A STUDY OF ELECTROMAGNET MOVING COIL GALVANOMETERS FOR USE IN ALTERNATING-CURRENT MEASUREMENTS

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CONTENTS

I. INTRODUCTION

The increasing use of alternating-current bridge and potentiometer methods of measurement emphasizes the need of an instrument for the detection and measurement of small alternating voltages. Instruments which have been used for this purpose are: The telephone receiver, the vibration galvanometer, the commutated direct-current galvanometer, the series and separately excited electrodynamometer, the vibration string galvanometer, and the vibration electrometer. Of these the telephone receiver and the vibration galvanometer are in most common use. The performances of the different instruments are so very different that some are more suitable for certain measurements than others. None of the instruments yet constructed has as good a performance as the sensitive moving coil galvanometers used in the corresponding direct-current measurements.

The separately excited electrodynamometer has been used to some extent as an alternating-current galvanometer, but usually the instruments were originally constructed for use in accurate measurements by deflection methods and therefore on account of the stiff suspensions, large coils, weak fields, etc., were not very sensitive. Rowland [1] used the electrodynamometer as a detector in various alternating-current bridge measurements of resistance, inductance, and capacity, but his instruments were not very sensitive. A more sensitive one has recently been designed by Palm.[2]

Stroud and Oates [3] were the first to use a moving coil galvanometer with an electromagnet excited by alternating current. This was found to be very sensitive and was used in bridge measurements of inductance and capacity. They observed effects due to the induced currents in the moving coil and in the frame on which the coil was wound and found that it was necessary to work with a false zero. The complete explanation of the effects observed was not given.

Sumpner and Phillips [4] designed iron-cored instruments and devised a number of methods of measurements, using them separately excited as detectors. Sumpner [5] gave special attention to the design of alternating-current electromagnets in which the magnetic flux is very nearly in quadrature with the impressed voltage. Moving coil instruments with such electromagnets were found to be useful in measurements by deflection methods.

H. Abraham [6] in 1906 described briefly an electromagnet moving coil galvanometer and showed that its performance depended upon the constants of the moving coil circuit. The effects of electromagnetic damping were observed and it was found possible by suitable adjustment of the constants of the moving coil circuit to make the motion aperiodic. He also observed that, on account of the induced current, the period and hence the sensitivity

[1] Physical Papers, pp. 294, 314; 1897.

[2] Zeit. Instr. **33**, p. 368, 1913; **34**, p. 281, 1914.

[3] Phil. Mag., **6**, p. 707; 1903.

[4] Proc. Phys. Soc., **22**, p. 395; 1908.

[5] Proc. Roy. Soc., **80**, p. 310, 1908; Jour. Inst. E. E., **36**, p. 421, 1905.

[6] Comptes Rendus, **142**, p. 993; 1906.

depended upon the reactance of the moving coil circuit. He found that, by compensating for the inductance of the moving coil by means of a suitable condenser in parallel with a resistance, the effect of the induced current could be made zero. This method of compensation for inductance was pointed out before by Sumpner[7] and applied to telephone circuits. The same method was used by Trowbridge at the University of Wisconsin in 1905 to compensate for the inductance of the moving coil of an electromagnet galvanometer. Rosa[8] used this method in 1906 to compensate for the inductance of the moving coil of an electrodynamometer.

The step from the separately excited electrodynamometer to the electromagnet moving coil galvanometer is a natural one in the development of a sensitive alternating-current galvanometer. The use of iron in the fixed coil of the instrument results in a marked increase in the strength of the magnetic field, so that the induced current is sufficient to affect the performance. It was therefore considered necessary for the proper design, construction, and use of such instruments to make a theoretical study taking these effects into account. The investigation has led to the construction of instruments having sensitivities much greater than those previously obtained and equal to those of the best permanent magnet moving coil galvanometers for direct-current measurements. The high sensitivity and good values for the other working constants result from the use of a strong magnetic field and a moving system properly designed for the conditions of use. Methods of testing and using have been worked out. The use of such instruments is not as simple as that of the telephone receiver or the vibration galvanometer, but it is believed that in many cases the gain in sensitivity and usefulness more than compensates for the trouble involved.

II. THEORY

1. ASSUMPTIONS

In order to determine the operation of electromagnet moving coil galvanometers of various kinds and under various conditions it was necessary to derive the general equation of motion of a coil carrying an alternating current in an alternating magnetic field. To do this it is necessary to consider all of the torques acting on the moving coil.

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[7] Journ. Soc. Tel. Eng., 16, p. 344; 1887.

[8] This Bulletin, 3, p. 43; 1907.

In most cases the moving coil is made of fine wire, so that the assumption of a linear moving coil circuit introduces no appreciable errors arising from the variation of the current distribution in the cross section of the wire. Therefore, we consider the electromagnet moving coil galvanometer to consist of a rigid linear circuit suspended so as to rotate through a small angle in an alternating magnetic field. The magnetic field will be assumed to be approximately uniform and to have no rotating components[9] around the axis, so that it is possible to turn the coil to a position in which the resultant total magnetic flux is always zero. This is the normal position of the coil and only small deflections from a reference position near this will be considered.

2. NOTATION

To show the relations between the intrinsic constants, which depend upon the construction, and the performance under certain conditions it will be convenient to use the following symbols. Unless otherwise stated the quantities are to be expressed in electromagnetic cgs units.

Symbol — Quantity

θ = Angular deflection from reference position.

t = Time.

K = Moment of inertia of moving system.

D = Moment of damping of moving system.

U = Moment of restoration acting on moving system.

$\omega = 2\pi$ times the fundamental frequency.

n = Degree of the harmonic ($n = 1$ for fundamental).

φ = Total instantaneous magnetic flux through the coil produced by the separate excitation.

G_n = Rate of change with deflection of the effective value of the n^{th} harmonic of the total flux.

α_n = Phase angle of lead of the n^{th} harmonic flux ahead of the fundamental flux.

θ_0 = Deflection for no flux through the coil.

i = Total instantaneous current through the moving coil.

e_1 = Instantaneous value of the impressed electromotive force in the moving coil circuit.

E_n = Effective value of the n^{th} harmonic of the impressed electromotive force.

β_n = Phase angle of lead of the n^{th} harmonic of the impressed electromotive force ahead of the fundamental flux.

e_2 = Instantaneous induced electromotive force in coil.

Z_n = Impedance of the complete moving coil circuit to current of n^{th} harmonic frequency.

γ_n = Phase angle of lead of the n^{th} harmonic of the current ahead of electromotive force producing it.

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[9] In general, the alternating field at any point in space may be considered as made up of two unidirectional alternating fields at right angles and a constant rotating field always perpendicular to one of these.

T=Complete undamped (free) period of the moving system.
T_1=Complete period on open circuit.
λ=Logarithmic decrement.[10]
λ_1=Logarithmic decrement on open circuit.
S_n=Sensitivity to n^{th} harmonic electromotive force.
S=Sensitivity when critically damped.
r=Resistance of moving system.
R'=Total resistance of moving coil circuit.
R=External critical resistance.

3. EQUATION OF MOTION

Since, in general, there will be harmonics in the magnetic field of the electromagnet depending upon the manner of excitation, the flux through the moving coil due to the excitation is expressed as

$$\phi = \sum_{n=1}^{n=\infty} \sqrt{2}G_n(\theta - \theta_o) \sin (n\omega t + \alpha_n) \tag{1}$$

or this can be written

$$\phi = \theta \sum_{n=1}^{n=\infty} \sqrt{2}G_n \sin (n\omega t + \alpha_n) - \theta_o \sum_{n-1}^{n=\infty} \sqrt{2}G_n \sin (n\omega t + \alpha_n) \tag{2}$$

Now the torque acting on the moving coil when the current i flows is given by[11]

$$i\frac{d\phi}{d\theta} = i \sum_{n=1}^{n=\infty} \sqrt{2}G_n \sin (n\omega t + \alpha_n) \tag{3}$$

On equating this to the sum of the moments due to angular acceleration, damping, and restoration we obtain the differential equation of motion

$$K\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + U\theta = i \sum_{n=1}^{n=\infty} \sqrt{2}G_n \sin (n\omega t + \alpha_n) \tag{4}$$

The current in the moving coil can be considered to be the result of two electromotive forces, the impressed electromotive force and the induced electromotive force. If the impressed electromotive force contains harmonics and if it is in synchronism with the magnetic field we can represent it by

$$e_1 = \sum_{n=1}^{n=\infty} \sqrt{2}G_n \sin (n\omega t + \beta_n) \tag{5}$$

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[10] The logarithmic decrement is here taken to be the natural logarithm of the ratio of one deflection to the following deflection in the opposite direction.

[11] The reason for neglecting the effect of the flux due to i is given on p. 40.

The induced electromotive force is

$$
\begin{aligned}
e_2 = -\frac{d\phi}{dt} = &-\theta \sum_{n=1}^{n=\infty} \sqrt{2}n\omega G_n \cos(n\omega t + \alpha_n) \\
&-\frac{d\theta}{dt} \sum_{n=1}^{n=\infty} \sqrt{2}G_n \sin(n\omega t + \alpha_n) \\
&+\theta_o \sum_{n=1}^{n=\infty} \sqrt{2}n\omega G_n \cos(n\omega t + \alpha_n)
\end{aligned}
\tag{6}
$$

giving for the total current

$$
\begin{aligned}
i = &\sum_{n=1}^{n=\infty} \frac{\sqrt{2}E_n}{Z_n} \sin(n\omega t + \beta_n + \gamma_n) \\
&-\theta \sum_{n=1}^{n=\infty} \frac{\sqrt{2}n\omega G_n}{Z_n} \cos(n\omega t + \alpha_n + \gamma_n) \\
&-\frac{d\theta}{dt} \sum_{n=1}^{n=\infty} \frac{\sqrt{2}G_n}{Z_n} \sin(n\omega t + \alpha_n + \gamma_n) \\
&+\theta_o \sum_{n=1}^{n=\infty} \frac{\sqrt{2}n\omega G_n}{Z_n} \cos(n\omega t + \alpha_n + \gamma_n)
\end{aligned}
\tag{7}
$$

Substituting in (3) we get for the moment of displacement

$$
\sum_{n=1}^{n=\infty} \sqrt{2}G_n \sin(n\omega t + \alpha_n)
\left[
\begin{aligned}
&\sum_{n=1}^{n=\infty} \frac{\sqrt{2}E_n}{Z_n} \sin(n\omega t + \beta_n + \gamma_n) \\
&-\theta \sum_{n=1}^{n=\infty} \frac{\sqrt{2}n\omega G_n}{Z_n} \cos(n\omega t + \alpha_n + \gamma_n) \\
&-\frac{d\theta}{dt} \sum_{n=1}^{n=\infty} \frac{\sqrt{2}G_n}{Z_n} \sin(n\omega t + \alpha_n + \gamma_n) \\
&+\theta_o \sum_{n=1}^{n=\infty} \frac{\sqrt{2}n\omega G_n}{Z_n} \cos(n\omega t + \alpha_n + \gamma_n)
\end{aligned}
\right]
\tag{8}
$$

From this it follows that the resultant torque due to the alternating current and magnetic field of frequency $\frac{\omega}{2\pi}$ is the sum of a constant torque (the average of the above) and a series of sinusoidal torques of frequency $\frac{\omega}{2\pi}$ and higher. If, as is usually the case, no even harmonics are present, the sinusoidal torques are of frequency $\frac{\omega}{\pi}$ and higher; that is, have frequencies equal to and greater than twice the fundamental frequency. Now, this is large compared with $1/T$, so that the motion will be very nearly the same as if a torque equal to the average of the above were acting.[12] The average torque as calculated from the instantaneous torque given above is

$$\begin{aligned}\sum_{n=1}^{n=\infty}\frac{G_n E_n}{Z_n}\cos(\alpha_n-\beta_n-\gamma_n)+\theta\sum_{n=1}^{n=\infty}\frac{n\omega\, G_n^2}{Z_n}\sin\gamma_n\\ -\frac{d\theta}{dt}\sum_{n=1}^{n=\infty}\frac{G_n^2}{Z_n}\cos\gamma_n \qquad -\theta_o\sum_{n=1}^{n=\infty}\frac{n\omega\, G_n^2}{Z_n}\sin\gamma_n\end{aligned}\tag{9}$$

We can now write the equation of motion as follows:

$$\begin{aligned}K\frac{d^2\theta}{dt^2}+\left(D+\sum_{n=1}^{n=\infty}\frac{G_n^2}{Z_n}\cos\gamma_n\right)\frac{d\theta}{dt}+\left(U-\sum_{n=1}^{n=\infty}\frac{n\omega\, G_n^2}{Z_n}\sin\gamma_n\right)\theta\\ =\sum_{n=1}^{n=\infty}\frac{G_n E_n}{Z_n}\cos(\alpha_n-\beta_n-\gamma_n)-\theta_o\sum_{n=1}^{n=\infty}\frac{n\omega\, G_n^2}{Z_n}\sin\gamma_n\end{aligned}\tag{10}$$

which is of the form

$$K\frac{d^2\theta}{dt^2}+D'\frac{d\theta}{dt}+U'\theta=A+B \tag{11}$$

where D', U' and B depend upon the constants of the galvanometer and the moving coil circuit and A depends upon the impressed electromotive force in the moving coil circuit. This is the familiar

[12] Actually the motion is the sum of the slow motion resulting from the average torque and a vibration containing all of the harmonics to some extent. It can be expressed as a Fourier's series, but the coefficients are very complicated functions of the intrinsic constants. The magnitude of the error in determining the deflection due to the blurring of the image on account of the vibration of the coil can be estimated by inspection of the equation of motion for the simple case of sine wave flux and impressed electromotive force which is

$$\begin{aligned}K\frac{d^2\theta}{dt^2}+\left(D+\frac{G^2}{Z}\cos\gamma\right)\frac{d\theta}{dt}+\left(U-\frac{\omega G^2}{Z}\sin\gamma\right)\theta=\frac{GE}{Z}\cos(\beta+\gamma)-\theta_o\frac{\omega G^2}{Z}\sin\gamma-\frac{GE}{Z}\cos(2\omega t+\beta+\gamma)\\ -\frac{d\theta}{dt}\frac{G^2}{Z}\cos(2\omega t+\gamma)+(\theta-\theta_o)\frac{\omega G^2}{Z}\sin(2\omega t+\gamma)\end{aligned}$$

The maximum values of the alternating torques are

$$\frac{GE}{Z},\ \frac{d\theta}{dt}\frac{G^2}{Z}\sin\gamma,\text{ and }(\theta-\theta_o)\frac{\omega G^2}{Z}$$

and the vibration produced by these can be calculated approximately by the equation

$$\psi'=\left(\frac{\pi/\omega}{T}\right)^2\psi$$

where ψ' is the amplitude of the vibration and ψ is the deflection which a constant torque equal to the maximum value of the alternating torque would produce. This formula is not correct when T approaches the value $\frac{\pi}{\omega}$; that is, near mechanical resonance. It is interesting to note that with a tight suspension and a small coil—that is, when T is small—the instrument becomes a "vibration galvanometer" which may be made to vibrate with twice the frequency of the magnetic field and hence be tuned to the frequency of the alternating torques.

equation of an oscillating system acted upon by the torques A and B and the particular solution shows that the motion may be either underdamped, aperiodic (critically damped or dead beat) or overdamped according as $(D')^2$ is less than, equal to, or greater than $4KU'$. The free period is

$$T = 2\pi\sqrt{\frac{K}{U'}} \tag{12}$$

This is the undamped period and equals the time required for the coil to deflect 99 per cent of the total deflection when critically damped. It is therefore a measure of the speed of the instrument. The logarithmic decrement is

$$\lambda = \frac{\pi D'}{\sqrt{4KU' - (D')^2}} \tag{13}$$

The torques A and B produce deflections from the equilibrium position equal to A/U' and B/U'.

The period not only depends upon the moment of inertia of the moving system and moment of restoration of the suspension, but also upon the constants of the moving coil circuit and the excitation and may be larger or smaller than the period without excitation. The damping is always larger with excitation. The torque B produces a deflection of the coil from its equilibrium position even when there is no impressed electromotive force. This deflection is proportional to the angle θ_0 and is due to the induced current in the circuit. The torque A which results from the impressed electromotive force produces a deflection depending upon the magnitudes of the harmonics in the excitation and in the electromotive force, upon the phase relations and upon the constants of the moving coil circuit.

4. SIMPLE CASE

For the simple ideal case of a sine-wave magnetic field and a sine wave of impressed electromotive force of the same frequency the equation (10) reduces to

$$\begin{aligned} K\frac{d^2\theta}{dt^2} + \left(D + \frac{G^2}{Z}\cos\gamma\right)\frac{d\theta}{dt} + \left(U - \frac{\omega G^2}{Z}\sin\gamma\right)\theta \\ = \frac{GE}{Z}\cos(\beta+\gamma) - \theta_0\frac{\omega G^2}{Z}\sin\gamma \end{aligned} \tag{14}$$

In some cases γ, if not already zero, is made so by the use of suitable impedance in the moving coil circuit. Under these conditions (14) becomes

$$K\frac{d^2\theta}{dt^2}+\left(D+\frac{G^2}{R'}\right)\frac{d\theta}{dt}+U\theta=\frac{GE}{R'}\cos\beta \tag{15}$$

This shows that when an electromotive force E is inserted in the moving coil circuit the coil deflects through an angle

$$\theta=\frac{GE}{R'U}\cos\beta \tag{16}$$

so that the deflection is proportional to the component of the electromotive force in phase with the magnetic field. The motion of the coil to this new position depends upon the resistance R' and G and if

$$\left(D+\frac{G^2}{R'}\right)^2=4\,KU \tag{17}$$

or

$$R'=\frac{G^2}{\sqrt{4KU}-D}=R+r \tag{18}$$

the motion is aperiodic.

The operating constants of importance when the electromagnet moving coil galvanometer is used as a detector in bridge or potentiometer measurements are the sensitivity S, the free period T, and the external critical resistance R. This sensitivity is defined as the deflection per unit electromotive force in the moving coil circuit when the motion is aperiodic and is expressed in millimeters deflection at 1 meter of the reflected ray per microvolt. The free period is in seconds and the resistance in ohms. The relations between these constants and the intrinsic constants follow from equations (12), (16), and (18), and are

$$S=200\,000\,\frac{\sqrt{4KU}-D}{GU} \tag{19}$$

$$T=2\pi\sqrt{\frac{K}{U}} \tag{20}$$

$$R=10^{-9}\frac{G^2}{\sqrt{4KU}-D}-r \tag{21}$$

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For these conditions the operating constants are independent of frequency and are the same as for the direct-current galvanometer.

In order to design an instrument for use as a detector in a certain kind of measurement it is necessary to be able to calculate the intrinsic constants from the required operating constants. The necessary relations are readily obtained from equations (19), (20), and (21), and are

$$K = 0.161 \frac{T^3}{S^2(r+R)} m \tag{22}$$

$$U = 6.4 \frac{T}{S^2(r+R)} m \tag{23}$$

$$G = 31\ 800 \frac{T}{S} m \tag{24}$$

where

$$m = 1 \pm \sqrt{1 - 0.99 \frac{S^2(r+R)}{T^2} D}$$

We see from this that the damping D can not be greater than $\frac{T^2}{0.99 S^2 (r+R)}$. Furthermore, the $\pm$ sign indicates that two designs are possible, one with all of the constants K, U, and G larger than the corresponding constants in the other design. The larger constants are the more easily obtained.[13]

5. EFFECTS OF REACTANCE

If the moving coil circuit is inductive we see from equation (14) that one effect is a deflection of the coil on account of the additional torque

$$-\theta_0 \frac{\omega G^2}{Z} \sin \gamma$$

If the circuit has inductance (γ negative) the coil turns toward the position of zero flux and if the circuit has capacity it turns in the opposite direction. The effect is zero, however, if θ_0 is zero; that is, if the equilibrium position of the coil with the circuit open is such that no flux links with it. A second effect of γ being different from zero is that the period is changed on account of the torque given by the term

$$-\theta \frac{\omega G^2}{Z} \sin \gamma$$

[13] For a discussion of the design of critically damped galvanometers see Wenner, this Bulletin, 13, 1916. Scientific Paper No. 273.

Thus if γ is negative (lagging current) the period is shortened on account of the increased restoring torque, and if γ is positive the period is lengthened. If the factor

$$\frac{\omega G^2}{Z} \sin \gamma \text{ becomes greater than } U$$

the coil is in unstable equilibrium and will, if displaced slightly from its zero position, turn suddenly toward a position at right angles to the field. Stroud and Oates[14] observed the shortening of the period, and the fact that the equilibrium position depends, in general, upon the impedence of the moving coil circuit. A third effect is the increase in damping, which for small angles of lead or lag is negligible.

The change in the restoring torque resulting from the reactance in the moving coil circuit causes a change in the period, damping, and sensitivity according to the following relations:

$$T = 2\pi \sqrt{\frac{K}{U - \frac{\omega G^2}{Z} \sin \gamma}} \tag{25}$$

$$\lambda = \frac{\pi \left(D + \frac{G^2}{Z} \cos \gamma\right)}{\sqrt{4K\left(U - \frac{\omega G^2}{Z} \sin \gamma\right) - \left(D + \frac{G^2}{Z} \cos \gamma\right)^2}} \tag{26}$$

$$S_1 = 200\,000 \frac{G}{Z\left(U - \frac{\omega G^2}{Z} \sin\gamma\right)} \tag{27}$$

These follow immediately from equations (12), (13), and (14), and give the performance of the electromagnet moving coil galvanometer when the motion is underdamped.

When the damping D and the resistance r are small, the equations (19), (20), and (21) can be put in the more convenient form

$$T = 2\pi \sqrt{\frac{K}{U}} \tag{28}$$

$$S = \frac{200\,000}{\pi} \frac{T}{G} \tag{29}$$

$$R = \frac{10^{-9}}{4\pi} \frac{G^2 T}{K} \tag{30}$$

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[14] Phil. Mag., 6, p. 707; 1903.

and if the U becomes $U - \frac{\omega G^2}{Z} \sin\gamma$ these become

$$T' = 2\pi\sqrt{\frac{K}{U - \frac{\omega G^2}{Z}\sin\gamma}} \tag{31}$$

$$S' = \frac{200\ 000}{\pi}\frac{T'}{G} = \frac{T'}{T} S \tag{32}$$

$$R' = \frac{10^{-9}}{4\pi}\frac{G^2 T'}{K} = \frac{T'}{T} R \tag{33}$$

showing that the period, sensitivity, and the external critical resistance change in the same proportion when the constants of the moving coil circuit are changed. We can therefore in this manner change the operating constants of the instrument without an actual change in the construction.

The intrinsic constants K, D, and U can readily be determined for any particular galvanometer by methods described later. G can also be determined by these methods for the value of the excitation current used and being approximately proportional to this current can be calculated for other values. If the electromagnet moving coil galvanometer is to be used at a certain frequency with apparatus having a known resistance and reactance between its "galvanometer terminals," it is possible by the use of equations (25), (26), and (27) to adjust Z, γ, and the excitation to give the most satisfactory operation under the conditions of use.

In the adjustment of Z and γ to the best values it is sometimes necessary to use series or parallel resistance, inductance, or capacity. Sometimes the circuit containing the small electromotive force to be measured has a reactance which is large compared with its resistance, for example, a bridge for measuring large inductances or small capacities. The general aim in these cases is to produce an electrical resonating system [15] so that the total reactance in the moving coil circuit is zero or such a value as will give the best operating constants. Obviously if such a system is used it is necessary to maintain the frequency constant.

[15] Wenner, Weibel, and Silsbee, this Bulletin, 12, p. 18; 1915.

6. EFFECTS OF HARMONICS

To return now to the more general case in which the motion is that given by equation (10) we see that the effects of harmonics in the magnetic field and the impressed electromotive force can be considered as additions to the various torques acting on the coil. The result of the presence of the harmonics is to change the equilibrium position for no impressed electromotive force, to change the period, to change the damping, and to give a deflection proportional to the sum of the products of the harmonic flux and harmonic electromotive force and the cosine of the angle between them. If a given n^{th} harmonic electromotive force be impressed, the maximum deflection produced as its phase is changed gives a measure of G_n/Z_n and therefore of G_n since Z_n can be measured. This furnishes a means of measuring the selectivity of the galvanometer. Usually the magnitudes of the harmonics are small compared with the fundamental. The selectivity can be still further increased by using an electrical resonating system in the moving coil circuit. The effect of this is to make the impedance to the harmonics very large compared with the impedance to the fundamental, so that G_n/Z_n and hence the sensitivity to the harmonics is very small. The use of such a device, however, decreases the deflection with no impressed electromotive force. These changes are very small since they diminish as G_n/Z_n diminishes.

7. DETERMINATION OF CONSTANTS

The constants Z_1, Z_2, Z_3, etc., and γ_1, γ_2, γ_3, etc., depend upon the resistance, inductance, and capacity of the moving coil circuit and can usually be easily measured. E_1, E_2, E_3, etc., and β_1, β_2, β_3, etc., depend upon the conditions of use. The intrinsic constants K, D, U, θ_0, G_1, G_2, G_3, etc., and α_1, α_2, α_3, etc., depend upon the construction and excitation, and can all be determined experimentally as follows:

(*a*) The period T_1 and logarithmic decrement λ_1 are measured on open circuit.

(*b*) The period T_2 and logarithmic decrement λ_2 are measured with the moving coil connected to a fairly noninductive resistance R_2 of such a value that the decrement is about 10 times that on open circuit.

(*c*) With the moving coil connected to a very high noninductive resistance R_1 the sensitivity S_n of the instrument to electromotive forces of the frequencies of the harmonics is determined for various phase relations of the harmonics with respect to the fundamental.

(*d*) The deflection of the coil when it is connected to an inductance gives θ_0 and by adjustment of the reference position of the coil—that is, by twisting the suspension slightly—θ_0 can be made zero.[16]

These data are sufficient for the calculation of all of the intrinsic constants under the conditions of excitation, etc., that exist during the measurements. The most important of the intrinsic constants can be calculated from the observed data by the use of the following formulas in the order given.

$$K = \frac{(r+R_2)\ T_1^4}{4\pi^4\ (r+R_1)^2 \sum_{n=1}^{n=\infty} S^2_n}\left(\frac{\lambda_2}{T_2} - \frac{\lambda_1}{T_1}\right)\left(1 - \frac{\lambda_1^2}{\pi^2}\right)^2 \tag{34}$$

$$U = \frac{4\pi^2}{T_1^2}\left(1 - \frac{\lambda_1^2}{\pi^2}\right)^{-1} K \tag{35}$$

$$D = \frac{4\lambda_1}{T_1} K \tag{36}$$

$$G_1 = S_1\ (r+R_1)\ U,\ G_2 = S_2\ (r+R_1)\ U, \text{ etc.} \tag{37}$$

The deduction of these relations is based upon the general equation of motion as given above (equation 10) and is as follows:

(*a*)

In this case the equation of motion becomes

$$K\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + U\theta = 0 \tag{38}$$

which gives for the period

$$T_1 = 2\pi\sqrt{\frac{K}{U}}\left(1 - \frac{D^2}{4KU}\right)^{-\frac{1}{2}} \tag{39}$$

and for the decrement

$$\lambda_1 = \frac{\pi D}{\sqrt{4KU}}\left(1 - \frac{D^2}{4KU}\right)^{-\frac{1}{2}} \tag{40}$$

[16] If a small variable mutual inductance be used with one coil in the moving coil circuit and the other in the fixed coil circuit, this adjustment need only be made approximately. Then adjustment of the mutual inductance will make the total flux through the moving coil circuit zero for the position of the moving coil on open circuit. With this adjustment made the vibration is also least, and the equilibrium position of the coil with no impressed electromotive force is independent of the impedance of the circuit. If the magnetic flux is proportional to the current, the adjustment is made when

$$M\ I = G\theta_0.$$

(*b*)

The equation of motion is now

$$K\frac{d^2\theta}{dt^2}+\left(D+\sum_{n=1}^{n=\infty}\frac{G_n^2}{r+R_2}\right)\frac{d\theta}{dt}+U_2\theta=0 \tag{41}$$

which we see by comparison with (10) is true if the resistance is sufficiently noninductive, so that the cosine differs but little from one and Z differs but little from R_2 at all frequencies for which G_n is at all appreciable. The moment of restoration may be different, as is likely even if the resistance is sufficiently noninductive to make the above assumption valid.

As in (*a*) above this gives the period and decrement as follows:

$$T_2=2\pi\sqrt{\frac{K}{U_2}}\left[1-\frac{\left(D+\sum_{n=1}^{n=\infty}\frac{G_n^2}{r+R_2}\right)^2}{4KU_2}\right]^{-\frac{1}{2}} \tag{42}$$

$$\lambda_2=\frac{\pi\left(D+\sum_{n=1}^{n=\infty}\frac{G_n^2}{r+R_2}\right)}{\sqrt{4KU_2}}\left[1-\frac{\left(D+\sum_{n=1}^{n=\infty}\frac{G_n^2}{r+R_2}\right)^2}{4KU_2}\right]^{-\frac{1}{2}} \tag{43}$$

(*c*)

With a very high noninductive resistance in the moving coil circuit the equation of motion is assumed to be

$$K\frac{d^2\theta}{dt^2}+D\frac{d\theta}{dt}+U\theta=\sum_{n=1}^{n=\infty}\frac{G_n E_n}{r+R_1}\cos(\alpha_n-\beta_n-\gamma_n) \tag{44}$$

Referring to (10) we see that the errors due to the omission of the damping term, the restoring term, and the deflecting term can be made as small as desired by making Z—that is, R_1—large. Further, the period, decrement, and zero can be observed and should check with the observations in (*a*). An observation under these conditions gives

$$S_n=\frac{\Delta\theta}{\Delta E_n}=\frac{G_n}{(r+R_1)U} \tag{45}$$

so we get

$$S_1=\frac{G_1}{(r+R_1)U},\ S_2=\frac{G_2}{(r+R_1)U},\ \text{etc.} \tag{46}$$

(*d*)

From (10) we see that the rest point with an inductance in the moving coil circuit and no impressed electromotive force is

$$\theta = \frac{\sum_{n=1}^{n=\infty} \frac{nG_n^2}{L}}{U + \sum_{n=1}^{n=\infty} \frac{nG_n^2}{L}} \theta_0 \tag{47}$$

and if $\sum_{n=1}^{n=\infty} \frac{nG_n^2}{L}$ is large compared with U, this becomes θ_0. When this is true the period is very short, since U' is now many times U.

In (*a*), (*b*), and (*c*) the observed quantities are T_1, λ_1, T_2, λ_2, R_2, R_1, S_1, S_2, S_3, etc., and from these we are to determine K, D, U, G_1, G_2, G_3, etc., by using equations (39), (40), (42), (43), and (46). From (39) and (40) we get by division and rearrangement

$$D = \frac{4K}{T_1} \lambda_1 \tag{48}$$

In a similar manner we get from (42) and (43)

$$D + \sum_{n=1}^{n=\infty} \frac{G^2_n}{r+R_2} = \frac{4K}{T_2} \lambda_2 \tag{49}$$

Substituting (48) in (39) gives

$$T_1 = 2\pi \sqrt{\frac{K}{U}} \left(1 - \frac{\lambda_1^2}{\pi^2}\right)^{-1/2} \tag{50}$$

so that

$$U = \frac{4\pi^2}{T_1^2} \left(1 - \frac{\lambda_1^2}{\pi^2}\right)^{-1} K \tag{51}$$

Using this in (45) we get

$$G_n = \frac{4\pi^2 S_n (r+R_1)}{T_1^2} \left(1 - \frac{\lambda_1^2}{\pi^2}\right)^{-1} K \tag{52}$$

Now, on substituting this and (48) in (49) we find that

$$K = \frac{(r+R_2) T_1^4}{4\pi^4 (r+R_1) \sum_{n=1}^{n=\infty} S^2_n} \left(\frac{\lambda_2}{T_2} - \frac{\lambda_1}{T_1}\right) \left(1 - \frac{\lambda_1^2}{\pi^2}\right)^2 \tag{53}$$

Which, together with (52), (51), and (48) give the relations desired.

The phase angles α_1, α_2, α_3, etc., can be determined from the observed phase changes necessary to give maximum (or zero, which is more accurate) deflection for the various harmonics.

From a few observations it is therefore possible to obtain all of the intrinsic constants and hence determine the performance of the electromagnet moving coil galvanometer under various conditions of use.

8. DISTURBING TORQUES

It has been assumed in the preceding development of the theory of the electromagnet moving coil galvanometer that the only torques acting were those contained in equation (10), which are: (1) The torque of reaction due to the acceleration; (2) the torque due to the constant damping D; (3) the torque due to the suspension; (4) the torque due to the current resulting from the impressed electromotive force; and (5) the torque due to the current resulting from the induced electromotive force.

Besides these torques there are, in general, others which we shall call disturbing torques. An analysis of these torques, using the results of the preceding considerations, shows the causes and indicates the means of making the disturbances small. These disturbing effects will now be considered in turn. Among them there are several which are similar in that their magnitude is proportional to the deflection from some position. The effect of such a torque is, in general, to change the period and cause a permanent deflection. For if we let the disturbing torque be $-u(\theta-\theta')$ and the torque of the suspension is $-U\theta$, we get for the total torque

$$-U\theta-u(\theta-\theta')=-(U+u)\theta+u\theta' \tag{54}$$

and if for simplicity we equate this to $K\frac{d^2\theta}{dt^2}$ we see that the resulting period is

$$T'=2\pi\sqrt{\frac{K}{U+u}}=2\pi\sqrt{\frac{K}{U}}\left(1+\frac{u}{U}\right)^{-1/2}=T_0\left(1+\frac{u}{U}\right)^{-1/2} \tag{55}$$

and if u/U is small, this becomes

$$T_0\left(1-\tfrac{1}{2}\frac{u}{U}\right) \tag{56}$$

Also we see that the new rest point is

$$\theta''=\frac{u}{U+u}\theta' \tag{57}$$

This is a position of stable equilibrium if u is between $-U$ and ∞. As u decreases approaching $-U$ the period becomes longer and for smaller values the coil is not in stable equilibrium.

One disturbing torque of this kind is that due to the electrostatic attraction between the coil and the case, pole faces, etc., due to a difference of potential. This torque is proportional to the square of the potential difference and to the rate of change of the capacity between the coil and the case with deflection. The coil will therefore tend to a definite position and the disturbing torque will be approximately proportional to the deflection from this position. Obviously this disturbance requires only one lead to the coil for its existence. This torque has been definitely identified in several instruments and it was found that a potential difference of 20 or 30 volts was necessary in order to cause serious disturbances. The coil and the case should therefore be maintained at approximately the same potential.

A similar torque is that due to the change of the self-inductance of the coil as it turns. In the equation (3) we have used only the flux due to the excitation. The total torque is really

$$i\frac{d\phi}{d\theta}+\frac{i^2}{2}\frac{dL}{d\theta} \tag{58}$$

Now the torque $\frac{i^2dL}{2d\theta}$ has been neglected. For the normal position of the coil the inductance will be a minimum, so that $\frac{d^2L}{d\theta^2}$ will be positive and for small angles $\frac{dL}{d\theta}$ is proportional to the deflection. Hence this torque tends to turn the coil out of the normal position and corresponds to u having a negative value. In sensitive instruments i is never large and $\frac{dL}{d\theta}$ is small, so that $\frac{i^2}{2}\frac{dL}{d\theta}$ is negligible compared with $i\frac{d\phi}{d\theta}$.

A very serious disturbing torque of this kind is that resulting from the fact that the materials of which the moving system is made have a finite susceptibility.[17] If magnetic impurities are present in the form of small particles, these will be acted upon by forces proportional to the gradient of H^2 and hence to H^2 since the field remains similar to itself as the intensity is varied. The same is true to a less extent for the materials of the coil with a

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[17] Ayrton and Mather, Phil. Mag., 42, p. 442, 1896; Anthony Zeleny, Phys. Rev., 23, p. 401, 1906.

susceptibility a little different from that of the surrounding air. Now, the resultant torque will obviously be zero for some position near the normal one, for usually the field is very nearly uniform and the lack of uniformity is symmetrical with respect to this position. This disturbing torque is very evident in sensitive instruments and, if care is not taken, may be much greater than that due to the suspension. The magnetic dirt which is usually on the surface can be removed by suitable treatment (p. 45). The magnetic field should be of suitable distribution. Thus, for a moving system whose parts are cylindrical, with axes parallel to the axis of rotation, a uniform field gives no torque. For other shaped systems the field should be radial, with its center on the axis of rotation of the coil.

A disturbing torque which is of this same kind is that resulting from the capacity current between turns of the moving coil. Let us replace this distributed capacity by a capacity localized at the terminals. Then it follows from equation (14) that in the simple case this would give a torque

$$\frac{\omega G^2}{Z} \sin \gamma \ (\theta - \theta_0) = \omega^2 G^2 C \ (\theta - \theta_0) \tag{59}$$

which is directly proportional to the capacity, to the square of the frequency, and to the square of the exciting current. The only remedy for this is to diminish the capacity between turns by the use of thicker insulation or to diminish G by using fewer turns. The effect of this torque is to diminish the total restoring torque; that is, to lengthen the period so that it is only necessary to change the external circuit constants to bring the control back to its original value. The change of rest point is zero if θ_0 is zero, which can be made so by adjustment of either the coil or the mutual inductance.

If there is an alternating potential difference between the case and one terminal of the coil, there will be capacity current flowing through the coil, at least in part, and if this current has a component in phase with the magnetism, a torque acts tending to deflect the coil. The torque is proportional to the potential difference and has been observed to be very large in a sensitive instrument when the potential difference was only 5 volts at a frequency of 2100 cycles per second.

Other disturbing torques result from leakage and capacity currents between the parts of the moving coil circuit and external

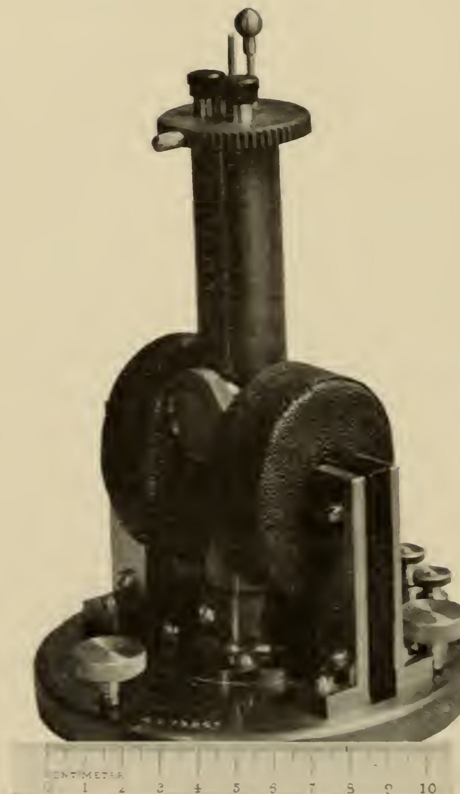

Fig. 1.—*Galvanometer B*

bodies which, on account of the inevitable dissymmetry in the two leads to the coil, flow through the coil. These disturbances are very serious at high frequencies when the capacity currents are large. The remedy for these and many of the other disturbing torques is the shielding scheme [18] shown in Fig. 2. The metallic shield and the one coil of the mutual inductance are maintained at a potential very nearly equal to the potential of the moving coil circuit.

The problem of the elimination of the mechanical disturbances is the same as for all instruments having delicate suspended systems, the difficulties increasing as the sensitivity increases. In extreme cases special attention is necessary in mounting the instrument.

In the electromagnet moving coil galvanometer it is sometimes necessary to use intense separate excitation, so that considerable

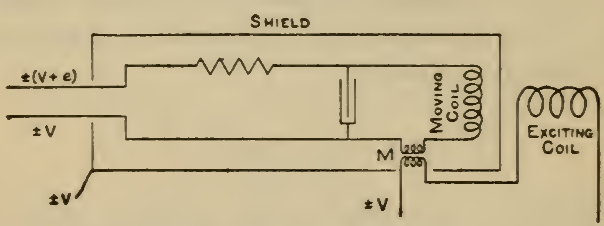

Fig. 2.—*Shielding arrangement*

heating is liable to occur. At low frequencies this occurs in the copper wire of the exciting coil or coils and at high frequencies the iron losses may be large. The exciting coils should always be separated from the moving system by a shield, so that disturbances caused by moving air will be minimized.

III. CONSTRUCTION AND PERFORMANCE

1. GENERAL DESIGN

The construction of an electromagnet moving coil galvanometer for use in alternating-current measurements differs but little from that of the sensitive moving coil galvanometer for direct currents. In place of the permanent magnet a laminated electromagnet is used. The moving system can be the same as for the direct-current galvanometer, with modifications suggested by the analysis of the disturbing torques. By using equations (22), (23), and (24), it is easy to calculate the values of the moment of

[18] Price, Phil. Mag., 42, p. 150, 1896; White, Phys. Rev., 25, p. 334, 1907.

inertia K, the moment of restoration U, and the moment of displacement G necessary for an instrument which is to be used with apparatus having a resistance between "galvanometer" terminals about equal to R and which is to have a sensitivity S and a period T. The resistance r and the damping are assumed to be zero in this preliminary calculation. A consideration of the construction of such an instrument from the material available then allows estimates of the probable values of r and D to be made, so that more accurate values of the intrinsic constants can be computed.

In the construction of the moving system it is desirable to have the resistance of the coil as low as possible and the material as free from magnetic impurities as possible. The use of thin insulating material (enameled wire) is desirable, for this keeps the moment of inertia low. Furthermore, but little insulation is needed. However, in case the instrument is to be used at high frequencies the use of thin insulation increases the capacity between turns and hence the corresponding disturbing torque. A large mirror adds to the moment of inertia, but some sacrifice is necessary if accurate readings are to be made easily. Much can be gained by the use of well-made mirror of good glass. The resistance of the suspensions should be kept reasonably low. Short, flat suspensions made by rolling small (0.02 mm) copper wire have been found satisfactory.

The construction of the electromagnet should be such as to give a field of the desired strength and shape with as little power loss and distortion of the wave shape as possible. The strength of the field is limited by the magnetic properties of the materials used in the coil and also by the source of excitation available. At high frequencies special attention is necessary to keep the losses due to hysteresis and eddy currents low. It is also necessary to design the exciting winding so that the higher voltages required at the higher frequencies cause no trouble. It is desirable to have the winding made up of several coils which can be connected in series or parallel as the case demands.

Usually the reluctance of the air gap is large compared with that of the iron and also low inductions (500–2000 gauss) are used so that the production of harmonics in the magnetic flux depends upon the source of excitation. Since the inductance of the fixed coil is large compared with the resistance, the exciting current and therefore the magnetic flux will contain fewer harmonics than the impressed electromotive force. The magnitude of the harmonics can be still further reduced by the use of a resonating system. A large inductance in series and a suitable condenser in parallel with the coils improves the wave shape of the magnetic flux and hence increases the selectivity of the instrument.[19]

In order to shield the moving system of the galvanometer from leakage and electrostatic effects, it is convenient to surround it with a metallic case which is connected to some point in the testing circuit such that the difference of potential between it and the coil is always very small. This shield can also be the support for the adjustment and arrestment devices, terminals, etc. The iron core of the fixed coils should be connected electrically to the shield.

Several sensitive galvanometers have been designed and built following the ideas given above. The description of the construction and the performances are given on the following pages.

2. GALVANOMETER A

The moving system of this instrument consists of a coil 8 mm wide and 12 mm high which has 27½ turns of 0.08 mm (No. 40 American wire gage) single silk-covered copper wire and a mirror 1 cm in diameter and 0.6 mm in thickness. The upper and lower suspensions are each about 5 cm long and were made from 0.015 mm copper wire rolled flat.

The coil was suspended in an air gap 10 mm long between pole-faces 8 mm square. The core of the fixed coil was made of laminations of good silicon steel. The exciting coil consisted of 1560 turns of 0.51 mm (No. 24) enameled copper wire. With 120 volts at 60 cycles per second on this winding the current was 0.93 ampere and the performance was as shown in Table 1. Here C is the capacity of the condenser connected to the terminals of the moving coil, T is the period of the instrument, R is the external critical resistance, and S_{60}, S_{180}, and S_{300} are the sensitivities to the fundamental, the third and the fifth harmonics, respectively. The prediction of aperiodic motion is borne out, but it is noted that the external resistance necessary to produce it is low.

TABLE 1

Performance of Galvanometer A

C	T	R	S_{60}	S_{180}	S_{300}
μf	seconds	ohms	mm/μv	mm/μv	mm/μv
0	5.6	0.1	13.5	0.13	0.015
2	8.0	5.1	16.0	....	....

[19] Ryan, Trans. Am. Inst. E. E., 20, p. 1419; 1903.

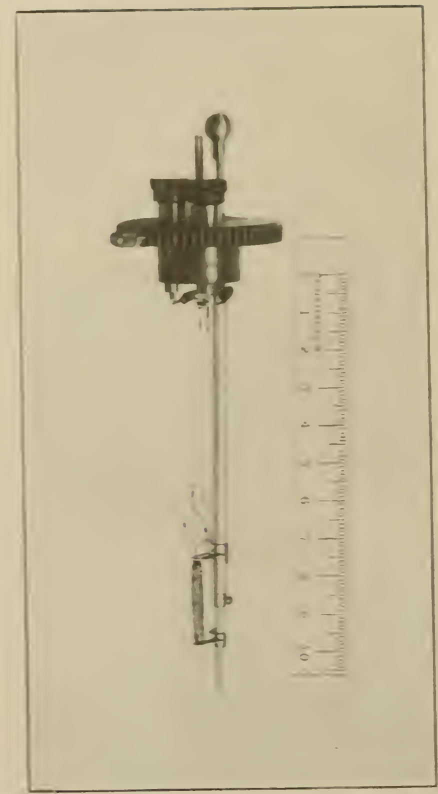

Fig. 3.—*Moving system of galvanometer B*

Approximate values of the intrinsic constants of this instrument are as follows:

$$K=0.022,\ D=0.001,\ U=0.028,\ G_1=28\,000,\ G_2=280,\ G_3=30,\ \text{and}\ r=14\times 10^9.$$

3. GALVANOMETER B

This instrument was intended for use with apparatus in which the resistance between the terminals to which the galvanometer is to be connected is of the order of 100 ohms. A period of only a few seconds, a sensitivity of several millimeters per microvolt, and aperiodic motion were required so that it was necessary to use a light coil. The coil consists of 100 turns of 0.08 mm (No. 40) enamelled copper wire wound on a wood frame to make a coil 2 cm long, 0.3 cm wide, and 0.1 cm thick. The wire is insulated with cellulose acetate which is very thin. The coil was treated with potassium copper chloride to remove the magnetic impurities and it was then washed, dried, and dipped into collodion, which bound the parts firmly together. The two end wires of the coil are brought out at the top instead of in the usual way in order not to have a large portion of the total magnetic flux through the core linking the moving coil circuit. One of these wires is soldered to a large copper wire, fixed to the wood frame of the coil, and to which a mirror 0.8 cm in diameter is attached. To the end of this large copper wire is soldered the supporting suspension, a flat copper strip similar to those used before. To the other end wire is soldered another strip, which is left loose and runs approximately parallel to the supporting suspension 0.3 cm away. The complete moving system is shown in Fig. 3. The fibre plug at the top fits into the case of the instrument as shown in Fig. 1. The moving system is easily removed, to be replaced by another of different constants.

The coil swings in a gap 0.6 cm long in the laminated steel core 2 by 0.8 cm in section. There are two exciting coils each wound with 650 turns of 0.72 mm (No. 21) enamelled copper wire, and the coils are arranged for either series or parallel connection.

TABLE 2

PERFORMANCE OF GALVANOMETER B

f	Excitation		C	T	R	S
~/second	volts	ampere	μf	seconds	ohms	mm/μv
0	2.6	0.4	0.00	1.7	25	2.3
0	3.9	.6	.00	1.4	60	1.2
60	29.0	.4	.00	1.9	30	2.6
60	29.0	.4	.22	4.2	100	5.7

The performance of this instrument under certain conditions of use can be seen from Table 2. The intrinsic constants were found to be as follows:

$$K=0.006,\ D=0.0009,\ U=0.05+0.2\ I^{2},\ G=115\ 000\ I,$$
$$r=25\ \times\ 10^{9}\ \text{and}\ c=0.07.$$

Here I is the exciting current in amperes (coils in series). The restoring force is seen to depend upon the excitation showing that there are magnetic impurities in the moving system. On account of the thin insulation used the capacity between turns is large and the quantity c is the capacity in microfarads which, if connected to the terminals, would affect the period by an amount equal to the effect of the distributed capacity. The resistance of the two fixed coils in series is 6.4 ohms and the inductance is 0.18 henry. The insulation is such that 120 volts can be used for excitation.

4. GALVANOMETER C

The fixed system of this galvanometer is the same as that of Galvanometer B. The moving system consists of a coil of 75 turns of 0.08 mm (No. 40) single silk-covered copper wire wound to form a coil 2 cm long, 0.5 cm wide, and about 0.15 cm thick, and a mirror 0.8 cm in diameter. This coil was made without a form by winding the wire over two copper wires held parallel and about 2 cm apart. It is necessary to remove the twist in the wire before winding, otherwise the coil will warp when taken off of the winding device. The turns of wire were bound together by means of silk fibers. Small copper end pieces are attached to hold the mirror and to serve as connections between the suspensions. After these were attached the coil was treated as before to remove the magnetic dirt and was then washed, dried, and dipped in collodion. The coil is rigid enough for the purpose.

The performance of this instrument is very satisfactory, the capacity effect being small and the coil being very free from magnetic impurities.[20]

In order to check the theory approximately the quantities K, D, U, and G were measured by the method already described. This was done for various values of the exciting current and fre-

[20] After first assembling this instrument its period with the field on was found to be much shorter than without the field, showing the coil to be magnetic. It was assumed that this was due to magnetic dirt picked up by the coil in the assembling. The coil was then treated again and replaced and the above results obtained. The total change in the restoring force when this coil is placed in a field of 4000 gausses is 0.02 dyne cm per radian, which is strikingly small. For Galvanometer B the change is 0.2 dyne cm per radian for the same field strength.

quency. The results are given in Table 3. It appears that the quantities K and U are real constants to the accuracy consistent with that of the determination of T_1, T_2, and λ_2. Further, the damping D and the moment of displacement G are constant for a given excitation. G is roughly proportional to the excitation current as it should be. D increases with excitation in such a manner as to indicate the presence of a damping due to eddy currents.

TABLE 3

Results of the Determination of the Intrinsic Constants of Galvanometer C at Various Excitations and Frequencies

I ampere	0.2	0.5	1.0	0.2	0.5	1.0	0.2
f~/seconds	0	0	0	60	60	60	300
OBSERVED QUANTITIES							
T_1 seconds	1.65	1.55	1.51	1.69	1.68	1.65	1.68
λ_1	.05	.07	.12	.05	.06	.10	.05
T_2 seconds	1.70	1.65	1.65	1.73	1.67	1.65	2.30
λ_2	.28	.53	.74	.38	.68	.77	.44
$(r+R_2)$ ohms	115.	305.	715.	67.	214.	650.	88.
$(r+R_1)$ S mm/μ ampere	35.	74.	125.	33.	75.	135.	33.
COMPUTED QUANTITIES							
K	0.01	0.009	0.009	0.01	0.012	0.011	0.011
D	.0011	.0016	.0029	.0011	.0016	.0027	.0012
U	.14	.15	.16	.14	.17	.16	.15
G	25 000	56 000	99 000	23 000	63 000	105 000	25 000

$r = 17$ ohms.

5. GALVANOMETER D

This instrument is the best of several designed for use as detectors of alternating electromotive forces of frequencies up to 2000 cycles per second. It was thought that if the same good performance could be obtained at these frequencies, the range of usefulness of the electromagnet moving coil galvanometer would be considerably extended. A great many difficulties have been encountered and, although a reasonably satisfactory instrument has been obtained, the conditions for satisfactory performance are probably too special to allow of any great use.

One of the first difficulties met with when the electromagnets were excited with alternating currents of these frequencies was a continual vibration of the coil when it was closed on a circuit of fairly low impedance. This vibration sometimes was a continuous one of constant amplitude and sometimes of varying amplitude indicating a phenomenon similar to beats. It was soon noticed that this vibration about the axis was accompanied by a back and forth motion of the coil in the gap. The disturbance increased with frequency, excitation, and with diminution of the impedance in the moving coil circuit. It was decided that this disturbance was the result of the coil being in a nonuniform field so that the back and forth vibration caused a periodic change in the value of G, and therefore in the apparent restoring torque, such that if the frequency of this vibration was twice that of the rotational vibration the motion would be maintained. If the frequency was different from this, beats occurred. A rough explanation of the phenomenon, which in detail is very complicated, can be seen by referring to Fig. 4, which represents the conditions during one complete rotational vibration of the coil. For simplicity the induced current is assumed to be that due to a condenser of capacity C. The resulting torque $G^2 \omega^2 C\theta$. (see p. 41) acts on the coil and as the coil vibrates in the nonuniform field G changes in value. In the figure x is the displacement of the coil from its equilibrium position. The arrows indicate the magnitude and the direction of the torque and it is seen to aid the vibration; that is, it causes energy to be given to the system so that the vibration may be maintained. If the circuit is inductive, the rotational vibration is maintained when the phase of x is shifted 180° so that as the coil deflects away from the reference position it is moving into a weaker instead of a stronger field.

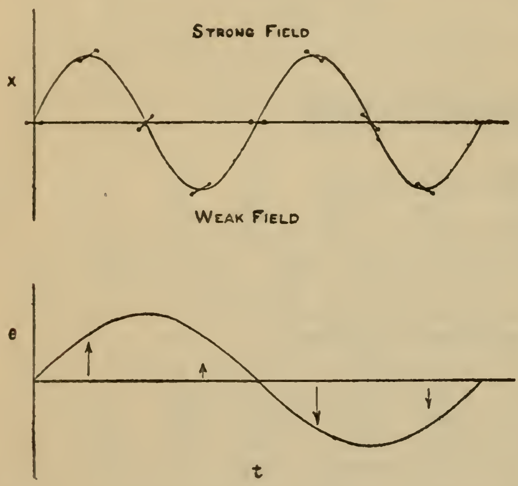

Fig. 4.—*Diagram showing the vibration of a coil in a nonuniform alternating magnetic field*

In order to eliminate this disturbance the period of vibration of the coil back and forth in the gap was made small by the use of short suspensions. More care was also taken to make the magnetic field uniform. These precautions were found to be sufficient so that there is no serious vibration of the coil of Galvanometer D at 2100 cycles per second even when the moving coil circuit is very inductive.

Referring to equation (14) we see that as the frequency is increased the term

$$\frac{\omega G^2}{Z} \sin \gamma$$

increases if γ is not zero so that there is a change in the total restoring torque and in the rest point. In order to obtain satisfactory performance this effect must be kept small so that it is necessary to keep γ small. At low frequencies the inductance can be compensated for by the use of a series resistance in parallel with a condenser as explained above. This compensation can be made to hold for a fairly large range of frequencies but at the higher frequencies such is not the case if the resistance is to be kept reasonably low. The frequency must therefore be maintained constant for satisfactory operation.

In the design of an alternating-current electromagnet for use as a frequency of 2000 cycles per second the losses due to hysteresis and eddy currents have to be considered. Since an air gap is necessary in which to swing the coil, there is considerable leakage so that the magnetic induction in the core may be several times the field strength required in the gap. It was found by the measurements of the apparent resistance and inductance[21] of iron-cored coils that the losses could be computed with sufficient certainty from data obtained at low frequencies.

A rectangular core 5.9 by 8.0 cm outside with a cross section of 1.5 by 1.7 cm was used. This was built up of laminations of silicon steel 0.18 mm thick, the laminations being insulated from each other by thin paper. An exciting coil of 500 turns of 0.64 mm (No. 22) copper wire is wound on one of the long sides of the rectangle. In the middle of the other side are cut two gaps 0.18 cm long and 0.75 cm apart in which the coil swings.

The moving coil of this galvanometer is rectangular in shape of mean area 0.8 by 2.5 cm. It is wound with 71½ turns of 0.08 mm (No. 40) single silk-covered copper wire. The wire is wound on a copper and mica frame made up as shown in Fig. 5. The copper pieces are covered with silk and after the coil is wound the

[21] These were measured by means of a Wheatstone bridge, three arms containing simply resistance and the fourth containing the test coil in series with a condenser. A telephone receiver was used as a detector and by adjustment of the resistances and the capacity very good balances could be obtained for various currents and frequencies. From the resistances, capacity, and frequency the apparent resistance and inductance could be computed. Then knowing the dimensions, number of turns, etc., the loss for various inductions could be computed.

small copper extensions are bent down and hold the wire in place. The mirror is attached as shown. The suspensions of this instrument are about 1 cm long and are of the flat copper strip used before. The lower suspension is soldered to a small flat copper spring so that the suspensions can be held tight.

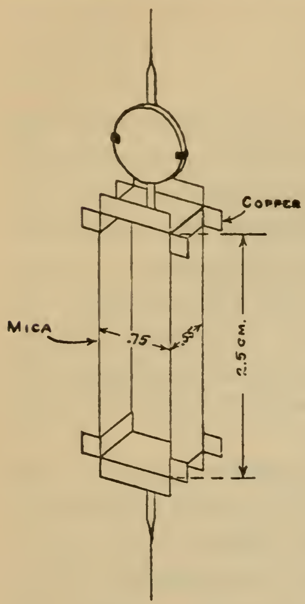

Fig. 5.—*Diagram showing construction of moving system of galvanometer D*

Table 4 shows the performance of this instrument under various conditions of use. *W* is the power required for excitation, *C* is the capacity of the parallel condenser used in controlling the period, and S_i is the current sensitivity. With a current of 0.5 ampere the motion is aperiodic when the external resistance is 16 ohms. At the low frequencies such excitation is available but at the high frequencies this represents considerable power and was not obtainable from the generators available. The performance with the excitation obtained was satisfactory however. The sensitivity is high and the motion is not far from being aperiodic. The air damping is larger in this instrument than in the others on account of the larger coil and closer proximity of the pole faces. This accounts for the reasonably satisfactory damping at the high frequencies where the electromagnetic damping is not large.

TABLE 4

Performance of Galvanometer D

f	I	W	C	T	R′	S	S_i
~/second	ampere	watts	μ f	seconds	ohms	mm/μ v	mm/μ ampere
0	0.490	1.0	0.00	2.4	16	1.10	38.
430	.260	3.4	.00	3.3	23	.90	38.
430	.260	3.4	.10	2.5	76	.23	22.
2100	.068	1.2	1.00	1.7	11	.09	2.7
2100	.066	1.1	1.00	2.4	22.5	.12	5.
2100	.063	.9	.25	2.4	42.5	.08	4.9
2100	.063	.9	.05	3.1	100	.07	8.3
2100	.151	6.8	1.00	2.2	23.5	.23	9.8
2100	.198	12.6	1.00	2.4	23.5	.37	16.
2100	.210	14.2	.05	1.5	94	.058	6.6
2100	.210	14.2	.05	4.2	100	.43	51.

The intrinsic constants of this instrument are

$$K=0.075,\ D=0.03,\ U=0.6,\ G=200\ 000\ I \text{ and } r=19\times 10^{9}.$$

The direct-current resistance of the exciting coil is 4 ohms. The apparent resistance to alternating current increases with current and frequency approaching the value 320 ohms at 2100 cycles per second and 50 ohms at 430 cycles per second for currents above 0.1 ampere. For currents less than this the resistance seems to approach the direct-current value as the current approaches zero. The apparent inductance increases from the value 0.06 henry for small currents to the value of 0.1 henry for currents above 0.1 ampere. The apparent inductance is independent of frequency indicating that the eddy currents are not large.

IV. USES

1. DETECTOR

In general, the purpose of a detector in null methods of electrical measurement is to show when the current or the electromotive force in a certain circuit is zero or does not change when certain changes are made in the testing current or voltage. Such conditions are brought about by an adjustment of some part or parts of the measuring apparatus. For example, in the measurement with a Wheatstone bridge using alternating current the inductance and resistance in one arm of the bridge might be adjusted to a balance; that is, until the detector shows that the current in the galvanometer circuit is independent of the testing current. The purpose of a detector can therefore be considered to be measure roughly the difference of potential between two terminals of apparatus having a certain impedance. Most of the measurements are of this kind, but there are some in which the current flowing in a circuit of high impedance (leakage) is to be measured or detected. In any case besides the sensitivity the user is interested usually in the time required for an observation. A reasonably short period and reasonably deadbeat motion is required.

It is seen from the preceding discussion that the performance of the electromagnet moving coil galvanometer as a detector is similar to that of the direct-current moving coil galvanometer. If one is dealing with sinusoidal current or electromotive force to be detected and noninductive circuits, the only difference is due to the fact that the small electromotive force or current may be out of phase with the magnetic field. Since the deflection is proportional to the component of the electromotive force in phase with the magnetic field, zero deflection means only that the electromotive force, if it is detectable, is in quadrature with the magnetic field. In order, therefore, to be sure that the electromotive force is zero, the phase of the excitation is changed. With noninductive circuits the phase of the excitation can be adjusted once for all so that the maximum sensitivity is obtained. Under these conditions the magnetic field is in phase with the unbalanced electromotive forces which are always in the same phase. However, in this case, it is only necessary that the magnetic field be approximately in phase with the testing current, for since the phase of the unbalanced electromotive forces is always the same as that of the testing current, only its magnitude can be changed and hence zero deflection means zero electromotive force..

Usually in alternating-current measurements we have to do with apparatus having both resistance and reactance so that the unbalanced electromotive force may be of any phase whatsoever. There are usually two adjustments, the "resistance" and the "inductance" adjustments. A small change in one results in a change in the unbalanced electromotive force with a phase approximately in quadrature with the change produced by the other adjustment. Now if the phase of the excitation be such that a large change in one adjustment produces no change in deflection, it is possible by shifting the phase of the excitation 90 electrical time degrees to make the one adjustment independent of the other. When such an adjustment of the phase of the magnetic field is not made the balance must be obtained by successive approximations. This is illustrated in Fig. 6. Here ϕ_1 and ϕ_2 are the two phase positions of the magnetic field, R and L are the phases of the resistance and inductance adjustments, respectively, and E is the small electromotive force to be reduced to zero. Suppose to start with, the excitation is at ϕ_1 we will now make some adjustment which reduces the deflection to zero; that is, makes E in quadrature with ϕ_1. To do this we may make either a resistance or an inductance adjustment. Suppose, first, we make an inductance adjustment which carries the end of E to a. Now shift the excitation to ϕ_2 and make a resistance adjustment which brings us to b, and so on. After a number of adjustments the deflection will be reduced to a negligible amount for both phase positions of the excitation. But suppose a resistance adjustment to be made first. On following this out it is seen that the electromotive force to be made zero increases. The advantage of having the adjustments independent is shown by comparing Fig. 6 with Fig. 7.

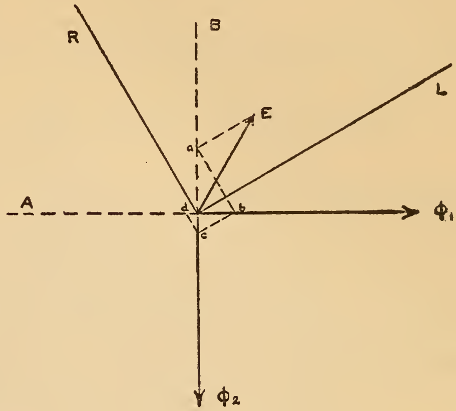

FIG. 6.—*Vector diagram showing adjustments with quadrature excitation. Adjustments not independent*

It is obvious that either the phase of the magnetic field or of the testing current can be changed. Various phase-shifting devices can be used. A phase shifter of the rotating-field type is very convenient since any phase relation is obtainable which allows the adjustments to be made absolutely independent irrespective of the phase of the testing current. Any polyphase supply furnishes a definite shift of phase so that adjustment to zero is possible. Sometimes when the testing current is small it can be shifted by means of suitable inductance coils or condensers.

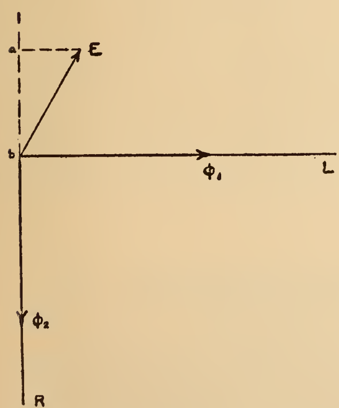

FIG. 7.—*Vector diagram showing adjustments with quadrature excitation. Adjustments independent*

The precautions necessary in the use of an electromagnet moving coil galvanometer as a detector have been discussed with reference to the disturbing torques. The most important precaution is the maintaining of the coil and the metallic shield at the same potential. At the higher frequencies this is particularly true. Fig. 8 shows a good scheme of connections for use when the galvanometer is used as a detector for a Wheatstone bridge measurement. The metallic shield around the moving coil is electrically connected to a metal tube through which the leads are carried to the control apparatus. This apparatus, consisting of a parallel and a series resistance, a condenser, the mutual inductance, and a reversing switch, is surrounded by a metallic shield (wire netting) which is connected to the tube from the galvanometer and to the divided resistance as shown. By means of the divided resistance the shield is maintained at a potential near that of the galvanometer branch points of the bridge. The reversing switch in the galvanometer circuit allows the main test current to be left

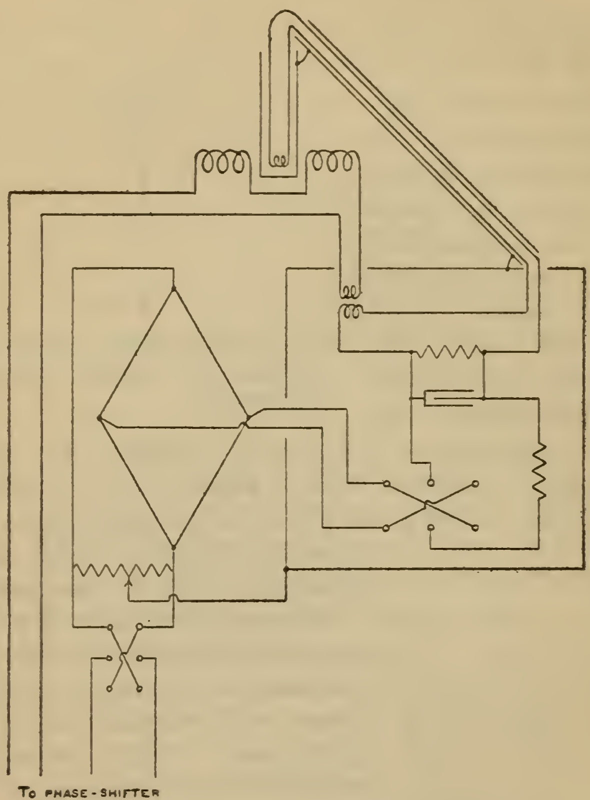

FIG. 8.—*Diagram of connections with galvanometer used as a detector in alternating current bridge measurements*

constant during the balance. If on reversal the rest point of the galvanometer does not change, the difference of potential between the leads from the bridge is zero. Usually the opening of the moving coil circuit causes a deflection on account of the change in the induced current, as pointed out above, so that this scheme is feasible only if the mutual inductance is adjusted to make $\theta_0 = 0$ or if the impedance of the bridge is high compared with resistance shunting the galvanometer. Reversal of the current is the most desirable method of testing for a balance, for then the impedance of the moving coil circuit remains constant and a permanent change in deflection can result only from an electromotive force which reverses when the test current is reversed. In case the balance depends upon the frequency, reversal may cause a sudden motion of the coil due to the differences in the rates of decay and growth of the currents furnishing the balancing electromotive forces. Care is also necessary if stray electric or magnetic fields are present.

In some measurements it is desirable that the detector be sensitive to currents of the fundamental frequency only. This is the case when sinusoidal supply is not available or when harmonics are introduced by the apparatus so that the comparatively large harmonic electromotive forces must not cause deflections which will mask the deflection produced by the fundamental

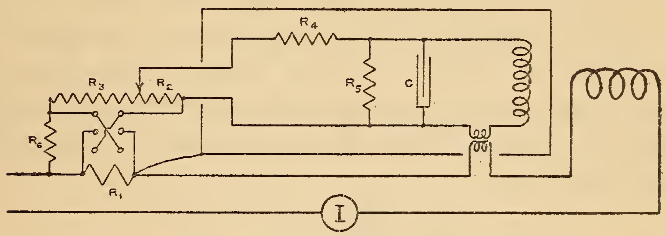

FIG. 9.—*Diagram of connections used in calibration of galvanometer*

unbalanced electromotive force. The use of electrical resonating systems in the circuits has been discussed above (pp. 34 and 43). These, together with care as to excitation, will usually be sufficient to give ample selectivity. Constancy of the frequency is essential.

2. OTHER USES

The electromagnet moving coil galvanometer is inherently an instrument of the galvanometer type inasmuch as it is not feasible to know with precision the constants depending upon the separate excitation, which data would be needed if the current in the moving coil were to be measured accurately. For approximate measurements the instrument can be calibrated by using connections as shown in Fig. 9. A small electromotive force approximately in phase with the magnetic field is obtained from the drop in potential across the low resistance r_1 by using the divided resistances r_2 and r_3. The resistances must be sufficiently noninductive at the frequency used. With C, r_4, r_5, and r_6 adjusted to the values to be used in the measurement the change in the deflection on reversing the electromotive force is noted. A simple calculation then gives the calibration factor. The resistance r_6 is very high in comparison with r_1 and is used to maintain the resistances r_2 and r_3 and hence the moving coil circuit at the potential of the shield when the switch is open. This is very necessary at high frequencies on account of the capacity current which would flow through the moving coil due to the capacity between the resistances r_2 and r_3 and the surroundings. If the apparatus containing the small electromotive force to be measured is available it should be inserted so that the moving coil circuit will be the same during the calibration as during the measurement.

The electromagnet moving coil galvanometer can be used to measure inductance in terms of capacity and vice versa by a resonance method in which the inductance and capacity are inserted in series in the moving coil circuit. For this measurement the moving coil is displaced from its position of zero induced electromotive force so that $\theta_0 = 0$. Now suppose that either the inductance or the capacity be adjusted until there is no deflection on closing the circuit. Then if the excitation be sinusoidal we must have from (11)

$$\frac{\sin \gamma}{Z} = 0 \tag{60}$$

which means that

$$L\omega = \frac{1}{\omega C}. \tag{61}$$

The sensitivity is determined by θ_0, G, and the magnitude of the series resistance R' and from (11) can be shown to be determined by the equation

$$\theta = \theta_0 \frac{\omega G^2}{(R')^2 U}\left(L\omega - \frac{1}{\omega C}\right) = \theta_0 \frac{\omega G^2}{(R')^2 U} X. \tag{62}$$

By referring to the data on the instruments it is seen that a difference corresponding to 1 microhenry can be detected at 60 cycles per second. Therefore we see that a small reactance can be measured by itself, and equation (62) shows that the deflection is proportional to the reactance and is in one direction if it is inductive and is in the other if it is condensive reactance.

This study is part of a general investigation of galvanometers being carried on by Dr. Frank Wenner and the author, and the experience gained in the study of sensitive permanent magnet moving coil galvanometers for direct current has been especially valuable. I am much indebted to Dr. Wenner for his hearty cooperation in this work.

V. SUMMARY

(1) The electromagnet moving coil galvanometer has been studied and the equation of motion obtained, from which its performance under various conditions can be determined. The performance as a detector is very similar to that of the direct-current moving coil galvanometer in that the motion of the coil may, in general, be of any degree of damping, depending upon the external circuit and in that the deflection is proportional to the impressed electromotive force. In the electromagnet moving coil galvanometer the period, as well as the damping, depends upon the constants of the external circuit being shortened by inductance and lengthened by capacity. Further, the deflection produced depends upon the component of the electromotive force in phase with the excitation.

(2) The relations between the operating and the intrinsic constants are given, so that the design and construction of instruments for any particular purpose are facilitated.

(3) A method of determining the intrinsic constants of electromagnet moving coil galvanometers has been devised. This gives from observations on the performance, under easily obtained working conditions, the moment of inertia, moment of damping, moment of restoration, etc., so that the performance of the instrument under other conditions can be predicted.

(4) An analysis of the disturbances has been made and it has been shown that the best procedure in the use of a sensitive electromagnet moving coil galvanometer is to surround the moving coil and moving coil circuit up to the testing apparatus by a metallic shield, which is maintained at a potential equal to that of the moving coil circuit. This can be done by connecting the shield to a point on the testing circuit having the same potential as the moving coil.

(5) The description and performance of four sensitive instruments are given and show results as expected from the theory. The sensitivity of these instruments is much greater than that of the telephone receiver at the lower frequencies. It is greater than that of the vibration galvanometer and is equal to that of the best direct-current moving coil galvanometers. The instru-

ments have been found to give very satisfactory performance at low frequencies. One instrument has been used at the frequency of 2100 cycles per second and, although very sensitive, many precautions have to be taken for satisfactory operation.

WASHINGTON, April 4, 1916.