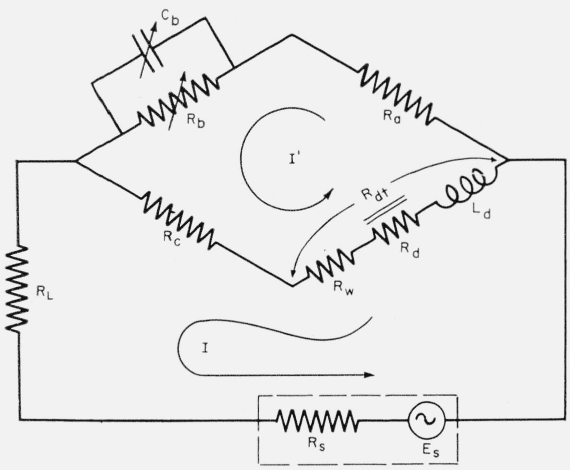

FIGURE 4. *Equivalent-circuit diagram of the circuit shown in figure 3.*

exciting current to be nonsinusoidal. This causes the voltage, V_d, across bridge arm d to have a nonsinusoidal wave form. If the bridge is balanced at fundamental frequency, we have

$$\frac{V_{d1}}{V_{c1}}=\frac{R_a(1+j\omega R_b C_b)}{R_b}, \tag{3}$$

where V_{d1} and V_{c1} are the fundamental components of the voltage drops across arms d and c. As $V_c=I_cR_c=I_dR_c$, the impedance of arm d at fundamental frequency is

$$Z_{d1}=\frac{V_{d1}}{I_{d1}}=\frac{R_aR_c}{R_b}+j\omega R_aR_cC_b. \tag{4}$$

It is evident from eq (4) that arm d is equivalent to a series combination of resistance and inductance, such as R_{dt} and L_d in figure 4.

The power associated with arm d is the time average of the product of the voltage across d and the current in d, or, since the contributions of the various harmonics may be separated, we can write

$$P_{dt}=\int_t V_dI_ddt=\int_t V_{d1}I_{d1}dt+\int_t V_{d2}I_{d2}dt+\ldots,$$

$$P_{dt}=P_{dt\ 1}+\sum_{h>1} P_{dt\ h}, \tag{5}$$

where

$P_{dt\ 1}$=power associated with the fundamental frequency,
$P_{dt\ h}$=power associated with each higher harmonic, denoted by h.

From eq (4) we find

$$P_{dt\ 1}=I_{d\ 1}^2\left(\frac{R_aR_c}{R_b}\right)=I_{d\ 1}^2(R_d+R_W).$$

Now, by the principle of the conservation of energy,

$$\left.\begin{aligned}P_{s1}&=P_{dt\ 1}+P_{e1}\\P_{s2}&=P_{dt\ 2}+P_{e2}\\\ldots&=\ldots+\ldots\\P_{sh}&=P_{dt\ h}+P_{eh}\end{aligned}\right\}, \tag{6}$$

where

P_{sh}=active power supplied by the source at the harmonic frequency h,
$P_{dt\ h}$=power associated with the arm d at the harmonic frequency h,
P_{eh}=power dissipated in resistance R_e at the harmonic frequency h, where $R_e=R_c+R_L+R_s$.

As the voltage, E_s, of the source has a pure sinusoidal wave form,

$$\left.\begin{aligned}P_{s1}&=P_{dt\ 1}+P_{e1}\\0&=P_{dt\ h}+P_{eh}\text{ for }h>1\end{aligned}\right\}. \tag{7}$$

Therefore, for any harmonic greater than 1,

$$\left.\begin{aligned}P_{dt\ h}&=-P_{eh}=-I_{c\ h}^2R_e\\P_{dt\ h}&=-I_{d\ h}^2R_e\end{aligned}\right\}. \tag{8}$$

From eq (8) it is concluded that arm d is *delivering* rather than *absorbing* harmonic power.

The total power dissipated in the iron is

$$P_d=P_{dt}-P_{wh}.$$

Using eq (5), we may write

$$P_d=P_{dt\ 1}+\sum_{h>1}P_{dt\ h}-R_wI_{d1}^2-R_w\sum_{h>1}I_{dh}^2.$$

From eq (8),

$$P_{dt\ h}=-I_{dh}^2R_e,$$

and as

$$P_{dt\ 1}=I_{d1}^2\ R_d+I_{d1}^2R_w,$$

$$P_d=I_{d1}^2R_d-R_e\sum_{h>1}I_{dh}^2-R_w\sum_{h>1}I_{dh}^2.$$

Or, because

$$R_p=R_w+R_c+R_L+R_s,$$

$$P_d=I_{d1}^2R_d-R_p\sum_{h>1}I_{dh}^2. \tag{9}$$

As the current in R_a and R_b is negligible compared to the current in arms $_c$ and $_d$,

$$I_{d1}=I_1,$$

where I_1 is the fundamental component of the exciting current in the primary loop. Equation (9) may then be written

$$P_d=I_1^2R_d-R_p\sum_{h>1}I_h^2. \tag{10}$$

This equation has been derived on the assumption that the resistance R_p (i. e., R_w, R_c, R_L, and R_s) is independent of current and frequency. At power frequencies this assumption introduces no significant errors with respect to R_w, R_c, and R_L. However, the resistance of the power source should be examined. This was done for the equipment used for this investigation, and it was found that changes of R_s due to changes of current and frequency were small enough to be neglected.

In words, eq (10) means that the power dissipated in the iron core is the difference between the apparent power delivered to it by the source at fundamental-frequency current, and the power dissipated in the resistive elements of the circuit due to harmonic-frequency currents. It was noted earlier that the term $I_1^2R_d$ is usually taken to represent P_d, the power loss in the iron core. The second member of the right-hand side of eq (10) may be viewed as a correction term. This correction term is small in cases where the harmonic currents are small, but it will be shown later that at high flux densities, the correction term may be several times as large as the true value of P_d. In other words, the result would be in error by several hundred percent if the correction was neglected. This applies to any method of determining core loss in terms of the in-phase component of impedance at fundamental frequency, including bridges and other alternating-current potentiometers.

Consideration of eq (10) brings out a fact that has not been universally recognized. That is, the value of the core loss, P_d, depends on the values of R_L and R_s, resistive elements that *are not included in any one of the four arms of the "bridge."* Specifically, the resistance of the source enters into the determination of P_d, as does the resistance of the leads, etc., used to connect the source to the bridge.

For the case of a coil containing ferromagnetic material undergoing symmetrically cyclic magnetization, even-order harmonics do not appear, and eq (10) may be simplified to

$$P_d=I_1^2R_d-\sum_{h=3}^{k}I_h^2(R_w+R_c+R_L+R_s), \qquad (11)$$

where h can have only odd integral values. That is, $h=2n+1$, where n is a positive integer.

A concept due to Peterson [9] may also be used in the analysis of circuits containing nonlinear elements.

In Peterson's method, the equivalent circuit for the nonlinear element (iron-cored coil) would consist of a resistance, an inductance, and a series of "harmonic generators." The analysis on the basis of this concept would lead to the same power equation, eq (11). The harmonic correction term would be interpreted as the power returned to the circuit by the harmonic generators.

The above concepts are also valid when applied to nonlinear resistances and impedances, as well as to coils containing ferromagnetic materials. In general, the equivalent circuit for nonlinear elements may be represented as consisting of energy-*dissipating* elements plus energy-*delivering* elements.

3. Experimental Procedure

3.1. Simultaneous Measurements

The calibrated wattmeter is very satisfactory and accurate at power frequencies, and it is believed to measure the true core loss, even with a distorted current.[3] The bridge circuit used in this investigation was designed to include a wattmeter as a reference instrument. This wattmeter was of the electrodynamic type, and had been calibrated at several power factors and for a frequency range of 60 to 300 cps. The bridge circuit is shown in figure 5. The current coil of the wattmeter is connected

[3] There is some doubt concerning the accuracy of the wattmeter under conditions of extreme distortion. There is evidence that the core loss indicated by the wattmeter is too small. Losses ound in actual experience are somewhat higher [10].

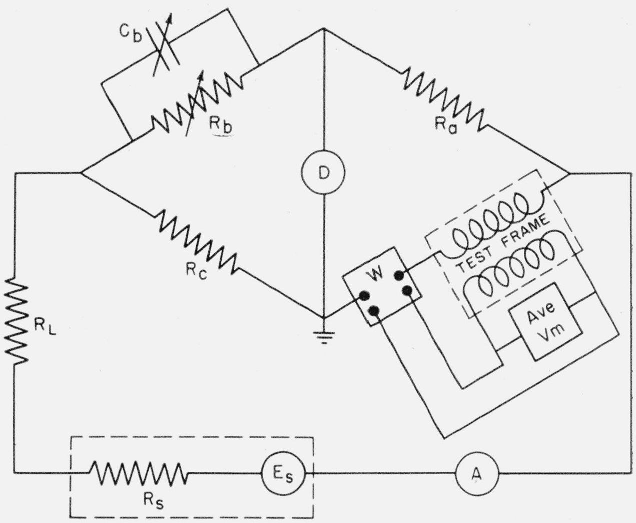

FIGURE 5. *Circuit for measuring core loss with bridge and wattmeter simultaneously.*

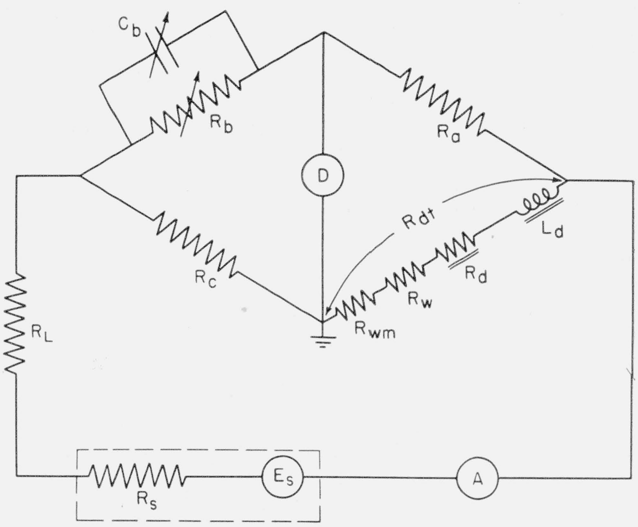

FIGURE 6. *Equivalent-circuit diagram o f the circuit shown in figure 5.*

in series with the primary winding of the test frame; the potential coil is connected across the secondary winding, in parallel with the average-indicating voltmeter.

This circuit has several distinct advantages for the study of bridge measurements. Measurements can be made on a specimen by using the wattmeter and bridge simultaneously, with exactly the same induction in the specimen, and with the same exciting current having identical wave form. Furthermore, the instrument losses [4] and form factor are the same for both methods, and for purposes of comparison, it is not necessary to correct for them. Thus, this circuit permits a direct comparison of the two methods under identical conditions.

The equivalent circuit for this arrangement is shown in figure 6, in which

R_{wm} = resistance of the current coil of the wattmeter,

R_w = d-c resistance of the primary winding of the frame,

R_d = measured increase in the resistance of the test frame winding, resulting from the power losses,

L_d = measured inductance of the bridge arm containing the test frame.

3.2. Equipment

Power was supplied to the circuit by a source consisting of a 200-w amplifier driven by a low-distortion oscillator. This source had a voltage wave form with a total distortion of less than 0.2 percent when delivering a current of 2 amp to a resistive load. With the source connected to the bridge circuit, the apparent voltage distortion did not exceed 1.3 percent, even though the *current* distortion was more than 50 percent in some cases. The increase in apparent distortion is attributed to the I_hR_s drop.

The measurements reported in this paper were made at a frequency of 68.8 cps. It was desired to avoid the power frequency of 60 cps and its harmonics, in order to eliminate interference, pickup from stray magnetic fields, etc.

A 25-cm Epstein frame, built according to the specifications of the American Society for Testing Materials [1], was used in the measurement of core loss. The primary and secondary windings each had 700 turns.

A mutual inductor was used to compensate for air flux, in the manner described in [1]. This inductor is not shown in any of the figures. The resistance of its winding is included in R_w. As a result of the use of this inductor, all values of induction are the intrinsic values, denoted by the symbol B_i.

A high-quality wave analyzer was used as the bridge-balance detector. For this purpose, it was tuned to the fundamental frequency. Then, with bridge balance established, the wave analyzer was connected across R_c, and successive measurements were made of the fundamental and harmonic voltage drops. To attain the accuracy required for these measurements, special methods of calibration and correction were used.

The bridge components, R_a, R_b, and C_b, were high-quality decade boxes, with accurately known values and negligible residual reactances.

R_c was a manganin standard resistor, capable of carrying the large currents used, with accurately known resistance, negligible residual inductance and capacitance, and a temperature coefficient of resistance of less than 10 ppm/° C.

[4] Power is dissipated in the potential coil of the wattmeter, and in the average-indicating voltmeter. This power is included in the quantity measured by both the wattmeter and the bridge.

3.3. Specimens

Each specimen weighed approximately 500 g and consisted of strips 3 cm wide and 28 cm long. The five types of material listed in table 1 were used.

TABLE 1. *Identification of specimens*

AISI designation	Grade	Approximate Si content
		%
M-15	Transformer	3.5 to 5.0
M-19	do	3.5 to 5.0
M-22	Dynamo	2.0 to 3.5
M-36	Electrical	0.5 to 2.0
M-43	Armature	0.5 to 2.0

3.4. Measurements

In order to determine core loss by this method, it is necessary to measure the quantities appearing in eq (11), modified to include R_{wm}.

$$P_d = I_1^2 R_d - \sum_{h=3}^{k} I_h^2 (R_w + R_{wm} + R_c + R_L + R_s) \quad (12)$$

I_1 is determined from the drop across R_c at fundamental frequency, measured with the wave analyzer.

R_d is given by

$$R_d = R_{dt} - (R_w + R_{wm}),$$

a relation that is evident in figure 6. R_{dt} is given by the bridge-balance equation. $(R_w + R_{wm})$ was measured carefully under various conditions of current and temperature, and appropriate values were used in the equation above.

The value of the term $\sum_{h=3}^{k} I_h^2$ was determined in two ways. In one method, each harmonic current, I_h, was determined from the corresponding voltage drop across resistor R_c, measured with the wave analyzer tuned to the appropriate frequency. The magnitude of each harmonic current was squared, and the sum taken. In the alternative method, the rms value of the current was measured by a calibrated rms-indicating ammeter connected in the supply leads to the bridge, and the fundamental-frequency current, I_1, was determined from the I_1R_c drop, as in the first method. As the square of the rms current equals the sum of the squares of the component currents,

$$\sum_{h=3}^{k} I_h^2 = I_{rms}^2 - I_1^2.$$

R_p is the sum of R_w, R_{wm}, R_c, R_L, and R_s. The combined resistance of R_w, R_{wm}, R_c, and R_L was measured and used in the same manner as $R_w + R_{wm}$.

R_s is more difficult to measure with accuracy. However, as the output voltage of the power source has a negligible amount of distortion, the impedance of the source, Z_s, may be determined by measuring the harmonic voltage drop across the terminals, and also the harmonic current. If the inductive reactance, ωL_s, of the source is sufficiently small, the ratio R_s/Z_s is approximately unity, and the value of R_s can be determined from the more easily measured quantity, Z_s. It was found that changes of Z_s with changes of frequency were small for the source used. Hence it was concluded that L_s was small.

As an independent check on these measurements, however, a method was developed that measures both resistance and inductance of the source while it is under operating conditions. For the power source used, the value of R_s obtained by this method was the same, within experimental error, as the value of Z_s obtained with the voltmeter-ammeter method. Also, the value obtained for L_s was very small, which is consistent with previous findings.

4. Results

An example of the measurements and calculations required for the comparison of core losses as obtained by the bridge and wattmeter methods is given in table 2.

Table 3 gives the magnitudes of the power terms for the individual harmonics for three grades of materials. It can be seen that in some cases, harmonics as high as the 13th produce a power term of measurable magnitude. However, as flux density decreases, the harmonic power terms decrease also, and become negligible at some intermediate flux density. At and below this flux density, the bridge will give accurate results without applying any correction for harmonic power losses.

Table 4 is a condensation of the results of measurements on five grades of material. A graphical presentation is given in figure 7.

It is evident from an inspection of the curves and the final column of table 4, that the two methods are in excellent agreement.

TABLE 2. *Example of measurements* [a]

Material: Silicon steel, type M15. Effective weight of sample: 0.903 lb. Induction: 15.0 kilogausses. Frequency: 68.8 cps.

Resistance:	Current	Current²
Ohms	*amp*	*amp²*
$R_a = 59000$	$I_1 = 1.699$	2.887
$R_b = 42060$	$I_3 = 0.983$	0.966
$R_c = 2.515$	$I_5 = 0.301$	0.091
$R_{dt} = 3.528$	$I_7 = 0.110$	0.012
$(R_w + R_{wm}) = 1.298$	$I_9 = 0.082$	0.007
$R_d = 2.230$	$I_{11} = 0.029$	0.001
$(R_w + R_{wm}) = 1.298$	$I_{13} = 0.027$	0.001
$R_c = 2.515$	$I_{rms} = 1.992$	3.968
$R_s = 0.295$		$(I_{rms}^2 - I_1^2) = 1.081$
$R_L = 0.184$		$\sum_{h=1}^{13} I_h^2 = 3.968$
$R_p = 4.292$		
		$\sum_{h=3}^{13} I_h^2 = 1.078$

B_{max}	$I_1^2 R_d$	$\sum_{h=3}^{11} I_h^2 R_p$	$(I_{rms}^2 - I_1^2) R_p$	Core loss	
				Bridge	Wattmeter
Kilogausses	*w*	*w*	*w*	*w/lb*	*w/lb*
15.0	6.438	4.627	---------	2.006	1.990
15.0	6.438	---------	4.640	1.991	1.990

[a] No corrections were made for instrument losses or waveform errors. These are identical for the two methods and do not affect the validity of the comparison.

TABLE 3. *Relative magnitudes of individual harmonic power terms*

AISI designation	B_i	$I_1^2 R_d$	$I_3^2 R_p$	$I_5^2 R_p$	$I_7^2 R_p$	$I_9^2 R_p$	$I_{11}^2 R_p$	$I_{13}^2 R_p$	$\Sigma I_h^2 R_p$
	Kilogausses	*w*	*w*	*w*	*w*	*w*	*w*	*w*	*w*
M-15	14.99	6.438	4.146	0.391	0.0519	0.0288	0.0031	0.0030	4.627
	14.19	2.878	1.143	.239	.0163	.0038	.0027	--------	1.405
	13.11	1.276	0.1054	.0284	.0066	.0010	--------	--------	0.141
	11.94	0.922	.0135	.0021	.0004	--------	--------	--------	.016
M-22	15.76	6.142	3.548	.3090	.0347	.0153	.0022	.0013	3.911
	14.82	2.995	0.896	.1191	.0094	.0007	--------	--------	1.026
	13.35	1.738	.1199	.0125	.0014	--------	--------	--------	0.134
	12.06	1.344	.0365	.0029	--------	--------	--------	--------	.039
	10.85	1.086	.0153	.0010	--------	--------	--------	--------	.016
M-43	16.41	10.254	3.480	.387	.0315	.0137	.0016	.0006	3.912
	15.42	6.125	0.796	.157	.0082	.0005	--------	--------	0.960
	14.15	4.251	.1320	.0378	.0040	--------	--------	--------	.174
	12.85	3.242	.0261	.0086	--------	--------	--------	--------	.036

TABLE 4. *Results obtained by the wattmeter and bridge for five grades of material* [a]

AISI designation	Gage	Effective weight	B_i	$I_1^2R_d$	$\Sigma I_h^2R_p$	Power loss		Δ
						Bridge	Wattmeter	
		lb	*Kilogausses*	*w*	*w*	*w/lb*	*w/lb*	%
M-15	29	0.903	14.99	6.438	4.627	2.005	1.990	0.8
			14.19	2.878	1.405	1.631	1.614	1.1
			13.11	1.276	0.141	1.256	1.262	−0.5
			11.94	0.922	.016	1.003	0.996	.7
			10.79	.725	.005	0.797	.797	.0
			9.64	.587	.002	.648	.642	.9
M-19	29	0.744	15.32	5.366	3.677	2.272	2.278	−.3
			14.36	2.478	0.994	1.996	1.966	1.5
			13.09	1.342	.163	1.586	1.560	1.7
			12.80	1.220	.117	1.484	1.480	0.3
			11.58	0.906	.033	1.174	1.157	1.5
			10.09	.659	.012	0.870	0.861	1.0
			8.69	.484	.004	.646	.646	0.0
M-22	26	0.836	15.76	6.142	3.911	2.668	2.674	−.2
			14.82	2.995	1.026	2.355	2.330	1.1
			13.35	1.738	0.134	1.918	1.914	0.2
			12.06	1.344	.039	1.561	1.555	.4
			10.85	1.086	.016	1.280	1.268	.9
			8.62	0.707	.004	0.841	0.837	.5
M-36	26	0.877	15.74	8.022	3.913	4.684	4.720	−.8
			14.59	4.153	1.089	3.493	3.511	−.5
			13.88	3.027	0.418	2.974	2.964	.3
			13.03	2.323	.123	2.508	2.508	.0
			11.88	1.835	.032	2.055	2.052	.1
			10.64	1.506	.012	1.703	1.687	.9
			9.46	1.197	.005	1.359	1.345	1.0
			8.46	0.968	.003	1.100	1.094	0.5
M-43	24	0.895	16.41	10.254	3.912	7.084	7.020	.9
			15.42	6.125	0.960	5.769	5.725	.8
			14.15	4.251	.174	4.554	4.468	1.9
			12.85	3.242	.036	3.581	3.574	0.2
			11.52	2.537	.009	2.824	2.815	.3
			10.29	2.007	.004	2.237	2.234	.1

[a] These data are not corrected for instrument loss or form factor.

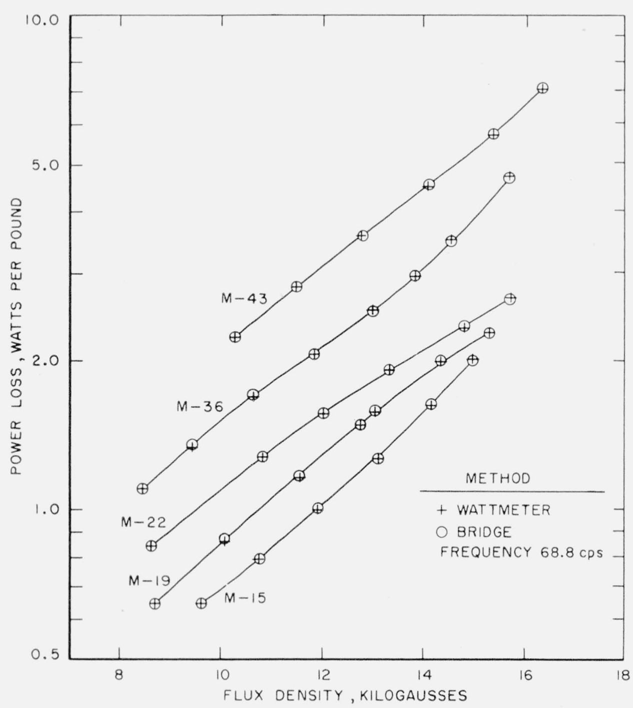

FIGURE 7. *Comparison of results obtained by the bridge and wattmeter for five grades of materials.*

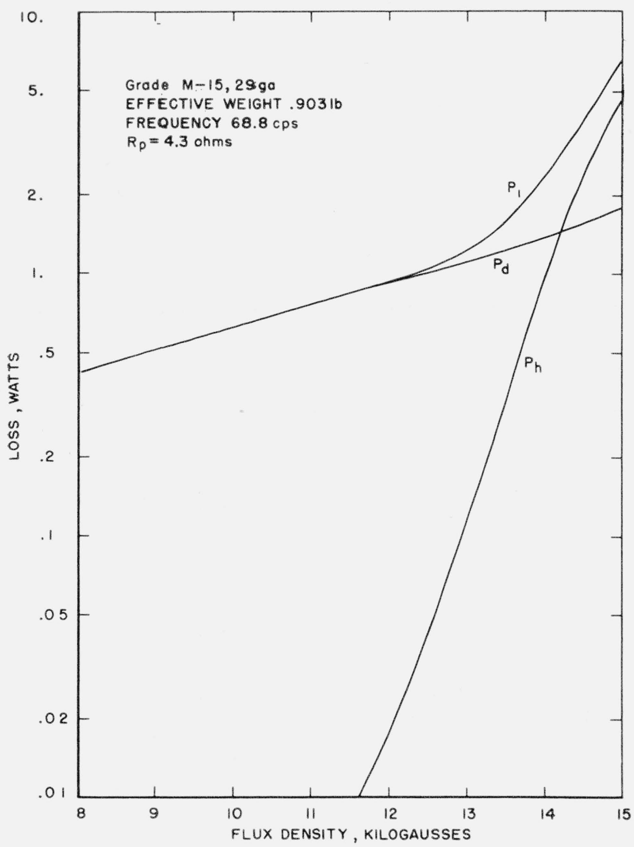

FIGURE 8. *P_1, P_h, and P_d versus flux density for one specimen.*

5. Discussion

Figure 8 shows graphically the relative magnitudes of the terms of the power equation, eq (12), for one typical set of conditions. If we let

$$P_1 = I_1^2 R_d,$$

the power supplied to the iron specimen at fundamental frequency, and

$$P_h = \sum_{h=3}^{k} I_h^2 R_p,$$

the power dissipated in the resistive elements of the circuit at higher harmonic frequencies, then eq (12) becomes

$$P_d = P_1 - P_h.$$

P_h may be considered to be a correction term, to be applied to the measured loss, P_1, in order to obtain the core-loss value, P_d. The bridge method, as generally used, yields the value P_1.

It can be seen that the magnitude of P_h increases very rapidly as high inductions are reached, becoming equal in magnitude to the core loss, P_d, at about 14.2 kilogausses, in this example.

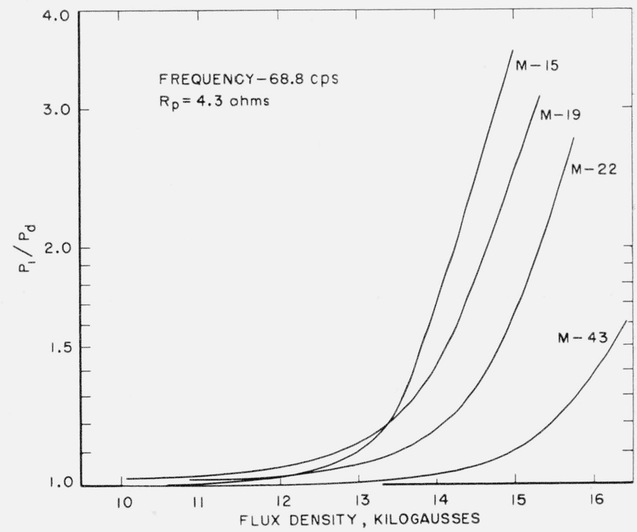

FIGURE 9. *The ratio P_1/P_d versus flux density for four grades of materials.*

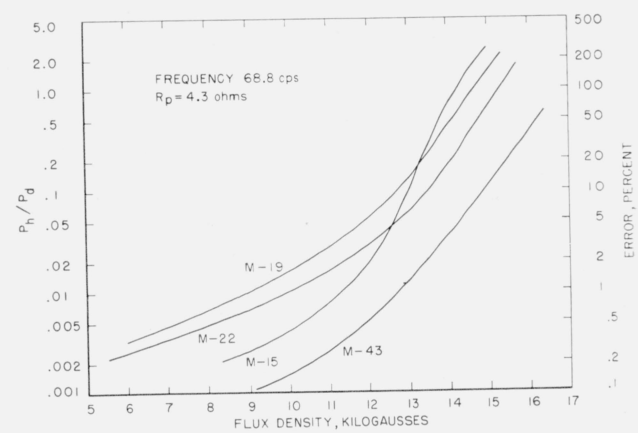

FIGURE 10. *The ratio P_h/P_d versus flux density for four grades of materials.*

In figure 9, the ratio P_1/P_d is shown as a function of flux density for four grades of material. Note that the general shape of the curves is the same, although the flux density at which the ratio reaches a given value varies with the material. In general, the lower the silicon content, the higher the flux density that can be reached before the ratio P_1/P_d exceeds any given value.

As P_1 becomes large with respect to P_d, errors in the measurement of P_1 are multiplied in the determination of P_d. For example, an error of 2 percent in the value of P_1 when the ratio P_1/P_d is 3.0 will yield an error of 6 percent in the value of P_d, assuming that there is no error in the measurement of P_h.

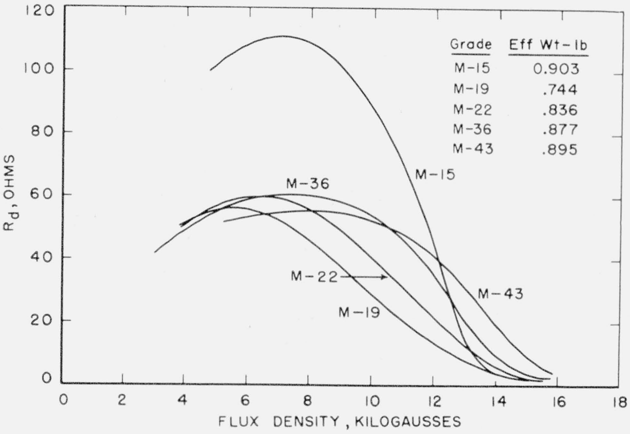

FIGURE 11. *R_d versus flux density for five grades of materials.*

Of course, as P_1/P_d increases, P_h is also increasing, and the need for accuracy in the value for P_h is increased.

In figure 10, the ratio P_h/P_d is shown as a function of flux density. This curve is closely related to figure 9, as $P_h/P_d=(P_1/P_d)-1$. However, it is interesting to note that $(P_h/P_d)\times 100$ is the percentage by which the value of P_d would be in error if P_h were not taken into account. This percentage is used as the ordinate at the right side of figure 10.

We see again that, in general, for a given value of error, the lower-silicon materials can be taken to higher inductions. For example, if we consider the case where the bridge is used without correcting for the harmonic power dissipated, and for the error to be 5 percent or less, the grade M–15 material can be taken to about 12.5 kilogausses, while the grade M–43 specimen could be taken to over 14 kilogausses. If the M–15 specimen were measured at 14 kilogausses, the error would be 70 percent.

Figures 8, 9, and 10 show clearly that the harmonic power conversion must be taken into consideration in order to make accurate measurements with the bridge at higher inductions. Figure 7 and table 4 show that, if this correction is made, excellent accuracy is possible.

It is obvious that every parameter should be examined with a view to reducing the magnitude of the correction term and increasing the accuracy with which it can be measured.[5]

In order to reduce P_h, the resistance of the circuit, including that of the source, should be made as small as possible.[6] Ordinarily, it is desirable to work with bridge arms that are approximately equal, but in this case it is better to make R_c as low as possible. R_a and R_b should be high, in order to limit the current, and therefore the dissipation, in them.

R_d, and therefore P_1, can be measured more accurately if it is large with respect to R_w and R_{wm}, which are measured along with R_d. Figure 11 depicts the change of R_d with respect to flux density. As high inductions (14 to 15 kilogausses) are reached, the value of R_d decreases to less than 10 ohms. When it has become as small as 2 ohms, the accuracy required in the values of R_w and R_{wm} is difficult to attain.

It is interesting to note that when the bridge is used alone, without the wattmeter current coil included in the bridge arm, R_{wm} will be eliminated, and the results should have improved accuracy. In order to show the validity of the method presented in this paper, the bridge was used under conditions known to be disadvantageous to it, and it is expected that results obtained with optimized conditions will exceed in accuracy those reported here.

6. Summary and Conclusions

The use of an alternating-current bridge for measuring the total core losses of ferromagnetic materials has some advantages over the conventional wattmeter method, such as a wider frequency range and better sensitivity. However, it has been found that the bridge method is subject to errors that may be as large as several hundred percent if the effect of distortion in the current is neglected. A correction for this effect may be derived by considering the equivalent circuit for the ferromagnetic material to consist of nonlinear elements of resistance and inductance. This method requires that the resistance of the entire circuit be determined, including the apparent resistance of the power source. A method of measuring the resistance of the power source under operating conditions was developed.

A circuit was devised by which measurements of core loss can be made by the bridge and wattmeter simultaneously. By so doing, discrepancies due to differences in frequency or induction are obviated, and comparisons can be made at identical values of frequency and induction.

Excellent agreement between the results of measurements with the two methods was obtained for inductions as high as those for which the wattmeter is considered to be accurate.

It is concluded, therefore, that even for measurements at higher inductions than have hitherto been considered feasible, the accuracy of an a-c bridge is comparable with that of the wattmeter, provided that the corrections for harmonic components in the exciting current are determined and applied with care.

It is also concluded that the results of the analysis as presented are valid when applied not only to bridge circuits, but also to other electric circuits that contain nonlinear resistances and impedances.

[5] For the sake of consistency, all the results shown were made with the same circuit resistance.

[6] P_h would also be small for large R_p, but this case is of no practical importance because the voltage of the source would have to be made very high in order to attain high flux densities. In addition, the suppression of harmonics in the exciting current would result in extreme distortion in the secondary voltage.

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