"CENTER OF GRAVITY" AND "EFFECTIVE WAVE LENGTH" OF TRANSMISSION OF PYROMETER COLOR SCREENS, AND THE EXTRAPOLATION OF THE HIGH TEMPERATURE SCALE

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Some 10 years ago Waidner and Burgess [1] called attention to the shift in effective wave length of the color screens used with optical pyrometers, when the temperature of the source sighted upon changes. Pirani [2] has contributed two papers upon this subject, and very recently a paper by Hyde, Cady, and Forsythe [3] has appeared.

Formerly it has been assumed that the effective wave length is the so-called "center of gravity" of the luminosity curve. Hyde, Cady, and Forsythe have given a new definition of the mean effective wave length between any two temperatures, which has a real physical basis. They have defined the effective wave length of the glass used between these two temperatures as that wave length for which the ratio of the two radiation intensities corresponding to the two temperatures exactly equals the ratio of the integral luminosities through the screen for the same two temperatures.

In the following paragraphs the difference between the center of gravity and the true effective wave length is discussed and a very simple and accurate method is derived for obtaining the true effective wave length.

[1] Waidner and Burgess, this Bulletin, 3, p. 175; 1907.
[2] M. v Pirani Verh. d. Phys. Ges., 15, pp. 836–38; 1913; 17, pp. 47–62; 1915.
[3] Hyde, Cady, and Forsythe, Phys. R., 6, pp. 70, 74; 1915; also Astrophys, J., 42, pp. 294–304; 1915.

Let
$$J_\lambda = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda\theta}} = \text{Wien's law.}$$
$$T = f(\lambda) = \text{transmission of color screen.}$$
$$V = \varphi(\lambda) = \text{visibility.}$$

T and V are functions of λ only, but the analytical forms of the functions, at least in the case of $T = f(\lambda)$, are unknown. Consider two temperatures, θ_1 and θ_2.

$$\text{(1)} \qquad \begin{cases} \text{for } \theta_1 & J_1 = c_1 \lambda^{-5} e^{-c_2/\lambda\theta_1} \\ \text{for } \theta_2 & J_2 = c_1 \lambda^{-5} e^{-c_2/\lambda\theta_2} \end{cases}$$

$$\text{(2)} \qquad J_1/J_2 = e^{\frac{c_2}{\lambda}\left(\frac{1}{\theta_2} - \frac{1}{\theta_1}\right)}$$

Since λ may have any arbitrary value from o to ∞ the ratio J_1/J_2 may have any chosen value by properly choosing λ. Let L_1 = luminosity at temperature θ_1 and L_2 = luminosity at temperature θ_2.

$$L_1 = \int_0^\infty JTVd\lambda = \int_0^\infty J(\lambda, \theta_1)\, T(\lambda)\, V(\lambda)\, d\lambda$$
$$L_2 = \int_0^\infty J(\lambda, \theta_2)\, T(\lambda)\, V(\lambda)\, d\lambda$$
$$\text{(3)} \qquad L_1/L_2 = \frac{\int_0^\infty J(\lambda, \theta_1)\, T\, V\, d\lambda}{\int_0^\infty J(\lambda, \theta_2)\, T\, V\, d\lambda}$$

L_1 and L_2, and hence the ratio L_1/L_2, can be determined by graphical integration. Let the ratio be any definite number. It is possible to choose λ in (2) such that $J_1/J_2 = L_1/L_2$. Call this value of $\lambda = \lambda_L$. Substituting in (2):

$$\text{(4)} \qquad \text{mean } \lambda_L = \frac{c_2\left(\frac{1}{\theta_2} - \frac{1}{\theta_1}\right)}{\log L_1 - \log L_2} \qquad \text{(natural logarithms).}$$

This is the definition, expressed analytically, of the mean effective wave length over a given temperature range, proposed by Hyde, Cady, and Forsythe. Instead of referring the effective wave length to a given temperature range it may be referred to a definite temperature by letting the two temperatures approach one another. Let θ_1 and θ_2 approach θ as a limit. The right term of (4) is indeterminate but can be evaluated by the ordinary rule of L'Hospital, observing in the differentiation of the denomi-

nator that $L=\int_0^\infty TVJd\lambda$ where only J is a function of θ. Then for the limit $\theta_1=\theta_2=\theta$ the following value of λ_L is obtained:

$$\lambda_L=\frac{\int_0^\infty TVJ\,d\lambda}{\int_0^\infty \frac{TVJ}{\lambda}d\lambda} \tag{5}$$

This is the true effective wave length of the glass for a temperature θ. Define the wave length corresponding to the line which bisects the area of $\int_0^\infty TVJd\lambda$ as λ_g; that is, the axis of the center of gravity of the luminosity curve, so chosen that $\int_0^\infty TVJd\lambda = 2\int_0^{\lambda_g} TVJd\lambda$. Then by definition of the center of gravity:

$$\lambda_g=\frac{\int_0^\infty TVJ\,\lambda d\lambda}{\int_0^\infty TVJ\,d\lambda} \tag{6}$$

Having drawn the luminosity curve on any arbitrary scale whatever, paying no attention to actual values of the scale of ordinates (L), only relative values of L needing consideration, λ_g can be determined with a planimeter having two indicating dials, one of area and the other of moments, such as the Amsler integrator No. 1. Surprising accuracy is obtainable in measuring λ_g. This is evident when one considers that the luminosity curve for a good red glass extends only from about $\lambda=0.600$ to 0.750μ. Since all errors are actually based on the difference of these two wave lengths, a low precision in the determination of the center of gravity as far as the curve is concerned still means a very high precision in the determination of the absolute value of λ_g. Four or five significant figures are obtainable.

The method of dertermining λ_L suggested by Hyde involves the log ratio of the areas of two luminosity curves, and when λ_L is expressed by equation (5) its determination requires the ratio of the areas of two curves. In general, the area of luminosity curves can not be measured by a planimeter with an accuracy much better than 0.5 per cent, and since a direct mathematical integration is impossible because $T=f(\lambda)$, certainly, and $V=f(\lambda)$, possibly, are not expressible by any exact usable equation, it is

evident that great precision is not readily possible in the determination of λ_L by such a method—probably not better than three significant figures when great care is taken.

Inspection of equation (5), however, shows a very curious relation. Suppose that instead of plotting λ versus JTV as ordinates, the ordinary luminosity curve, we plot JVT/λ as ordinates. The line of the center of gravity of this curve is given by the following expression:

$$\lambda \text{ (center gravity)} = \frac{\int_0^\infty TVJ d\lambda}{\int_0^\infty \frac{TVJ}{\lambda} d\lambda} \tag{7}$$

which is identical with (5).

Therefore, the true effective wave length of the pyrometer glass is the wave length corresponding to the center of gravity of the curve $f(\lambda) = JVT/\lambda$ versus λ, i. e., the luminosity at any wave length divided by the wave length. The true value of λ_L is slightly different from the value of λ_g. The center of gravity of the curve JVT/λ versus λ can be determined with great accuracy so that a curve of $\lambda_L = f(\theta)$ may be easily obtained by this method. Whether such high mathematical accuracy is of value is a point open to question since the transmission of the glass can not be measured extremely accurately and visibility curves appear very different for different observers. But in the case of the transmission, absolute values are not required, merely the form of the curve needs to be known. And in the case of the visibility, only a short range of the curve is important, and over this range the slopes of the visibility curves of various observers are in good agreement.

The fact that the effective wave length of a red pyrometer screen is altered by change of temperature of the source sighted upon is accordingly well recognized, but the effect which this shift may have upon the accuracy of temperature measurements has never been considered. The present paper concerns the type of pyrometer, such as the Holborn-Kurlbaum instrument, in

which red glass screens are used in the ocular for producing as monochromatic illumination as possible. This instrument may be calibrated at the so-called fixed points of the temperature scale. On account of deterioration of the filament of the standard lamp, the maximum temperature at which the lamp can be safely operated is not greater than 1400° C and in general 1300° C. Two methods are used for measuring temperatures above this range. One involves decreasing the intensity of the source by means of neutral absorption glasses and the other by use of a sector disk. The effective wave length in either system shifts toward the green with increasing temperature. The best "neutral" absorption glasses are far from being neutral.[4] The transmission coefficient of the best glass obtainable is low in the blue, green, and yellow, and increases very rapidly in the red, with a maximum transmission in the early infra-red.

When strictly monochromatic radiation of wave length λ is used the following relation is obtained from Wien's law, where $\theta =$ true absolute temperature of the source, $S =$ observed absolute temperature when the sector or screen of transmission T is used in front of the source, and $c_2 = 14\ 500$ micron degrees.

$$\frac{1}{\theta} - \frac{1}{S} = \frac{\lambda \log T}{c_2 \log e} = A \tag{8}$$

The factor $\lambda \log T/c_2 \log e$ is a constant $= A$. When the above equation is applied for radiation which is not strictly monochromatic A is no longer a constant as generally assumed but is a function of θ, because when a sector disk is employed, λ, the effective wave length of the red glass screen is a function of θ, and when an absorption glass is used, the value of $\log T$ changes rapidly with variations in θ. Experimentally, the quantity A is determined by sighting upon a source at constant temperature and measuring θ and S. But such a calibration can be made over a very limited temperature range only, because the maximum value of θ is determined by the highest temperature at which it

[4] Dr. Nutting has informed me that he has succeeded in producing strictly neutral absorption screens from dye materials. These screens should prove of great value for use with optical pyrometers.

is safe to operate the pyrometer lamp, and the minimum value is determined by S, which rapidly becomes so low that no accuracy is possible in matching the pyrometer filament against the source. The allowable range in θ by this method for a transmission factor much less than 1 per cent is not greater than 200° C. Another method of obtaining A directly can be used. This consists in sighting upon a black body immersed in a metal or other material at its melting point. The temperature of the melting point is θ and the apparent or measured temperature is S. Suitable materials having high melting points which are known with sufficient accuracy to justify their use as fixed points are platinum, palladium, and possibly diopside. The construction of a black body to operate at the temperatures of the melting point of platinum and palladium is extremely difficult.

The highest fixed temperature which the Bureau has found convenient to use in standardizing absorption glasses is the diopside melting point. The method of obtaining a black body at this temperature has been described elsewhere.[5] However, whether the platinum, palladium, or the diopside melting point is used as the highest working temperature, the range of temperature over which an absorption glass can be calibrated is too small to determine how the quantity A varies with θ.

Two types of glass have been considered, one the Jena red transmission screen 2745, and the other the best obtainable black screen, Jena F 3815 with the laboratory reference number 4248B. A slightly better red filter is the Jena F 4512, but the difference between this and Jena 2745 is not great enough to change materially the conclusions obtained with the somewhat poorer glass. The monochromatic transmission coefficients through a portion of the visible spectrum were carefully measured with a silver strip cube spectrophotometer and a Brodhun adjustable sector disk. The observations are represented graphically in Fig. 1. The accuracy obtained with the low transmission black glass was possible only after eliminating stray light in the spectrophotometer system. For this reason the silver strip cube was found very much superior to the ordinary Lummer-Brodhun or Martens-

[5] Kanolt, this Bulletin, **10**, p. 5; 1914.

König spectrophotometer. In order to apply equation (5) to the determination of the effective wave length of the pyrometer system when (*a*) one red glass and (*b*) two red glasses were used in the ocular, the value of J_λ was computed from Wien's law for the absolute temperatures 1000, 1200, 1400, 1600, 1800, 2000, 2500, 3000, 3500. The factors for the visibility were obtained as a mean

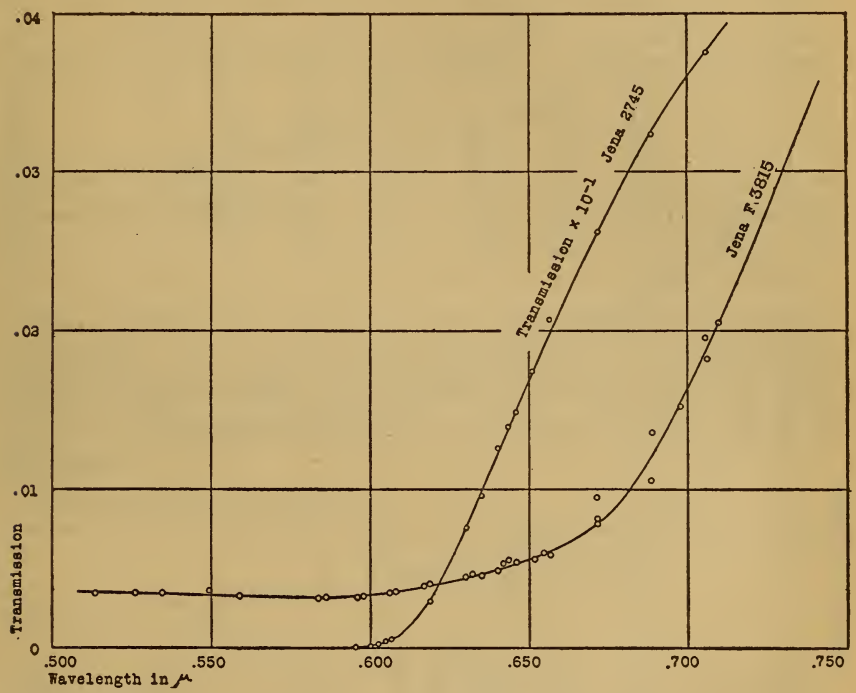

FIG. 1.—*Transmission coefficients of Jena red glass 2745 and Jena "neutral" glass F 3815. Room temperature. Thickness, 2745=3.20 mm. F 3815=0.52 mm*

of the data of Nutting,[6] Ives,[7] and Hyde and Forsythe.[8] The values of $\frac{JVT}{\lambda} = f(\lambda, \theta)$ were computed and the center of gravity λ_L of each of these curves, $f(\lambda, \theta)$ versus λ, was determined directly with an Amsler integrator. The following table illustrates the

[6] Nutting, Phil. Mag., 29, p. 306; 1915.
[7] Ives, Phil. Mag., Dec.; 1912.
[8] Hyde and Forsythe, Phys. R., 6, p. 70; 1915.

shift of wave length, λ_L, with change of temperature, and the data are represented graphically by Fig. 2. To show the difference between λ_L and the wave lengths corresponding to the center

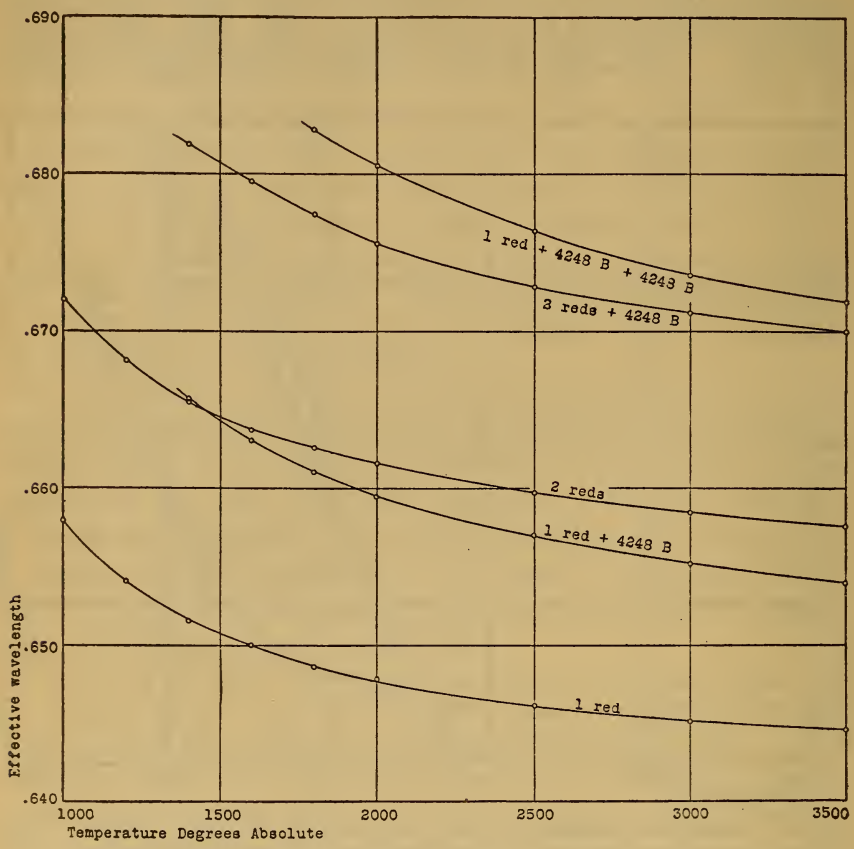

FIG. 2.—*Effective wave lengths of pyrometer glasses as a function of the temperature of the source*

of gravity λ_g of the luminosity curves, *JVT* versus λ, which, incorrectly, have been frequently considered to be the effective wave lengths of the glass, these values also are given in Table 1.

TABLE 1

"Effective Wave Length" and "Center of Gravity" of Red Screen Jena 2745 Used With and Without Neutral Screen Jena F 3815 (Reference No. 4248B)

[Thickness red screen=3.20 mm; thickness black screen=0.52 mm]

Temperature of source	1 red glass		2 red glasses		1 red+4248B		2 reds+4248B	
	λ_L	λ_g	λ_L	λ_g	λ_L	λ_g	λ_L	λ_g
1000 °abs	0.6578	0.6587	0.6720	0.6729	.....	.....	.....	.....
1200	.6541	.6548	.6681	.6690	.....	.....	.....	.....
1400	.6515	.6522	.6655	.6661	0.6657	0.6673	0.6818	0.6831
1600	.6500	.6506	.6637	.6644	.6630	.6645	.6796	.6807
1800	.6487	.6490	.6625	.6628	.6610	.6624	.6774	.6783
2000	.6478	.6486	.6615	.6619	.6595	.6609	.6756	.6767
2500	.6462	.6467	.6597	.6600	.6570	.6582	.6728	.6741
3000	.6451	.6456	.6584	.6589	.6552	.6563	.6712	.6721
3500	.6445	.6446	.6574	.6581	.6539	.6552	.6700	.6710

It is seen from Fig. 2 that the curve λ_L versus θ is approximately a hyperbola. An equation of the type $\lambda_L = a + b/\theta + c/\theta^2$ where the term c/θ^2 is exceedingly small fits the observations almost exactly. That the relation is not precisely hyperbolic is evident from the following:

Differentiating equation (5)

$$\frac{d\lambda_L}{d\theta} = \frac{c_2}{\theta^2}\left[1 - \frac{\int_0^\infty JVTd\lambda \int_0^\infty \frac{JVT}{\lambda^2}d\lambda}{\left(\int_0^\infty \frac{JVT}{\lambda}d\lambda\right)^2}\right]$$

$$= \frac{c_2}{\theta^2}(1 - \lambda_L/\lambda_c)$$

where λ_c is the center of gravity of the curve TVJ/λ^2 versus λ. If λ_L/λ_c were a constant, then $\lambda_L = a + b/\theta$, the equation of a hyperbola. Actually λ_L/λ_c shows an extremely small variation with θ. It is interesting to note that values of $\lambda_L = f(\theta)$ suggest the form of an equation representing the luminosity of any system.

Thus,

$$\frac{d \log L}{d\theta} = \frac{c_2}{\theta^2}\frac{1}{\lambda_L}.$$

When $\lambda_L = a + b/\theta$, the equation of luminosity and temperature takes the form derived by Nutting.[9]

If the carbon filament of the lamp of the Holborn-Kurlbaum pyrometer emits approximately black body radiation so that when it is matched against a source at a temperature θ the temperature of the filament is also θ, neglecting reflection losses at the lens surfaces, the match will always be an exact color match as well as intensity match. (Actual deviations from this condition introduce only a second order effect.) Such being the case, as far as temperature measurements are concerned, no errors are introduced by the shift in wave length of the system. It is only when the absorption glass or sector disk is employed that errors occur. When the absorption glass, for example, is used and a photometric match effected, the light from the source at a high temperature passing through the absorption glass and the red glass screen in the ocular is matched against the light from the lamp, at a lower temperature, passing through the red screen only. An exact color match can not be made in this case although the difference in color of the two fields is usually too slight to cause serious trouble. The match is obtained when the luminosity from the source through the black glass and the red glass equals the luminosity from the lamp through the red glass. Let T = transmission coefficient of the red glass, T' the transmission coefficient of the black glass, θ the temperature of the source, S the temperature of the lamp (apparent or measured temperature), L_θ the luminosity of the source, and L_s the luminosity of the lamp. Then

$$L_\theta = \int_0^\infty TT'VJ(\lambda,\theta)d\lambda$$

$$L_s = \int_0^\infty TVJ(\lambda, S)d\lambda$$

and the condition for a match is:

$$L_\theta = L_s$$

Curves were constructed of L_θ and L_s versus θ and S and values of θ and S obtained, by interpolation, for various values of $L_\theta = L_s$.

[9] This Bulletin, 5, p. 306; 1908

Hence, knowing θ and S, the quantity A or absorption factor can be determined by the relation $1/\theta - 1/S = A$. It is found, however, that interpolation of these plotted curves is somewhat inaccurate unless the values of L are computed for a great many temperatures. The method discussed below was used and is to be preferred for determining A.

Analogous to the Wien equation (8) for monochromatic radiation, the following equation applies when the condition of monochromatism is not realized.

$$\lambda'_L \log Tr = c_2\left(\frac{1}{\theta} - \frac{1}{S}\right)\log e = Ac_2 \log e \tag{9}$$

where λ'_L is the effective wave length of the complete system, lamp + red screen and source + black screen + red screen, and Tr is the effective transmission of the black screen. The effective transmission of the black screen will depend upon the temperature of the source and upon the number of red glass screens used in the ocular. Its value is determined by the ratio of the luminosity, L'_θ, of the black body source at a given temperature, through the black glass and the red glass, and the luminosity, L_θ, with the black glass removed, thus:

$$Tr = \frac{L'_\theta}{L_\theta} = \frac{\int_0^\infty TT'VJ(\lambda, \theta)d\lambda}{\int_0^\infty TVJ(\lambda, \theta)d\lambda} \tag{10}$$

Also since $L'_\theta = L_S$

$$Tr = \frac{L_S}{L_\theta} = \frac{\int_0^\infty TVJ(\lambda, S)d\lambda}{\int_0^\infty TVJ(\lambda, \theta)d\lambda} \tag{11}$$

Values of L'_θ and L_θ are obtained by computing and integrating the luminosity curves for any arbitrary temperatures, and hence Tr is found without interpolation of the curves $L = f(\theta)$.

It can be shown that λ'_L, the effective wave length of the complete system is the arithmetical mean of the effective wave lengths of the two components of the system. The effective wave lengths of the system, source + black screen + red screen, were obtained as the centers of gravity of the curves $TT'VJ/\lambda$ versus λ (cf Fig. 2 and Table 1).

The method, then, for computing A is as follows: In equation (9) assume any value of A approximately near the correct value and compute approximate values of S for various values of θ. From curves Fig. 2 obtain the effective wave lengths corresponding to the temperature S of the lamp observed through the red screen, and the temperature θ of the furnace observed through the black glass + the red glass. The mean of these values is λ'_L. The value of log Tr is obtained from equation (10). Whence from (9) the equation $A = \lambda'_L \log Tr/c_2 \log e$ may be solved for the quantity $A = f(\theta)$. Table 2 shows the variation of A with the temperature of the source for absorption glass 4248B. These values are presented graphically by Fig. 3. Table 2 also includes the effective transmission coefficients of the black screen. The marked variation of the transmission with the temperature of the source is of interest and also the dependence of the transmission upon the number of screens used in the ocular. Thus the addition of 1 red glass in the ocular actually increases the effective transmission of the black glass about 30 per cent. This table emphasizes the importance of a thorough study of the transmission screens used in photometry. A screen calibrated with a lamp burning at a definite watts per candle can not be used without applying corrections to the transmission factor when the lamp is burning at any other efficiency.

TABLE 2

Absorption Glass Jena F 3815

[$A = 1/\theta - 1/S$ and transmission coefficient as a function of θ]

θ	1400	1600	1800	2000	2500	3000	3500	
$Tr \times 10^5$	696	670	658	636	612	601	589	1 red glass
$-A \times 10^7$	2265	2274	2276	2287	2297	2300	2305	
$Tr \times 10^5$	912	867	849	828	787	768	749	2 red glasses
$-A \times 10^7$	2190	2207	2210	2217	2232	2239	2248	

The above discussion applies equally well when a sector disk is used to cut down the luminosity of the source. In this case the transmission factor is constant, but the effective wave length depends upon the temperature and number of red glasses used in the ocular. Table 3 gives the values of A obtained with a sector having an opening of 2.7°.

TABLE 3

Sector disk with 2.7° opening

[$A=f(\theta)$]

θ	1400	1600	1800	2000	2500	3000	3500	
$-A\times10^7$	2205	2201	2196	2192	2187	2183	2181	1 red glass
$-A\times10^7$	2254	2248	2243	2239	2233	2228	2226	2 red glasses

The variation of A with the temperature of the source when a sector disk is used is also represented by Fig. 3. For the combina-

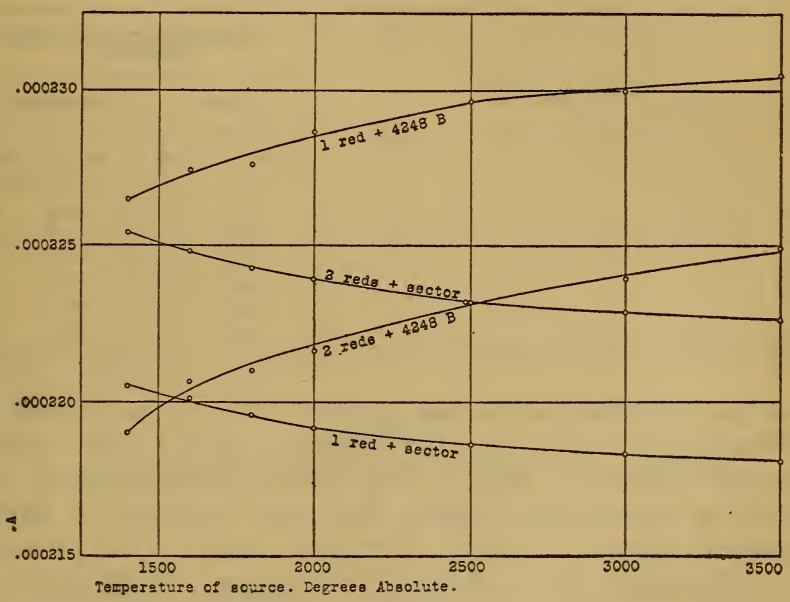

FIG. 3.—*A as a function of θ*

tion of red glass and black glass, A increases with increasing temperature and for the combination of red glass and sector disk, A decreases with increasing temperature. If the absorption glass and sector disk are calibrated at about the most favorable point possible, the melting point of diopside, and the values of A thus obtained are assumed constant with temperature, as has always been the assumption, measurements upon a source at a temperature of 3500° absolute will be 30° to 40° low when the absorption

glass is used, and about 20° high when the sector disk is used. The difference in the measurements by the two methods will amount to 50° or 60°.

The errors to be expected in measurements of this kind are shown by Table 4. The values are based on the favorable calibration temperature of about 1650° absolute.

TABLE 4

Absorption Glass 4248B and Sector Disk of Opening 2.7°

[Calibration temperature about 1650° abs.]

Temperature θ° abs.	True temperature minus observed temperature			
	4248B		Sector disk	
	1 red	2 reds	1 red	2 reds
1200	− 5	− 5	+ 2	+ 2
1600	− 1	− 1	0	0
2000	+ 4	+ 5	− 3	− 3
2400	+11	+13	− 7	− 8
2800	+19	+23	−12	−14
3200	+27	+34	−17	−20

Actually it would not be safe to operate the pyrometer lamp at a temperature which, with the absorption factors of the above absorption glass and sector, would permit measurements of the source at a temperature higher than 2500° absolute. A denser absorption glass or a sector of smaller opening must be used. But if the transmission of the glass or sector is much less, it is impossible to calibrate them. The general practice has been to use two absorption glasses, which have been calibrated independently, and to assume that the "constant" A for the two glasses used together is the sum of the constants when calibrated separately. Table 5 and Fig. 4 show the values of A for two absorption glasses of the type 4248B, used together. The values of A for the glasses used separately are added and given by the third line of the table. The "constant" obtained by the separate calibration "constants" of such glasses at the diopside point, added

together, is given in the fourth line. This latter value represents

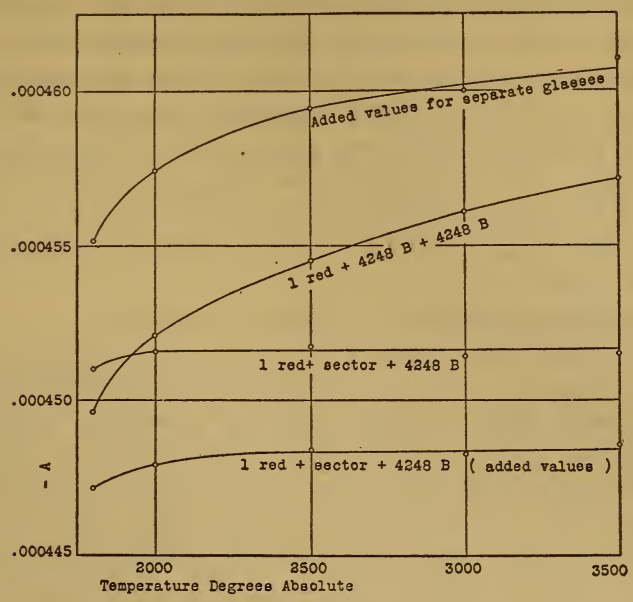

FIG. 4.—*A as a function of θ*

that obtained by the method which has been quite generally heretofore assumed correct.

TABLE 5

Use of Two Black Glasses of the Type 4248B With 1 Red Glass

θ	1800	2000	2500	3000	3500
$-A \times 10^7$ (correct values)	4497	4521	4545	4561	4572
Values obtained by adding A for each glass	4552	4574	4594	4600	4610
Added value of A at θ=1650	4550	4550	4550	4550	4550

The use of a combination of sector disk and absorption glass would very likely prove highly satisfactory for extreme temperatures. Fig. 4 shows the values of *A* for a temperature range 1800° to 3500° absolute, for the combination of one red glass, the black glass 4248B, and a 2.7° opening disk. The variation of *A* is negligible above 2000°, but the calibration of such a system is again difficult. If the sector and absorption glass are calibrated separately and the separate values of *A* added, the values thus

obtained are too low, as indicated by Fig. 4. In the present case, however, if the sector is calibrated alone, with two red glasses in the ocular, at any temperature, and the absorption glass is calibrated with one red glass in the ocular, at the same temperature, the sum of the values obtained separately is equal to the value of A for the combination one red glass + sector + black glass, at all temperatures. The addition of the extra red glass in the calibration of the sector is simply to bring the effective wave length of the system nearer to that of the combination one red glass + one black glass.

Experimental confirmation of above discussion:

The difference in the value of $A = 1/\theta - 1/S$ when one red glass is used and when two red glasses are used in the ocular is very pronounced with either a sector disk or absorption glass. This difference may amount to 50° at 2000° if the value of A determined for one red glass in the ocular is used for computing temperatures measured when two red glasses are in the ocular, and vice versa. Or if the pyrometer filament is matched through the black glass or sector against a source at, say, 2000°, using one red glass, the match is completely destroyed when a second red glass is added to the ocular, and vice versa.

The fact that A is a function of the temperature which, with increasing temperature of the source, increases for the absorption glass and decreases for the sector disk, may be easily shown qualitatively. Decreasing the temperature of the source obviously must increase the value of the effective wave length of either system. Instead of increasing the effective wave length by decreasing the temperature, the same effect may be produced by the addition of one red glass in the ocular. Obtain a match at a high temperature with the sector and one red glass. Add one red glass. The match is destroyed and the filament will appear darker than the field. If the same process is repeated with a sector, the filament will appear brighter than the field. A sector disk having one opening of 3.6° was accessible, which could be rotated at 3000 revolutions per minute. But the luminosity is so high when only one red glass is used, at a temperature of the source of 2000°, that 3000 revolutions per minute is below the critical speed and serious flickering prevents accurate matching.

Two Glan nicol prisms nearly crossed were therefore substituted for the sector. Since the Glan nicol is practically nonselectively absorbing, its effect is precisely that of the sector disk. The following table shows the apparent temperatures obtained for a match when the source was about 1700° C:

Ocular	1 red glass	2 red glasses
	°C	°C
Absorption glass	1109	1126
Nicols	1095	1087

Quantitative proof that $A = f(\theta)$.

Referring to Table 4, it is seen that the sector disk or nicols should give a higher temperature reading than the absorption glass when all are calibrated at a low temperature and used with a source at a high temperature. The sector disk of 3.6° opening, the nicols, and the absorption glass were calibrated on a source at about 1450° absolute. Measurements throughout were made by two observers working independently. Table 6 shows that the difference in temperatures of a source at about 2000° measured by the two methods is of serious importance. If it had been possible to maintain the temperature of the source at 2500° to 3000°, instead of 2000°, differences of two or three times this magnitude would have resulted.

TABLE 6

Experimental Data on Use of Sector Disk, Nicol Prisms, and Absorption Glass for Determination of High Temperatures

Observer	Calibration temperature	Red glasses	Sector	Nicols	Absorption glass	Sector or nicols-absorption glass
	Deg. abs.		Deg.	Deg.	Deg.	Deg.
COF	1450	2	1828	.....	1810	18
PDF	1450	1	1986	.....	1979	7
PDF	1450	1	1998	.....	1989	9
PDF	1450	2	2002	.....	1984	18
PDF	1450	1	.....	1968	1955	13
PDF	1450	1	.....	1970	1958	12
PDF	1450	1	.....	1974	1961	13
COF	1400	1	.....	1971	1962	9

Use of two and more absorption glasses. That the separate values of A determined for several absorption glasses separately can not be added to obtain the value of A for the glasses when used together has been noted by Waidner and Burgess [10] in their investigation of the temperature of the arc. Recomputing their data for four glasses showing the same absorption the following table is obtained:

Glasses	Observed A	Added values of A
1	−.0001112	−.0001112
2	2209	2224
3	3283	3336
4	4332	4448

The above data were taken without careful reference to the calibration temperatures, and hence they can not be considered quantitatively. In general, however, the value of A for several glasses used together is less than the sum of the values of A for the glasses used separately. This effect has nothing to do with multiple reflection between absorption glasses, as multiple reflection is usually of negligible importance.

The explanation lies in the change of the transmission coefficient of each successively added absorption glass, although all the glasses are identically alike. The quantity A is proportional to $\lambda'_L \log Tr$. The addition of each absorption glass moves the effective wave length of the entire system toward the red. This increases the value of λ'_L and would cause abnormally high values of A. But the transmission coefficient of each successively added absorption glass is greater than that of the glass used immediately preceding. Hence, the value of $\log Tr$ for any definite temperature of the source decreases numerically for each successive glass. The relative decrease in $\log Tr$ is greater than the relative increase in λ'_L, and thus the observed values of A are smaller than the added values of the single glasses.

[10] Waidner and Burgess (this Bulletin, 1), p. 116; 1904.

SUMMARY

An equation has been derived representing as a function of the temperature of the black body source sighted upon, the "effective wave length" of any pyrometer color screen or system of color screens, or of color screens used in combinations with sector disk, nicol prisms, or absorption screens. Attention is called to the difference between the "effective wave length" and the "center of gravity" of the luminosity curve, as the so-called center of gravity of the color screen has been in the past sometimes considered the true effective wave length. When an absorption glass, nicol prisms, or sector disk is used with a pyrometer employing light which is not strictly monochromatic, such as the Hoborn-Kurlbaum instrument, for the object of reducing the brightness of the sources sighted upon, the relation between the true temperature θ and the measured temperature S is of the form $1/\theta - 1/S = A$ where A has been heretofore considered a constant. Actually, it is shown, A varies rapidly with change in temperature of the source. The consideration of A as a constant may introduce errors as great as 40° or 50° at 3500° if the absorption device is calibrated in the usual manner. When two or more absorption glasses are used together, it is not permissible to assume that the value of A for the entire system is equal to the sum of the values of A for the glasses used separately. It is shown that the combination of a sector disk and absorption glass gives a value of A which is practically independent of θ.

In conclusion, acknowledgment is due C. O. Fairchild and T. R. Harrison, of this laboratory, for assisting in the measurements and for graphically integrating some 150 curves.

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