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An Instrument for the Rapid Production of a Decimal Series of Potentials and Its Application to Ballistic Measurements[1]

By Howard S. Roberts[2] and Harvey L. Curtis

An instrument for producing decimal subdivisions of a chosen potential was developed for calibrating a cathode-ray oscillograph, often within a few milliseconds after the ballistic record had been obtained. The maximum potential could be chosen as any integral multiple of 10 millivolts, with a maximum of 100 millivolts. To permit this choice, ten 10-ohm coils were connected in series, and a current of 1 milliampere was sent through them. Then the drop in potential over each coil was 10 millivolts, and the maximum output potential was the product of 10 millivolts and the number of coils included between the output terminals. The decimal subdivisions of the chosen potential were obtained by varying the current in the ten 10-ohm coils in steps of 0.1 milliampere from 0 to 1.0 milliampere by shunt resistances. The calibration started with zero voltage when the shunt resistance was zero, and the shunt resistance increased in such steps that the current in the 10-ohm coils increased in 0.1 milliampere steps and hence the output voltage in decimal steps of the chosen maximum potential. The shunt resistances were inserted by opening switches that normally short-circuited them. For oscillograph calibration these switches were opened in succession by a falling weight.

The numerical data used for illustration are based on decimal subdivision of the maximum available potential. The general case is treated, however, showing that a calibrator of this type may be used for any desired number of potential steps.

I. Introduction

An instrument was required in a ballistic investigation[3] for the rapid production of a decimal series of potential steps of known values. This was used in connection with the calibration of apparatus recording on a cathode-ray oscillograph. It was important that the operator should be able to adjust the mximum potential to correspond approximately to that expected from the operation of the apparatus and that this adjustment should not affect the ratios of the potentials delivered by the different steps. As the calibration might have to be completed in about 10 milliseconds, sliding contacts were inadmissible because of the probability of introducing thermal electromotive forces, and even the closing of a contact was considered undesirable because of the possibility of vibrations affecting the constancy of the contact resistance. An instrument was developed that met all the requirements by having the rapid changes in potential result from the opening of contacts. The maximum potential for any particular calibration could be predetermined as any integral multiple of 10 mv from 10 to 100 mv. This maximum was automatically subdivided by the calibrating apparatus into steps, usually decimal steps. This instrument was very successful, and the principles employed are herein extended to higher voltages.

[1] The information in this article was, in part, obtained through a transfer of funds from the Office of Scientific Research and Development under the supervision of the National Defense Research Committee.

[2] Mr. Roberts, now deceased, was a member of the staff of the Geophysical Laboratory.

[3] See the following reports by the National Bureau of Standards and the Geophysical Laboratory to the National Defense Research Committee for the illustration of the usefulness of the instrument herein described: NDRC Report No. A 229 (OSRD No. 2019), pp. 26 and 59. NDRC Report No. 323 (OSRD No. 4986), p. 48. NDRC Report No. A 460 (OSRD No. 6531), p. 14. Also see NBS Report, restricted, to Navy on 90-mm gun (NAR-ORD No. 430).

II. Description of Instrument

The potential produced by the instrument is the fall in potential of a known current in a known resistance. The circuit arrangement can be most easily described by reference to figure 1, in which the shunt contacts (*C1* to *C10*) are shown as being opened in succession by a falling weight, the diagram showing *C1* and *C2* already opened, *C3* at the point of being opened, and *C4* to *C10* to be opened in order as the weight falls. The battery, of electromotive force *E*, is the source of the current in the circuit, which includes the standardizing resistance, P_0, the adjustable resistance, P_a, and the decade and shunt resistances connected in parallel. The decade resistance, *R*, consists of 10 equal coils, *R1* to *R10*. The shunt resistance consists of coils *S1* to *S9* (*S10* is infinite) having values such that the opening of each contact changes the current in the decade resistance [4] by the same number of milliamperes. The standardizing resistance, P_0, has a value that is numerically a decimal multiple of the value of the

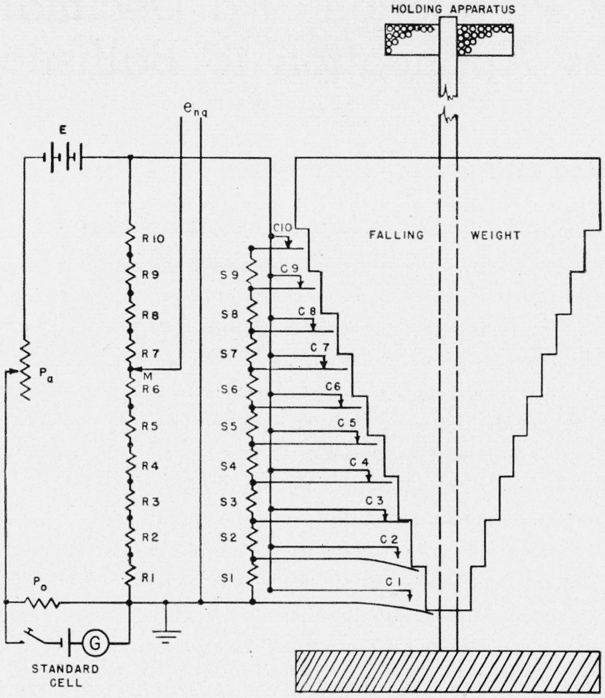

FIGURE 1. *Oscillograph calibrator giving decimal steps of voltage at millisecond intervals by opening contacts with a falling weight.*

electromotive force of the standard cell, this numeric being 1,000 for the instruments actually constructed. The adjustable resistance, P_a, is, with the shunt resistance infinite (weight down), varied until the fall in potential produced by the current I_0 in P_0 equals the electromotive force of the standard cell as shown by zero deflection of the galvanometer when the key in series with it is closed. Under this condition the current, I_0, normally 1 ma, produces in the decade resistances *R1*, *R2*, etc., which are normally 10 ohms each, a potential drop of 10 mv in each coil. By so connecting the movable contact, *M*, that *n* of the decade coils are included between it and ground, the delivered voltage, e_{nq}, when the shunt contact *C10* is open, is 10*n* mv. When using the instrument the value of *n* is so chosen that 10*n* corresponds approximately to the maximum voltage of the apparatus to be calibrated.

To vary the value of the delivered potential in decimal steps, the shunt resistances *S1*, *S2*, . . . *S9* (*S10* is infinite resistance) are constructed with such values that the current in the decade resistance is varied from 0 to 1 ma in steps of 0.1 ma as the contact switches *C1* to *C10* are opened in succession. When all the contacts are closed, the decade resistance is short-circuited so that its current is zero, and hence the delivered potential is zero. When *C1* is opened, *S1* is in parallel with the decade. By giving a proper value to *S1*, the current in the decade resistance will be 0.1 ma, so that the delivered voltage is one-tenth of 10*n*, or *n* mv. Likewise, when *C1* and *C2* are open, the current in the decade becomes 0.20 ma, and the delivered voltage becomes 2*n* mv. Opening the contacts in succession increases the current in the decade resistance in the decade steps, and likewise increases the delivered potential. If the decade contact is attached at the *n*th position, and if the contacts from 1 to *q*, inclusive, are open, the potential delivered, e_{nq}, is *nq* mv.[5]

III. Values of Resistances

The values of shunt resistances *S1*, *S2*, . . . *Sq* . . . *S9* depend on the value of *E*, or, what is equivalent, on the value of *P*, which equals

[4] The resistance arrangement is similar to a Waidner-Wolff resistance element. See Mueller and Wenner, J. Research NBS **15**, 477 (1935) RP842.

[5] Although the delivered potential has decimal steps, it is not possible to obtain every integral value between 1 and 100 mv. For example, values corresponding to the prime numbers between 10 and 100 cannot be obtained. Neither can those values in which one of these prime numbers is a factor. In addition, 75, 84, 96, and 98 mv cannot be obtained.

P_0+P_a. When the current is adjusted by varying P_a until there is zero deflection of the galvanometer, the shunt contacts are all open, so that the current in the decade resistances is the same as in P_0 and P_a. When the weight is raised so that all contacts are closed, the decade resistance is short-circuited so that the current in it is zero. The shunt resistances *S1*, *S2*, . . . *Sq* have such values that, as the contacts *C1*, *C2*, . . . *Cq* are opened, the current in the decade resistances increases in steps of one-tenth of I_0, the current I_0 being the current in the decade resistance R when all contacts are opened. By algebraic equations it can be shown (See appendix 1) that

$$Sq=\frac{RP}{R+P}\,\frac{10}{(10-q)(11-q)}=\frac{PRI_0}{E}\,\frac{10}{(10-q)(11-q)},$$

Values of Sq are given in table 1 for two different values of E with R constant and for two values of R with E constant.

The preceding description has assumed that both the decade resistance and the shunt resistance have ten steps. However, there is no magic in the number ten. The same principles can be used when the number of steps in either or both resistances is larger or smaller than ten. As used in the ballistic laboratory, the decade resistance always consisted of ten coils, but the number of shunt coils was usually five or six.

TABLE 1. *Shunt resistances in potential apparatus*

$$Sq=\frac{PR}{R+P}\,\frac{10}{(10-q)(11-q)}$$

q	$\frac{10}{(10-q)(11-q)}$	Values of Sq in ohms		
		E=4.5 v R=100 ohms P=4, 400 ohms	E=6 v R=100 ohms P=5, 900 ohms	E=6 v R=1,000 ohms P=5, 000 ohms
1	1/9	10. 86	10. 93	92. 59
2	5/36	13. 58	13. 66	115. 74
3	5/28	17. 46	17. 56	148. 81
4	5/21	23. 28	23. 41	198. 41
5	1/3	32. 59	32. 78	277. 78
6	1/2	48. 89	49. 17	416. 67
7	5/6	81. 48	81. 94	694. 44
8	5/3	162. 96	163. 89	1, 388. 89
9	5	488. 89	491. 67	4, 166. 67
10	∞	∞	∞	∞

IV. Applied Electromotive Force

There are two disadvantages to the method: one, the odd values of the shunt resistances, and the other, the effect of changes in the value of the applied electromotive force, E. The odd values of resistances are not serious, since coils can be constructed that are correct to 0.1 percent and that will retain their values. The effect of a changing value of E demands more consideration, since commercial cells, used to produce E, change their electromotive force with time. The results in table 1 show that the change in E from 4.5 to 6 v requires only a change in the shunt resistances of 0.6 percent. The change in the electromotive force of the battery may be observed by noting the departure of P_a from that used in the calculation of the shunt resistances. If the computation was made with E=4.5 v, I_0=1 ma, and R=100 ohms so that P=4,400 ohms, it follows from equations developed in appendix 2 that if the error in the delivered potential is to be less than 0.1 percent, the decrease in P can be more than 200 ohms. In other words, the voltage of E, used as 4.5 v in computing the shunt resistance, could decrease to 4.28 v without changing any value of the delivered potential by as much as 0.1 percent. Three commercial dry cells that have an initial voltage of 4.5 v can therefore be used.

V. Applications

The instrument has applications in ballistic measurements. One is the oscillograph calibration of the deflections produced by piezoelectric gages, and another, a similar calibration when the deflections are produced by a velocimeter. In both applications, the calibrating instrument impresses on the oscillograph a series of voltages, the maximum value of which corresponds approximately to the maximum produced by the ballistic instrument. As currently designed, the maximum voltage from the velocimeter is many times that from the piezoelectric gage, so that a calibrator to be used with a piezoelectric gage cannot be used with such a velocimeter.

1. Calibration of Piezoelectric Gage

The piezoelectric gage develops a quantity of electricity that is proportional to the force applied to the gage electrodes. This force may be externally applied to the electrodes of the gage, or may result from the acceleration of the gage. The gage electrodes are connected to the deflecting plates of an oscillograph, and the voltage applied to the oscillograph is controlled by introducing a shunt capacitance. After the force on the gage is removed, the calibrator inserts between the gage and the shunt capacitor a series of known voltage steps. The use of a shunt capacitance and the insertion of calibration potentials make possible the determination, from the deflections of the oscillograph, the quantity of electricity on the gage at any instant during its use, and does not require the measurement of the capacitance of the gage or its leads. A diagram of the electrical connections is shown in figure 2.

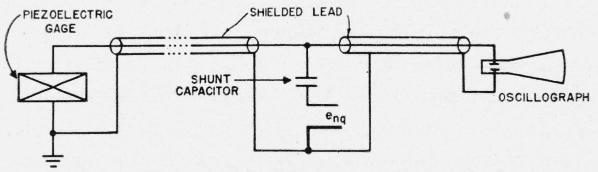

FIGURE 2. *Circuit of piezoelectric gage with indicated connection to the calibrating voltage,* e_{nq}.

The calibrating potentials are inserted between the gage and the shunt capacitor, but very close to the capacitor. When the ballistic measurement is being made, the calibrating potential is zero, since the shunt contacts of the calibrator are closed, and hence there is no current in the decade resistance, R (see fig. 1). The only effect of the resistance in the calibrating circuit while the record is being made is to delay the rise in potential on the capacitor, C, when a charge is suddenly produced on the gage. The effect of this delay is negligible.

2. Calibration of Velocimeter

A second application is the calibration of an oscillograph when used with a velocimeter in which is induced an electromotive force proportional to a velocity. For this application it is necessary to introduce a switch, operated by the falling weight of the calibration apparatus, that disconnects the velocimeter and connects the calibrator. Such a switch, with its electrical connections, is shown in figure 3. Adjustments are such that when the falling weight deflects the lower strip, thus opening the contact between the upper and lower strips, the upper strip deflects down and makes contact with the middle strip. The center and lower strips are relatively stiff and the upper one quite flexible. When silver contacts are used, there is no difficulty in making adjustments, so that the normal contact resistance is negligibly small.

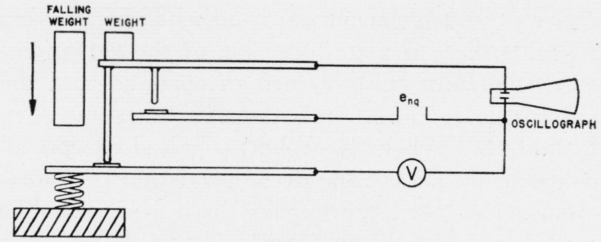

FIGURE 3. *Switch operated by a falling weight to disconnect the velocimeter, V, from the oscillograph and then to connect to it the calibrating voltage,* e_{nq}.

In deriving the equation for determining the velocity from the oscillograph deflection, the following symbols are used:

E=induced electromotive force
D_v=deflection of oscillograph produced by E
e_{nq}=calibration potential applied
D_c=oscillograph deflection produced by e_{nq}.

Assuming that the deflections of the oscillograph are proportional to the applied voltage and that the velocity is proportional to the induced electromotive force,

$$V=kE=ke_{nq}D_v/D_c,$$

where k, the factor connecting V and E, is determined by a laboratory standardization [6] and is here taken as a known constant. As the deflection of the oscillograph may not be strictly proportional to the applied voltage, it is important that the calibrating potential, D_c, should cover nearly the same voltage range as the induced voltage, D_v, so that a method of interpolation can be employed if found necessary.

VI. Falling Weight Apparatus for Operating Switches

Figure 4 is a drawing of the falling weight and switches of the calibrator used in the ballistic investigation. It was designed to calibrate two oscillographs simultaneously, with five contacts in each of two independent sets. As the contacts are normally concealed, the insulating plate holding the front set of contacts is not shown in the drawing, but the plate and contacts are identical with those shown at the back. The three-point switches (see fig. 3) shown at left in the front set of contacts and at right in the back were used

[6] NDRC Report 229 (OSRD No. 2019) p. 39.

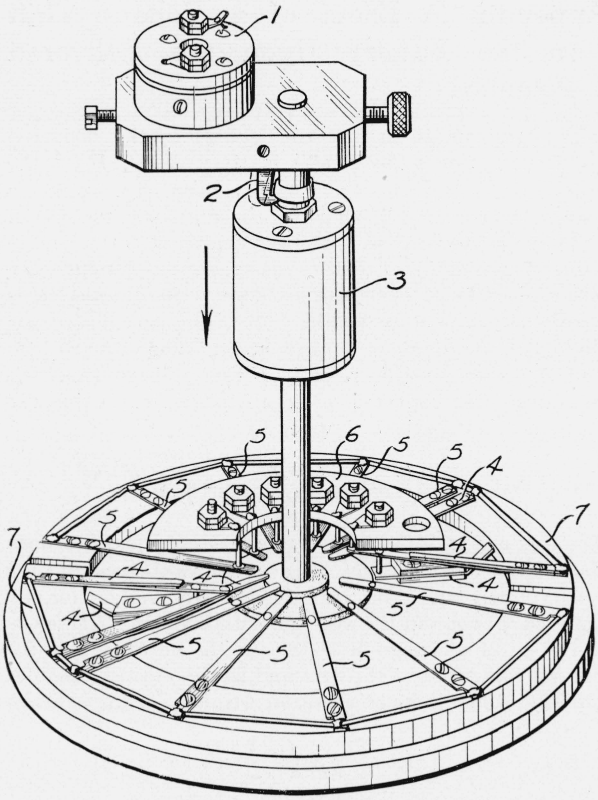

FIGURE 4. *Switches and falling weight of an oscillograph calibrator.*

1, Magnet; 2, holding latch; 3, weight; 4, three-point switch; 5, switches to open contacts in succession; 6, insulating plate; 7, insulating ring.

only with the velocimeter. They were opened about 5 msec before the opening of the first calibrating switch, and in no case did the vibrations of a switch persist for 5 msec. Each of the five contact switches (shown complete at the back) consists of a bronze strip attached to an insulating ring, and a silver stud attached to an insulating plate. With the falling weight raised, a silver contact point on each strip is pressed against its silver stud. Each stud was about 1 mm longer than the one preceding it—the shortest being next to the three-point switch. The shunt resistances are connected between the binding posts on the insulating plate and the common terminal on the insulating ring. All the switches were operated by the cylindrical falling weight. This was held above the contacts by a latch that opened when the releasing magnet was energized. The height was so adjusted that the falling weight operated the first contact a few milliseconds after the completion of the ballistic phenomenon. The falling weight apparatus is shown in figure 5.

The purpose of the calibrator in connection with a force, an acceleration, or a velocity that varies with time, is either to insure that the deflections of the oscillograph are proportional to the potential applied to its plates or to provide data by which the relationship can be evaluated graphically or analytically for all values of the applied potential. In practice, a calibration usually consists of five or six steps for the calibration voltage, e_{nq}. If the results of the calibration show that the deflection is not proportional to the applied voltage, but varies systematically, analytical methods can be used to express the relationship. In any case graphical methods can be employed. A useful method is to plot values of the ratio of the applied potential to the observed deflection as ordinates and values of the deflection as abscissa. Then for an observed deflection, the corresponding ratio can be read from the curve.

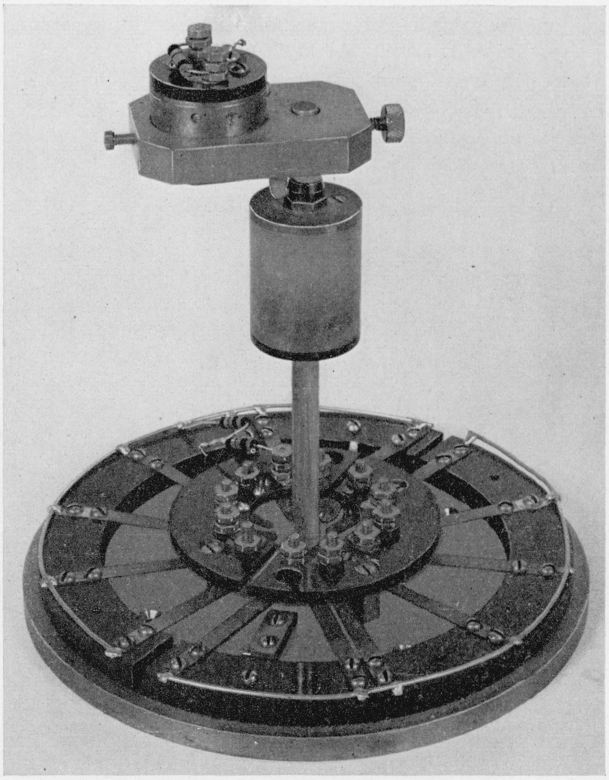

FIGURE 5. *Switches and falling weight of an oscillograph calibrator.*

Appendix 1. Derivation of Formula for Computing Shunt Resistances

The nomenclature is in accordance with that shown shown in figure 1.

E =electromotive force of battery
n =general integer in connection with decade resistance
q =general integer in connection with shunt resistance
e_{nq}=delivered potential (n-coils in decade resistance, q coils in shunt resistance)
P $=P_a+P_0$=control resistance
R $=R1+R2+\ldots+R10$, the decade resistance
$Rn=R1+R2\ldots+Rn$
Sq =resistance introduced into shunt circuit by opening contact q when all contacts of lower number are open
Q $=S1+S2+\ldots+Sq$, the shunt resistance when contacts 1 to q are open
I_0 =current in control resistance and in the decade resistance when the shunt resistance is infinite. This is normally 1 ma.
I_q =current in control resistance when shunt resistance is Q
iq =current in decade resistance when shunt resistance is Q
jq =current in shunt resistance when shunt resistance is Q

The problem is to find the value of S_q that will increase I_q by 0.1 ma when contact q is opened.

Note that when $q=0$, $Q=0$, and when $q=10$, $Q=\infty$. As $I_q=i_q+j_q$, $j_q=i_qR/Q$, and $i_q=qI_0/10$, it follows that

$$I_q=qI_0(R+Q)/10Q.$$

But

$$I_q=\frac{E}{P+\frac{RQ}{R+Q}}, \text{ and } I_0=\frac{E}{P+R}$$

Substituting in the preceding equation,

$$\frac{R+Q}{P(R+Q)+RQ}=\frac{q(R+Q}{10Q(R+P)}.$$

Solving

$$Q=\frac{RP}{R+P}\,\frac{q}{10-q}.$$

The value of Sq is $Q-(S_{q-1}+\ldots+S1)$ but $(S_{q-1}+\ldots+S1)$ can be obtained by substituting $q-1$ for q in the above equation for Q. Making this substitution and subtracting

$$Sq=\frac{RP}{R+P}\,\frac{10}{(10-q)(11-q)}.$$

Although this derivation appears to indicate that R can have any value from 0 to ∞, useful values of R are quite limited. If R is too small, the resistance of the contacts and connecting wires introducs troublesome corrections. If R is too large, the time constants of the circuit to which it is connected may be adversely affected.

Appendix 2. Effect of a Voltage Drift in the Battery Upon the Delivered Potential

The problem is to find the change in the delivered voltage, e_{nq}, for a small drift in the voltage, E, of the battery that furnishes the current, when this current is maintained constant by changing the adjustable resistance.

The shunt resistances have been computed for a definite value of E and I_0, so that e_{nq} has the desired steps only when both E and I_0 remain constant. In practice, I_0 is always adjusted to a definite value, but E will decrease slightly in the course of a few months. The problem is to find the permissible decrease in E that will not affect e_{nq} more than the expected observational error. Using the symbols and equations of appendix 1.

$$e_{nq}=R_ni_q=\frac{QR_nI_q}{R+Q}=\frac{QR_nE}{P(R+Q)+RQ}=\frac{QR_nI_0E}{E(R+Q)-R^2I_0}.$$

The first and last members of this series of equalities show the relationship, in terms of constant resistances and the constant current, I_0, between the delivered voltage for any value of n or q and the voltage of the battery. To show the effect of a change in voltage of the battery, take the natural logarithm of the first and last members, then obtain the differential of these logarithms. Thus

$$\frac{de_{nq}}{e_{nq}}=\frac{dE}{E}-\frac{dE(R+Q)}{E(R+Q)-R^2I_0}.$$

Substituting vaules for I_0 and Q, as given in appendix 1,

$$\frac{de_{nq}}{e_{nq}}=-\frac{dE}{E}\left[\frac{R(10-q)}{10P}\right].$$

The most unfavorable condition is when $q=1$. Then if de_{nq}/e_{nq} be less than 10^{-3}, with $E=4.5$ v and $R=100$ ohms, the permissible decrease, dE, in E is 0.22 v. But if E decreased by 0.22 v, I_0 can be maintained at 1 ma only by decreasing P by 220 ohms. Hence when P, which is 4,400 ohms with new cells, becomes as small as 4.180 ohms, the primary cells should be replaced by new ones.

Appendix 3. Derivation of the Calibration Factor for a Piezoelectric Gage

The following nomenclature corresponds to that used in figure 2 and accompanying description:

G=quantity of electricity developed on the gage by a force resulting from pressure or acceleration
C_s=capacitance of shunt capacitor
C_g=capacitance of gage
C_l=capacitance of shielded lead connecting gage to capacitor
V=potential produced by G on gage, leads and capacitor in parallel
e_{nq}=calibrating potential
V_g=potential across gage and leads in parallel when calibrating potential is e_{nq} and force is zero

D_f=deflection of oscillograph when force is F and $e_{nq}=0$

D_c=deflection of oscillograph when $F=0$ and e_{nq} is applied by the calibrator.

When a force, F, is applied to the gage and the calibrating potential is zero,

$$F=kG.$$

The factor k is determined by measuring[7] the charge developed by a known force (or acceleration) and is here assumed to be known.

When the force or acceleration is applied during a ballistic measurement, the charge developed is distributed on the shunt capacitor, the leads, and the gages, according to the capacitance of each, since they are connected in parallel. The potential of each is V, so that

$$V=aD_f=\frac{G}{C_g+C_l+C_s}.$$

[7] See NDRC Report A229 (OSRD No. 2019) p. 60; NDRC Report No. 323 (OSRD No. 4986) p. 47; NAR-ORD Report No. 430 p. 5q.

The factor, a, must be determined or procedures developed by which it can be eliminated. When the force or the acceleration becomes zero and the calibration voltage, e_{nq}, is applied, the oscillograph is connected to measure the potential, V_g, across the gage and its leads, which are now connected in parallel, and the combination in series with the shunt capacitor. Then

$$V_g=\frac{e_{nq}C_s}{C_g+C_l+C_s}=aD_c.$$

From these equations it follows that $F=ke_{nq}\,C_sD_f/D_c$. Hence the proportionality factor by which the deflection of the oscillograph is multiplied to give the force or acceleration is $ke_{nq}C_s/D_c$. In deriving this, the assumption was made that the factor, a, is a constant throughout the calibration range. When this is not the case, because of maladjustment of instruments, the value of a, for any given value of D_f, can be obtained from the calibration data treated either analytically or graphically.

WASHINGTON, March 3, 1948