A VARIABLE SELF AND MUTUAL INDUCTOR

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CONTENTS

1. NECESSITY FOR DEVELOPING THE INSTRUMENT

In the work of testing current transformers at this Bureau the need arose for a variable inductor,[1] to have a range of variation of about 1 millihenry and to carry 5 amperes continuously. It was also necessary that the resistance should be as low as practicable consistent with reasonable dimensions. As no instrument meeting the requirements was on the market, the design of an apparatus of this kind was undertaken.

The type of instrument which was devised as a result of this investigation is not limited in its application to the work of testing current transformers, but is applicable wherever it is necessary to vary the self-inductance of a circuit or the mutual inductance between two circuits while keeping the resistance constant. It was therefore considered desirable to give a description of the apparatus and to compare its performance with that of other instruments now available for this class of work.

[1] The expression "variable inductor" is suggested for such instruments as being preferable to the usual wordings "variable inductance," "variable standard of inductance," or "variable standard of self-induction." It is felt that the word "inductance" should be reserved for the property of the apparatus and should not be used for the apparatus itself. A similar tendency is seen in the growing use of the word "resistor" to signify a coil, grid, or other device used because of its resistance.

2. EXISTING INSTRUMENTS AND THEIR LIMITATIONS

The first commercial variable inductor was the one designed by Ayrton and Perry,[2] in which the fixed coil and the moving coil are each wound as belts on concentric spherical surfaces. While this inductor would give the required values of resistance and inductance if wound with wire of suitable size, it has the drawback of being non-astatic. The coils inclose a large area, and when current flows through them a very appreciable magnetic field is set up which may affect other parts of the circuit. Conversely, the variation of magnetic fields set up by other parts of the circuit will induce disturbing electromotive forces in the coils. In either case errors of appreciable amount may arise unless care is taken to keep individual instruments at a sufficient distance apart.

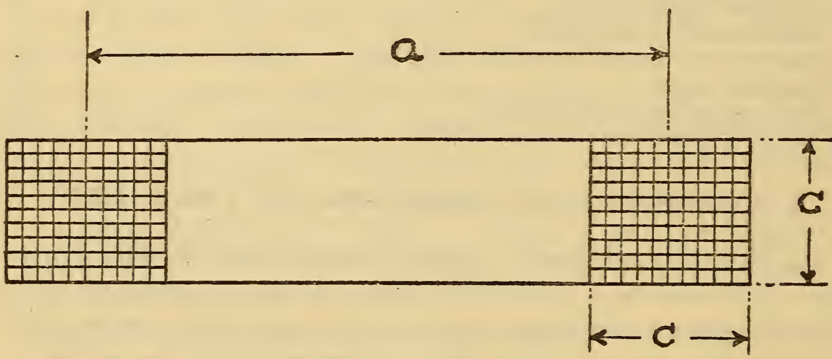

FIG. 1.—*Coil of Maxwell proportions for maximum time constant*

Two other commercial forms were available, namely, the Mansbridge,[3] and a form similar to it in external appearance, originally made by Nalder Bros. & Co. Each of these instruments consisted of two circular plates of hard rubber, the lower plate containing two fixed coils, while the upper plate, containing two similar coils, could be turned about a central pivot carried by the lower plate. The scale divisions of the Nalder instrument were very unequally spaced, and the time constant of the Mansbridge instrument, in the size in which it is regularly manufactured, was too low for the given special purpose.

[2] Electrician (London), 34, p. 546; 1895.

[3] United States Patent No. 893340, issued July 14, 1908, to G. F. Mansbridge.

3. PRELIMINARY BASIS FOR THE DESIGN

As a starting point in the new design, use was made of the proportions given by Maxwell[4] for a circular coil of square cross section to give the maximum value of time constant for a given mass of wire. Such a coil is shown to scale in Fig. 1. Maxwell stated that for maximum time constant the mean diameter *a* should equal 3.7 times the side *c* of the winding cross section.[5] The original idea was to use two sets of interleaved[6] coils, each set when in the position of maximum inductance to be equivalent to a single coil of the Maxwell proportions. The arrangement is shown diagram-

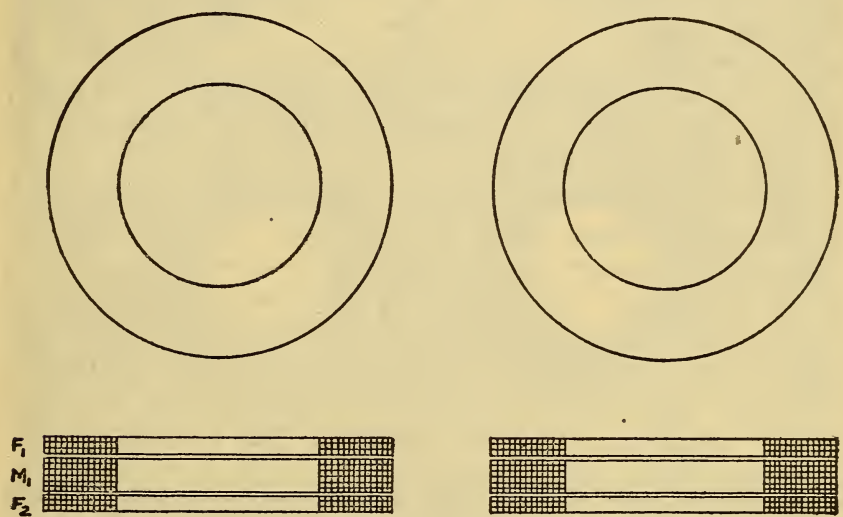

Fig. 2.—*Sets of interleaved coils as used in experimental variable inductor*

matically in Fig. 2, where F_1 and F_2 are the fixed coils of the left-hand group and M_1 the moving coil. By making M_1 of twice the axial thickness of either fixed coil, so that it contained as many turns as F_1 and F_2 together, the inductance of the group with M_1 opposing the fixed coils has as low a value as can be obtained, except by subdivision of the group into five, seven, nine, etc., sections. While this further subdivision is possible, it was thought best for mechanical considerations to limit the number of coils in each group to three.

[4] Maxwell, Electricity and Magnetism, 3d ed., 2, pp. 345–346.

[5] It has recently been shown that for maximum time constant the mean diameter *a* should be three times the side *c*. See paper by Shawcross and Wells, Electrician, 75, p. 64; 1915.

[6] This interleaving of the coils was felt from the start to be an important feature for reasons given on p 579 in the discussion of the Nalder and the Mansbridge inductors.

4. SCALE LAW OF CIRCULAR COILS

As there was reason to fear that the scale law of such a variable inductor might not be satisfactory, a wooden frame carrying two such sets of coils was constructed to test this point experimentally, since to do so by mathematical analysis would be very difficult.[7] The apparatus is shown in Fig. 3 and the curve A in Fig. 4 shows the manner of variation of the inductance when the moving coils are turned through 180°. It will be seen that at two angular positions the rate of change of the inductance is zero, and hence in the vicinity of these points no scale divisions can be placed. Also, between the positions 75° and 110° the inductance decreases with increasing angle, while over the rest of the range the reverse is true. The scale of such an instrument would therefore not be figured between, say, 60° and 120°, and its use would be very inconvenient, especially when making settings in balance methods without looking at the scale.

Experiments were made with the two groups of circular coils at different distances apart. It was found that when the two groups were brought almost into contact a much better scale law was obtained. However, such an arrangement leaves no room for the shaft about which the coils must rotate, and further improvement was sought by modifying the form of the coils.

5. SCALE LAW OF LINK-SHAPED COILS

It was very desirable to secure a uniform scale, while keeping each group of coils as close to the Maxwell proportions as possible. By empirical reasoning the cause of the unfavorable part of curve A, Fig. 4, was ascertained, and the conclusion was reached that the defect might be eliminated by elongating the coils into an approximately elliptical form with the longer axes perpendicular to the line joining the centers of the two coil groups. Trial with a rough model gave encouraging results. The new curve did not show the undesirable maximum and minimum of curve A. The inductance increased continually as the moving coil was turned from the initial position, but the curve was such as to give scale divisions of very unequal length in different parts of the range.

A large number of experiments were now made, in which the form of the coil, the spacing of the coil groups, and the angle

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[7] A. Campbell, Phil. Mag., 6th series, 15, p. 167; 1908. S. Butterworth, Phil. Mag., 6th series, 31, p. 443; 1916.

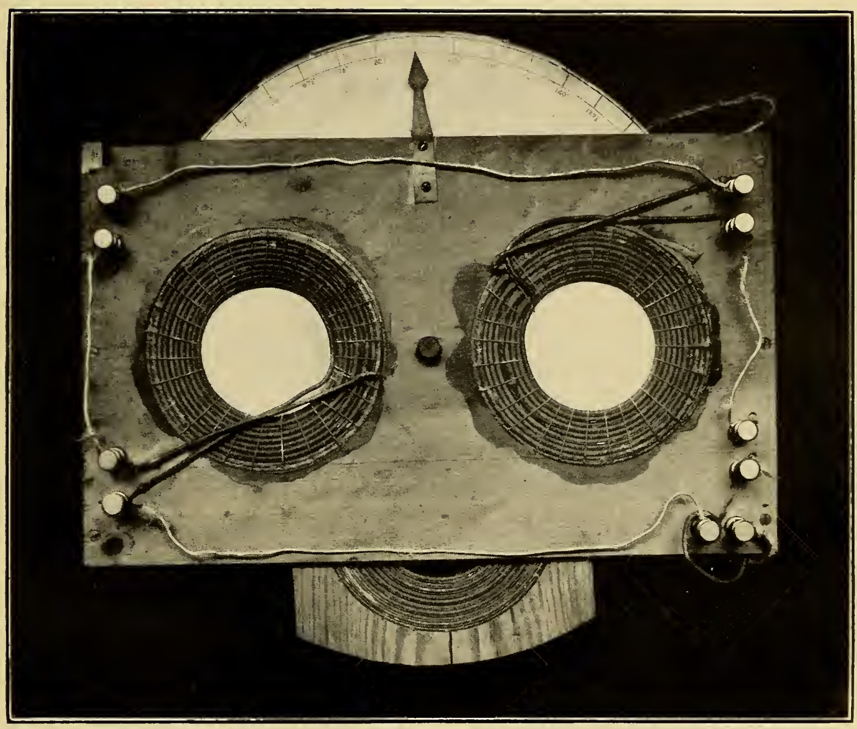

FIG. 3.—*Experimental variable inductor with circular coils*

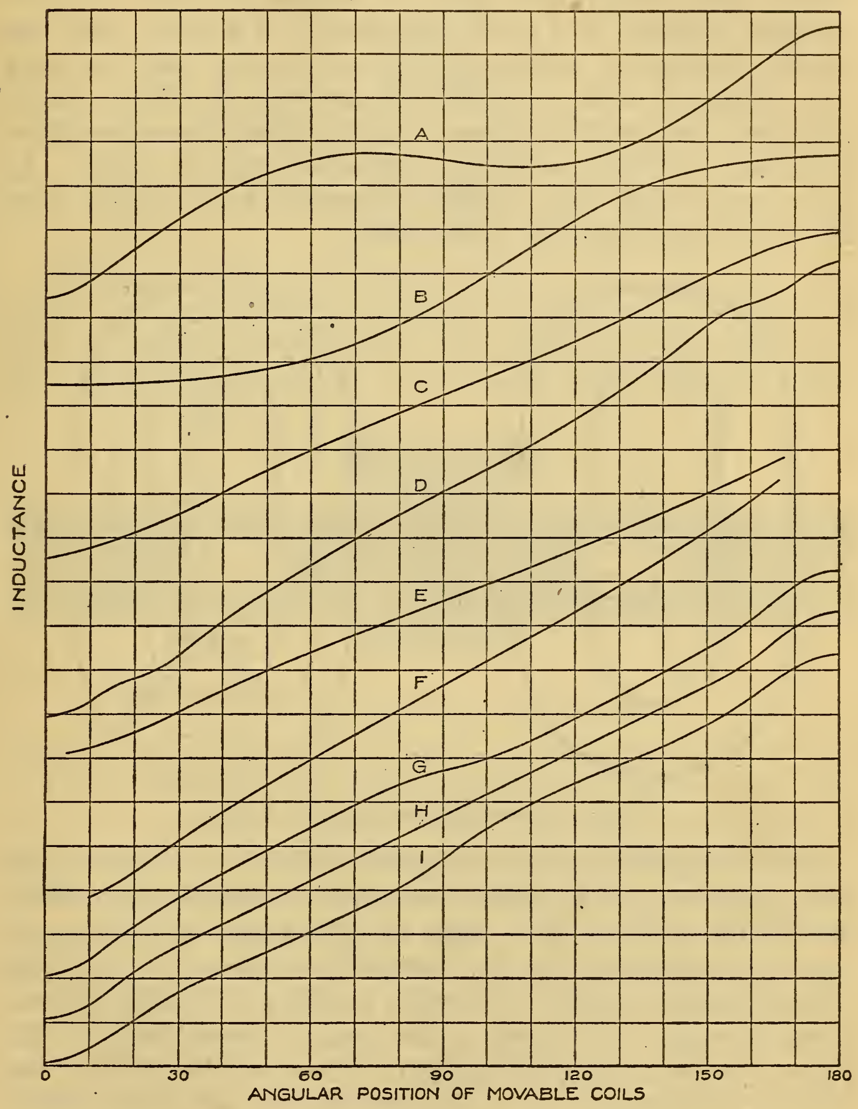

FIG. 4.—*Curves showing manner of variation of inductance of several forms of variable inductor*

between the longer axes of the coil and the line joining the centers of the coil groups were each varied separately, a curve being determined for each case. While several arrangements were found which would give scales as uniform as those of other variable inductors in use, the advantages[8] of a linear scale (one having divisions of uniform length) are so great that the work was continued until a compact arrangement of coils of simple geometric form and high time constant was determined which gave this result, as closely as physical limitations seem to permit. In the course of this work valuable suggestions were received from Dr. F. W. Grover and Dr. H. L. Curtis.

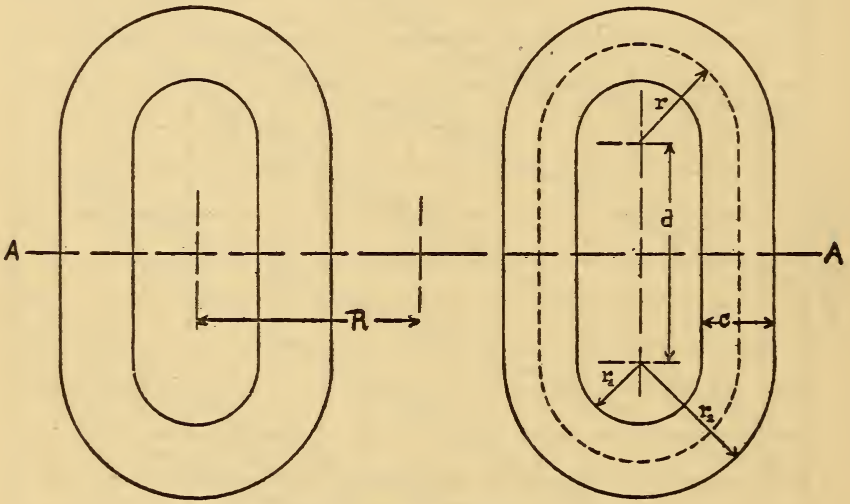

FIG. 5.—*Plan of link-shaped groups of coils*

The arrangement is shown in plan in Fig. 5. The link-shaped coil groups may be thought of as made by cutting the circular coil groups in halves by a plane at right angles to the plane of the coils, separating the two halves by a distance d, and then joining the cut surfaces by straight portions of winding to complete the link. The depth of the group, perpendicular to the paper, equals c, the radial breadth. The centers of the two coil groups are separated by a distance $2R$. To secure uniformity of the scale requires a particular value of R, the half distance

[8] These advantages are, in part, as follows: The scale is easier to make and it is easier to interpolate between divisions. In checking the instrument from time to time, fewer points need to be tested. If a small displacement of the scale or of the fiducial mark occurs, a small error is introduced which is of nearly the same absolute value for all points of the scale, while for an instrument having scale divisions of very unequal length the same displacement may make the absolute error in some parts of the scale much greater than in others.

between the centers of the coil groups. The relative values were determined to be as follows, r being the mean radius of the circular part of the coil.

$$c = 0.78r$$
$$d = 2.2r$$
$$R = 2.26r$$

From these relations it follows that—

$$r_1 = 0.61r$$
$$r_2 = 1.39r$$

Referring to Fig. 4, the three curves G, H, and I show the effect of a variation of dimension R on the scale law. Curve H is for the normal value of R ($=2.26r$) and gives a linear scale over the angular range of 30° to 150° movement from the initial

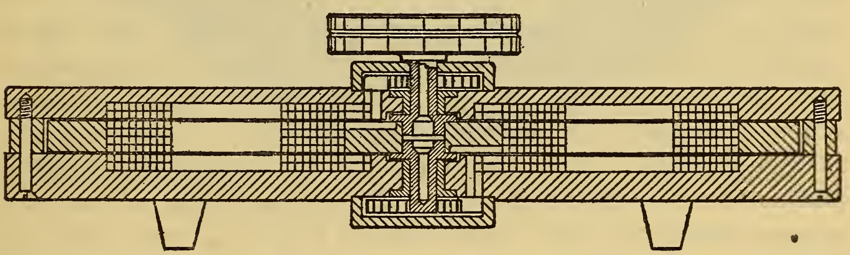

FIG. 6.—*Cross-sectional view of variable inductor*

position. The curve must become horizontal at 0° and 180°, and it will be noted that the change of slope from 30° to 0° and 150° to 180° is gradual. Curve G shows the effect of increasing R by 6 per cent. Since the length of a scale division at any part of the curve is proportional to the reciprocal of the slope of the curve, the divisions would be about twice as large at the 90° position as over the range from 30° to 70° or from 110° to 150°. A similar condition exists in curve I, for which R was 6 per cent smaller than normal value, but in this case the scale divisions are about three-fourths as long at 90° as in the other two parts of the scale above mentioned.

Curves B to F of Fig. 4 are given for comparison. B is for a Campbell mutual inductor,[9] and was determined from measure-

[9] A Campbell, Phil. Mag., 6th series, 15, pp. 155–171; 1908. In this instrument the difficulty shown by curve A is avoided by using a circular moving coil eccentrically pivoted between two fixed circular coils which are at a relatively large distance apart. This arrangement, however, is not suitable for use as a variable self inductor, because it makes the mutual inductance so small in comparison with the self inductance. This would result in a very small ratio of maximum inductance to minimum inductance.

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ments of the scale of the instrument. *E* and *F* are for the lower and higher ranges of a Mansbridge variable inductor, and were plotted from a facsimile scale given in a circular issued by the (American) maker. Curves *C* and *D* are for variable inductors of Ayrton-Perry form, made by an American firm.

6. DESCRIPTION OF THE NEW INSTRUMENT

Using the proportions given on page 575, a variable inductor was constructed by Joseph Ludewig, of the instrument shop of this Bureau. Fig. 6 shows a cross section along the line *AA* of Fig. 5. The value of *r*, the mean radius of the circular part of the coils, is 36.8 mm. The coils are mounted in three hard-rubber disks, of which the inner one is provided with bronze spindles which turn in brass bushings in the upper and lower plates respectively. The two outer disks are held together by six screws and separating pieces and form the body of the instrument. The diameter of these disks is 35.5 cm (14 inches). The bronze spindles of the movable disk also serve to carry current to and from the moving coil. Heavy copper spirals protected by hard-rubber caps carry the current to the spindles. It was not considered feasible to carry current through the bearing, as a variable and uncertain resistance is very bad in the secondary circuit of a current transformer and makes serious trouble in most work for which a variable inductor is used. Stops are provided to limit the motion of the inner coil to 180°.

The actual number of turns of wire is shown in Fig. 6. For example, each fixed coil is wound in nine layers of two turns each. The wire is stranded, being made of seven insulated copper wires of 0.8 mm. diameter (No. 20 B. & S. gage), the conductor so formed having a cross-sectional area of 3.6 mm^2. The design was made on the basis of a range of variation of 1.1 millihenrys. The uniformity of the scale may be seen from Fig. 7, which shows the inner disk alone. The scale is figured in microhenrys, each division being 5 microhenrys. The useful part of the scale is from 125 to 1225 microhenrys. From 325 to 1025 microhenrys the scale is uniformly divided to within the limits of the measurement of the inductance. It is not possible to secure an arrangement of coils which will give a uniform scale through the entire 180°, as this requires either that the fixed coils and the movable coils would have to coincide at 0° and 180° or each coil group would have to contain an infinite number of infinitely thin sections.

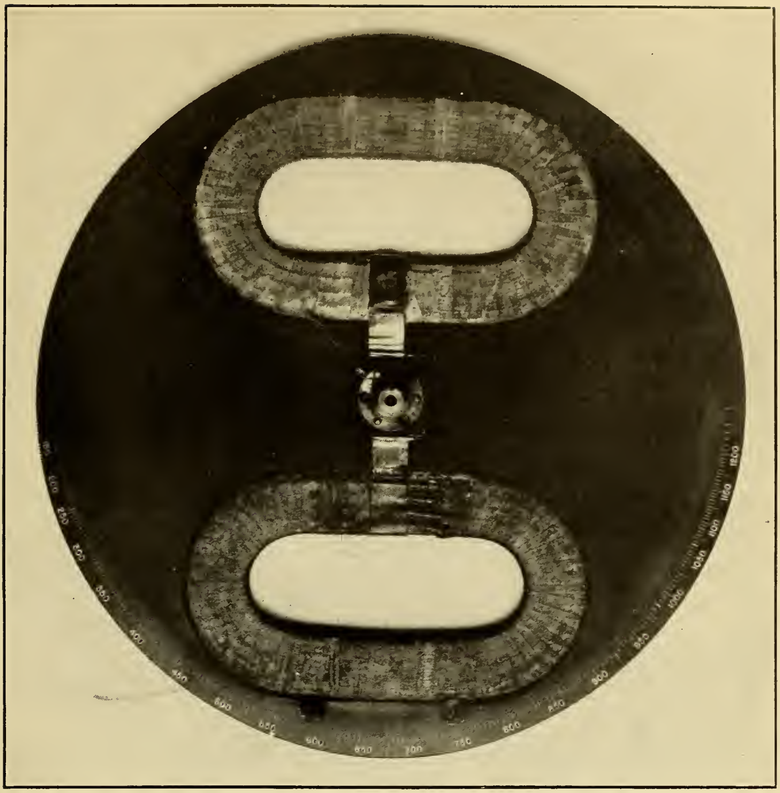

FIG. 7.—*Inner disk of variable inductor, about ¼ size*

FIG. 8.—*Variable inductor, about ¼ size*

However, it will be noticed that below 325 and above 1025 microhenrys the length of the scale divisions decreases gradually until the points 150 and 1175 are reached. There are no points within the working range where abrupt change of scale law occurs, and this fact assists greatly in making an accurate scale and in interpolating between adjacent scale divisions.

The four fixed coils are connected in series and brought out to two binding posts, and the two movable coils in series to another pair of binding posts. The scale is figured to read the self-inductance with all the coils in series. By using the fixed coils and the movable coils as primary and secondary the instrument becomes a variable mutual inductor. To avoid confusion, a scale for reading the values of mutual inductance was not added. The mutual inductance is found [10] by subtracting 669 microhenrys from the reading and dividing the remainder by 2. A table of values of mutual inductance corresponding to readings on the self-inductance scale may be used.

With the coils in series the resistance is 0.35 ohm and the time constant for maximum inductance setting is 3.4 milliseconds.[11] The time constant of the Ayrton-Perry inductor, which occupies several times the space required by the form herein described, is about 4.5 milliseconds, and that of the Mansbridge is 1.6 milliseconds. In comparing time constants of different types of inductors, it should be remembered that a fair comparison must take into account the relative space occupied by each instrument. The time constant for any model may be increased by increasing the dimensions.

The complete instrument is shown in Fig. 8. The upper fixed disk is cut away to show the scale and has a fiducial mark. The instrument has been in use for several years and has been satisfactory except that trouble has occurred at times from the sticking of the moving disk. At the time the instrument was planned hard rubber was known to be liable to gradual change of form,

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[10] In any such arrangement of two sets of coils whose mutual inductance can be varied by change of relative position,

$$L = L_1 + L_2 \pm 2M_{12}$$

where L is the total self-inductance, L_1 is the self-inductance of one set of coils alone, L_2 that of the other, and M_{12} the mutual inductance of one set on the other. When the coils are so placed that M_{12} is zero, $L = L_1 + L_2 = C$, a constant for the given apparatus. From this it follows that for any position of the coils

$$\pm M_{12} = \frac{L - C}{2}$$

[11] This value would be greater if solid wire were used, as would be the case for smaller wire for ordinary frequencies, because the space factor of the stranded conductor is low. The conductor being rather large, it was thought best to strand it as a precaution against change of inductance with frequency.

but it seemed to be the best material for the purpose when making a single instrument by machining the disks to shape. The use of modern molded insulating materials gives promise of satisfactory performance in this respect. Referring to Fig. 6, the connections are such that the two sets of fixed coils have opposite magnetic polarities, as viewed from above, and the two moving coils also have opposite polarities. This astatic construction tends to reduce trouble from stray magnetic fields.

7. DESIGN PROCEDURE

In the design of the windings to give a definite maximum inductance the general procedure was as follows. The effect of mutual inductance between the left-hand group of coils (Fig. 6) and the right-hand group contributes 3 per cent to the total self-inductance of all the coils in the position for maximum inductance, but this may be neglected and each group be computed to give one-half of the maximum value of self-inductance required. Each group is considered as a coil of square cross section which may be imagined as having been originally wound as a circular coil and then flattened into the link form. For design purposes, where an accuracy of 1 or 2 per cent suffices, it may be stated that the inductance of the link-shaped coil will bear the same ratio to the inductance of the circular coil as the area inclosed by the mean turn of the link-shaped coil bears to the area inclosed by the mean turn of the circular coil.[12] The inductance of the circular coil is conveniently calculated with sufficient accuracy by Maxwell's approximate formula.[13]

However, a simpler design procedure may now be based on the observed performance of the instrument here described, and from its constants new values of dimensions and of number of turns of wire may be computed to meet any given requirements. First, the time constant may require to be changed. This may be accomplished by changing all the dimensions, while keeping the same proportions. The factor m, by which all the dimensions must be multiplied, may be found from the relation that for coils of similar form wound with wire having the same space factor the time constant varies directly as the square of a given linear dimension and is independent of the size of the wire. The inductance, however, varies as the first power of a given dimension, hence the inductance of the new set of coils will be m times the inductance

[12] This ratio for the instrument here described is 0.830.

[13] Maxwell, Electricity and Magnetism, 3d ed., 2, pp. 345–346.

of the original set.[14] This value of inductance will not be what is wanted, in general, and the size of the wire to be used [15] to give the desired inductance may be found from the relation that for a coil occupying a given space the inductance varies directly as the square of the number of turns of wire in the coil.

It is a good plan to check the computed results by making a temporary wooden form in which can be wound a trial coil of the computed size of wire occupying the same space as the three coils composing one-half the windings of the instrument; that is, two fixed coils with the moving coil between them. This coil should have an inductance equal to one-half of the maximum value of inductance desired in the finished instrument at the upper end of the range.

8. OTHER INSTRUMENTS OF SIMILAR FORM

This instrument was designed and constructed about five years ago. Since then a number of similar ones have come to our notice. One designed by Rendahl [16] and made by the Gesellschaft für drahtlose Telegraphie has two D-shaped coils in a circular hard-rubber plate above which turns a similar plate and set of coils. The coils were wound in sections so that the range may be varied by changing the connections. Very similar instruments were patented by Ferrié and by Einstein.[17] We have no information concerning the scale law of these instruments or the relative dimensions of their parts. They have an inherent defect which is also found in the Nalder [18] and Mansbridge forms, namely, that a slight axial displacement of the disks from their normal relative position will appreciably change the inductance. Such a displacement may result from wear or loosening of the bearing, or from change of form of the disk.[19] In the construction described in this paper, the disks holding the two sets of fixed coils are rigidly connected. A slight axial displacement of the movable disk will not affect the inductance, because the change caused by its movement away from one fixed disk is compensated by the change caused by its approach toward the other.

[14] This assumes that the number of turns of wire is not altered. If all the dimensions of a coil are increased to m times their original value, the size of the wire remaining unchanged, the inductance increases to m^5 times its original value.

[15] The wire must of course be large enough to carry the desired current.

[16] Markau, Die Telephonie ohne Draht, pp. 96–98; 1912.

[17] Nesper, Jahrbuch der drahtlosen Teleg. und Teleph., 2; pp. 326–339; 1908–9.

[18] The variable inductor referred to in this paper as the Nalder is no longer made by Nalder Bros. & Co.

[19] Hard rubber (ebonite) yields slowly to stress, behaving somewhat like a very viscous liquid. Its high electrical resistance and the ease with which it can be machined have caused it to be very widely used in spite of its defects. It has the drawback of suffering great depreciation of its surface insulation when exposed to light, moisture, and dust.

9. DESIRABLE FEATURES IN A VARIABLE INDUCTOR

The desirable features to be obtained in a variable inductor are given below, on the assumption that the instrument is used to give variable self-inductance.

1. High time constant (ratio of inductance to resistance) for the space occupied by the instrument. This should be based on the maximum value of the inductance at the upper end of the useful part of the scale and not on any greater value which is beyond the useful part of the scale.

2. Large ratio of the maximum inductance to the minimum inductance. Neither of these values must be outside the useful part of the scale.

3. Astaticism.

4. Linear scale.

It is probable that the instrument herein described fulfills requirements 1 and 4 about as well as can be done. Some gain in time constant may be possible by starting with coils having Maxwell proportions as corrected by Shawcross and Wells,[20] since this would give a slightly more compact coil as the starting point. The improvement may not be great enough to warrant the labor of determining by trial the relative dimensions for uniform scale, though if such a scale is possible, the labor would doubtless be much less than that required in determining the proportions given in this paper.

Requirement 2 can be more fully met, though at some sacrifice of 1, by reducing the axial thickness of the coil. Experiments were made with coils of one-half the thickness given by the proportions of page 575. It appeared to be feasible to increase the ratio of maximum to minimum to about 18 or 20, but this gain is offset by a loss of about 40 per cent in the time constant.

Requirement 3 could be more fully met by a design using four sets of coils spaced 90° apart around the circular disks, but this would reduce the scale to one-half its present length.

It is therefore believed that the present design most nearly balances conflicting requirements, and that the labor of developing the other forms would only be justified if unusual requirements were to be met.

WASHINGTON, May 26, 1916.

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[20] Shawcross and Wells, Electrician, 75, p. 64; 1915.