

# Response Characteristics of Temperature-Sensing Elements for Use in the Control of Jet Engines

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The rate at which a temperature-sensing element located in the gas stream of a jet engine responds to sudden changes in temperature is of great practical importance in the control and operation of such an engine. The factors that determine rate of response are discussed, and the significance of the characteristic time is emphasized. It is shown that laboratory determinations of characteristic time must be made under simulated engine conditions in which the rate of heat transfer by forced convection is the controlling factor. Apparatus used at the National Bureau of Standards for measuring characteristic times is described, and typical results are presented. The rate of response of a device in a jet engine varies greatly with engine speed and with the altitude of flight, so that satisfactory performance of a temperature-actuated control system can be expected only if the sensing element responds with sufficient rapidity under starting conditions and at the flight ceiling.

## I. Introduction

The importance of accurate temperature measurements in the development and operation of gas turbines and jet engines is well recognized. Such measurements determine important operating characteristics of the power plant and permit the engine designer to single out for special attention those components wherein changes would be most effective in improving performance.

Of commensurate importance is the application of temperature-sensing devices in the control of jet engines. An acceptable control system must prevent the engine speed from increasing at an excessive rate and protect critically stressed parts from being overheated. Since a change in fuel-air ratio is normally accompanied by a simultaneous change in gas temperature, adequate protection of the power plant requires a control that will act very suddenly to limit the rate of change of gas temperature during acceleration and to reduce the gas temperature whenever it exceeds a safe value. If the control is temperature-actuated, the sensing element must therefore respond rapidly when subjected to a change in temperature.

Protecting against overheating of the part that is considered most likely to fail mechanically might be achieved by attaching a temperature-sensing element directly to such part. Usually, however, most highly stressed parts are rotating, so that it is extremely difficult to make satisfactory temperature measurements of such parts. Even if this installation problem were solved, the instrument so applied would respond but slowly to the stimulus (i. e., a change in gas temperature) that causes engine speed to change. Thus it is logical to base the control system upon the temperature of the high-velocity gas, since such a system can be made to protect against overheating, to prevent too rapid acceleration, and to establish the power output of the engine. Obviously, these essential controls can be achieved only if the temperature sensing element and the auxiliary parts constituting the control system can be made to respond very rapidly to changes in gas temperature.

In the course of projects dealing with temperature-

sensing devices applicable in gas turbines and jet engines, sponsored at the National Bureau of Standards by the Air Matériel Command and by the Bureau of Ships, many misconceptions have come to light concerning the rate of response of such devices to sudden changes in temperature. These do not involve the basic theory of response, which has been treated clearly by Harper [1],<sup>1</sup> but indicate some confusion as to the significance of measured values and as to the proper application of such values.

This paper is concerned particularly with the rates of response of interest in the control of turbine-type engines, where the temperature-sensing element is normally immersed in flowing gas. Various factors that determine the rate of response under such circumstances are discussed, the relative importance of these factors is indicated, and laboratory equipment and methods used at this Bureau for determining response characteristics are described. Previous papers in this field [2, 3, 4, 5] deal with other applications, and therefore do not emphasize some of the factors that are specific to power plants, particularly those for aircraft.

## II. General Considerations

An object immersed in a gas at temperature  $T_g$ , retained by walls at temperature  $T_w$ , eventually attains a steady temperature,  $T_2$ . At this steady state the object gains and loses heat at equal rates, and its actual temperature  $T_2$  is seldom identical with either  $T_g$  or  $T_w$ . The object may be a temperature-sensitive element initially at any given temperature  $T_1$ , which is introduced suddenly within the aforementioned environment. The rate at which its temperature changes from  $T_1$  to  $T_2$  may now be measured and is found to be a function of its surface area ( $S$ ), mass ( $M$ ), heat capacity ( $C$ ), and the rate of heat transfer with the surroundings. This may be expressed as follows:

$$dT/dt = (S/MC)f(k, T, T_g, T_w), \quad (1)$$

<sup>1</sup> Figures in brackets indicate the literature references at the end of this paper.

in which  $T$  is its temperature at time,  $t$ , after its environment is changed, and  $k$  is a generalized coefficient of heat transfer such that  $f(k, T, T_g, T_w)$  describes completely the heat transfer to and from the element.

For every specific element, the heat-transfer function in turn governs the rate at which the element responds to sudden changes in environment. Although this function cannot be evaluated theoretically for the completely general case, present purposes are served by considering special cases in which the heat transfer is by one single method at a time, because, in most practical applications including that in engines, the rate of transfer by one method is so preponderant that the effects of all others are negligible by comparison.

### III. Factors Influencing Response in Gas Streams

#### 1. Heat Transfer by Convection

In the transfer of heat by fluid convection, motion of the molecules continuously brings new particles in contact with the surface of the object being heated or cooled. A distinction is made between natural convection, in which the motion of the medium is due to differences in density resulting from differences in temperature, and forced convection, in which the motion of the medium is produced by mechanical means. The transfer of heat by conduction through gas is included in the transfer by convection, so that it need not be treated separately. In any case, the rate of transfer of heat by conduction is small, compared with that by convection.

Consider, first, the case in which radiation can be neglected. This could be approximated closely if the object were provided with an efficient radiation shield. Under these conditions the wall temperature is without effect, and  $T_2 = T_g$ . Thus eq 1 may be written

$$dT/dt = (S/MC)f(h_c T, T_2), \quad (2)$$

in which  $h_c$  is the coefficient of heat transfer by convection. Also, the rate of heat transfer by convection varies directly as the difference in temperature (Newton's law of cooling), so that  $f(h_c, T, T_2)$  becomes  $(T_2 - T)f(h_c)$ .

The value of  $h_c$  for forced convection is essentially independent of temperature, so that for this case eq 2 becomes

$$dT/dt = h_c(S/MC)(T_2 - T). \quad (3)$$

It has been shown experimentally [6, 7] that, for natural convection, the coefficient may be expressed as

$$h_c = k_1(\Delta T)^{1/4}, \quad (4)$$

in which  $k_1$  is a constant and  $\Delta T$  is the temperature excess of the gas over the element heated. Thus, for this case eq 1 becomes

$$dT/dt = k_1(S/MC)(T_2 - T)^{5/4}. \quad (5)$$

#### (a) Forced Convection

This is the case of real interest when considering the response of temperature-sensing elements immersed in the gas stream of jet engines, because the rate of heat transfer by forced convection far exceeds that by any other method in such applications.

With  $\tau$  substituted arbitrarily for the quantity  $MC/Sh_c$  eq 3 integrates to

$$-\log_e T_2 - T = t/\tau + \text{constant}, \quad (6)$$

and, since  $T = T_1$  at  $t = 0$ ,

$$(T - T_1)/(T_2 - T_1) = \Delta T/\Delta T_0 = 1 - e^{-t/\tau}, \quad (7)$$

in which  $\Delta T$  is the change in temperature that the element has experienced at time  $t$ ,  $\Delta T_0$  is the total change to which it is subjected, and  $e$  is the base of Napierian logarithms.

From eq 7 it is apparent that the quantity  $\tau$  must have the dimensions of time, and that at time  $t = \tau$ , the ratio  $\Delta T/\Delta T_0 = 1 - 1/e = 0.632$ . Thus  $\tau$  is the time required for the object to undergo 63.2 percent of the total change in temperature to which it is subjected instantaneously. The time  $\tau$ , so defined, is generally referred to as the time constant or "characteristic time" of the element, and this expression will be adhered to throughout the remainder of this paper. A similar concept of the time constant appears frequently in the fields of mechanics and electricity.

The characteristic time of a temperature-sensing element has a number of interesting physical interpretations. Referring to a specific installation, both  $\Delta T_0$  and  $\tau$  are independent of time, and eq 7 can be differentiated to give

$$d(\Delta T)/dt = (\Delta T_0/\tau)e^{-t/\tau}. \quad (8)$$

Equation 7 may also be written

$$T_2 - T = \Delta T_0 e^{-t/\tau}. \quad (9)$$

Dividing eq 9 by eq 8 gives

$$(T_2 - T)/[d(\Delta T)/dt] = \tau. \quad (10)$$

Equation 10 shows that the characteristic time may also be defined as the time that would be required for the object to change from temperature  $T$  at time  $t$ , to temperature  $T_2$ , if the rate of change  $[d(\Delta T)/dt]$  of temperature prevailing at time,  $t$ , were held constant. This interpretation of the characteristic time is shown graphically in figure 1, in which the curve represents eq 7, plotted in terms of the dimensionless ratios  $\Delta T/\Delta T_0$  and  $t/\tau$ . It will be noted that if a tangent is drawn to the curve at any point and extended to  $\Delta T/\Delta T_0 = 1$  (or  $T = T_2$ ), the horizontal component of such a tangent is equal to  $\tau$ .

The quantity  $\tau$  was introduced above as a substitute for the quotient  $MC/Sh_c$ . Among these quantities,  $M$ ,  $C$ , and  $S$  are physical characteristics

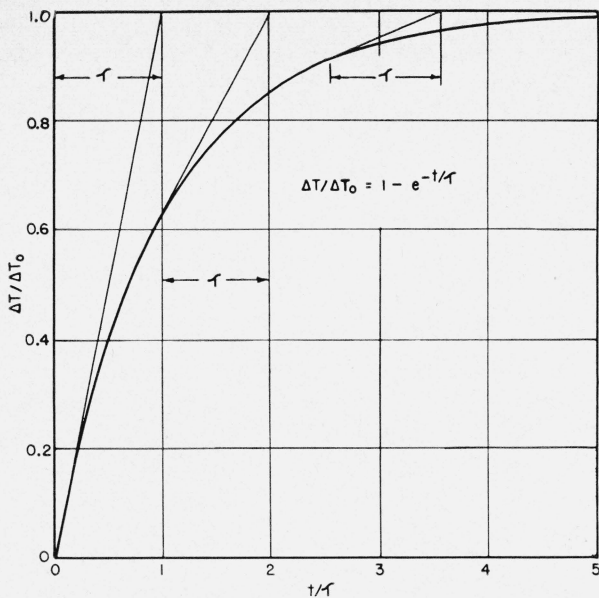


FIGURE 1. Physical interpretation of characteristic time.  
Heat transfer by forced convection only.

of the object, and if the coefficient of heat transfer by forced convection ( $h_c$ ) were independent of flow conditions, then the characteristic time of a particular object would be a single-valued property. Actually  $h_c$  varies in a complex manner, for which no satisfactory theoretical treatment has been evolved. However, comprehensive analyses of available data indicate that, for a given object,  $h_c$  is essentially dependent only on the mass rate of flow. Thus, the characteristic time of an object immersed in gas flowing at velocities attained in jet engines is essentially independent of the temperature and of the temperature difference to which it is subjected, and the same value of  $\tau$  applies whether the object is being heated or cooled. For instruments used in flowing gases, a numerical value of characteristic time is significant only when the mass flow rate is specified, but no statement as to the magnitude or direction of the temperature change is required. For example, it is meaningful to say that a particular device has a characteristic time of 2 sec at a mass flow rate of five lb/sec ft<sup>2</sup>.

Since the characteristic time of any object varies directly as its mass and inversely as its surface area, it is apparent that  $\tau$  will be decreased by any process that increases the surface area proportionately greater than the mass. Consequently, any method of increasing the surface area without changing the mass will reduce  $\tau$ , but such means for reducing  $\tau$  are practicable only within the limits established by the ability of the element to withstand the stress to which it is subjected.

#### (b) Natural Convection

It is known that the rates of response of instruments designed for use in jet engines have been measured in apparatus in which they gained or lost heat

primarily by natural convection and radiation. The following sections are included merely to show why values measured in this way cannot be significant, so far as applications in engines are concerned.

If the expression  $Sk_1/MC$  is represented by  $\beta$ , eq 5 may be written

$$dT/dt = \beta(T_2 - T)^{5/4}, \quad (11)$$

which may be integrated to give

$$t = [4\beta/(T_2 - T_1)^{1/4}] \{ [(T_2 - T_1)/(T_2 - T)]^{1/4} - 1 \}. \quad (12)$$

It can be seen from eq 12 that the rate at which an object will respond to a sudden change in temperature, when heat is transferred solely by natural convection, is a function of the temperature interval over which the heating takes place. This is shown graphically in figure 2, in which  $t/4\beta$  and  $\Delta T/\Delta T_0$  are plotted for the single value  $T_1 = 500^\circ \text{ F}$  and for three values of  $\Delta T_0$ .

Since the response due to natural convection depends upon the particular temperature conditions encountered, whereas that due to forced convection does not, the characteristic times of the same device under these two sets of operating conditions cannot be related.

## 2. Heat Transfer by Radiation

Consider the special case in which heat transfer to or from a relatively small body, such as a thermocouple junction, takes place by radiation only, so that there is no transfer of heat by convection,  $T_g$  has no effect, and  $T_2 = T_w$ . As an example, a junction might be moved suddenly from a cool to a hot location within a vacuum furnace. The transfer of heat in this case takes place in accordance with the Stefan-Boltzmann law, and eq 1 becomes

$$dT/dt = (S\sigma\epsilon/MC)(T_2^4 - T^4) = \alpha(T_2^4 - T^4), \quad (13)$$

in which  $\sigma$  is the Stefan-Boltzmann constant,  $\epsilon$  is the surface emissivity of the junction, and  $\alpha$  stands for the quotient  $S\sigma\epsilon/MC$ .

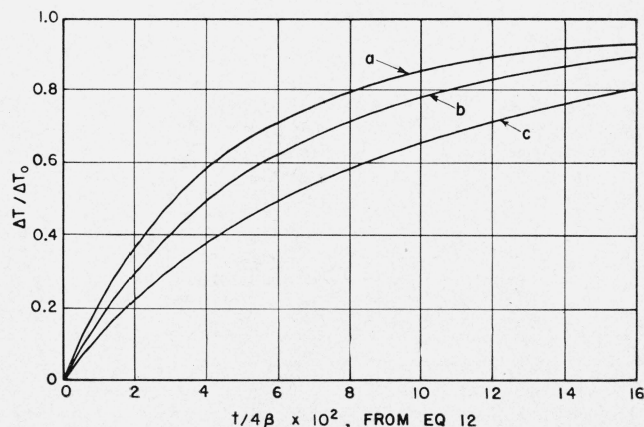


FIGURE 2. Graph of equation 12.

Heat transfer by natural convection only. Curve a,  $\Delta T_0 = 1,500^\circ \text{ F}$ ; curve b,  $\Delta T_0 = 500^\circ \text{ F}$ ; curve c,  $\Delta T_0 = 100^\circ \text{ F}$ .



Integration of eq 13 gives

$$t = \frac{1}{4\alpha T_2^3} \left[ \log_e \frac{(T_1 - T_2)(T + T_2)}{(T_1 + T_2)(T - T_2)} + 2 \tan^{-1} (T/T_2) - 2 \tan^{-1} (T_1/T_2) \right] \quad (14)$$

Examination of this equation shows that the time-temperature relation, and hence also the rate of response, depends markedly on the following: (a) whether the junction is being heated or cooled; (b) the temperature level; and (c) the temperature interval.

Item (a) can be illustrated by considering that, in one case, a junction heated from 500° to 2,000° R, and in another it is cooled from 2,000° to 500° R. The quantity  $4\alpha t$  is plotted against temperature in figure 3 for each of these cases. Although the initial

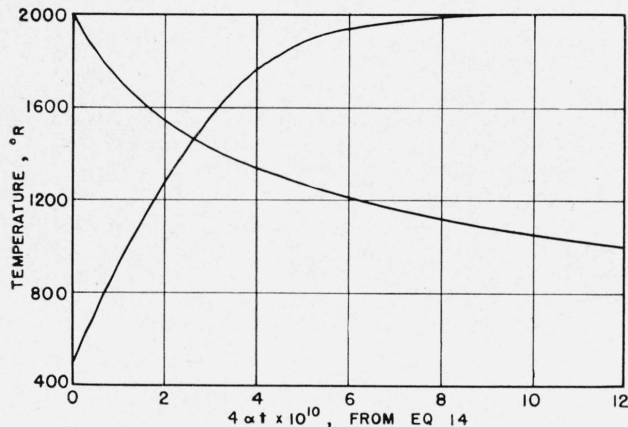


FIGURE 3. Graph of equation 14.  
Heat transfer by radiation only.

rates of temperature change are the same in the two cases, the time required for the temperature to change by a given percentage of the temperature difference is less during heating than it is during cooling.

Thus when most of the heat is transferred by radiation, the rate of response of a given junction depends on different factors and has entirely different characteristics than when the primary heat transfer is by convection.

#### IV. Conditions of Heat Transfer in Jet Engines

The gas flow rates that prevail in a jet engine cover a wide range and depend primarily upon the engine operating conditions and the altitude of flight. However, even during starting, heat is transferred to a device immersed in the gas stream predominately by forced convection. Since the various solid parts that retain the gas stream are normally cooler than the gas because of heat losses, a sensing element in the gas stream normally attains a steady temperature below that of the gas itself, because of radiation and conduction from the sensing element. Nevertheless, when the temperature of the gas is changed suddenly, the rate of change in the temperature of the sensing element is determined almost entirely by the rate of heat transfer by forced convection, and the effects

of radiation and conduction over the short interval of time involved are insignificant.

As the characteristic time of the instrument is a function of the operating conditions, as well as of the physical characteristics of the device itself, these conditions must be taken into account in devising laboratory equipment for measuring  $\tau$  and in using measured values in the design of control systems for engines. The mass flow rate through a given engine, and hence also the characteristic time of an instrument immersed in its gas stream, varies with the density of the ambient air, engine rpm, and to a lesser extent with air speed. Thus the over-all rate of response of the control system must be sufficiently high that adequate protection is provided at the lowest mass flow rates, which occur during static starts and at the flight ceiling. A system that protects the engine at sea level and high-power outputs may be utterly inadequate at low-power outputs and at high altitudes.

#### V. Laboratory Apparatus for Measuring Characteristic Time

Equipment used at this Bureau for studying the performance of temperature-sensing devices applicable in jet engines is shown schematically in figure 4, and the apparatus used for producing sudden changes in the temperature of the test instruments is shown in figure 5. As indicated in figure 4, a compressor supplies air to a single Jumo 004 turbojet engine combustor equipped with its normal fuel injector and spark plug. The gas temperature and flow rate at the combustor outlet are subject to control through a valve at the inlet to the compressor and by adjusting the fuel pressure. Exhaust gases from the burner pass through two 90° turns, through a perforated plate, and then through about 10 ft of straight pipe before reaching the test section. The latter has three convenient hatches for mounting instruments in the gas stream, which is at essentially uniform temperature and velocity over the central half diameter of the test section. Due to the shape of the duct system, no instrument is exposed to direct radiation from the flame.

A bleed line containing a butterfly valve provides an auxiliary control of the flow in the test section, independent of the operating conditions of the burner. Pressures observed with a pitot-static tube, together with the known value of gas temperature, permit calculation of the mass flow rate in the test section.

Two such systems have been used to date, and it is planned to maintain both for future studies of a similar nature. The air for one is supplied by a blower with a capacity of 4,000 ft<sup>3</sup> of free air a minute and that for the other by centrifugal compressors having a combined capacity of 10,000



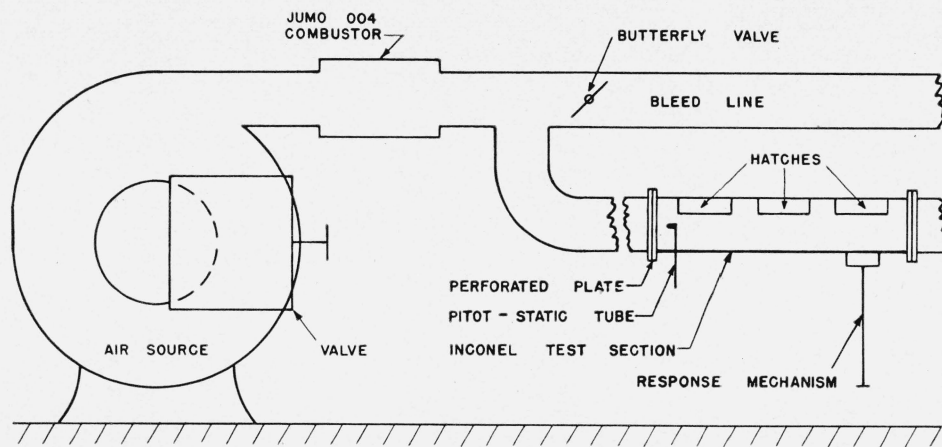


FIGURE 4. Diagram of test systems.

ft<sup>3</sup>/min. The systems have identical Inconel test sections, about 6 in. in diameter. The remainder of the former is of ordinary iron, which limits the operating temperature to about 1,600° F because of scaling. Most of the work on rates of response has been done in this system, in which the blower capacity and the drop in pressure through the burner and lines limit the mass flow rate to approximately 8 lb/sec ft<sup>2</sup>. Currently a second blower is being added in series, and all iron pipe downstream of the burner is being replaced with Inconel, in order that the operating range may be increased.

All of the second system beyond the Jumo combustor is of Inconel, and an afterburner is installed at the burner outlet. This system can be operated at temperatures up to about 2,000° F, and at mass flow rates up to 15 lb/sec ft<sup>2</sup> in the 6-in. test section. A test section 3 in. in diameter is available for use in life tests and in research where velocities up to 1,800 ft/sec are of interest.

For measuring the rate of response, the temperature of the gas surrounding the test instrument must be changed suddenly. This cannot be accomplished with sufficient rapidity by changing the burner operating conditions. It can be done, without altering the steady flow of exhaust gas in the test section, with the apparatus shown in figure 5. An Inconel tube, held in position around the test instrument by a release plate, provides a flow channel for cold air. Upon removing the release plate, the Inconel tube is removed suddenly by a spring, thus exposing the instrument to the hot gas of the main stream almost instantaneously. During the downward movement of the tube, the supply of cold air is stopped automatically. In this way a test instrument at a known, moderate temperature (controlled by the air rate through the Inconel tube) can be exposed to exhaust gas at any chosen temperature and mass flow rate within the capability of the test system.

The performance of a variety of thermocouples, resistance thermometers, and thermistors has been studied. In each case the experimental result was recorded with a direct-inking oscillograph, in the

form of an emf-time chart. The time scale can be had either by counting the strokes of the pen, which oscillates continuously at 60 cycles/sec, or from the known speed of the paper, which is moved by a synchronous motor.

An a-c amplifier was used with the oscillograph, so that it was necessary to use a "chopper" on the output of each thermocouple. With the resistance-type devices, both a-c and d-c bridges with choppers have been used with equal success. In any of these systems the amplitude of the recording pen is proportional to the amplified emf, so that the envelope of the record gives the impressed emf as a function of time. It is not necessary to know the amplification factor. The initial and final temperatures, if desired, can be measured more conveniently and more accurately with other indicating or recording instruments.

If the output of the sensing element varies approximately linearly with temperature, as is the case with thermocouples and resistance thermometers, the characteristic time can be read directly from the record. It is simply the time required for the amplitude to change from its initial value by 63.2 percent of the total change that it experiences.

A typical record of this type, together with a drawing of the envelope curve to an enlarged scale, is shown in figure 6. This particular record is for a No. 22 gage bare Chromel-Alumel thermocouple, the junction of which was formed by fusing the ends of the wires into a bead having a diameter approximately twice that of a single wire. Initial and final temperatures, measured with other instruments, were 300° and 1,200° F, respectively, and the mass flow rate was 4 lb/sec ft<sup>2</sup>.

In the case of thermistors, for which the resistance decreases approximately exponentially with increasing temperature, a satisfactory arrangement is to use the test unit as one arm of a bridge, in which the oscillograph indicates balance. Having determined the resistance-temperature relation for the test instrument, and having chosen the experimental values of  $T_1$  and  $T_2$  to be used in the test, the resistance corresponding to the temperature  $[T_1 + 0.632$

$(T_2 - T_1)$  is calculated and set up in a second arm of the bridge. The remainder of the test of a thermistor can then be conducted as described for thermocouples, but the oscillograph record now

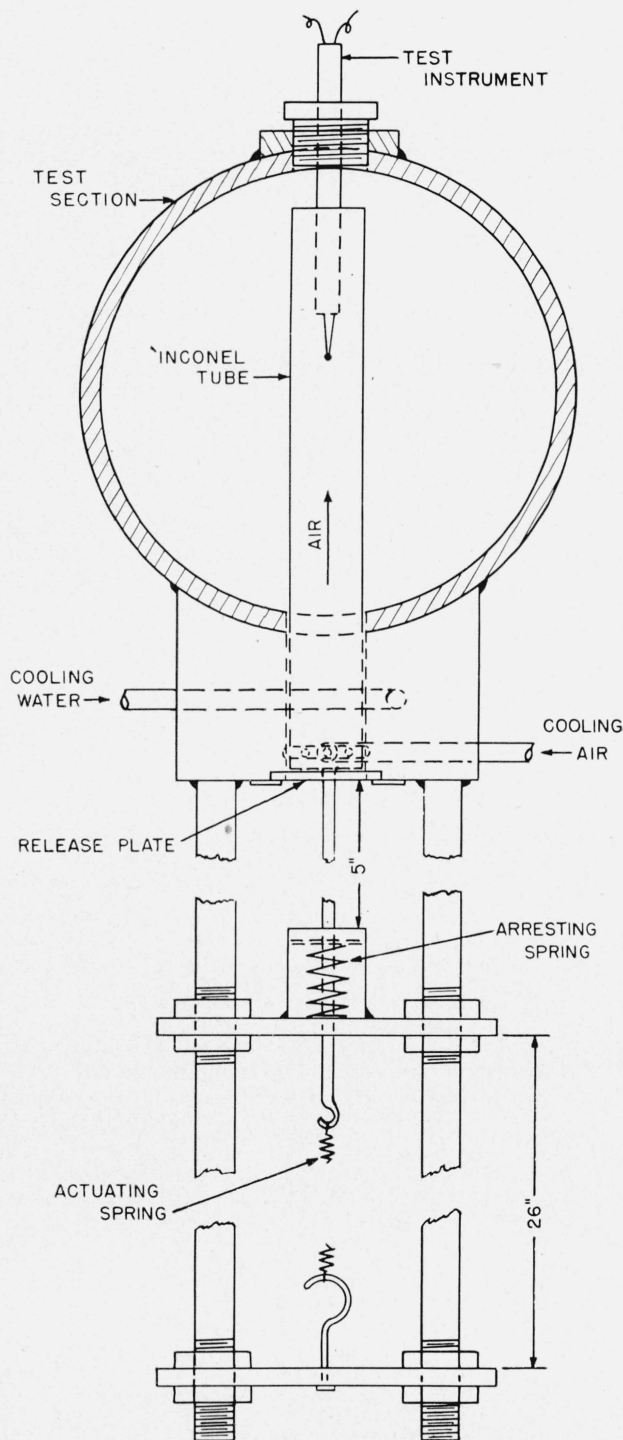


FIGURE 5. Equipment for producing sudden changes in temperature.

shows a maximum amplitude at the start of the test and a minimum amplitude at the characteristic time, as measured from the instant of exposure to hot gas.

A wide variety of instruments was studied, including sensing elements exposed directly to the gas stream, elements encased in metal and ceramic protection tubes, and elements imbedded in insulating materials such as quartz and beryllia. Characteristic times as low as 0.02 sec have been measured successfully, and there seems to be no upper limit imposed by the test system or recording equipment. For each instrument, from three to five records were taken at each of the flow rates 2, 4, and 6 lb/sec ft<sup>2</sup>. No results have been obtained at flow rates above 6.8 lb/sec ft<sup>2</sup>, primarily because the flow rates of greatest interest in engines are less than this.

Presentation of the results in detail would be useless without a complete description of each instrument, which is considered neither practicable nor

TABLE 1. Characteristic times of bare thermocouple junctions

Gage number of thermocouple wire	Characteristic time at mass flow rates (lb/sec ft <sup>2</sup> ) of—		
	2.2	4.2	6.8
18	2.2	1.6	1.2
22	1.3	1.0	0.8
28	0.54	0.41	.34
36	.17	.12	.10
40	.11	.08	.07
44	.07	.05	.04

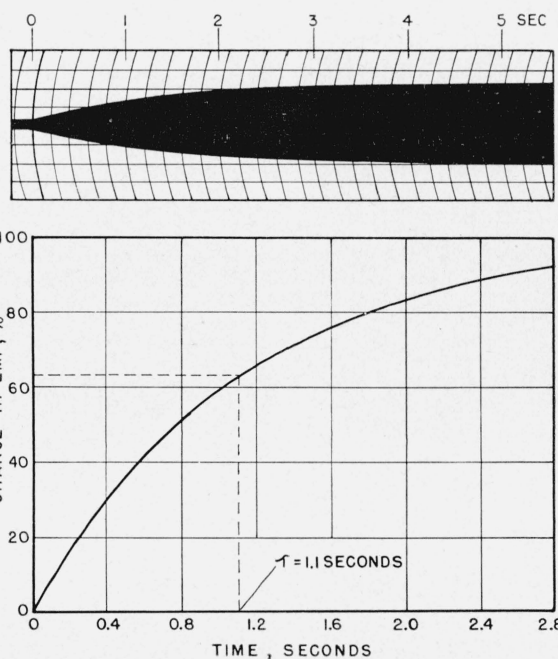


FIGURE 6. Typical record of the response of a Chromel-Alumel thermocouple.

worth while. Some typical results obtained for bare, untwisted Chromel-Alumel thermocouples are presented in table 1, merely to give an idea of the magnitudes of the characteristic times with which the designer of control equipment for jet engines must cope.

Examination of all the results obtained to date indicates that the effect of flow rate upon characteristic time is approximately the same for all instruments studied, and that the ratio of the characteristic times of any instrument at two flow rates is approximately proportional to the square root of the ratio of these mass flow rates.

From many measurements of characteristic time under experimental conditions, such that the principal heat transfer was by forced convection, it is apparent that the value observed for any particular instrument is essentially independent of the temperatures employed during the tests. Typical results illustrating this fact are shown in table 2. These results are for a No. 20 gage bare thermocouple, the junction of which was a bead of approximately twice the wire diameter, and for a mass flow rate of 6 lb/sec ft<sup>2</sup>.

TABLE 2. Effect of operating temperature range on observed characteristic time, when heat is transferred by forced convection

Run	Temperature		Characteristic time (flow rate 6 lb/sec ft <sup>2</sup> )
	T <sub>1</sub>	T <sub>2</sub>	
	°F	°F	sec
1.....	300	1,000	1.1
2.....	400	1,200	1.25
3.....	600	1,200	1.1
4.....	700	1,500	1.2
avg.....	-----	-----	1.16

The present equipment was designed primarily to subject the test junction to a sudden increase in temperature. By heating the test junction electrically while it was surrounded by the Inconel tube, then subjecting it to a stream of unheated air, characteristic times for the cooling process were obtained. Results agree with the prediction of the theory that applies for the case of heat transfer by forced convection, in that the characteristic time at a given mass flow is the same, whether the instrument be heated or cooled.

## VI. Conclusion

The characteristic time of a temperature-sensing element immersed in the gas stream of a turbine-type engine depends primarily on the mass flow rate, and is essentially independent of the temperature level, of the temperature difference over which it is measured, and of the direction of net heat flow. This is true primarily because the rate of heat transfer by forced convection is large compared with the rates of transfer by radiation and natural convection. Where radiation and natural convection are the controlling processes, the rate of response of an object to sudden changes in temperature depends greatly upon the actual temperatures involved. In the case of heat transfer by radiation only, different rates are obtained when the object is heated and cooled between the same two temperatures. Therefore, values of characteristic time that are significant in engine applications must be determined with equipment in which the mass flow rates approximate those prevailing in engines.

Apparatus of this kind is described in detail, and some typical results are presented. Results with a wide variety of instruments confirm the conclusions stated above.

The rate of response of a temperature-sensing element in the gas stream of an aircraft gas turbine varies greatly with the altitude of flight and with the engine speed. Adequate protection of the power plant requires a control system that will respond with the necessary rapidity during starting of the engine and at the flight ceiling. If this is accomplished, then the protection provided at all other altitudes and engine speeds should be more than adequate.

## VII. References

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