CONSTRUCTION OF PRIMARY MERCURIAL RESISTANCE STANDARDS

By F. A. Wolff, M. P. Shoemaker, and C. A. Briggs

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INTRODUCTION

The absolute unit of resistance is defined as 10^9 units of the c. g. s. electromagnetic system. Although depending solely on the measurement of length and time, it can not, at the present time, be determined to a higher degree of accuracy than the international unit of resistance can be realized by primary mercurial resistance standards constructed in accordance with the specifications of the London Conference on Electrical Units and Standards (1908).

Since the international unit is in almost exclusive use, concrete standards will continue to be employed for work of the highest precision, even though the unit thus defined should be found to differ appreciably from the absolute unit, as it is of the greatest importance, for purposes of comparison, that the same units be employed everywhere.

There are two possible methods of defining the international unit of resistance—(1) by adopting a particular standard, as has been done in the case of length and mass, and (2) by defining it in terms of the resistance, at a specified temperature, of a pure material of specified dimensions. If suitable materials were available, the first method would offer an ideal solution; but, while considerable improvements have been made in the construction of resistance standards and in the production of specially adapted resistance materials, much remains to be accomplished before such standards can be safely trusted to maintain their values indefinitely. The remaining alternative, best realized through the mercury unit proposed by Siemens in 1860, has therefore been internationally adopted. The dimensions of the mercury column have been twice modified so as to bring the mercury unit into closer agreement with the absolute ohm; but further changes, except possibly in the adoption of more rigorous specifications, are not likely to be made until the accuracy attainable in absolute measurement equals or exceeds that realized in the reproducibility of the mercury unit.

The London International Electrical Conference of 1908 was the first to draw a sharp distinction between the absolute and international units. The ohm was defined as having the value of 1,000,000,000 (10^9) in terms of the centimeter, and second, the ampere, volt, and watt being also defined in absolute units, but at the same time the conference recommended for adoption "a system of *international* units representing the above and sufficiently near for the purpose of electrical measurements, and as a basis for legislation." The unit of resistance, the realization of which is the object of this investigation, was defined as follows:

The *international ohm* is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area and of a length of 106.300 centimeters.

Certain details of the procedure for the construction of mercurial resistance standards, discussed elsewhere in this paper, were also adopted, in the form of specifications. The above value for the mass of mercury contained in a column 106.300 cm long corresponds to a cross section of 1 sq. mm if the density of mercury is taken as 13.5956 grams per cubic centimeter. This method of definition, first adopted by the Chicago congress, was chosen in order to eliminate the uncertainty due to experimental errors in the determination of absolute density, including, as it does, the error in the determination of the relation between the liter and cubic decimeter.

PART I. THEORETICAL

In practice the cross section and length of the mercury column are given by the dimensions of glass tubing, employed to confine the mercury, which can not be expected to agree exactly with the dimensions specified either in magnitude or in uniformity of cross section. The manner in which such variations are taken into account will be briefly described below.

A. THE METHOD OF CALCULATING THE RESISTANCE OF A GIVEN MERCURIAL COLUMN

The resistance and mass of a column of mercury of uniform cross section s_0 and of length L_0 are given by the formulæ:

$$R_0 = \rho_0 \frac{L_0}{s_0} \tag{1}$$

$$M_0 = D_0 L_0 s_0$$

in which ρ_0 and D_0 denote, respectively, the resistivity and density of mercury.

Multiplying, we have

$$R_0 M_0 = \rho_0 D_0 L_0^2$$

or

$$R_0 = \rho_0 D_0 \frac{L_0^2}{M_0} \tag{2}$$

But from the definition of the international ohm, $R=1$ when $L_0=106.300$ cm and $M_0=14.4521$ g.

Hence,

$$\rho_0 D_0=\frac{14.4521}{(106.300)^2}$$

and consequently,

$$R_0=\frac{14.4521}{(106.300)^2}\,\frac{L_0^2}{M_0} \tag{3}$$

This formula gives the resistance, in international ohms, of a column of mercury of uniform cross section, L_0 centimeters in length and M_0 grams in mass. Hence any departure from the mean cross section specified in the definition can be allowed for.

The above formula may also be written

$$R_0=\frac{14.4521}{(106.300)^2}\,\frac{L_0}{\frac{M_0}{L_0}} \tag{4}$$

in which the term $\frac{M_0}{L_0}$ denotes the mass of mercury per unit length of the tube. For a 1-ohm tube the ohm length is therefore given by the formula:

$$L_0=\frac{(106.300)^2}{14.4521}\,\frac{M_0}{L_0} \tag{5}$$

But $\frac{M_0}{L_0}$ is given by ratio of the mass in grams to the observed length in centimeters of *any* mercury column employed, if the tube be uniform in cross section (a correction being applied for the menisci at the ends of the column).

Hence,

$$L_0=\frac{(106.300)^2}{14.4521}\,\frac{M'_0}{L'_0} \tag{6}$$

in which L'_0 denotes the length in centimeters, and M'_0 the mass in grams, of any mercury column in a tube of uniform cross section.

Variable Cross Section.—The correction factor which must be applied for variations in cross section throughout the length of a given tube may be deduced as follows:

$$R'_0 = \rho_0 \int_0^{L_0} \frac{dl}{s}$$

$$M_0 = D_0 \int_0^{L_0} s\, dl \tag{7}$$

$$R'_0 M_0 = \rho_0 D_0 \int_0^{L_0} s\, dl \int_0^{L_0} \frac{dl}{s}$$

and hence

$$R'_0 = \rho_0 \frac{D_0}{M_0} \int_0^{L_0} s\, dl \int_0^{L_0} \frac{dl}{s} \tag{8}$$

By comparison with formula (2) for a tube of uniform cross section it will be seen that

$$R'_0 = R_0 \left\{ \frac{1}{L_0^2} \int_0^{L_0} s\, dl \int_0^{L_0} \frac{dl}{s} \right\} \tag{9}$$

The factor by which R_0, the resistance of a uniform column of the length L_0 and mass M_0, must be multiplied to obtain the resistance of a column of the same length and mass but of variable cross section, is known as the caliber factor. Denoting this by $(1 + C)$, we have

$$1 + C = \frac{1}{L_0^2} \int_0^{L_0} s\, dl \int_0^{L_0} \frac{dl}{s} \tag{10}$$

Since it is impossible to directly determine the cross section of a capillary tube from point to point to a sufficient degree of accuracy for the purpose in hand, we must content ourselves with an approximation based on a calibration of the tube. This will give the mean cross section of finite elements into which the tube may be divided. If the resistance of the whole column be computed

as the sum of the resistances of n cylindrical elements, each of length l, formula (9) becomes

$$R'_0 = R_0 \frac{1}{L_0^2}\left\{\Sigma_1^n sl \; \Sigma_1^n \frac{1}{s} l\right\} \tag{11}$$

or, since $nl = L_0$,

$$R'_0 = R_0 \frac{1}{n^2} \Sigma_1^n s \; \Sigma_1^n \frac{1}{s} \tag{12}$$

Hence,

$$1 + C = \left(\frac{1}{n}\Sigma_1^n s\right)\left(\frac{1}{n}\Sigma_1^n \frac{1}{s}\right) \tag{13}$$

If, as is usually the case, the particular length of tube having unit resistance and for which the caliber factor is sought, consists of n elements of length l and two fractional elements of length αl and βl and cross sections s_α and s_β, at the ends of the tube,

$$L_0 = (n + \alpha + \beta) l$$

$$\Sigma sl = l\left\{\Sigma_1^n s + \alpha s_\alpha + \beta s_\beta\right\}$$

$$\Sigma \frac{1}{s} l = l\left\{\Sigma_1^n \frac{1}{s} + \frac{\alpha}{s_\alpha} + \frac{\beta}{s_\beta}\right\}$$

and

$$1 + C = \left\{\frac{1}{n+\alpha+\beta}\left(\Sigma_1^n s + \alpha s_\alpha + \beta s_\beta\right)\right\}\left\{\frac{1}{n+\alpha+\beta}\left(\Sigma_1^n \frac{1}{s} + \frac{\alpha}{s_\alpha} + \frac{\beta}{s_\beta}\right)\right\} \tag{14}$$

The caliber factor is therefore given by the product of the mean cross section by the mean reciprocal cross section, each element being weighted in proportion to its length.

Since the s's enter into the numerator of the first series and into the denominator of the second series, the result is obviously independent, as it should be, of the unit in terms of which the cross sections are expressed.

The labor of calculation of the caliber factor can be simplified by expressing the cross sections of the separate elements as follows:

Let

$$\left.\begin{aligned} s_m &= s(1 + k_m) \\ s_\alpha &= s(1 + k_\alpha) \\ s_\beta &= s(1 + k_\beta) \end{aligned}\right\} \tag{15}$$

Then

$$\Sigma_1^n s + \alpha s_\alpha + \beta s_\beta = (n+\alpha+\beta)s\left\{1 + \frac{\Sigma_1^n k + \alpha k_\alpha + \beta k_\beta}{n+\alpha+\beta}\right\}$$

and since $\frac{1}{1+k} = 1 - k + k^2 - k^3$, etc.,

$$\Sigma_1^n \frac{1}{s} + \frac{\alpha}{s_\alpha} + \frac{\beta}{s_\beta} = (n+\alpha+\beta)\frac{1}{s}\left\{1 - \frac{\Sigma_1^n k + \alpha k_\alpha + \beta k_\beta}{n+\alpha+\beta} + \frac{\Sigma_1^n k^2 + \alpha k_\alpha{}^2 + \beta k_\beta{}^2}{n+\alpha+\beta} - \frac{\Sigma_1^n k^3 + \alpha k_\alpha{}^3 + \beta k_\beta{}^3}{n+\alpha+\beta} + \text{etc.}\right\}$$

Hence

$$1 + C = \left\{1 + \frac{\Sigma_1^n k + \alpha k_\alpha + \beta k_\beta}{n+\alpha+\beta}\right\}\left\{1 - \frac{\Sigma_1^n k + \alpha k_\alpha + \beta k_\beta}{n+\alpha+\beta} + \frac{\Sigma_1^n k^2 + \alpha k_\alpha{}^2 + \beta k_\beta{}^2}{n+\alpha+\beta} - \frac{\Sigma_1^n k^3 + \alpha k_\alpha{}^3 + \beta k_\beta{}^3}{n+\alpha+\beta} + \text{etc.}\right\} \quad (16)$$

Since $\frac{\Sigma_1^n k + \alpha k_\alpha + \beta k_\beta}{n+\alpha+\beta} s$ represents the average difference between s and the mean cross section of the particular portion of the tube to which the caliber factor is referred, it can be made zero or so small in comparison with unity by a suitable choice of s that the cross products (each including another very small term) obtained by multiplying the two factors are negligible. Moreover, the first-mentioned term enters with opposite signs in the two factors, so that it disappears from the product.

Hence,

$$1 + C = 1 + \frac{\Sigma_1^n k^2 + \alpha k_\alpha{}^2 + \beta k_\beta{}^2}{n+\alpha+\beta} - \frac{\Sigma_1^n k^3 + \alpha k_\alpha{}^3 + \beta k_\beta{}^3}{n+\alpha+\beta} + \text{etc.} \quad (17)$$

The accuracy of the caliber factor will depend not only on the accuracy of the observations, but also on the magnitude of the range and the freedom from abrupt discontinuities in the cross section of the tube, since in the calculation each of the elements is supposed to be of uniform cross section throughout its length. The length of the elements must therefore be so short that this must apply to a sufficient degree of approximation. In cases where the cross-section curve deduced from the observations is sufficiently continuous, a further approximation can be made by increasing the number of elements, each of the original elements being considered as made up of two or more having the mean cross sections given by the curve, or more simply by computing the ratio of the sum of the resistances of the separate elements,

regarding each as uniformly conical, with the terminal cross sections given by the curve, to their resistance computed as uniform cylinders, the ratio being applied as a correction to the caliber factor.

It must, however, be pointed out that variations within the elements, not following the general variations from element to element throughout the length of the tube, are not disclosed by the calibration, and, moreover, that the likelihood of the presence of such variations decreases as the range of the cross section of the tube decreases and as its general degree of uniformity increases.

B. THE DETERMINATION OF THE CALIBER CORRECTIONS

For the sake of completeness the following discussion of the methods of calibration and the reduction of the results is given, although they have been described a number of times, both in connection with the study of primary thermometers and of ohm tubes.

For simplicity consider a tube provided with an accurate graduation in the interval *op* which is to be calibrated.

Let v = mean volume per division between o and p.

v_{ab} = mean volume per division between a and b.

V_{ab} = volume between a and b.

s = mean cross section between o and p.

s_{ab} = mean cross section between a and b.

Then

$$v = cs \text{ and } v_{ab} = cs_{ab},$$

c depending upon the units in terms of which v and s are expressed.

If we put

$$V_{ab} = v\{(b-a) + (x_b - x_a)\}$$

Then

$$V_{op} = v\{p + (x_p - x_o)\}$$

But

$$V_{op} = v_p$$

Hence

$$x_p = x_o$$

If we put

$$x_o = o \text{ then } x_p = o$$

and

$$V_{oa} = v(a + x_a)$$
$$V_{ob} = v(b + x_b)$$

Obviously, vx_a, vx_b, and $v(x_b - x_a)$ represent the excess in volume of the respective intervals of the given tube over the corresponding intervals of a strictly uniform tube of the same mean cross section.

In addition,

$$V_{ab} = cs_{ab}(b-a) = v\{(b-a) + (x_b - x_a)\}$$

and since $$v = cs$$

$$s_{ab} = s\frac{(b-a) + (x_b - x_a)}{(b-a)}$$

or $$s_{ab} = s\left\{1 + \frac{x_b - x_a}{b-a}\right\} \tag{18}$$

The quantities x_a, x_b, etc., are called the caliber corrections of the tube referred to the interval *op*, and if determined for a number of equidistant points the relative mean cross sections are given bv the formula

$$s_m = s\left\{1 + \frac{x_{m+1} - x_m}{l}\right\}$$

If $$s_m = s(1 + k_m),$$

$$k_m = \frac{x_{m+1} - x_m}{l} \tag{19}$$

From either of these the caliber factor of the tube can be calculated subject to the limitations pointed out below.

1. METHODS OF CALIBRATION

Various methods have been devised for calibrating capillary tubing, principally in connection with thermometers. These are, of course, directly applicable to ohm tubes. The following brief description of the theory is based on the appendix to Benoit's[1] paper dealing with the construction of mercury resistance standards for the use of the French post office.

The methods of calibration may be classified as: Simple; complete; abbreviated; combinations of the foregoing.

In the simple calibration of a tube it is considered to consist of n sections of length l. The length of a mercury column, having approximately the length l, is determined when placed in each of

[1] Construction des Étalons Prototypes de Résistance Électrique, R. Benoit, Paris, Gauthier-Villars, 1885.

the sections. Correcting for the volumes of the two menisci, the relative cross sections are given approximately by the reciprocals of the corrected thread lengths.

In the complete calibration $(n-1)$ calibrating threads of lengths $\frac{1}{n}L$, $\frac{2}{n}L$ . . . . $\frac{n-1}{n}L$ are employed. The length of each thread is determined in all the possible positions, the ends being at or near the $n+1$ points for which the caliber corrections are sought. The number of observations is given by the formula $\frac{(n-1)(n+2)}{2}$, so that when n is large the process is long and tedious.

The abbreviated calibration, devised to minimize the labor involved in a complete calibration, consists in the employment of three or more threads of the lengths given above, each of which is measured in the same positions as in the complete calibration.

Combinations of the above methods are resorted to to further minimize the work, a calibration being first made by subdividing the tube into r sections each of which is further subdivided into $\frac{n}{r}$ sections by any of the methods above described.

The Simple Calibration.—Let x_1, x_2 . . . . x_{n+1} be the caliber corrections for $n+1$ equidistant points, p_1, p_2 . . . . p_{n+1} and let $p_1p_2=l$.

Let $(m+1)\ l+\Delta_{m+1}$ and $ml+\Delta_m$ be the observed positions of the ends of the thread in the position $p_m\ p_{m+1}$ and let $l+\lambda$ be the length which the calibrating thread would have if measured in a uniform tube of the same mean cross section as that of the given tube between the points p_1 and p_{n+1}.

Then

$$V_{o\,m+1}-V_{o\,m}=v\ (l+\lambda)$$

However,

$$V_{o\,m+1}-V_{o\,m}=v\{(m+1)\ l+x_{m+1}+\Delta_{m+1}\}-v\{ml+x_m+\Delta_m\}$$

Hence

$$x_{m+1}-x_m+\Delta_{m+1}-\Delta_m=\lambda$$

Or

$$x_m-x_{m+1}+\lambda=\Delta_{m+1}-\Delta_m=a_m \qquad (20)$$

a_m is obviously the excess of the thread length in the position $p_m\, p_{m+1}$ over the length of the subinterval l, and the x's, Δ's and λ must be expressed in terms of the same units.

The resulting n equations, employing the customary notation, are as follows:

$$\left.\begin{array}{l} x_1 - x_2 + \lambda \quad = a_1 \\ x_2 - x_3 + \lambda \quad = a_2 \\ \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ x_n - x_{n+1} + \lambda = a_n \end{array}\right\} \tag{21}$$

These are sufficient to determine the x's and λ, since two of the x's can be arbitrarily fixed (which is equivalent to fixing v). The usual practice is to make x_1 and x_{n+1} zero, in which case the caliber corrections are referred to the interval $p_1\, p_{n+1}$.

Making $x_1 = x_{n+1} = o$ and adding the equations we obtain

$$n\lambda = \Sigma_1^n a \text{ or } \lambda = \frac{1}{n}\Sigma_1^n a \tag{22}$$

Substituting in succession in the observational equations the values of the x's are found.

Second Approximation.—In equation (20) the caliber corrections for the ends of the calibrating thread are taken as being equal to the correction at the precise points for which they are sought. This equation should have been written

$$x_{m+\Delta_m} - x_{m+1+\Delta_{m+1}} + \lambda = a_m$$

so that if the caliber corrections are changing appreciably allowance must be made as follows:

$$x_{m+\Delta_m} = x_m + \left(\frac{dx}{dl}\right)_m \Delta_m$$

$$x_{m+1+\Delta_{m+1}} = x_{m+1} + \left(\frac{dx}{dl}\right)_{m+1} \Delta_{m+1}$$

Substituting

$$x_m - x_{m+1} + \lambda = a_m - \left\{\left(\frac{dx}{dl}\right)_m \Delta_m - \left(\frac{dx}{dl}\right)_{m+1} \Delta_{m+1}\right\} \tag{23}$$

By plotting as ordinates the values of the x's first found a first approximation caliber curve is obtained. From this the approximate values of $\left(\frac{dx}{dl}\right)$ can be obtained for the points $p_1 \ldots p_{n+1}$, so that the necessary correction can be applied to the a's in equations (21), and thus a second approximation to the x's and λ found. If the tube is very irregular, a third approximation, made in the same manner, may be required.

The Complete Calibration.—In the complete calibration as stated above, $n-1$ threads of lengths $\frac{1}{n}L, \frac{2}{n}L \ldots \frac{n-1}{n}L$ are employed, each thread being measured in all possible positions, the ends of the thread being, however, at or near the points $p_1\ p_2 \ldots p_{n+1}$. In this manner the following observational equations, expressed in the customary notation, are obtained:

$$\left|\begin{array}{cccccc}
x_1-x_2+\lambda_n=a_{1\,2} & x_1-x_3+\lambda_{n-1}=a_{1\,3} & \cdots\cdots & x_1-x_{n-1}+\lambda_3=a_{1\,n-1} & x_1-x_n+\lambda_2=a_{1\,n} \\
x_2-x_3+\lambda_n=a_{2\,3} & x_2-x_4+\lambda_{n-1}=a_{2\,4} & \cdots\cdots & x_2-x_n+\lambda_3=a_{2\,n} & x_2-x_{n+1}+\lambda_2=a_{2\,n+1} \\
\cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot & \cdots\cdots & x_3-x_{n+1}+\lambda_3=a_{3\,n+1} & \\
\cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot & \cdots & & \\
\cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot & \cdots & & \\
\cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot & \cdot\cdot & & \\
\cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot & \cdot & & \\
\cdot\ \cdot\ \cdot\ \cdot & x_{n-1}-x_{n+1}+\lambda_{n-1}=a_{n-1\,n+1} & & & \\
x_n-x_{n+1}+\lambda_n=a_{n\,n+1} & & & &
\end{array}\right| \qquad (24)$$

There are thus $\frac{1}{2}(n-1)(n+2)$ equations for the determination of $(n-1)$ x's (2 being arbitrary) and the $(n-1)$ λ's. Methods for the solution of these equations according to the principles of least squares have been given. After finding the values of the x's and λ's, the computed values of the a's can be determined. Denoting the difference between the observed and computed values by Δ, an amount less than the error of observation on account of the least square adjustment of the results, we find as the most probable error of a single observation, for a calibration into tenths,

$$r=.6745\sqrt{\frac{\Sigma\Delta^2}{36}} \qquad (25)$$

The probable errors of the x's, expressed in terms of r, are as follows:

$$\left.\begin{aligned} r\ (x_1) &= r\ (x_{11}) = 0 \\ r\ (x_2) &= r\ (x_{10}) = .416\ r \\ r\ (x_3) &= r\ (x_9) = .401\ r \\ r\ (x_4) &= r\ (x_8) = .387\ r \\ r\ (x_5) &= r\ (x_7) = .376\ r \\ r\ (x_6) &= .369\ r \end{aligned}\right\} \tag{26}$$

The Abbreviated Calibration into Tenths.—One of the most common of the many possible abbreviated methods of calibration and the principal method employed in the calibration of the four tubes here dealt with consists in limiting the number of threads to three, approximately four-tenths, five-tenths, and six-tenths of the length of the interval to be calibrated. The resulting observational equations are as follows:

$x_1-x_5+\lambda_7=a_{1\ 5}$	$x_1-x_6+\lambda_5=a_{1\ 6}$	$x_1-x_7+\lambda_5=a_{1\ 7}$
$x_2-x_6+\lambda_7=a_{2\ 6}$	$x_2-x_7+\lambda_6=a_{2\ 7}$	$x_2-x_8+\lambda_5=a_{2\ 8}$
. . . .	. . . .	.
. . . .	. . . .	.
. . . .	. . . .	$x_5-x_{11}+\lambda_5=a_{5\ 11}$
. . . .	$x_6-x_{11}+\lambda_6=a_{6\ 11}$	.
$x_7-x_{11}+\lambda_7=a_{7\ 11}$		

(27)

The 18 equations are solved by the method of least squares for the nine independent x's and the three λ's. The method of computation, which has been reduced to its simplest form, is given by by Broch and Guilleaume.[2]

In the abbreviated calibration into tenths

$$r = .6745\sqrt{\frac{\Sigma\Delta^2}{6}} \tag{28}$$

and the probable errors of the x's are as follows:

$$\left.\begin{aligned} r\ (x_1) &= r\ (x_{11}) = 0 \\ r\ (x_2) &= r\ (x_{10}) = .836\ r \\ r\ (x_3) &= r\ (x_9) = .864\ r \\ r\ (x_4) &= r\ (x_8) = .840\ r \\ r\ (x_5) &= r\ (x_7) = .616\ r \\ r\ (x_6) &= .416\ r \end{aligned}\right\} \tag{29}$$

[2] Guillaume: Traité Pratique de la Thermométrie de Précision, p. 75.

Since the probable error of a single observation, r, is fixed by the accuracy of the observations, it should be the same for the abbreviated and complete calibrations. The mean probable error of the nine x's (excluding x_1 and x_{11}) is 0.773 r, while for the complete calibration into tenths it is 0.392 r, being reduced in a somewhat greater ratio than computed from the number of observations.

The most economical method of calibration, which needs no further description, consists, as stated above, in making a complete, abbreviated or simple calibration of the entire length into a suitable number of sections, each of which is in turn calibrated into sufficiently short elements by one of the above-described methods. This introduces the problem of expressing the caliber corrections in each section, and given in terms of its mean cross section, in terms of the mean cross section of the entire interval.

2. TRANSFORMATION OF CALIBER CORRECTIONS FROM ONE SYSTEM INTO ANOTHER

Let y_1 y_2, etc., denote the caliber corrections in the first system and x_1 x_2, etc., the caliber corrections in the second system.

Denoting the points at the ends of the smaller section by a and b, the caliber corrections in the "x" system by x_a and x_b and in the "y" system by y_a and y_b and the mean cross sections in the two systems by s_x and s_y we have

$$s_x\{(b+x_b)-(a+x_a)\}=s_y\{(b+y_b)-(a+y_a)\} \tag{30}$$

Hence

$$\frac{s_y}{s_x}=\frac{(b+x_b)-(a+x_a)}{(b+y_b)-(a+y_a)}\equiv B \tag{31}$$

To find the caliber correction for any point in terms of the "y" system if it is given in terms of the "x" system, we have

$$\begin{aligned} y_m &= y_a+\int_a^m \frac{s-s_y}{s_y}\,dl \\ &= y_a-(m-a)+\frac{1}{s_y}\int_a^m s\,dl \\ &= y_a-(m-a)+\frac{s_x}{s_y}\{(m+x_m)-(a+x_a)\} \\ &= y_a+\frac{x_m-x_a+(1-B)\,(m-a)}{B} \end{aligned} \tag{32}$$

The cross section of the tube at a given point must of course be the same in the two systems

$$s_m = s_x\left(1 + \frac{dx_m}{dl}\right) = s_y\left(1 + \frac{dy_m}{dl}\right) \tag{33}$$

By means of the transformation formula (32) the caliber corrections in the "y" system can therefore readily be calculated from those on the "x" system (or vice versa), if the caliber corrections for the same two points are known in both systems. In most cases the "x" corrections are taken to represent those obtained from the calibration of the separate sections and the "y" corrections those referred to the whole length of the calibrated portion of the tube. The same method can be applied to the calculation of a new system of corrections referred to the particular portion of the tube selected as representing unit resistance. In most cases a simpler calculation will suffice for the latter purpose.

C. THE CORRECTIONS FOR THE TERMINAL BULBS

In accordance with the London specifications, connections for electrical comparisons must be made by the aid of spherical terminal bulbs 4 cm in internal diameter. In addition, the London conference adopted as the correction for the added resistance introduced by the terminal bulbs the formula:

$$R_{T_1} + R_{T_2} = \frac{0.80}{\pi 1063}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

ohm, r_1 and r_2 being expressed in millimeters. In this it is assumed that the correction factor is 0.80 and that the terminal cross section of the ohm tube is circular, the tube cylindrical, and its ends forming continuations of the internal surface of the terminal bulb, all of which may be closely approximated by careful selection of tubes and by proper construction.

The factor 0.80 seems to agree better with the results of actual measurement than 0.82, which was fixed as the most probable value by Lord Rayleigh. As far as reproducibility is concerned, any uncertainty in the value employed will affect the results by the same amount so long as the end correction bears an approxi-

mately constant ratio to the resistance contributed by the tube proper, the dimensions of the terminal bulb being specified. The possible variation due to an uncertainty of 0.01 in the factor is shown, for 1-ohm tubes, in Table 1.

TABLE 1

Ohm length	Correction for terminal bulbs	Error due to uncertainty of 0.01 in correction factor	Ohm length	Correction for terminal bulbs	Error due to uncertainty of 0.01 in correction factor
Centimeters	Microhms	Microhms	Centimeters	Microhms	Microhms
120	799.3	10.0	80	978.9	12.2
115	816.4	10.2	75	1011.0	12.6
110	834.8	10.4	70	1046.5	13.1
105	854.4	10.7	65	1086.0	13.6
100	875.5	10.9	60	1130.3	14.1
95	898.3	11.2	55	1180.6	14.8
90	922.9	11.5	50	1238.2	15.5
85	949.7	11.9			

Thus, an uncertainty of 0.01 in the assumed value of the correction factor would in itself introduce a discrepancy of five parts in one million between two tubes having lengths of 50 and 120 cm. Such differences in length are found among tubes of the same set, and might have to be taken into account in comparing the results of different investigators. In the interest of the highest accuracy of reproduction it would, therefore, seem desirable to specify that the mean length of tubes of a set should not depart from 100 cm. by more than would correspond to an uncertainty of one part in one million. This would allow a range of 15 cm and would not be too difficult to realize. A redetermination of the value of the correction factor seems, however, highly desirable for the particular type of terminal bulb adopted and should form the subject of a further investigation.

It also seems desirable, in the absence of any experimental data and in order to eliminate a further possible source of discrepancy, to more exactly specify the dimensions of the end surface of the tube. Since the principal part of the resistance is found close to the end of the capillary, it is evident that the effect of imperfec-

tions in the end bulb, particularly adjoining the end surface of the tube, will be lessened as the area of the latter is increased up to a certain limit.

The data for the calculation of the correction for the terminal bulbs are obtained from the relative cross-section curve deduced from the caliber corrections and from the mean cross section of the interval to which the calibration is referred. No error is introduced if the density of the mercury is taken as 13.5956, from which the mass given in the definition of the ohm is derived.

PART II.—EXPERIMENTAL DETAILS

The formula for calculating the resistance of the finished tube being given in terms of L_0, M_0, the caliber factor, and the correction for the terminal bulbs as above described, the construction of mercurial resistance standards may be regarded as comprising the following steps:

A. The selection of a suitable number of tubes of sufficiently uniform cross section, their straightening and annealing.

B. The calibration of the tubes (also the calibration of the graduations, if the measurements of thread length are referred to graduations on the tube).

C. The determination of the approximate ohm lengths, allowance being made for the corrections for terminal bulbs and caliber factor.

D. The selection of the points at which the tubes are to be cut, so that the terminal planes will lie where the cross sections can be most accurately interpolated.

E. The cutting of the tubes, grinding to approximate length, and polishing of the end surfaces. (Also tapering for the terminal bulbs).

F. The calculation of the caliber factors for the finished tubes.

G. The determination of M_0.

H. The determination of L_0.

I. The calculation of the corrections for the terminal bulbs.

J. The calculation of the resistance of the finished tubes.

K. The electrical intercomparison with secondary standards.

L. Results of electrical comparisons.

A. SELECTION OF THE TUBES

A number of lots of tubing were procured from the Jena Glass Works and subjected to a preliminary calibration. Some very good tubes were found, but as the probable error of the caliber factor decreases as the caliber factor itself decreases, and, as pointed out above, it was found desirable to have the ohm length approximately one meter, a special drawing of 59^{III} borosilicate tubing was made for the Bureau by Schott and Genossen, for which we hereby express our appreciation. As the tubes were to be provided with a millimeter graduation, 10 kilos were sent to M. Baudin, the Paris thermometer manufacturer, with instructions to select by preliminary calibration the four tubes most nearly answering specifications furnished, to anneal and straighten them, and to provide each with a millimeter graduation extending over an interval of 110 cm.

Four tubes of "Verre Dur" were also obtained through M. Baudin, who in this case superintended the drawing of the tubing himself. These have not yet been used, principally on account of their small external diameter, although having caliber factors somewhat less than the Jena tubes. The graduation of all the tubes was exceptionally uniform and the lines very sharp, their width not exceeding 50 microns, a result particularly difficult to accomplish with Jena 59^{III} glass.

B. CALIBRATION OF THE TUBES

The comparator used in the calibration of the tubes was an old one which, by slight alterations, was adapted to the purpose. It consists of a massive iron base supporting two short stout angle beams, carrying a pair of parallel horizontal brass bars 105 cm long on which are mounted two micrometer microscopes. The minimum distance between the microscopes is 7 cm, so that no direct observations on shorter threads could be made. The microscopes have an approximate focal length of 20 cm, and when focused on the base of the meniscus of a mercury column in the tube, the tip of the meniscus and the graduations of the tube were also in sufficiently good focus, so that observations could be made with the required accuracy without refocusing.

The micrometer value, checked frequently by comparison with a subdivided scale, was approximately 3.7μ per division of the head, or 370μ per turn. The tubes were mounted in five V-grooved supports aligned 22 cm apart on a steel T beam, and held in position by springs directly over each support. They were thus rigidly supported, and any residual curvature was reduced.

The tubes were cleaned and dried for each calibrating thread employed. The method consisted in first filling the tube with fuming sulphuric acid. After five minutes the acid was removed by suction and the tube washed with several hundred cc of double distilled water. This method was used throughout in the calibration of tube No. 1 and in the calibrations of the remaining tubes with 20-cm threads. In the subcalibration of tubes No. 2, No. 3, and No. 4 the above treatment was followed by washing with a 20 per cent solution of sodium hydroxide and again with double distilled water.

Drying was effected by drawing air, first passed through a calcium chloride bulb, through the tube for a sufficient length of time.

The calibrating threads of double distilled mercury were introduced by means of a glass tube drawn out to a capillary sufficiently small to permit its insertion into the bore. After adjusting the thread length, so as to minimize the second approximation correction, calcium chloride bulbs were attached to either end of the tube to prevent condensation of moisture within. The threads were brought into the desired positions by pressure applied to the rubber tubing by which the drying bulbs were attached.

A simple 20-cm calibration of each tube with two different threads was first made for the interval 5–105 cm. Readings were made for the successive positions at each end of the thread on both the tip and base of the meniscus as well as on the two adjacent millimeter divisions of the graduations; the observations were then repeated in the reverse order. The graduation corrections were applied to all 20-cm threads of the four tubes and also to all threads used in the calibration of tube No. 1.

The subcalibration of the 20-cm sections into tenths was made by taking 8, 10, and 12 cm threads and displacing them 2 cm at

a time over the 20-cm section, the observations being repeated in the reverse order. For the end sections the calibration was extended so as to include the 1, 3, 107, and 109 cm points. In the calibration of tube No. 1 the thread lengths were referred to the graduations as described above, but with tubes Nos. 2, 3, and 4, the microscopes were set at a fixed distance for a given thread and the slightly varying lengths expressed in terms of this fixed distance and the micrometer settings. The end sections, 0–25 and 85–110 cm, were also calibrated into tenths by using threads of 10, 12.5, and 15 cm in length. This was done to gain additional information about those portions of the tubes in which the cuts were to be located. The tubes were always recleaned and dried after a thread had been used in calibrating a single section.

To secure a suitable accuracy of the observations on the tip of the meniscus particular attention had to be given to the question of illumination. The desired result was obtained by the aid of an evenly illuminated ground glass plate, extending over the full width of the comparator, and white diffusing screens above, below, and behind the tubes. By cutting off the lateral illumination by means of two blackened metal disks about 4 cm in diameter, held 3 mm apart and each joined to a sleeve 1 cm long and of a diameter slightly larger than that of the ohm tubes, the meniscus appeared black and sharply defined against a white background, thus greatly increasing the ease and accuracy of the observations.

The correction for the menisci was calculated on the assumption that the bounding surfaces were spherical. The length of cylindrical columns having the same volume as a meniscus of height h in a tube of radius r, is given by the formula:

$$\Delta l = \frac{h}{2}\left(1 + \frac{1}{3}\frac{h^2}{r^2}\right)$$

For the mean meniscus height found, the term $\frac{1}{3}\frac{h^2}{r^2}$ amounts to approximately 0.04, and since the maximum variation did not exceed 0.1 mm, the average variation amounting to less than 0.03 mm, the factor was regarded as constant and the corrected thread length was calculated by adding to the length of the cylindrical portion 4 per cent of the mean observed meniscus height. The

latter was particularly uniform when the 20 per cent sodium hydroxide solution was used in cleaning the tube.

The temperature adjacent to the thread under measurement was always taken at the beginning and end of each set of observations for each position, and the whole set of observations on a given thread length in the several positions throughout a section reduced to a common temperature.

The variation in temperature of a thread throughout a complete set of observations was slight. The effect of the small progressive changes was, moreover, eliminated by the method of procedure, the observations being always repeated in the reverse order. The above remarks also apply to the variations in the temperature of the bar supporting the miscroscopes. From the results obtained the caliber corrections for the 25, 45, 65, and 85 cm points were first calculated, the corrections for the 5 and 105 cm points, limiting the interval to which the calibration was in each case referred, being put equal to zero.

The observations made for the subcalibration of these 20-cm sections and the 0–25 and 85–110 cm end sections were reduced by the least squares method as given by Guillaume.[3] The caliber corrections thus calculated referred to the particular sections of the tube. From these the corresponding caliber corrections in the interval 5–105 cm were determined. In each case a second approximation correction was applied and the caliber corrections recalculated.

The corrections for the 1 and 3 cm points were determined by the method given in Guillaume, the three threads employed in the calibration of the 5–25 cm section being also measured in the 1–9, 3–11, 1–11, 3–13, 1–13, and 3–15 positions. This enabled the corrections at the 1 and 3 cm points to be determined in terms of the x's in the 5–25 cm interval and the three λ's computed as indicated above, the general formula applicable being $x_m - x_n + \lambda = a$. The same procedure was followed for the determination of the 107 and 109 cm points.

The results are given in Tables 2 to 5 and the four curves following. The quantity, $\frac{s-s_m}{s_m} = \frac{y_{m+1} - y_m}{l}$ tabulated in the last column

[3] Guillaume, Traité Pratique de la Thermométrie de Précision, p. 75 et seq.

is denoted by k_m in formula (19), page 383, from which the excess of the caliber factor above unity for the separate sections of each of the four tubes is calculated. The close agreement of the results obtained by the first and second approximations as shown by Table 6 is to be attributed in part to the uniformity of the tubes, but mainly to the careful adjustment of the thread lengths employed, the same thread being used in only one section in the subcalibration into tenths. Attention is also called to the agreement between the 20 and 25 mm calibrations of the end sections. While at first sight it might appear that the values given by the former should *always* exceed those given by the latter, excluding experimental errors, it is obvious that the sign of the difference will depend in part on the location of the irregularities in the curves with respect to the particular subdivisions selected.

TABLE 2

Tube No. 1

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$	Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm			mm	mm	
1	−0.409	−0.220		33	−0.478	−1.731	
			+0.00375				−0.01065
3	− .240	− .145		35	− .612	−1.944	
			+ .00725				− .00675
5	.000	.000		37	− .669	−2.079	
			+ .00300				+ .00130
7	+ .154	+ .060		39	− .562	−2.053	
			− .00355				+ .00125
9	+ .177	− .011		41	− .458	−2.028	
			− .00995				+ .01095
11	+ .072	− .210		43	− .158	−1.809	
			− .01885				+ .00395
13	− .213	− .587		45	.000	−1.730	
			− .01505				
15	− .421	− .888		45	.000	−1.730	
			− .00665				− .00570
17	− .461	−1.021		47	− .254	−1.844	
			− .00155				− .00530
19	− .387	−1.052		49	− .500	−1.950	
			+ .00220				+ .00220
21	− .259	−1.008		51	− .598	−1.906	
			+ .00340				+ .00820
23	− .096	− .940		53	− .576	−1.742	
			+ .00010				+ .01145
25	.000	− .938		55	− .490	−1.513	
							+ .01355
25	.000	− .938		57	− .361	−1.242	
			− .00740				+ .01080
27	− .069	−1.086		59	− .288	−1.026	
			− .01280				+ .02145
29	− .247	−1.342		61	− .003	− .597	
			− .00850				+ .01080
31	− .337	−1.512		63	+ .070	− .381	
			− .01095				+ .00355
				65	.000	− .310	

TABLE 2—Continued

Tube No. 1—Continued

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$	Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm			mm	mm	
65	0.000	−0.310		0	0.000	−0.186	
			+0.00060				+0.00036
67	+ .132	− .298		2.5	+ .085	− .177	
			+ .00360				+ .00708
69	+ .324	− .226		5	+ .338	.000	
			+ .00285				+ .00268
71	+ .501	− .169		7.5	+ .480	+ .067	
			− .00435				− .00512
73	+ .533	− .256		10	+ .428	− .061	
			− .00555				− .01612
75	+ .542	− .367		12.5	+ .099	− .464	
			− .01750				− .01660
77	+ .309	− .717		15	− .243	− .879	
			− .01535				− .00512
79	+ .120	−1.024		17.5	− .295	−1.007	
			− .01015				+ .00004
81	+ .036	−1.227		20	− .219	−1.006	
			− .00445				+ .00196
83	+ .065	−1.316		22.5	− .095	− .957	
			− .00920				+ .00076
85	.000	−1.500		25	.000	− .938	
85	.000	−1.500		85	.000	−1.500	
			− .00645				− .00524
87	− .277	−1.629		87.5	− .397	−1.631	
			+ .00025				+ .00168
89	− .421	−1.624		90	− .623	−1.589	
			+ .00360				+ .00248
91	− .499	−1.552		92.5	− .829	−1.527	
			+ .00045				− .00032
93	− .639	−1.543		95	−1.104	−1.535	
			− .00030				+ .01316
95	− .793	−1.549		97.5	−1.046	−1.206	
			+ .01200				+ .01704
97	− .703	−1.309		100	− .892	− .780	
			+ .01905				+ .01148
99	− .474	− .928		102.5	− .876	− .493	
			+ .01230				+ .01972
101	− .379	− .682		105	− .655	.000	
			+ .01355				+ .02472
103	− .259	− .411		107.5	− .311	+ .618	
			+ .02055				+ .02380
105	.000	− .000		110	.000	+1.203	
			+ .02375				
107	+ .322	+ .475					
			+ .02355				
109	+ .641	+ .946					

TABLE 3

Tube No. 2

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$	Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm			mm	mm	
1	+0.227	+0.068		7	−0.096	−0.016	
			−0.00285				−0.00065
3	+ .091	+ .011		9	− .188	− .029	
			− .00060				+ .00180
5	.000	.000		11	− .232	+ .007	
			− .00080				+ .00365

TABLE 3—Continued

Tube No. 2—Continued

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$	Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm			mm	mm	
13	−0.239	+0.080		73	+0.261	+1.020	
			+0.00295				+0.01360
15	− .260	+ .139		75	+ .346	+1.292	
			+ .00460				+ .01155
17	− .248	+ .231		77	+ .392	+1.523	
			+ .00805				+ .00835
19	− .168	+ .392		79	+ .375	+1.690	
			+ .00830				+ .00470
21	− .081	+ .558		81	+ .284	+1.784	
			+ .00720				+ .00420
23	− .018	+ .702		83	+ .183	+1.868	
			+ .00490				.00000
25	.000	+ .800		85	.000	+1.868	
25	.000	+ .800		85	.000	+1.868	
			+ .00170				− .00520
27	+ .091	+ .834		87	+ .084	+1.764	
			− .00830				− .01185
29	− .018	+ .668		89	+ .031	+1.525	
			− .00135				− .01355
31	+ .011	+ .641		91	− .054	+1.254	
			+ .00145				− .01210
33	+ .097	+ .670		93	− .110	+1.012	
			− .00200				− .01170
35	+ .114	+ .630		95	− .157	+ .778	
			− .00410				− .01060
37	+ .088	+ .548		97	− .183	+ .566	
			− .00735				− .00555
39	− .003	+ .401		99	− .106	+ .455	
			− .00510				− .00435
41	− .048	+ .299		101	− .006	+ .368	
			− .00215				− .00665
43	− .035	+ .256		103	+ .049	+ .235	
			− .00110				− .01175
45	.000	+ .234		105	.000	.000	
							− .01180
				107	− .050	− .236	
45	.000	+ .234		109	− .073	− .446	− .01050
			+ .00395				
47	+ .101	+ .313		0	.000	+ .146	
			+ .00210				− .00568
49	+ .165	+ .355		2.5	− .207	+ .004	
			+ .00015				− .00016
51	+ .189	+ .358		5	− .276	.000	
			− .00110				− .00136
53	+ .189	+ .336		7.5	− .375	− .034	
			− .00135				+ .00040
55	+ .184	+ .309		10	− .430	− .024	
			− .00745				+ .00352
57	+ .057	+ .160		12.5	− .408	+ .064	
			− .01565				+ .00272
59	− .234	− .153		15	− .406	+ .132	
			− .00395				+ .00488
61	− .292	− .232		17.5	− .350	+ .254	
			+ .00340				+ .00912
63	− .202	+ .164		20	− .187	+ .482	
			+ .00900				+ .00748
65	.000	+ .016		22.5	− .066	+ .669	
							+ .00524
65	.000	+ .016		25	.000	+ .800	
			+ .00920				
67	− .002	+ .200					
			+ .01230				
69	+ .059	+ .446		85	.000	+1.868	
			+ .01475				− .00632
71	+ .168	+ .741		87.5	+ .082	+1.710	
			+ .01395				− .01320

TABLE 3—Continued

Tube No. 2—Continued

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$	Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm			mm	mm	
90	−0.009	+1.380		102.5	+0.094	+0.282	
			−0.01248				−0.01128
92.5	− .082	+1.068		105	+ .051	.000	
			− .01140				− .01168
95	− .128	+ .783		107.5	− .002	− .292	
			− .01000				− .00952
97.5	− .138	+ .533		110	.000	− .530	
			− .00512				
100	− .024	+ .405					
			− .00492				

TABLE 4

Tube No. 3

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$	Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm			mm	mm	
1	−0.320	−0.651		41	+0.109	+2.090	
			+0.01865				+0.00015
3	− .113	− .278		43	+ .070	+2.093	
			+ .01390				− .00140
5	.000	.000		45	.000	+2.065	
			+ .00530				
7	− .058	+ .106		45	.000	+2.065	
			+ .00245				+ .00085
9	− .172	+ .155		47	+ .012	+2.082	
			+ .00595				+ .00080
11	− .217	+ .274		49	+ .022	+2.098	
			+ .00675				+ .00365
13	− .246	+ .409		51	+ .090	+2.171	
			+ .00940				+ .00490
15	− .223	+ .597		53	+ .182	+2.269	
			+ .01245				+ .00630
17	− .139	+ .846		55	+ .302	+2.395	
			+ .01400				+ .00400
19	− .025	+1.126		57	+ .377	+2.475	
			+ .01215				− .00350
21	+ .053	+1.369		59	+ .302	+2.405	
			+ .00800				− .00470
23	+ .049	+1.529		61	+ .202	+2.311	
			+ .00575				− .00460
25	.000	+1.644		63	+ .106	+2.219	
							− .00505
25	.000	+1.644		65	.000	+2.118	
			+ .00580				
27	+ .074	+1.760		65	.000	+2.118	
			+ .00255				− .00115
29	+ .083	+1.811		67	− 1.06	+2.095	
			+ .00030				+ .00100
31	+ .047	+1.817		69	+ .032	+2.115	
			− .00110				− .00090
33	− .017	+1.795		71	+ .031	+2.097	
			+ .00085				+ .00560
35	− .042	+1.812		73	+ .161	+2.209	
			+ .00430				+ .00170
37	+ .001	+1.898		75	+ .212	+2.243	
			+ .00575				− .00440
39	+ .074	+2.013		77	+ .141	+2.155	
			+ .00385				− .00690

TABLE 4—Continued

Tube No. 3—Continued

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$	Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm			mm	mm	
79	+0.019	+2.017		5	+0.347	0.000	
			−0.00325				+0.00512
81	− .029	+1.952		7.5	+ .227	+ .128	
			− .00005				+ .00368
83	− .012	+1.951		10	+ .071	+ .220	
			− .00025				+ .00648
85	.000	+1.946		12.5	− .016	+ .382	
							+ .00856
				15	− .050	+ .596	
85	.000	+1.946					+ .01288
			− .00450	17.5	+ .022	+ .918	
87	+ .105	+1.856					+ .01328
			− .01480	20	+ .104	+1.250	
89	+ .003	+1.560					+ .00992
			− .01145	22.5	+ .102	+1.498	
91	− .031	+1.331					+ .00584
			− .01010	25	.000	+1.644	
93	− .039	+1.129					
			− 0.1275				
95	− .100	+ .874		85	.000	+1.946	
			− .01365				− .00640
97	− .179	+ .601		87.5	+ .043	+1.786	
			− .01320				− .01436
99	− .249	+ .337		90	− .114	+1.427	
			− .00670				− .00976
101	− .188	+ .203		92.5	− .156	+1.183	
			− .00575				− .01232
103	− .108	+ .088		95	− .263	+ .875	
			− .00440				− .01424
105	.000	.000		97.5	− .417	+ .519	
			− .00200				− .01044
107	+ .156	− .040		100	− .476	+ .258	
			− .00135				− .00540
109	+ .325	− .067		102.5	− .407	+ .123	
							− .00492
				105	− .327	.000	
0	.000	− .849					− .00180
			+ .01992	107.5	− .168	− .045	
2.5	+ .246	− .351					− .00144
			+ .01404	110	.000	− .081	

TABLE 5

Tube No. 4

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$	Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm			mm	mm	
1	+0.183	+0.268		17	+0.205	−0.054	
			−0.00830				−0.00795
3	+ .059	+ .102		19	+ .089	− .213	
			− .00510				− .00405
5	.000	.000		21	+ .050	− .294	
			− .00085				− .00315
7	+ .026	− .017		23	+ .032	− .357	
			− .00335				− .00365
9	+ .002	− .084		25	.000	− .430	
			+ .00105				
11	+ .066	− .063		25	.000	− .430	
			+ .00290				− .00700
13	+ .167	− .005		27	+ .038	− .570	
			+ .00340				− .01865
15	+ .278	+ .063		29	− .161	− .943	
			− .00585				− .01205

TABLE 5—Continued

Tube No. 4—Continued

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$	Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm			mm	mm	
31	−0.225	−1.184		89	−0.039	−2.153	
			−0.00520				+0.01585
33	− .152	−1.288		91	+ .013	−1.836	
			− .00805				+ .01680
35	− .135	−1.449		93	+ .083	−1.500	
			− .01425				+ .01790
37	− .245	−1.734		95	+ .176	−1.142	
			− .01145				+ .01765
39	− .297	−1.963		97	+ .264	− .789	
			− .00335				+ .01960
41	− .185	−2.030		99	+ .390	− .397	
			− .00220				+ .01135
43	− .052	−2.074		101	+ .354	− .170	
			− .00630				+ .00715
45	.000	−2.200		103	+ .234	− .027	
							+ .00135
45	.000	−2.200		105	.000	.000	
			− .01390				+ .00310
47	− .123	−2.478		107	− .200	+ .062	
			− .01185				+ .00715
49	− .205	−2.715		109	− .319	+ .205	
			− .00880				
51	− .224	−2.891		0	.000	+ .412	
			− .00875				− .01256
53	− .244	−3.066		2.5	− .230	+ .098	
			− .00465				− .00392
55	− .180	−3.159		5	− .244	.000	
			− .00970				− .00152
57	− .219	−3.353		7.5	− .198	− .038	
			− .01350				− .00260
59	− .333	−3.623		10	− .178	− .103	
			− .00650				+ .00236
61	− .307	−3.753		12.5	− .035	− .044	
			− .00060				+ .00396
63	− .162	−3.765		15	+ .149	+ .055	
			+ .00025				− .00568
65	.000	−3.760		17.5	+ .091	− .087	
							− .00828
65	.000	−3.760		20	− .032	− .294	
			− .00700				− .00200
67	− .250	−3.900		22.5	+ .002	− .344	
			− .00280				− .00344
69	− .418	−3.956		25	.000	− .430	
			+ .00460				
71	− .437	−3.864		85	.000	−2.641	
			+ .00850				+ .01116
73	− .380	−3.694		87.5	− .010	−2.362	
			+ .01065				+ .01424
75	− .280	−3.481		90	+ .056	−2.006	
			+ .00140				+ .01676
77	− .363	−3.453		92.5	+ .184	−1.587	
			+ .00675				+ .01760
79	− .340	−3.318		95	+ .333	−1.147	
			+ .01230				+ .01848
81	− .207	−3.072		97.5	+ .504	− .685	
			+ .01085				+ .01616
83	− .102	−2.855		100	+ .617	− .281	
			+ .01075				+ .00952
85	.000	−2.640		102.5	+ .566	− .043	
							+ .00172
85	.000	−2.640		105	+ .323	.000	
			+ .01170				+ .00320
87	− .029	−2.406		107.5	+ .117	+ .080	
			+ .01265				+ .00680
				110	.000	+ .250	

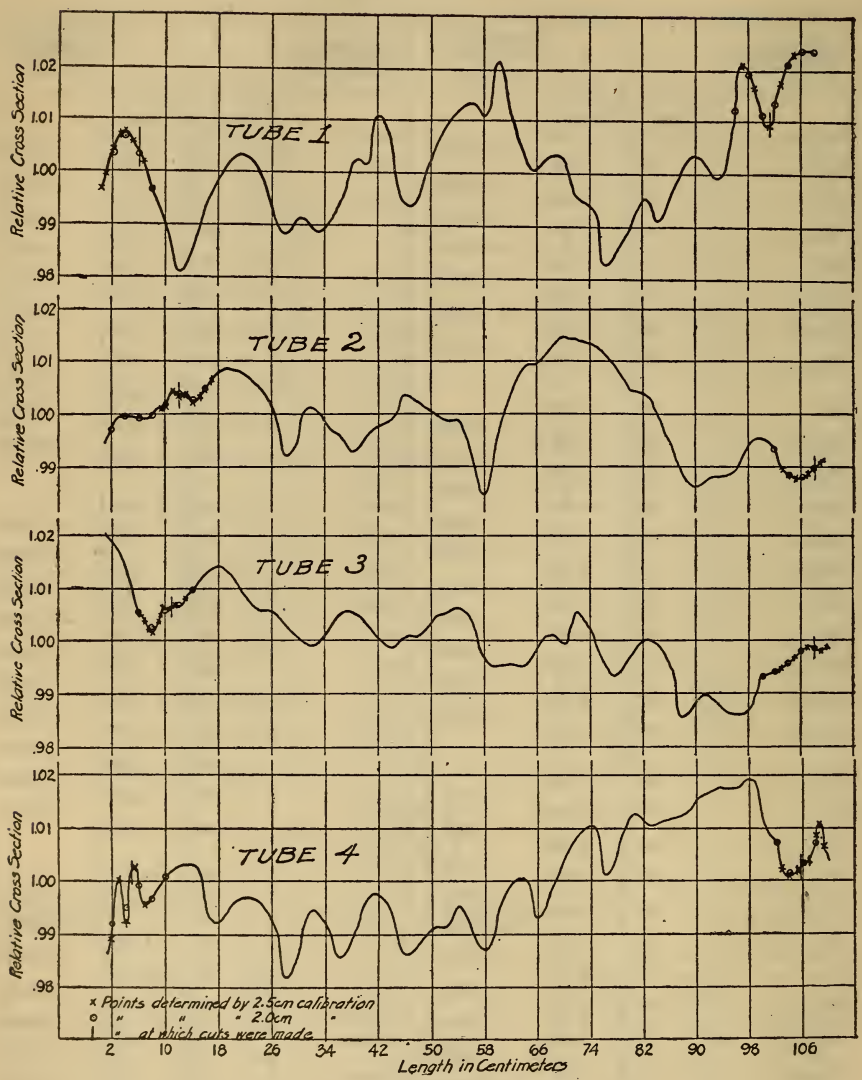


FIG. 1.—*Calibration curves*

TABLE 6

Corrections for Lack of Uniformity (In Parts per Million).

Tube	Interval	20-mm calibration		25-mm calibration	
		First approximation	Second approximation	First approximation	Second approximation
No. 1	5—25	15.27	15.54	15.06	15.21
	25—45	14.54	14.26		
	45—65	22.49	22.73		
	65—85	16.15	16.63		
	85—105	24.78	25.95	24.65	25.12
	5—105	93.23	95.11		
No. 2	5—25	5.16	5.10	5.28	5.27
	25—45	3.74	3.67		
	45—65	8.72	8.71		
	65—85	21.31	21.25		
	85—105	20.38	19.85	19.37	19.72
	5—105	59.31	58.58		
No. 3	5—25	15.84	15.81	15.72	15.58
	25—45	2.20	2.20		
	45—65	3.46	3.50		
	65—85	2.35	2.31		
	85—105	22.02	22.13	21.59	21.75
	5—105	45.87	45.95		
No. 4	5—25	3.40	3.42	3.53	3.69
	25—45	20.98	20.75		
	45—65	16.96	16.76		
	65—85	13.96	13.79		
	85—105	40.21	39.91	39.82	39.71
	5—105	95.51	94.63		

A rapid method of calibration employed at the National Physical Laboratory[4] consisted in making five separate series of measurements of 5-cm threads. Each tube was provided with a reference mark, and the measurements were usually made over an interval 10 to 15 cm beyond the limits selected for use.

The first series of readings consisted in the linear measurements (by the aid of a simple comparator) of a thread of mercury, approximately 5 cm in length, in successive 5-cm intervals. The

[4] F. E. Smith, "On the Construction of some mercury standards of resistance," etc. Phil. Trans., A. 204, pp. 66–67; 1904.

intervals were usually designated 0–5, 5–10, 10–15, etc. The second series consisted in determining the thread lengths for the positions 1–6, 6–11, 11–16, etc., and similarly for the third, fourth, and fifth series, approximately six hours being required for the five series of measurements.

The caliber factor was computed from each series of measurements and also from the combined data of the entire set of measurements. Separate corrections were applied for conicality. There is no data given as to the order of agreement of the caliber factor as separately determined.

A preliminary simple calibration of each of the four Bureau of Standards tubes was made in order to determine the approximate magnitude of their caliber factors. The following readings on 5-cm threads (made with the aid of a hand reading glass only) together with the computation of the caliber factors of each tube are given in Table 7.

While the caliber factors of tubes No. 2 and No. 3, thus determined agree very closely with those given in Table 6, we feel that the larger discrepancies shown in the case of tubes No. 1 and No. 4 have justified the more accurate calibration made. Table 8 is a summary of four separate calculations of the caliber factor of each tube and is self-explanatory.

TABLE 7

Observational Data from a Simple Calibration of the 4 B. S. Tubes together with the Computation of Their Caliber Factors

Interval	Tube No. 1				Tube No. 2			
	Thread length in centimeters			$\frac{1}{l}$	Thread length in centimeters			$\frac{1}{l}$
	Down	Up	Mean		Down	Up	Mean	
5– 10	5.01	5.00	5.005	0.1998002	5.04	5.03	5.035	0.1986097
10– 15	5.09	5.08	5.085	.1966568	5.01	5.015	5.012	.1995211
15– 20	5.03	5.02	5.025	.1990050	5.00	5.00	5.000	.2000000
20– 25	5.00	5.00	5.000	.2000000	5.00	5.00	5.000	.2000000
25– 30	5.04	5.04	5.040	.1984127	5.03	5.04	5.035	.1986097
30– 35	5.06	5.05	5.055	.1978239	5.03	5.04	5.035	.1986097
35– 40	5.01	5.02	5.015	.1994018	5.055	5.06	5.058	.1977066
40– 45	4.96	4.965	4.962	.2015316	5.06	5.05	5.055	.1978239
45– 50	5.03	5.03	5.030	.1988072	5.015	5.02	5.018	.1992826
50– 55	4.97	4.97	4.970	.2012072	5.045	5.04	5.042	.1983340
55– 60	4.95	4.93	4.940	.2024291	5.09	5.08	5.085	.1966568
60– 65	4.96	4.96	4.960	.2016129	5.02	5.02	5.020	.1992032
65– 70	4.99	4.99	4.990	.2004008	4.975	4.98	4.978	.2008839
70– 75	5.00	5.01	5.005	.1998002	4.96	4.95	4.955	.2018163
75– 80	5.08	5.07	5.075	.1970443	4.98	4.985	4.982	.2007226
80– 85	5.04	5.03	5.035	.1986097	5.02	5.02	5.020	.1992032
85– 90	5.00	5.01	5.005	.1998002	5.09	5.09	5.090	.1964637
90– 95	4.98	4.99	4.985	.2006018	5.10	5.10	5.100	.1960784
95–100	4.91	4.91	4.910	.2036660	5.075	5.075	5.075	.1970443
100–105	4.92	4.91	4.915	.2034588	5.08	5.075	5.078	.1969279
			100.007	4.0000702			100.679	3.9734976
	$\frac{100.007 \times 4.0000702}{400} = 1.0000876$				$\frac{100.679 \times 3.9734976}{400} = 1.0000598$			

TABLE 7—Continued

Interval	Tube No. 3				Tube No. 4			
	Thread length in centimeters			$\frac{1}{l}$	Thread length in centimeters			$\frac{1}{l}$
	Down	Up	Mean		Down	Up	Mean	
5– 10	4.93	4.91	4.920	0.2032520	4.99	5.00	4.995	0.2002002
10– 15	4.89	4.88	4.885	.2047083	4.97	4.97	4.970	.2012072
15– 20	4.87	4.86	4.865	.2055498	5.02	5.01	5.015	.1994018
20– 25	4.89	4.89	4.890	.2044990	4.98	4.99	4.985	.2006018
25– 30	4.90	4.90	4.900	.2040816	5.04	5.04	5.040	.1984127
30– 35	4.915	4.93	4.922	.2031694	5.00	5.01	5.005	.1998002
35– 40	4.91	4.91	4.910	.2036660	5.03	5.04	5.035	.1986097
40– 45	4.94	4.94	4.940	.2024291	5.00	5.00	5.000	.2000000
45– 50	4.93	4.93	4.930	.2028398	5.04	5.03	5.035	.1986097
50– 55	4.90	4.90	4.900	.2040816	5.02	5.02	5.020	.1992032
55– 60	4.945	4.93	4.938	.2025111	5.03	5.03	5.030	.1988072
60– 65	5.95	4.95	4.950	.2020202	4.98	4.98	4.980	.2008032
65– 70	4.94	4.94	4.940	.2024291	4.99	5.00	4.995	.2002002
70– 75	4.905	4.91	4.908	.2037490	4.95	4.94	4.945	.2022245
75– 80	4.95	4.95	4.950	.2020202	4.95	4.955	4.952	.2019386
80– 85	4.94	4.95	4.945	.2022245	4.92	4.92	4.920	.2032520
85– 90	4.99	4.985	4.988	.2004812	4.91	4.92	4.915	.2034588
90– 95	4.98	4.99	4.985	.2006018	4.89	4.89	4.890	.2044990
95–100	4.985	4.99	4.988	.2004812	4.89	4.89	4.890	.2044990
100–105	4.95	4.95	4.950	.2020202	4.94	4.94	4.940	.2024291
			98.604	4.0568151			99.557	4.0181581
	$\frac{98.604 \times 4.0568151}{400} = 1.0000455$				$\frac{99.557 \times 4.0181581}{400} = 1.0000894$			

TABLE 8

Calculated Caliber Factors, 5 to 105 cm Sections

How calculated	Tube No. 1	Tube No. 2	Tube No. 3	Tube No. 4
From eye observations on 5-cm threads	1.0000876	1.0000598	1.0000455	1.0000894
From abbreviated calibration as made, first approximation	1.0000931	1.0000593	1.0000458	1.0000955
From abbreviated calibration, second approximation	1.0000951	1.0000586	1.0000459	1.0000946
From relative cross section of 4-cm sections, second approximation curves	1.0000844	1.0000539	1.0000428	1.0000871

C. DETERMINATION OF THE APPROXIMATE OHM LENGTHS

The approximate ohm length of each tube was derived from the mean of three determinations, made as follows:

The tube under observation was first cleaned and dried in the usual manner and the micrometer microscopes set very nearly 100 cm apart. A thread of mercury of approximately this length was then introduced and observations made on both the base and tip of the meniscus at each end of the column in the 2.5 to 102.5, 5 to 105, and 7.5 to 107.5 cm intervals and on the adjacent graduations. These observations were then repeated in the reverse order. The thread lengths were thus referred to the graduations and the micrometer settings. The temperature at each end of the tube was recorded for each position of the thread and all thread lengths reduced to a common temperature. The mercury was then drawn out and carefully weighed and the ohm length at 0° C. derived from the following formula, after applying suitable temperature corrections:

$$L_0 = \frac{(106.3)^2}{14.4521} \frac{M}{L}$$

The resulting lengths in centimeters of the graduations, after correcting for the terminal bulbs and applying the caliber corrections, are given in Table 9.

TABLE 9

Approximate Ohm Lengths

Tube	Ohm length
	cm
1	95.15
2	96.32
3	97.02
4	100.66

D. LOCATION OF CUTS

Knowing the ohm lengths of the tubes, the locations of the cuts were so chosen that the terminal planes would lie where the cross section of the bore could be most accurately interpolated. This was done in each case by selecting two points on the cross-section

curve, the proper distance apart, where the curvature was most free from irregularities.

E. CUTTING, GRINDING, AND POLISHING

The tubes were cut at points 1 mm outside of the selected points, to allow for a possible irregular fracture. After grinding down to approximately the right points they were tapered for the end bulbs and cleaning and drying connections, the tapers being the same so as to make all fittings interchangeable. The ends were then given an optical polish. The details of the grinding and polishing are as follows:

To prevent the chipping of the bore, which must be very carefully guarded against on account of its possible influence on the correction for the terminal bulbs, the ends of the tube were plugged with a slightly tapering rod of the same kind of glass set in with sealing wax, the slight heating required being local.

The tube was inserted in a holder provided with a number of supports to prevent flexure of the tube. This holder, consisting of a brass tube, was clamped vertically between two sets of rollers, one of each set being carried by springs pressing the holder against the companion rollers mounted directly from the massive frame of the apparatus. The lower end of the holder was supported at one end of a lever, to the other end of which weights could be applied for counterbalancing. The holder was, therefore, free to move in the direction of the axis of the tube, the end of which was pressed lightly upward against the grinding or polishing surface. This had a bearing surface on a steel ring, mounted from the frame and capable of adjustment so as to be at right angles to the axis of the holder. The grinding plate was of brass turned plane in a lathe. The motion in grinding was irregular, the grinding plate being rotated, and any lateral motion being limited by a ring (screwed to the plate) of slightly smaller diameter than the opening of the steel ring. The polishing plate was constructed as shown in Fig. 2. The pitch surface, *a*, about 0.5 mm thick, was applied by covering the central metal surface with the melted material, the ring limiting the motion having been removed, so that by pressing the plate on a plane glass surface a plane polish-

ing surface was secured in the same plane as the bearing surface of the tool.

The polishing was continued until the surface was plane to within one-half wave length (0.3 micron), as shown by the interference fringes between the surface and an optically polished glass plate resting on the same, light from a helium tube being employed for testing. Tests were also made which showed that the end surfaces were in each case approximately at right angles to the axis of the tube.

The actual location of the cuts with respect to the graduations was determined as follows: The fixed distance between the two microscopes, set at approximately 12 cm apart, was determined by taking a double set of observations on the graduations, using the micrometers. The end of the tube was then brought under one microscope and the position of the other microscope with respect to the graduations noted. Repeated observations were made in each case; the mean values obtained and referred to the graduations are given in Table 10.

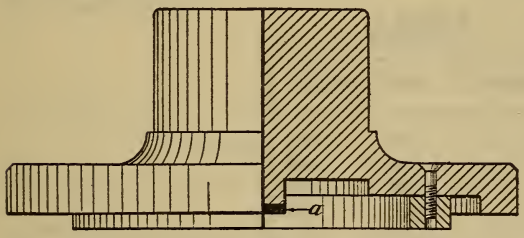


Fig. 2.—*Polishing plate used in grinding and polishing ends of tubes*

TABLE 10

Location of Cuts

Tube	0 end	100 end	Length
	cm	cm	cm
No. 1.....	5.976	101.055	95.079
No. 2.....	12.037	108.297	96.260
No. 3.....	10.996	107.997	97.001
No. 4.....	4.993	105.695	100.702

F. CALCULATION OF THE CALIBER FACTORS FOR THE FINISHED TUBES

Having determined the location of the terminal planes, the caliber factors for the finished tubes were determined from the formula

$$1+C=\left\{1+\frac{\Sigma_1^n k+\alpha k_\alpha+\beta k_\beta}{n+\alpha+\beta}\right\}$$

$$\left\{1-\frac{\Sigma_1^n k+\alpha k_\alpha+\beta k_\beta}{n+\alpha+\beta}+\frac{\Sigma_1^n k^2+\alpha k^2{}_\alpha+\beta k^2{}_\beta}{n+\alpha+\beta}-\frac{\Sigma_1^n k^3+\alpha k^3{}_\alpha+\beta k^3{}_\beta}{n+\alpha+\beta}\right\} \quad (16)$$

The values of α and β are given below:

TABLE 11

Tube	α	β
1.....	0.5120	0.0275
2.....	.5185	.6485
3.....	.0020	.4485
4.....	.0035	.3475

In order to make the effect of the term $\frac{\Sigma_1^n k+\alpha k_\alpha+\beta k_\beta}{n+\alpha+\beta}$ negligible, corrections were applied to the k's in Tables 2 to 5 which are expressed in terms of the mean cross sections in the interval 5–105, to refer them to the mean cross sections of the finished tubes. The simplified formula (17) becomes as shown on page 381,

$$C=\frac{\Sigma_1^n k^2+\alpha k^2{}_\alpha+\beta k^2{}_\beta}{n+\alpha+\beta}-\frac{\Sigma_1^n k^3+\alpha k^3{}_\alpha+\beta k^3{}_\beta}{n+\alpha+\beta}$$

The caliber factor computed by the above formula is based on the assumption that the resistance of the tube is equal to the sum of the resistances of the separate elements, each regarded as cylinders. As a further approximation they may be taken as uniformly conical, having terminal areas given by the cross-section curve. From these areas a correction factor for conicality[5] expressing the mean ratio of the resistance of the conical to the

[5] Smith, Trans. Roy. Soc., 204, p. 40; 1904.

corresponding cylindrical elements was computed. The part contributed by each element is given by the very approximate formula

$$\frac{R_o'}{R_o} = 1 + \frac{1}{12} d^2$$

in which d denotes the ratio of the terminal cross sections. This correction is not included in Tables 2 to 5 and has only been computed for the finished tubes.

A sample computation of the caliber factor of Tube No. 1 is given in Table 12. The k^2 and k^3 terms are taken directly from tables, so that, as will be seen, the computation has been very much simplified.

TABLE 12

Caliber Factor, Tube No. 1

Interval	$k_o = \frac{s-s_o}{s_o}$	$k = \frac{s-s_m}{s_m}$	k^2	k^3
			m	m
5.976–7	+0.00230	+0.00305	(4.8)	(0.0)
7–9	− .00355	− 0280	7.8	.0
9–11	− 0995	− 0920	84.6	.8
11–13	− 1885	− 1810	327.6	− 5.9
13–15	− 1505	− 1430	204.5	− 2.9
15–17	− 0665	− 0590	34.8	− .2
17–19	− 0155	− 0080	.6	.0
19–21	+ 0220	+ 0295	8.7	.0
21–23	+ 0340	+ 0415	17.2	+ .1
23–25	+ 0010	+ 0085	.7	.0
25–27	− 0740	− 0665	44.2	− .3
27–29	− 1280	− 1205	145.2	− 1.7
29–31	− 0850	− 0775	60.1	− .5
31–33	− 1095	− 1020	104.0	− 1.1
33–35	− 1065	− 0990	98.0	− .9
35–37	− 0675	− 0600	36.0	− .2
37–39	+ 0130	+ 0205	4.2	.0
39–41	+ 0125	+ 0200	4.0	.0
41–43	+ 1095	+ 1170	136.9	+ 1.6
43–45	+ 0395	+ 0470	22.1	+ .1
45–47	− 0570	− 0495	24.5	− .1
47–49	− 0530	− 0455	20.7	− .1
49–51	+ 0220	+ 0295	8.7	.0
51–53	+ 0820	+ 0895	80.1	+ .7
53–55	+ 1145	+ 1220	148.8	+ 1.8
55–57	+ 1355	+ 1430	204.5	+ 2.9
57–59	+ 1080	+ 1155	133.4	+ 1.5
59–61	+ 2145	+ 2220	492.8	+10.9
61–63	+ 1080	+ 1155	133.4	+ 1.5

TABLE 12—Continued

Interval.	$k_o=\frac{s-s_o}{s_o}$	$k=\frac{s-s_m}{s_m}$	k2	k3
			m	m
63–65	+0.00355	+0.00430	18.5	+ .1
65–67	+ .00060	+ 0135	1.8	.0
67–69	+ 0360	+ 0435	18.9	+ .1
69–71	+ 0285	+ 0360	13.0	.0
71–73	− 0435	− 0360	13.0	.0
73–75	− 0555	− 0480	23.0	− .1
75–77	− 1750	− 1675	280.6	− 4.7
77–79	− 1535	− 1460	213.2	− 3.1
79–81	− 1015	− 0940	88.4	− .8
81–83	− 0445	− 0370	13.7	− .1
83–85	− 0920	− 0845	71.4	− .6
85–87	− 0645	− 0570	32.5	− .1
87–89	+ 0025	+ 0100	1.0	.0
89–91	+ 0360	+ 0435	18.9	+ .1
91–93	+ 0045	+ 0120	1.4	.0
93–95	− 0030	+ 0045	.2	.0
95–97	+ 1200	+ 1275	162.6	+ 2.1
97–99	+ 1905	+ 1980	392.0	+ 6.6
99–101	+ 1230	+ 1305	170.3	+ 2.2
101–101.055	+ 0965	+ 1040	(.3)	(.0)

$\Sigma k^2 = 4127.6^m \quad \Sigma k^3 = +9.0^m$

Length of tube $= 95.079$ cm

Number of 2 cm sections $= 47.54$

$$C. = \frac{4127.6}{47.54} - \frac{9.0}{47.54} = 86.82 - .19 = 86.63 \text{ millionths}$$

Table 13 gives a summary of the data obtained for the four tubes. The part contributed by the k^3 term is very small and it was therefore unnecessary to compute the next term. The probable errors of the caliber factors for the finished tubes were not computed. They will be given, for the interval 5–105 cm, in a separate paper on "The probable error in the determination of the caliber factors of ohm tubes," by F. A. Wolff, to be published in this Bulletin. The computed probable errors found, which of course do not take into account the irregularities not disclosed by the calibrations, are in every case less than one part in two million. These values are sufficiently close, since the lengths of all the tubes differed but a few centimeters from a meter.

TABLE 13

Caliber Factor for Finished Tubes

[Corrections in parts per million.]

Tube	"k^2" term	—"k^3" term	Correction for conicality of elements	Caliber factor 1+C
1	86.82	—.19	+2.62	1.00008925
2	64.63	—.02	+1.32	1.00006593
3	45.82	+.05	+0.86	1.00004673
4	94.31	—.30	+1.86	1.00009587

G. DETERMINATION OF M_0

The methods available for the determination of the mass of mercury required to fill the tube between selected points may be briefly described as follows:

First. A mass of mercury approximately equal to that corresponding to the ohm length is introduced into the tube, preferably before cutting, in the form of a continuous column, the length of which is determined usually in a number of positions near the points selected from preliminary measurements. The correction for the menisci, computed on the assumption of spehrical bounding surfaces, more or less uncertain, must be applied. If measurements are not made at 0°, errors may be introduced through errors in the coefficients of thermal expansion employed for reduction as well as in the temperature measurements.

From the mass of the column, the corrected length at 0°, together with the relation between the mean cross section of the particular interval to that of the finished tube as given by the calibration curve, the mass of mercury required to fill the tube between the selected points may be calculated.

Attempts have been made to eliminate the meniscus correction by flattening the menisci [6] by means of ebonite plugs introduced into the capillary, but this can not be accomplished to a sufficient degree of accuracy for the precision now attainable in ohm work.

[6] Huchinson and Wilkes, Phil. Mag., 28, p. 17; 1889.

Second. This method consists in carefully filling the tube, closing one end by a plane surface and stroking off the excess mercury at the other end. As first carried out, the tube was filled in a trough of mercury and then the mercury contained in the tube was separated from the rest by covering the ends still under the mercury with pieces of leather fastened to the fingers.

This method was perfected at the Reichsanstalt, and at the same time very perfect exhaustion preliminary to filling was provided for. The desirability of such exhaustion would seem apparent in view of what is known of the condensation of moisture and gases on glass, for even though the adsorbed material is not perfectly removed by exhaustion without heating, which is not permissible on account of thermal hysteresis, variations in the thickness of the film should be reduced as the thickness itself is reduced.

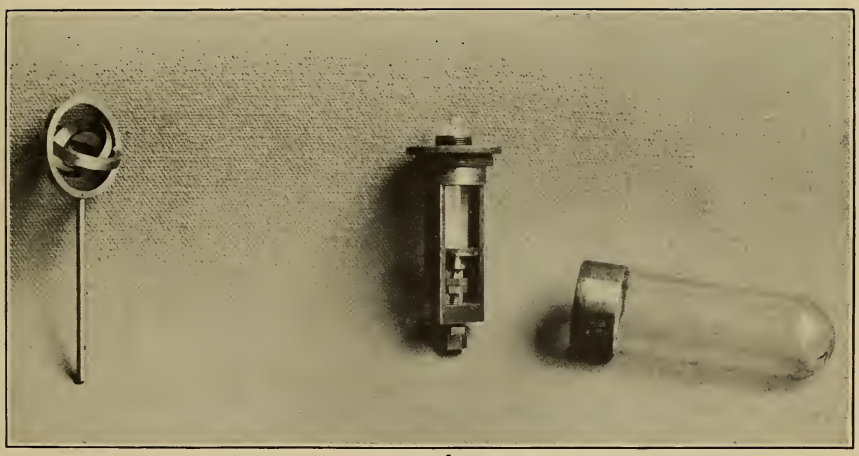
Third. This method, devised by Smith [7] at the National Physical Laboratory, is a modification of the first method, and consists in rendering the meniscus correction negligible and at the same time reducing the influence of errors in the measurement of thread length by means of capillary extensions coupled to the ends of the tube. The mass of mercury required to fill the tube is equal to that required to fill the tube plus extensions diminished by that required to fill the extensions alone, the differences in the readings of the capillaries being allowed for.

The method is, therefore, similar in principle to the method here employed for the determination of the lengths of the finished tubes.

Details of the Reichsanstalt Method.—The mass determinations employed in the calculation of the resistance of the four tubes were all made by the Reichsanstalt method.

The tube, after being cleaned and dried in the usual way, was placed in an inclined rack, the lower end being closed by a plane glass plate, *b*, Fig. 3, clamped to it by means of a hexagonal-headed steel screw with an intermediate ball-and-socket joint, the screw being threaded through the lower part of a steel yoke, which was in turn screwed to a steel collar cemented to the tube about 2½

[7] Smith, Nat. Phys. Lab. Collected Researches, Vol. V, pp. 149–157; 1909.



a *b* *c*

FIG. 3.—*Parts used in the determination of M_0 by Reichsanstalt method. a, Stroking-off plate in Gimbal mounting. b, Plane-polished plate mounted by ball and socket in steel yoke, used to seal lower end of tube. c, Tube used to protect b from water of ice bath*

centimeters from the end. The whole was made secure by suitable lock nuts and submerged in double-distilled mercury, contained in a cylindrical glass reservoir free to rotate about the axis of the ohm tube. At the bottom of this reservoir was a fixed steel plate with a hexagonal hole to engage the head of the screw carrying the plane polished plate. The upper end of the tube was connected to a Geryk pump by means of a ground-glass extension and a mercury seal. After exhausting the tube for 15 minutes sufficient mercury to fill it was admitted by rotating the mercury reservoir. The plate was then reclamped to the end of the tube in a similar manner.

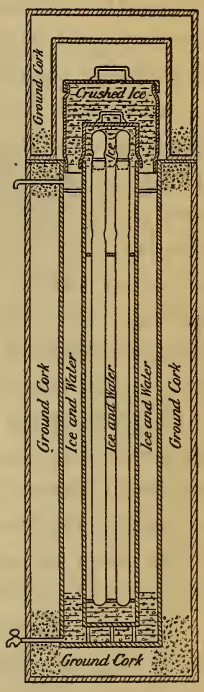


FIG. 4.—*Ice bath used in the determination of M_0 by the Reichsanstalt method*

After disconnecting the tube from the pump and removing it from the rack, the lower end was inclosed in a glass vessel, *c*, Fig. 3, screwed to the upper part of the yoke. To exclude moisture, this was then dipped in liquid rubber cement. The upper end of the tube was protected by a small ground-glass cap. The filled tube was then placed vertically within a specially constructed ice bath. This bath, Fig. 4, consisted of glass tubes 110 cm long and 5 cm in diameter, provided with constrictions near the top which served as supports for split rubber stoppers fastened to the ohm tubes to hold them in position. A tube was filled with distilled water to within 1 or 2 cm of the top, and after introducing an ohm tube a layer of finely shaved ice was added and the tube closed with a ground-glass cap. Around the glass tubes were two cylindrical copper baths, each of which was provided with a separate cover, so that the inclosed ohm tube could be completely surrounded with a double layer of ice and water. The outer copper wall was protected by a layer of cork placed between it and the heavy wooden box and cover inclosing the whole.

After allowing the tube to remain in the ice bath sufficiently long to come to temperature equilibrium, the top was uncovered

and the excess of mercury removed by stroking off with a plane polished glass plate in a gimbal mounting, *a*, Fig. 3. Condensation of moisture was eliminated by directing currents of dry, cooled air at both the end of the tube and the plane plate for a few seconds immediately before stroking off. Suitable illumination was employed, and in nearly every case interference fringes could be seen during the stroking-off process. After replacing the cap the tube was removed from the bath and allowed to come to room temperature before transferring the mercury to a porcelain crucible.

The Weighings.—The weighings for the determination of M_0 were all made in a basement room without windows, especially fitted up for use in silver voltameter work, in which connection it has been more fully described.[8] In one end an inner room was constructed with walls of hollow terra-cotta tiling and a double door and small double windows for the observing telescopes, and with openings for the rods and levers operating the balances. Temperature changes within the room were therefore found to be very small, even from day to day.

The weighings were made on a 200-gram Stückrath balance, so designed that the transposition of load and weights, as well as the manipulation of multiple riders and the other operations, are accomplished from a distance. There was, therefore, no necessity of the observer entering the inner room after the load and weights were put upon the balance pans.

Two sets of weights were used, one a set of one-piece gold-plated brass weights ranging from 1 to 50 grams, made by Gurley, and the other a set of platinum weights ranging from 0.0001 to 1 g, made by Christian Becker. Both sets were very carefully calibrated in the weights and measures division of the Bureau before and after making the series of weighings, and only one of the brass weights used was found to have changed by as much as 0.05 mg, the remaining ones have remained constant to 0.02 mg. None of the platinum weights changed by more than 0.002 mg.

The method of double weighings was used, the equilibrium points being determined by observing on the image of a scale reflected by a mirror on the balance beam and a fixed right-angle prism. The sensibility of the balance was determined for every

[8] The Silver Voltameter, this bulletin, 9, p. 174; 1913.

weighing and was approximately 37–38 scale divisions per milligram, so that in view of the constancy of the temperature the weighings may be considered correct to 0.01 mg, as was also shown by check weighings occasionally made.

The mercury to be weighed was collected in porcelain crucibles and for the purpose of weighing was transferred to a particular crucible, the weight of which was determined before and after each set.

The temperature, relative humidity, and barometric pressure were always recorded and corresponding buoyancy corrections applied.

Six fillings for each of the four ohm tubes were first made. In Table 14 the separate values of M_0, the deviations from the mean value, and the mean probable error of the result are given.

TABLE 14

Results of M_0 Determinations, Reichsanstalt Method

Tube	Filling	M_0 in grams	Deviation from the mean in milligrams
No. 1	1	11.574772	+0.028
	2	825	+ .081
	3	621	− .123
	4	799	+ .055
	5	738	− .006
	6	712	− .032
Mean		11.574744	± .054
No. 2	1	11.864017	+ .056
	2	3953	− .008
	3	3997	+ .036
	4	3920	− .041
	5	4015	+ .054
	6	3862	− .099
Mean		11.863961	± .049
No. 3	1	12.042690	− .025
	2	818	+ .103
	3	767	+ .052
	4	670	− .045
	5	680	− .035
	6	664	− .051
Mean		12.042715	± .052

TABLE 14—Continued.

Tube	Filling	M_0 in grams	Deviation from the mean in milligrams
No. 4	1	12.974942	− .008
	2	4976	+ .026
	3	4838	− .112
	4	4922	− .028
	5	5027	+ .077
	6	4994	+ .044
Mean		12.974950	± .049

Average deviation from mean:

Tube No. 1 ±4.7 parts per million.
Tube No. 2 ±4.1 parts per million.
Tube No. 3 ±4.3 parts per million.
Tube No. 4 ±3.9 parts per million.

Probable error of result:

Tube No. 1 ±1.73 millionths.
Tube No. 2 ±1.43 millionths.
Tube No. 3 ±1.44 millionths.
Tube No. 4 ±1.40 millionths.

Supplementary Fillings.—In making the above fillings for the determination of M_0 only slightly more mercury than required to fill it was admitted into the tube. In filling a tube for electrical comparison it was, however, necessary to pass enough additional mercury through it to fill one of the end bulbs. The fillings for M_0 and for the electrical comparisons were thus made under slightly different conditions, and to test the effect, if any, of the sweeping action upon the surface film produced by a relatively large volume of mercury passing through the tube, six subsequent fillings for mass determination were made on tube No. 3. The upper end of the tube, placed in the inclined rack, was connected to a bulb approximately 4 cm in diameter, which was in turn connected to the Geryk pump. When filling the tube enough mercury was drawn through it to fill this bulb also. The results of six determinations by this method are given in Table 15. As will be seen, the difference found was negligible.

TABLE 15

Supplementary Fillings, Tube No. 3, P. T. R. Method

Filling	M_0 in grams	Deviation from mean in milligrams
1	12.042585	−0.136
2	803	+ .082
3	711	− .010
4	725	+ .004
5	766	+ .045
6	737	+ .016
Mean	12.042721	± .049

Average deviation of the mean = ±4.1 parts per million.
Probable error of result = ±1.70 millionths.

Check Determinations of M_0 by Smith's Method.—The Smith method of determining M_0, as previsously stated, involves the two operations of filling the extensions alone and of filling the tube with the extensions attached, the difference in the masses of mercury required being the mass contained in the tube.

Some difficulty was experienced with this method, as the fillings frequently showed irregular effects when submitted to reduced pressure, even when made after very perfect exhaustion. Most of the difficulty was eliminated by modifications of the form of coupling first employed. The agreement of individual results in the fillings of the extensions alone is, however, not as good as that obtained by the Reichsanstalt method, so that only a relatively few check fillings were made.

The first form of coupling used was of the ball-and-socket pattern employed by Smith.[9] In the present case, however, they were made of steel with the sleeves threaded to screw onto the steel collars cemented to the tubes, instead of being secured by small set screws seated in dimples ground near the ends of the tube.

The coupling was provided with a mercury seal made as shown in Fig. 5; *a* is a glass cylinder with opening *d*, provided with a ground-glass cap; *b* and *b* are annular disks of rubber, while

[9] Smith, Nat. Phy. Lab. Collected Researches, Vol. V, p. 153; 1909.

c and *c* are steel cup-shaped pieces threaded to engage the collars of the tube and hold the rubber disks firmly against the ends of the glass seal.

The extensions (Fig. 6) were of Jena 59^{III} glass of 0.158 mm bore for the greater part of their length, but terminating in a bore of very approximately the same size as that of the ohm tubes. Each extension was graduated in millimeters over 3 cm of the vertical portion. The lower ends were ground to the same taper as that of the ohm tubes to permit using the same cleaning and drying fittings, and the end sections were ground and polished to a plane surface. The upper ends were tapered and ground to fit the connecting bulbs *a* and *b* as shown, while *c* and *d* are further ground glass extensions used in making connections to the exhaust pump. Both sides of *a* and *b* are provided with mercury seals.

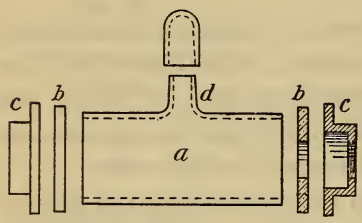


Fig. 5.—*Mercury seal for steel coupling; used in determination of M_0 by Smith's method*

The coupled extensions were mounted within a suitable Dewar vessel which in turn rested upon a small turntable. This arrangement made it possible to read the mercury columns from opposite sides thus eliminating the effects of parallax.

In filling the coupled extensions, the bulbs *a* and *b* are brought into place and the extensions *c* and *d* connected to the pump, bulb *a* being completely filled with mercury beforehand. After exhausting 15 minutes, the pressure at *c* is momentarily increased enough to start the mercury through the capillary. After filling, the bulb *a* is removed and extension *d* connected to the house vacuum and enough mercury withdrawn from the tube to bring the level of the two ends of the thread within the graduated portion of the capillaries. A cathetometer was used in making the observations, simply as a matter of convenience, the observing telescope turning about a vertical axis so as to bring into view either end of the mercury column as required.

Readings were taken with the mercury column alternately under atmospheric pressure and then under reduced pressure until a series of several observations were made, the relative positions of the ends of the column being shifted for each set of readings. The last two observations were made at normal pressure. In many cases, when under reduced pressure, the mercury rose in the the extensions. When this rise was appreciable the filling was rejected and a new one made. The true mean height of the ends of the mercury thread was taken as the mean of the heights at normal pressure in the two positions. After completing a set

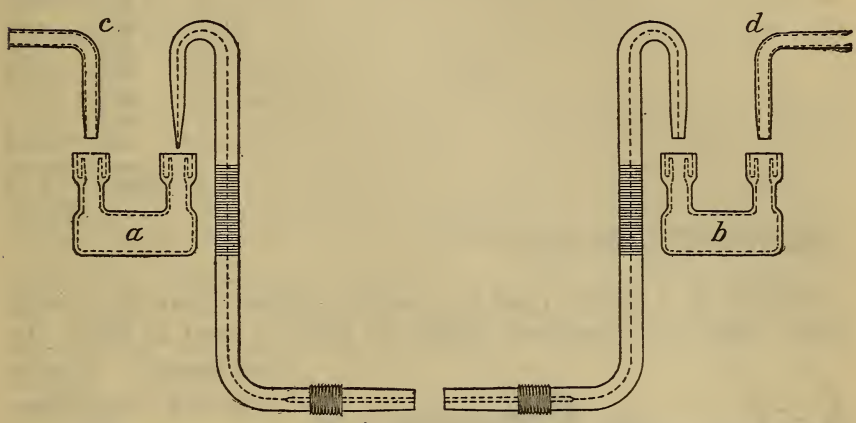


FIG. 6.—*Ohm tube extensions and fittings; used in determination of M_0 by Smith's method*

of observations the mercury was drawn off into a clean bulb by aid of the house vacuum. Weighings were made in the same manner and on the same balance as were the fillings made by the Reichsanstalt method. Thirteen fillings were made all of which were reduced to correspond to a definite height of mercury in the capillaries. The correction factors were determined from data obtained by a careful calibration of the capillaries (approximate cross section 0.158 sq mm) before sealing them onto the larger portions of the extensions. The results of the fillings are given in Table 16.

TABLE 16

Fillings of Extensions

Filling	Mass in grams	Deviation from mean in milligrams
1	1.125238	−0.26
2	467	− .03
3	341	− .16
4	624	+ .13
5	515	+ .02
6	553	+ .06
7	521	+ .02
8	394	− .10
9	585	+ .09
10	732	+ .24
11	377	− .12
12	620	+ .12
13	483	− .01
Mean	1.125496	± .10

Average deviation from mean=88.8 parts per million.
Probable error of result=22.5 millionths.

Fillings of a whole tube by Smith's method were first made when using two couplings similar to the one used in filling the extensions. These proved unsatisfactory, owing to fact that the introduction of additional joints multiplied the effect of reduced pressure on the mercury column. In many cases each end of the thread rose several centimeters, the original mean height not being recovered when again under normal pressure. A universal coupling was therefore devised. This is shown in Fig. 7; *a* is a ring to which are fastened the two cylindrical threaded parts *b* and *c*, riveted so they are free to move in planes at right angles to each other; *c* and the thin

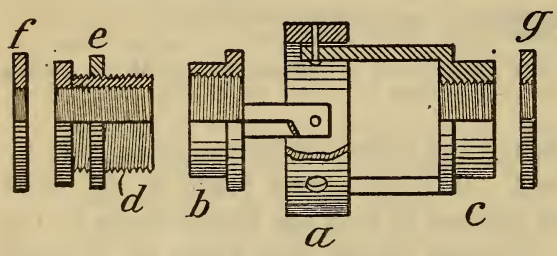


FIG. 7.—*Universal steel coupling for tube and extension or for extensions alone; used in the determination of M_0 by Smith's method*

double-threaded cylinder *d* are designed to engage the collars of the tubes and extensions; *d* also engages the part *b* of the coupling and is provided with a milled head and a lock nut *e*, while *f* and *g* are additional lock nuts. In making a coupling, the lock nuts *f* and *g*, together with the several parts of the seal, are first placed in their proper order beyond the collars of the tube and extension; *c* is then screwed onto the collar of the ohm tube until the end of the tube is about at the center of the ring *a*. The lock nut *g* is then made fast; *d* is made to engage *b* and the collar of the extension at the same time. By turning *d* the end of the extension is brought to the center of *a* and against the end of the tube. The lock nuts *e* and *f* are then brought into place, after which the seal is assembled.

In trying out these new couplings the same difficulty was experienced as when using the old ones. A relatively large quantity of air appeared to be trapped about the ends of the tube, which caused a rise of the ends of the mercury column when under reduced pressure. As the seating of the face of the tube against that of an extension appeared to be all that could be desired, it was thought that the difficulty lay in the fact that the metal of the coupling was too close to the joint of the tubes to allow of the intimate contact of the mercury with the glass at that point. Couplings of the same design but of larger diameter were therefore made and were used with greater success. Any small rise of the mercury in the capillaries, under reduced pressure, was succeeded by the columns recovering their mean height when again under normal pressure. Fillings were made in the same ice bath used when making electrical comparisons, the operations being the same as when making the fillings of the extensions. Six satisfactory fillings were made of tube No. 3, the results of which are shown in Table 17.

TABLE 17

Check Fillings, Tube No. 3, Smith's Method

Filling	M_0 in grams, tube and extensions	Deviation from mean in milligrams
1	13.168216	+0.089
2	8293	+ .166
3	8312	+ .185
4	7827	− .300
5	7965	− .162
6	8147	+ .020
Mean	13.168127	± .154
M_0 of end sections	1.125496	
M_0 of tube	12.042631	

Average deviations from the mean=11.7 parts per million, tube and extensions.
Probable error of result=4.0 millionths.
Probable error of result, tube alone=4.6 millionths.

A comparison of the determinations of M_0 for tube No. 3, as given in Tables 14 and 17, shows that the average deviation from the mean of six fillings and the probable errors of the results are greater in the Smith method than in the Reichsanstalt method of determination.

Effect of Pressure on Filling.—A calculation was made of the effect of internal pressure on the mass of mercury contained in an ohm tube of Jena 16^{III} glass when in a vertical position. In such a case the observed M_0 would be too large by approximately 4.7 parts per million, due to the internal pressure alone. This is reduced by 0.3 parts per million, due to the external pressure exerted by the surrounding water of the ice bath, making the resultant M_0 too large by 4.4 parts per million. As there was no complete data available on the constants of Jena 59^{III} tubing, such as that used in the present investigation, no correction for the effect of pressure was applied to the M_0 as determined by the Reichsanstalt method, although the correction is probably approximately the same. In the investigations at the Reichsanstalt no correction of this kind was applied.

The Effect of Capillary Pressure in Stroking Off.—At the Reichsanstalt, when making the M_0 determinations, both a plane plate

and one with a small groove along half its diameter were used in stroking off. A practically uniform difference of 0.15 mg in the fillings was noted, the lower values being found with the use of the grooved plate. The above difference corresponded to 17.8, 20.3, and 17.8 parts per million, respectively, for the three tubes XI, XIV, and 114. No allowance was made for capillary pressure at the Bureau, although an ungrooved plate was used throughout. However, the plate received some accidental scratches upon it through usage. An error in stroking off of 0.15 mg would make the calculated resistance of the four tubes too small by approximately 12.5 parts per million.

The resultant effect of possible slight heating in stroking off is uncertain, and it requires excess internal pressure to fill the space around the periphery of the capillary with mercury. These effects should be further investigated.

Accuracy of Some of the Fillings Made at the Reichsanstalt and at the National Physical Laboratory.—At the Reichsanstalt 10 fillings were made of each of the 1-ohm tubes XI and XIV, and six fillings of the 1-ohm tube 114. The average deviations of the individual fillings from their means, as calculated from available data,[10] were ±6.1, ±11.6, and ±6.7 parts per million, respectively, the mean average deviation for the 3 tubes, 26 fillings, being ±8.1 parts per million.

At the National Physical Laboratory 3 to 5 fillings were made on each of 11 tubes, 46 fillings being made in all. The measurements were made before the tubes were cut, the method employed consisting of comparing a column of mercury (the length and position of which very nearly coincided with the x and y limits as derived from the calibration) directly with a standard meter. The mean average deviation of all tubes, as calculated from data in published tables,[11] was ±8.1 parts per million. Omitting two tubes, the ohm length of which exceeded 1 meter and for which the length measurements were less accurate, the corresponding mean average deviation for the 9 tubes was ±6.4 parts per million.

[10] Wiss. Abh. P. T. R., Vol. 2, pp. 458 and 465; 1895; Vol. 3, pp. 130, 1900.
[11] Collected Researches, N. P. L., Vol. 1, pp. 170 and 171.

H. DETERMINATION OF L_0

The problem of determining the length of the ohm tubes reduces practically to that of comparing the two types of length standards, the usual type in which the standard length is given by the distance between two lines, and the now less common type where it is given as the distance between end surfaces. These two types of standards can be compared by various methods of which only three were considered.

Direct Sighting.—In this method the micrometer settings are made on some point on the outer edges of the plane end faces of the standard and then compared with settings made on a line standard. The length of the bore in the case of the ohm tube would be given by the average of values obtained with the tube orientated in various positions. This method may be modified by sighting on the edge of the bore instead of the outer edges of the end faces, being more direct but also more troublesome. In some ohm tubes previously constructed by other investigators the end faces were slightly curved to facilitate making observations on the bore.

In the first method a difficulty arises in that the micrometer settings are made on points identified by sharply contrasting fields of illumination, thus introducing the possibility of a personal error.

In sighting directly on the edge of the bore [12] an error due to a displacement of the image would in general result from a dissymmetry of the optical system of the observing microscope, since only part of the objective is used in forming the image, the other half being "stopped" by the tube, while in making settings on the line standard the whole of the objective is employed. This error is not in any sense eliminated by giving the end surfaces a small curvature.

The Optical Method.—This method makes use of the property of a reflecting plane that an object and its image are equidistant from the plane. The principle is utilized by bringing a fine hair or set of points near the face of the end standard and setting the

[12] This may be greatly facilitated by inserting a slightly tapering plug into the capillary, the settings being made on the line of demarcation between the tube and plug.

cross wires symmetrically with respect to the object and image, thus locating the plane. Possibilities of error due to dissymmetry of the optical system are also introduced in this method as above described.

The Contact Method.—A modification of the contact method, devised by L. A. Fischer of this Bureau and applied by him with much success in comparing line and end standards,[13] when adapted for use with the ohm tubes, was found so satisfactory that it was used throughout.

The method consists in applying end pieces or extensions to the end faces of the standard. These are provided with ruled lines upon which the micrometer settings may be directly made. This reduces the end standard or ohm tube to a line standard the length of which is equal to the length to be determined plus the added length due to the extensions. The amount added can be accurately determined by directly abutting the contact faces of the extensions and determining the distance between the lines.

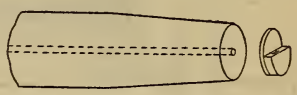


FIG. 8.—*End of tube and platinum-iridium end piece; used in the determination of* L_0

The end pieces used were those employed by Mr. Fischer in the work above referred to and were constructed as shown in Fig. 8. Each consisted of a lug of platinum-iridium of semicircular cross section, fastened to one face of a circular plate of the same material 6.5 mm in diameter, the other face of the plate serving as the contact area. The surface of the lug bearing the rulings, upon which the settings were made, was at right angles to the plane of the disk and intersecting it along a diameter. The average distance of the ruling from the surface of the end standard was approximately 0.8 mm, thus reducing to a minimum errors caused by any lack of parallelism of the end surfaces.

The extensions were held in place, as shown in Fig. 9, by the pressure of a flat spiral spring exerted through a small cupped brass lug mounted in the center of the spring. The spring was carried on an arm projecting from a circular nut screwing on to the steel collar of the tube. This nut was secured by a jam nut which

[13] Bull. Phil. Soc. (Wash.), 13, 241; 1898.

formed part of a casing to protect the ends from the ice and water of the bath in which the comparisons were made. This consisted of a cylinder of brass tubing closed at one end and threaded at the other. The threaded portion screwed into a flange of the jam nut above mentioned, thus inclosing the end of the tube. The lines on the extensions were viewed through a tube of smaller diameter fastened at right angles to the side of the brass cylinder, long enough to project well above the water level of the comparing bath.

Method of Mounting the Tubes.—In order to correct for the very slight residual curvature of the tubes, they were mounted in accurately aligned V-shaped supports, against which they were firmly held by the pressure of springs. The supports were clamped to one of the two adjustable T-shaped bars of the comparator box, provided for the support of standards under comparison, and were about 11 cm apart except in case of the end supports, which were adjusted to accommodate the tube under measurement.

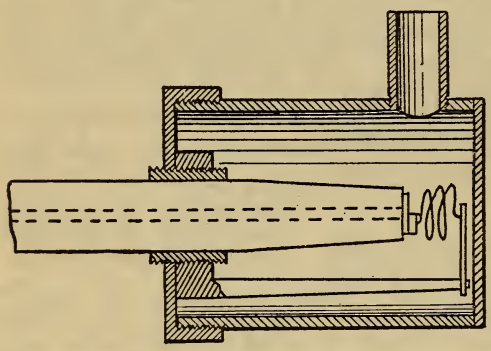


FIG. 9.—*Mounting for end piece and casing; used to protect the ends from ice and water of the comparing bath*

The Comparison Standard.—The comparison standard employed was an Invar bar of H section, known as meter 39. This, together with a companion bar belonging to the University of Wisconsin, was purchased from the Société Genevoise in January, 1903. The ruled surface has a high polish, and the lines are very fine and sharp. The bar was ruled throughout its length to millimeters, the terminating millimeters, which lay outside of the 0 and 100 cm marks, being further subdivided to tenth millimeters. In order to determine the length of tube No. 4, which slightly exceeded the length of the graduated portion, an additional line was ruled on the bar by R. Y. Ferner, of this Bureau.

The bar was calibrated and its length carefully determined when received. As changes with time have been observed in other nickel-steel standards, including the one purchased by the University of Wisconsin, the length was redetermined by comparison with platinum-iridium national prototype meter No. 21, after the completion of the measurements on the ohm tubes. The old calibration was used in determining the value of the subintervals used in the ohm tube comparisons and corrected for the new value of length found. The previous calibration did not include the terminating millimeters, necessitating the determination of the values of the interval 100.0–100.1 cm and its subdivisions. This, together with the determinations of the additional interval required for tube No. 4, will be described below. In order to protect the lines upon which observations were made from the ice and water of the comparing bath, a small brass tube, Fig. 10, was held vertically over the rulings by a brass plate through which it passed. This plate fitted over the top of the bar and held the lower end of the tube close to the surface of the graduations, the top of the tube extending above the water line. Before putting this device in place the lower end of the tube was supplied with an extension of soft wax, which, when the tube was in place, filled the gap between the tube end and the ruled surface. By perfecting this wax seal by means of a hot wire, the water was effectively prevented from entering. During observations, as is customary, the bar rested on two supports placed at the 23 and 77 cm points.

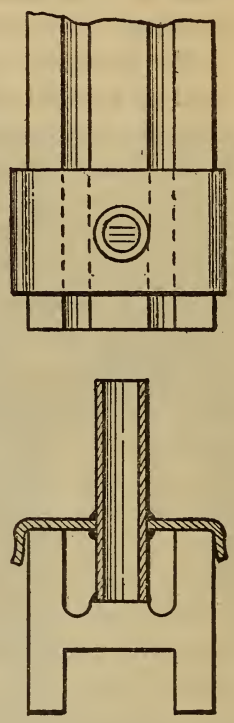


FIG. 10. — *Brass tube mounted on meter bar to protect lines under observation from ice and water of the comparing bath*

The comparator employed was the precision comparator, Fig. 11, of the Bureau, designed by L. A. Fischer. As this is to be described in detail at some future date, a brief outline only will be given here.

The micrometers of the comparator are mounted on heavy plates which are clamped to a beam supported at either end by concrete piers, having separate foundations. One end only of the beam is attached to its supporting pier, the other end resting freely on a plate set in the top of the other pier. The metal parts, including the micrometer screws, are composed of nickel steel.

The standards under comparison were mounted on two independent parallel supports provided with means for horizontal and vertical adjustments at either end. These supports were carried in the heavy metal inner compartment of a double brass box which rested on an open frame on the floor of the outer box, the latter being provided with leveling screws resting on the comparator carriage. The carriage ran on two rails mounted on a pier having an independent foundation. Its movement was perpendicular to the length of the standards, so that they could be brought successively beneath the micrometers by a slight shifting back and forth of the carriage, course and fine adjustment being provided. The inner and outer boxes were provided with lids through which apertures were cut for observations and to admit the keys which operated the various adjustments of the mountings. These lids, however, were used only in the comparisons of meter 21 and meter 39, made in air and at room temperature.

The Illumination.—The illumination for each microscope was secured by the light reflected from an incandescent lamp by a small totally reflecting prism. The latter was immediately beneath the observing microscope, covering one-half the field, and was carried on an arm extending from a sleeve on the microscope tube.

The sleeve could be rotated, and adjustments were provided for placing the prism in any desired position.

Preparation of the Ohm Tubes and Their Comparison with Length Standards.—The extensions were applied to the carefully cleaned ends of the ohm tube in the following manner: With the holders screwed on the collars of the tube, the distance of the spring from the end faces was adjusted so that a small pressure would be applied to the end pieces when in position. The tube was then placed in the mounting employed in its calibration and rotated into the

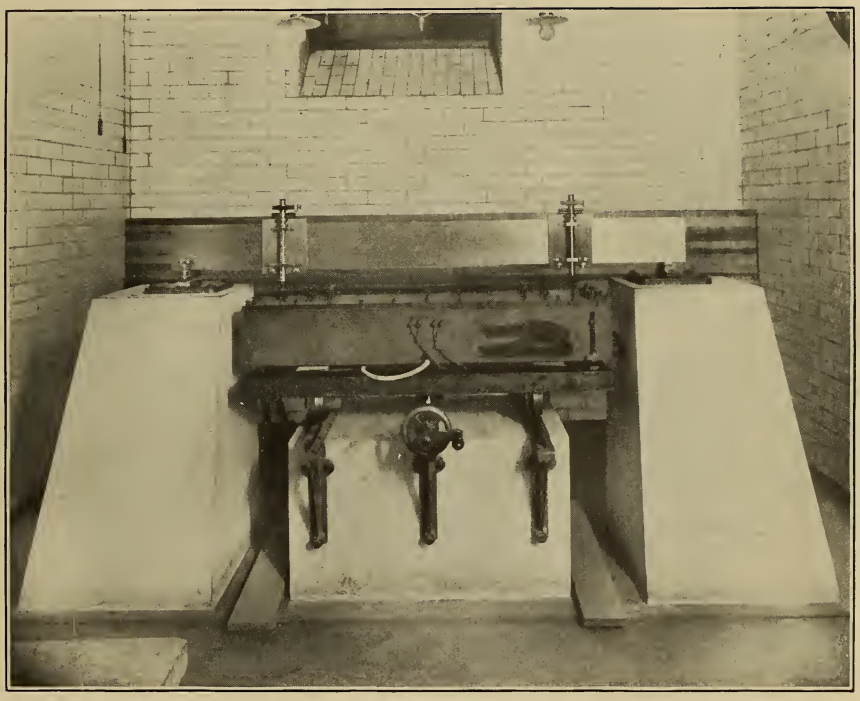


FIG. 11.—*Precision comparator used in the determination of the lengths of the tubes*

position in which it was to be compared. After a careful inspection of the tube ends and extensions for lint and dust particles, the extensions, which were handled throughout by small forceps, were applied to the tube. They were then centered by eye, the operation being greatly facilitated by illumination from the opposite end of the tube leaving a bright annulus around the extension. The ruled surfaces of the extensions were then made parallel by means of two paper targets placed on a line parallel to the axis of the tube, the proper orientation being determined by the targets, and their images being in line with the centers of the respective extensions. The adjustments were repeated when necessary until satisfactory in all particulars. The protecting boxes were next attached, wax being applied to the threads to insure waterproofing and the tube then placed in the mounting previously described.

The surface of the standard was prepared for observation by cleaning with benzene to remove traces of wax and oil that might have been left from a previous comparison and in order to give the ruled surface uniform treatment.

The approximate lengths of the tubes and extensions were separately obtained. The combined length was compared with an interval on the meter bar determined by the distance between one of the millimeter divisions toward the zero end and one of the lines of the subdivided millimeter at the 100-cm end. In this way the difference read on the micrometers was reduced to a minimum. The protecting tubes were then placed over the lines selected and the bar was placed on its supports in the comparator box.

After spacing the microscopes the surface of the bar was adjusted to be perpendicular to the axes by means of the leveling screws on the outer comparator box, the illumination in this position being a maximum. With the bar properly orientated it was approximately adjusted for observations. The tubes and meter bar were compared at 0° C, maintained by immersing them in a mixture of ice and water. Fine ice was placed directly about the tube and bar, being heaped to a height of several centimeters, except where it would have interfered with the observations and adjustments, while coarser ice was carefully packed in the space between the sides and bottoms of the inner and outer boxes. The water level

was maintained at a height of several centimeters above the standard. The ice was replenished as needed, and the excess of water was drained off from time to time.

In order to insure certainty in the temperature, an interval of at least 15 minutes was allowed to elapse after completing the icing before the observations were begun.

There was a tendency for atmospheric moisture to condense upon the surfaces under observation, thus obscuring the lines. This was effectively disposed of by blowing a current of dried air against the surfaces, after which they would remain clear for a considerable time.

The final adjustments were made after the icing was completed. The microscopes were spaced so that the distance between them was approximately the average of the two intervals compared. The standard and tube were then put in alignment so that the center of the lines came in the center of the microscope field.

In order to use the micrometers to the best advantage the bars were adjusted endwise until in passing from an observation on the tube to an observation on the bar the readings would shift from one side to about an equal distance on the other side of the center of the microscope field.

The focusing was done by raising or lowering the support of the standard under observation by means of the adjusting screws provided. The proper focus was obtained when the parallax, produced by oscillating the sleeve upon which the illuminating prism was mounted, was a minimum. Though this had a natural tendency to disturb the microscopes the results derived indicated that this effect was negligible.

The microscopes were focused preceding each group of four simultaneous observations, the two observers interchanging positions for the last two readings. The comparator carriage was then shifted, bringing the other standard in position, the microscopes were focused, and readings taken as before, the observers coming back to their original position for the last two readings. A comparison consisted of the average of three groups of observations on the ohm tube alternated with two groups on meter 39.

The difference between the two standards was obtained from the average of a set of six comparisons. Alternate sets were made

with the initial positions of the observers interchanged, in order to eliminate as far as possible personal errors in focusing and setting.

The results indicated that the personal error was quite small, the mean difference of the means of alternate comparisons not exceeding one-tenth micron. Comparisons were made in four positions of the tube about its axis 90° apart, necessitating dismounting the tube, recleaning the contact surfaces, resetting the extensions, and remounting the tube.

The final length was taken as the mean of values obtained for the four positions.

The length of meter 39 was determined following the work on the ohm tubes. This was done by comparisons made with National Prototype meter No. 21. The comparisons were made at temperatures of 20° C and 0° C in the same general manner as above described. The measurements at the higher temperature were made in order to check the temperature coefficient upon which the calibration depended. The comparisons at 0° C were made in two positions, meter 21 being turned end for end in the last half of the observations, and included 16 sets of observations. In the observations on the bar at the higher temperature the general routine in such work was followed, the bars being compared in eight relative positions, one being duplicated. Four observations were taken for each position and, in order to minimize temperature errors, several hours were allowed to elapse after placing the bars in position before observations were made. The comparisons therefore extended over several days.

The results showed that an appreciable change, amounting to approximately 5 microns, had taken place in the total length of the bar since its original calibration in January, 1903. There was also found a slightly different temperature coefficient, corresponding to a relative change of approximately 1½ microns for a range of 20° C. Changes of a similar character had been previously observed at the Bureau in case of the nickel-steel meter submitted for verification by the University of Wisconsin. The corrections of the graduations employed were recomputed on the assumption that the observed change in length was uniformly distributed throughout the bar.

The comparisons for the determinations of the values of the intervals −0.9 to 0.0 cm and 100.0 to 100.1 cm, the subdivisions of the latter, together with the check comparisons on the extensions, were made at room temperature on a Geneva Society dividing engine. The objects to be compared were aligned on the carriage of the dividing engine, the microscopes being adjusted so that the left-hand lines of the intervals compared came near the center of the field. Two micrometer readings were taken and the carriage was then shifted, bringing the right-hand lines into the center of the field. Four readings were then taken on each of the lines and the carriage again shifted to the left-hand lines, two readings being taken in this last position. In this manner the differences read on the micrometers were small, and errors due to progressive changes were also reduced. The standard intervals used were selected intervals on nickel-steel decimeter No. 24.

The interval −0.9 to 0.0 cm was determined by comparisons with intervals 0.0 to 0.9 cm and 0.1 to 1.0 cm of decimeter No. 24 and the results reduced to 0° C.

The value of the interval 100.0 to 100.1 cm was obtained by comparisons with the first 9-mm intervals of decimeter No. 24, the result also being reduced to 0° C.

The abbreviated method used in the calibration of the ohm tubes was employed in the subcalibration of this interval. In this the 0.4 mm, the 0.5 mm, and the 0.6 mm intervals were each intercompared by the aid of the corresponding fixed intervals selected on the subdivided millimeter of decimeter No. 24. From the data thus obtained and the length at 0° C of the whole interval the corrections of the subdivisions were computed.

The correction for the extensions was frequently checked by measurements made on the dividing engine at room temperature. The results agreed within the limits set by the errors of observation. It was found that the interval was unaffected by a slight relative displacement of the two abutting faces.

Correction for the Extensions.—To eliminate errors due to the variation in the appearance of the lines as viewed through different microscopes and under different illumination, the final

correction for the extensions was determined on the precision comparator in order to secure the same conditions which obtained in the measurement of the ohm tubes. The same general plan employed in the comparisons above described was adopted. The extensions and standard scale were mounted on either end of the same support in the comparator box, the shifting being accomplished by the aid of the adjustment provided for moving the support longitudinally. The comparison intervals selected were the five 2-mm intervals constituting the first centimeter of decimeter No. 24.

In the determination of the correction for the extensions it was highly important to include an optical constant, due to the proximity of the lines to the end plates in the same manner as it occurred in use with the tubes. This was realized by shifting the support so that the settings on the extensions always came in the central region of the field, the difference being taken up on the other micrometer. The final value was obtained from observations made in the manner previously described with the position of extensions and standard interchanged for half the readings.

The micrometer constants were redetermined, following the work on the tubes, by comparisons made with the half millimeters of the subdivided millimeters of decimeters No. 24 and No. 43, one scale being placed beneath each microscope. Observations were made simultaneously on both microscopes, observers changing positions in the same order as in the length comparisons. After a series of observations the decimeters were interchanged and readings taken as before. The constants were thus determined under conditions obtaining in the comparisons of the ohm tubes and extensions.

Summary of Results, Length Measurements.—The results obtained in the length measurements are shown in Tables 18 to 23, which also give the component and resultant probable errors, calculated from the observational data and from the probable errors of the comparison standards.

The probable error of the National Prototype meter No. 21 was assumed to be 0.20 μ, which is regarded as a liberal allowance. The probable error of the recomparison of meters 39 and 21 at

0° C was 0.15 μ. The probable error in the meter interval of meter 39 was therefore taken as 0.25 μ, and this was prorated according to the length of the subinterval employed.

As no probable errors had been assigned to the corrections of decimeters 24 and 43, calibrated at the International Bureau, liberal values were assumed.

No recalibration of the subdivisions of meter 39 was made, the corrections employed being derived from the calibration made in 1903, from which the probable errors of the graduations were also derived. On tubes 1, 2, and 3 the probable error in the determination and subcalibration of the interval 100.0 to 100.1 cm is also involved. Tube No. 4 includes the probable error in the determination of the interval −0.9 to 0.0 cm. Owing to the small differences between the measured lengths of the tubes and extensions and the comparison intervals the error due to the uncertainty in the micrometer values is sensible only in the constant for the extensions and in the measurements on tube No. 4. In the latter case the difference happened to be very approximately equal to that dealt with in the determination of the constant for the extensions, and consequently no error due to the micrometer value is introduced in the length of the tube itself.

In the following table are given the computed probable errors contributed by the comparison standard for each of the four tubes:

TABLE 18

Probable Errors of Comparison Intervals

	Tube 1	Tube 2	Tube 3	Tube 4
	μ	μ	μ	μ
Graduation at 0 cm end of meter 39	0.17	0.33	0.30	0.15
Graduation at 100 cm end of meter 39	.04	.08	.09	.00
Error of meter 39	.24	.24	.24	.25
Resultant error of comparison interval	.30	.42	.39	.29

In Tables 19 to 23 are given the set means and final values obtained in the determination of the correction for the extensions and of the corrected lengths of the tubes. In these tables the component and resultant probable errors are also given.

TABLE 19

Summary of Length Measurements

Set:	Value of Correction for Extensions Value at 0°
1	1629.01 μ
2	1629.09 μ
3	1629.31 μ
4	1629.49 μ
Mean	1629.22 μ

Average deviation from mean	±0.18 μ
Probable error of observation of mean	±0.07 μ
Probable error of correction, including probable error of comparison scale and micrometer value (tubes 1, 2, and 3)	±0.15 μ
Probable error of correction, including probable error of comparison scale (tube No. 4)	±0.14 μ

TABLE 20

Tube No. 1, Differences Measured by Micrometer

Position:	Tube+Extensions—Comparison Interval Observed differences
1	−2.82 μ
2	−2.68 μ
3	−3.43 μ
4	−3.17 μ
Mean	−3.02 μ

Average deviation from mean		±0.28 μ
Probable error of mean		±0.12 μ
Comparison interval, meter 39	4.8—100.05 cm	
Correction at 4.8 cm		− 4.98 μ
Correction at 100.05 cm		−49.64 μ
Correction to interval 4.8 cm to 100.05 cm		−44.66 μ
Corrected comparison interval	95.245534 cm	±0.30 μ
Corrections for extensions	− .162922 cm	±0.15 μ
Micrometrically measured difference	− .000302 cm	±0.12 μ
Corrected length at 0° C	95.082310 cm	±0.36 μ

TABLE 21

Tube No. 2, Differences Measured by Micrometer

Tube+Extensions—Comparison Interval

Position:	Observed differences
1	−18.92 μ
2	−18.40 μ
3	−18.70 μ
4	−18.45 μ
Mean	−18.62 μ

Average deviation from the mean	± 0.19 μ
Probable error of mean	± 0.08 μ
Comparison interval, meter 39	3.6 cm—100.03 cm
Correction at 3.6 cm	− 3.95 μ
Correction at 100.03 cm	−50.24 μ
Correction to interval 3.6 cm to 100.03 cm	−46.29 μ
Corrected comparison interval	96.425371 cm±0.42 μ
Corrections for extensions	− .162922 cm±0.15 μ
Micrometrically measured difference	− .001862 cm±0.08 μ
Corrected length at 0°C	96.260587 cm±0.45 μ

TABLE 22

Tube No. 3, Differences Measured by Micrometer

Tube+Extension—Comparison Interval

Position:	Observed differences
1	−62.03 μ
2	−62.53 μ
3	−62.07 μ
4	−63.25 μ
Mean	−62.47 μ

Average deviation from the mean	± 0.42 μ
Probable error of mean	± 0.19 μ
Comparison interval, meter 39	2.9—100.07 cm
Correction at 2.9 cm	− 4.30 μ
Correction at 100.07 cm	−50.32 μ
Correction to interval 2.9 cm to 100.07 cm	−46.02 μ
Corrected comparison interval	97.165398 cm±0.39 μ
Corrections for extensions	− .162922 cm±0.15 μ
Micrometrically measured difference	− .006247 cm±0.19 μ
Corrected length at 0°C	96.996229 cm±0.46 μ

TABLE 23

Tube No. 4, Differences measured by Micrometer

Tube+Extensions—Comparison Interval

Position:	Observed differences
1	−369.63 μ
2	−371.70 μ
3	−371.73 μ
4	−371.25 μ
Mean	−371.08 μ

Average deviation from the mean	± 0.72 μ
Probable error of mean	± 0.33 μ
Comparison interval, meter 39	−0.9−100.0 cm
Correction at −0.9 cm	+0011.81 μ
Correction at 100.0 cm	−50.19 μ
Correction to interval −0.9 cm to 100.0 cm	−0062.00 μ
Corrected comparison interval	100.893800 cm±0.29 μ
Correction for extensions	− .162922 cm±0.10 μ
Micrometrically measured difference	− .037108 cm±0.33 μ
Corrected length at 0°	100.693770 cm±0.45 μ

Discussion of Results, Length Measurements.—From the above tables it will appear that the probable error of the final length in no case exceeds 0.50μ. Moreover, except in the case of tube No. 4, the magnitude of the probable error is largely determined by the probable error of the comparison interval. It must be pointed out that the errors of comparison include errors due to variation in mounting the extensions, to lack of parallelism of the end faces of the tubes, and to variations in curvature of the tube between the supports when mounted in the four different positions. The error due to the curvature of a tube into a circular arc is given to a very high degree of approximation for any practical case by $\frac{8}{3}\frac{h^2}{l^2}$, h denoting the height of the arc above the chord and l the length of the cord. With 10 supports, 11 cm apart, the error due to an elevation of 0.1 mm of the middle point would correspond to an error of 2.2 parts per million in the length of a section. Since there were nine sections, the variation in the average curvature would determine the error of the result, but close agreement of the values obtained for the four positions in which the tubes were measured indicates that this source of error must have been quite small, particularly since the tubes had been straightened as previously mentioned and the slight residual curvature further reduced by the mounting in which they were measured.

I. TERMINAL BULBS

The terminal bulbs were constructed as shown in Fig. 12. They were made spherical, and very approximately 4 cm internal diameter. At one end of and on a line with a horizontal diameter of the bulb, a ground-glass extension was provided for the seating of the tapered end of the ohm tube. This extension was made by sealing on a short piece of thick-walled tubing with the outer

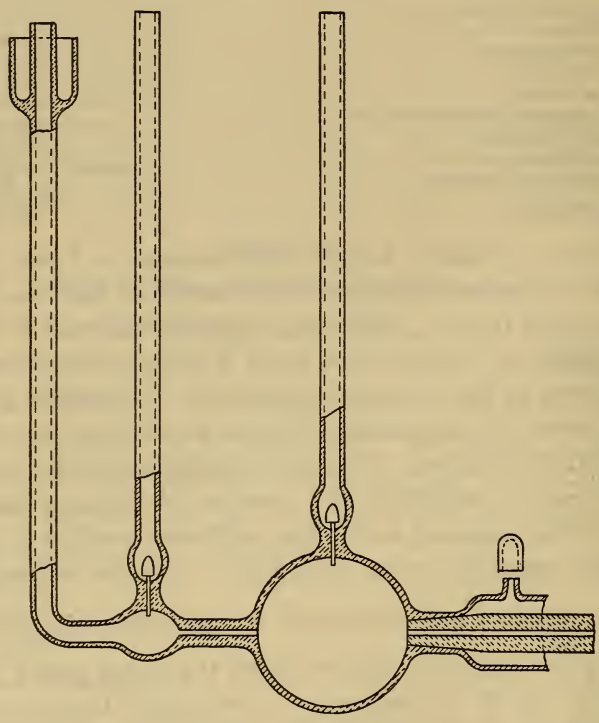


FIG. 12.—*Type of terminal bulb (exhaust end), forming part of a completed mercurial resistance standard*

half of its length enlarged so as to form part of a mercury seal. The grinding was then very carefully done, so that when completed the taper and diameter permitted the end of the seated tube to be flush with and to practically form part of the inner surface of the bulb. To the upper side of the seal portion of this extension was fused a short tube, provided with a ground-glass cap, serving for the introduction of mercury.

Diametrically opposite the above opening to the bulb, a piece of capillary tubing with a bore somewhat larger than that of the ohm tube was fused on. The central portion of this capillary was enlarged to provide for the current terminal. Beyond the enlargement, the capillary terminated in a tube of larger bore bent vertically upward to serve as an exhausting or filling tube. The potential terminal was attached to the bulb at the upper end of a vertical diameter.

The current and potential terminals were made by sealing in short platinum wires pointed at the inner ends and carrying conical copper lugs at the outer ends. Care was taken in sealing in these terminals to preserve the spherical contour of the bulbs. They were protected from the water of the ice bath by glass tubing of about 4-mm internal diameter, and of sufficient length to extend well above the surface of the bath. These protecting tubes were sealed onto the bulbs and were slightly enlarged about the copper lugs.

The above figure shows a bulb provided with an exhaust tube terminating in a ground seat and mercury seal for the connection to the exhaust pump. The companion bulb to the above is provided with a filling tube consisting of a thick-walled capillary, or a tube with a number of constrictions, bent downward for a few centimeters at the top and drawn out to a fine point before exhausting.

A distinct advantage of the type of bulb described lies in the fact that there is but one glass joint and this is protected by a mercury seal.

The resistance contributed by the terminal bulbs was calculated on the assumption that the length of the mercury column is increased at each end by eight-tenths of the radius.

The corresponding resistance is, therefore, given by the formula:

$$R_{T_1}+R_{T_2}=\frac{1}{1063}\left(\frac{0.8r_1}{\pi r_1^2}+\frac{0.8r_2}{\pi r_2^2}\right)$$
$$=\frac{0.8}{\pi 1063}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

r_1 and r_2 being expressed in millimeters.

The mean cross sections of the finished tubes are given by the L_0 and M_0 tabulations above, together with the density of mercury, which is taken as 13.5956 at 0° C.

The calibration data and the caliber and cross section curves were, however, all referred to the interval 5 cm to 105 cm, so that it was necessary to first determine the mean cross section in the above interval and from this the mean cross section at the ends.

The data employed in these calculations are given in Table 24.

TABLE 24

Data for the Calculation of the Correction for the Terminal Bulbs

	Tube No. 1	Tube No. 2	Tube No. 3	Tube No. 4
Caliber correction at lower end (mm)	+0.042	+0.050	+0.274	±0.000
Caliber correction at upper end (mm)	− .675	− .363	− .048	+ .010
Ratio of mean cross section of finished tube to mean cross section 5 cm–105 cm	.999246	.999571	.999669	1.000010
Ratio of cross section at lower end to mean cross section 5 cm–105 cm	1.00302	1.00334	1.00648	1.00196
Ratio of cross section at upper end to mean cross section 5 cm–105 cm	1.00968	.98978	.99875	1.00297
Mean cross section of finished tube (sq mm)	.895392	.906531	.913211	.947774
Mean cross section in interval 5 cm–105 cm (sq mm)	.896067	.906920	.913514	.947765
Cross section at lower end (sq mm)	.898773	.909949	.919434	.949623
Cross section at upper end (sq mm)	.904741	.897652	.912372	.950580
Correction for terminal bulbs in international ohms	.0008943	.0008933	.0008873	.0008712

The above values of the corrections for the terminal bulbs are correct to 0.000001 ohm as estimated by the accuracy of the calibration data, which, however, do not include irregularities within the subsections into which the tube is divided. The uncertainty arising therefrom, however, is believed to be small.

J. THEORETICAL RESISTANCES OF THE FINISHED TUBES

The theoretical resistances of the finished tubes can now be computed from the measured lengths, the mass of mercury required to fill the tubes and the corrections for the observed variations in cross section and for the terminal bulbs.

These fundamental constants together with the calculated resistances are summarized in Table 25.

TABLE 25

Theoretical Resistances of Finished Tubes

	Tube No. 1	Tube No. 2	Tube No. 3	Tube No. 4
L_0 in centimeters	95.082310	96.260589	96.996229	100.693770
M_0 in grams	11.574744	11.863961	12.042715	12.974950
Caliber factor	1.0000892	1.0000659	1.0000467	1.0000959
Resistance at 0°C of tube proper	.9990593	.9989882	.9992404	.9995525
Correction for terminal bulbs in international ohms	.0008943	.0008933	.0008873	.0008712
Total resisiance in international ohms	.9999536	.9998815	1.0001277	1.0004237

To the above values certain corrections are still to be applied, but owing to the absence of reliable data for their calculation, they have merely been discussed (p. 456).

K. THE ELECTRICAL MEASUREMENTS

The electrical comparisons of the ohm tubes with auxiliary wire standards were all made by a modification of the Thomson bridge method.

The wire standards were five in number and were of the small type previously described,[14] the cases being filled with oil and hermetically sealed to eliminate changes in resistance due to variations in atmospheric humidity. Two of them, Nos. 3939 and 3940, were constructed by Otto Wolff in accordance with specifications furnished, while the others were constructed at the Bureau.

The comparisons were made by substitution (see Fig. 13), each standard being in turn substituted in the same bridge arm, S, the auxiliary standard, R, in the opposite arm being an open 1-ohm Wolff coil, readjusted to have a resistance approximately 0.12 per cent higher than its nominal value, so that the shunt required for balancing the bridge would always be on the auxiliary standard, and so that it would be unnecessary to employ shunts of very high value.

The wire standards, together with a 100:100 ohm main ratio coil and a 10:10 ohm auxiliary ratio coil, were contained in a well-stirred, thermostatically controlled, and electrically heated oil

[14] A New Form of Standard Resistance, this bulletin, 5, pp. 413–434; 1908–9.

bath. In the comparisons they were connected in the desired manner by copper blocks mounted on suitable stands and provided with mercury cups, with the exception that the connection bridged over by the auxiliary ratio arms consisted of a short No. 10 copper wire, *L*, the ends dipping into the mercury cups,

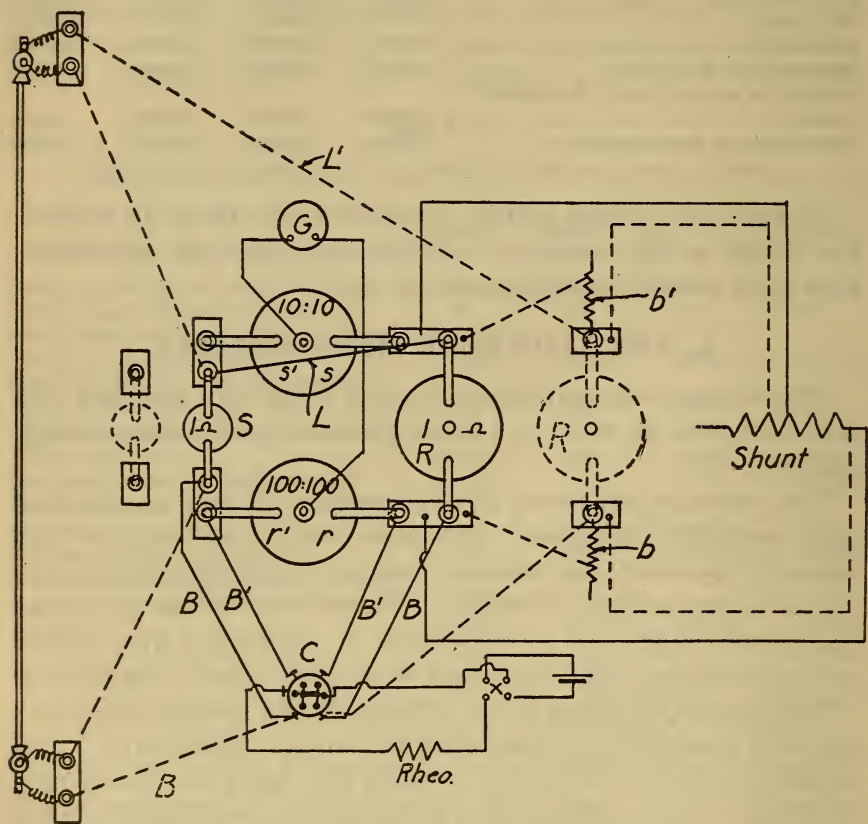


FIG. 13.—*Diagram of connections employed in making intercomparisons of mercurial and wire standards*

serving as potential terminals of the standards. When not in use the five reference standards were mounted on a metal stand, so as to be always completely surrounded by oil. The ratio coils were selected so as to obtain a very exact equality between the main and auxiliary ratios, thus eliminating any necessity of balancing the auxiliary arms. The main and auxiliary ratios were within one

millionth of each other and differed from unity by but 13 parts in 1 000 000.

As before stated, the bridge was balanced by adjusting the shunt on the auxiliary comparison standard, R, until the galvanometer, G, permanently connected to the bridge, gave no deflection on reversal of the test current. G is a special high sensitivity moving coil galvanometer and was employed throughout. Its external critical resistance is approximately 50 ohms, and the deflection obtained for a difference in balance of one hundred parts per million amounted to 50 or 60 mm, with the test current of 0.035 ampere employed, so that comparisons could be easily made to within a few parts in 10 million.

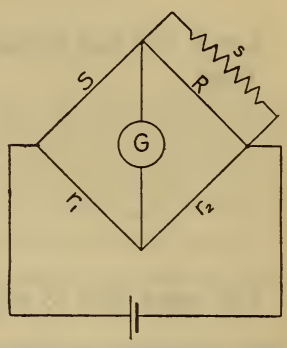


FIG. 14.—*Diagram of simple bridge with shunt*

The battery connections were normally applied to the potential terminals, adjacent to the main ratio arms, of the auxiliary and reference standards under comparison. By the aid of a mercury commutator, C, the connections could be shifted to the terminals of the main ratio coil, and the correction for the inequality of the connecting resistances from the latter to the standards could be interpolated from the deflection resulting on reversing the commutator. Since the resistances in the main ratio arms were approximately 100 ohms and the connecting resistances were themselves small, the correction never exceeded a few parts in 10 million.

Computation of the Results.—In a simple bridge, Fig. 14, let r_1 and r_2 denote the resistances of the main ratio arms; R that of the auxiliary standard; S_1, S_2, etc., that of the standards to be compared; and s_1, s_2, etc., that of the respective shunts applied to R to produce a balance.

Then

$$\frac{S_m}{\left(\frac{s_m R}{R+s_m}\right)}=\frac{r_1}{r_2}$$

or

$$S_m=\frac{s_m R}{R+s_m}\,\frac{r_1}{r_2}$$

As the direct calculation by the above formula is quite laborious, and as only the relative values of the standards under comparison are required, the computation was simplified by expressing the results in terms of the decrease in the resistance of the auxiliary standard produced by the balancing shunt.

The above formula may be written:

$$S=\frac{r_1}{r_2}\left(R-\frac{R^2}{R+s}\right)$$

or

$$S=\frac{r_1}{r_2}R-\frac{r_1}{r_2}\frac{R^2}{R+s}$$

Let $R=(1+\alpha)\,R_0$ and $s=m\,R_0$

Then

$$\frac{R^2}{R+s}=\frac{1+2\alpha+\alpha^2}{1+m+\alpha}R_0$$

$$=\frac{1+2\alpha+\alpha^2}{1+\frac{\alpha}{1+m}}\,\frac{R_0}{1+m}$$

The error due to neglecting α^2 in the numerator and $\frac{\alpha}{1+m}$ in the denominator is represented by

$$\left(\alpha^2-\frac{\alpha}{1+m}\right)\frac{R_0}{1+m}$$

and is readily shown to be negligible in all practical cases.

Hence

$$S=\frac{r_1}{r_2}R_0(1+\alpha)-\left(\frac{R_0}{1+m}+\frac{2\alpha}{1+m}R_0\right)\frac{r_1}{r_2}\equiv K-D$$

The first term of the expression which is denoted by K is constant, and its exact value is not required to obtain the relative values of standards substituted in the arm S. Moreover, since the second term, denoted by D, is a correction term, $\frac{r_1}{r_2}$ may be put equal to the nominal ratio in estimating its balance. In the special case here dealt with R_0 is equal to 1 ohm and $\frac{r_1}{r_2}$ differs only very slightly from unity, so that $D=\left(\frac{1}{1+m}+\frac{2\alpha}{1+m}\right)$. The first part is

given directly by reciprocal tables, and the second part is so small that it can be assumed constant for a particular standard, the slight differences in m found in different comparisons producing negligible effects. Disregarding the second term entirely would, however, have produced a maximum error only slightly exceeding 1 part in 1 million even in the comparison of the mercury standards.

The results of a series of comparisons made in the manner above described may be written

$$S_1 = K - D_1$$
$$S_2 = K - D_2$$
$$\cdot \quad \cdot \quad \cdot$$
$$S_n = K - D_n$$

Hence, assuming the value of one of the coils, or the mean value of a number of them, K is obtained. This value of K, substituted in the observational equations, gives the value of all standards substituted in the given bridge arm on the particular basis to which the assumed values are referred (in this case the mean of the 5 coils employed). The differences between the values thus computed and the assumed values of the separate coils, found from day to day and tabulated below, gives a measure both of the accuracy of the observations and of the relative constancy of the coils.

TABLE 26

Relative Constancy of Wire Standards During their Intercomparison with the Mercury Standards

Date	No. 3	No. 7	No. 8	No. 3939	No. 3940
1911	m	m	m	m	m
February 10	+0.8	−0.2	−0.6	+1.3	+0.4
February 13	+1.0	− .3	− .8	.0	+ .4
February 14	+ .9	− .6	− .4	.0	.0
February 15	+ .7	− .9	− .4	.0	+ .7
February 16	+ .6	− .6	− .6	+ .5	+ .4
February 18	+ .8	− .4	− .4	− .1	+ .5
February 21	+ .9	− .8	− .7	+ .1	+ .5
February 23	+1.0	− .6	− .5	+ .1	+ .3
February 24	+1.0	− .2	− .6	− .2	.0
February 25	+1.0	− .2	− .6	− .3	+ .1
February 27	+ .8	.0	− .9	− .4	+ .4
February 28	+1.0	− .2	− .6	− .4	− .1
March 1	+1.0	− .3	− .8	+ .4	.0
March 2	+1.0	− .1	− .8	− .1	+ .1
March 3	+1.0	− .4	− .8	− .1	+ .2
March 4	+ .8	− .2	− .8	− .2	+ .5
March 6	+ .8	− .2	− .8	− .2	+ .5
March 8	+1.0	− .2	− .8	− .1	+ .3
March 9	+1.0	− .1	− .7	− .1	− .1
March 16	+1.1	.0	− .7	− .1	− .1
March 17	+1.2	− .1	− .7	− .2	− .3
March 18	+1.1	− .1	− .7	− .1	− .1
March 20	+1.0	− .3	− .4	− .1	+ .1

Table 27 gives the temperature coefficients (in the general formula $R_t = R_{25}\{1 + \alpha\,(t-25) - \beta\,(t-25)^2\}$) as well as the values of the wire standards used in the intercomparisons, and covers the period from December 1, 1910, to June 14, 1912. The first five standards were used in the 1911 measurements. Two supplementary series of measurements were made in 1912, in which coils Nos. 3939 and 3940 were replaced by Nos. 4 and 15.

TABLE 27

One Ohm Wire Standards

[Excess in millionths over nominal value in international ohms]

Coil	α	β	Dec. 1, 1910	Feb. 17, 1911	Mar. 11, 1911	Jan. 11, 1912	Mar. 1, 1912	May 4, 1912	June 14, 1912
3	+10.8	−.48	+ 63	+ 63	+ 63.5	+ 66.5	+ 66.5	+ 66.5	+ 68
7	+ 9.2	−.49	+ 65	+ 64	+ 65	+ 65.5	+ 64.5	+ 64	+ 65.5
8	+ 9.2	−.50	+ 70	+ 70	+ 69.5	+ 69	+ 68	+ 68.5	+ 68
3939	+13.0	−.46	+ 99	+ 99	+ 99	+ 97.5	+ 99	+ 99	+100
3940	+13.1	−.45	+ 98	+ 97.5	+ 99	+ 98.5	+100	+101	+101
Mean	.....	.....	+ 79.0	78.7	+ 79.2	+ 79.4	+ 79.6	+ 79.8	+ 80.5
4	+ 8.2	−.46	+ 31	+ 31	+ 31.5	+ 29.5	+ 28	+ 28	+ 28.5
15	+ 6.0	−.50	+153	+155.5	+158	+167.5	+166.5	+168.5	+168

As stated above, the close equality of the ratios of the main and auxiliary arms of the bridge and the relatively low resistance of the connecting link between the two standards eliminated any error due to the link. The correction arising from the inequality of the connecting resistances in the main ratio arms was calculated from the deflection produced by shifting the battery connections as described above, a double deflection of 5 mm corresponding to a correction of one part in ten million. The observed deflection rarely exceeded 3 mm.

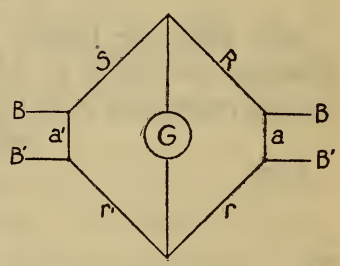


Fig. 15.—*Diagram of simple bridge with connecting resistances*

The considerably larger inequalities met with in the measurements of the mercury standards made it desirable to introduce compensating resistances in the opposite arms of the bridge.

The following discussion relates to the principles governing the method of adjusting the compensating resistances, so as to eliminate the correction involved, or to reduce it to such a small value that it can be computed with adequate accuracy:

Consider a simple Wheatstone bridge as shown in Fig. 15, in which S is the standard to be measured, R the standard in terms of which it is to be expressed, r and r' ratio arms, and a and a' con-

necting resistances, one or both adjustable, between the ratio arms and the standards under comparison.

$$\begin{aligned} \text{Let } S &= m\,R\,(1+x) \\ r &= n\,R \\ r' &= mn\,R\,(1+\alpha) \\ a &= c\,R \\ a' &= mc'\,R \end{aligned}$$

Then if the bridge be balanced (by preliminary adjustment of one of the connecting resistances) with the battery applied at B′ B′

$$\frac{S+a'}{R+a}=\frac{r'}{r}$$

Substituting

$$\frac{m\,R\,(1+x)+mc'\,R}{R+cR}=\frac{mn\,R\,(1+\alpha)}{nR}$$

From this it follows that

$$x=\alpha+(1+\alpha)\,c-c'$$

Next shift the battery connections from B' B' to BB and rebalance the bridge by adjustment of R to R′, keeping the connecting resistances a and a' unchanged and

$$\text{let } R'=R\,(1+\beta)$$

Then

$$\frac{S}{R'}=\frac{r'+a'}{r+a}$$

or,

$$\frac{mR(1+x)}{R(1+\beta)}=\frac{mnR(1+\alpha)+mc'R}{nR+cR}$$

On simplification we obtain,

$$1+x=(1+\beta)(1+\alpha)-(1+\beta)\frac{(1+\alpha)c-c'}{n+c}$$

In the first position of the battery connections, the error due to ignoring the connecting resistances would have made x too small by $(1+\alpha)c-c'$; in the second position the value found for x would have been too large by $(1+\beta)\frac{(1+\alpha)c-c'}{n+c}$. The ratio of the second to the first of these quantities is $\frac{1+\beta}{n+c}$.

From the above relations the following conclusions can be drawn:

(1) If the connecting resistances are entirely compensated,

$$x=\alpha,\ (1+\alpha)c=c' \text{ and } \beta=0.$$

That is,

$$\frac{S}{R}=\frac{r'}{r}=\frac{a'}{a}=m(1+\alpha)$$

In this case the bridge is in balance for both positions of the battery connections without adjusting R. Moreover, it is the only condition under which the connecting resistances are completely eliminated, as could have been seen directly from Fig. 15, by supposing the battery and galvanometer connections interchanged, there being in the above case no differences in potential between the pairs of terminals $B'B'$ and BB.

(2) The ratio $\frac{r}{R}$, designated by n, must be greater than unity in order that the error due to the connecting resistance may be reduced by rebalancing in the manner described. That is, the battery connections, when adjusting the bridge balance by varying one of the connecting resistances, must be applied so that the resistances a and a' are included in the arms of lower resistance. In other words, the connecting resistances are adjusted in such a manner as to make the ratio of the connecting resistances approximate the ratio of the bridge arms having the higher resistance. They can not, by the above process, be adjusted to the ratio of the arms of lower resistance.

(3) The original error, due to the presence of the connecting resistances remaining after the adjustment of one of them to balance the bridge with the battery applied at $B'B'$, is determined solely by the difference $x-\alpha$, the relative difference in balance if there be no connecting resistances. Rebalancing by adjustment of R (or S) with the battery applied at BB, the above error is reduced in the ratio $\frac{1+\beta}{n+c}$, or since β and c are generally small in comparison, respectively, with unity and n, the error is virtually reduced in the ratio $\frac{1}{n}$.

(4) The repetition of the above process, first rebalancing with the battery at $B'B'$, by adjustment of a or a', and then again rebalancing with the battery at BB by adjustment of R, will each time reduce the error in the approximate ratio $\frac{1}{n}$.

(5) In the special case in which $n=1$, the error in x in the two positions of the battery will be $(x-\alpha)$ and $-\frac{1+\beta}{1+c}(x-\alpha)$. That is, they will be approximately of the same magnitude but of opposite signs if β and c are small in comparisons with unity. For $n=1$, the method of eliminating the connecting resistances therefore fails.

(6) When $n \gtreqless 10$, the steps need only be repeated a very few times to eliminate the connecting resistances entirely, since $(x-\alpha)$ rarely exceeds 0.001. Moreover, in the preliminary adjustments the balance need not be exact, so that the final adjustment is quickly attained.

To apply these conclusions to the case of the Thomson bridge, it need only be remembered that the Thomson bridge is equivalent to a simple bridge having arms $R+K\frac{s}{s+s'}$, and $S+K\frac{s'}{s+s'}$, Fig. 16, where K is the resistance between the points bridged by the auxiliary ratio arms, and s and s' are the total resistances in the auxiliary ratio arms.

The connecting resistances a and a' must in the first instance be adjusted, so as to have the same ratio as r and r', no choice being left, as the ratio arms are larger than the resistances under comparison in practically all cases where the Thomson bridge is employed.

The procedure employed in the comparison of the mercury and wire standards was briefly as follows:

The wire standard, S (Fig. 13), was replaced by the mercury standard, connections to the bridge being made through the leads as shown. The auxiliary 1-ohm standard R was shifted to the position as indicated by the dotted lines and connected to the ratio arms by the adjustable low resistances[15] b and b' consisting of movable amalgamated copper rods in porcelain tubes containing mercury.

[15] Phys. Rev., 32, p. 614, 1911; 33, p. 215, 1911.

A rough balance of the bridge, with the link L' in, was first made by adjusting the shunt on R, the battery being applied at BB. The battery connections were next shifted [16] to B' B' by means of the commutator C, and the bridge rebalanced by adjustment of the mercury resistance b. With the battery again in the normal position and the link L' out, a balance was obtained by adjusting b'. The above operations were repeated and the final balance obtained by adjusting the shunt. After the approximate value of the resistance of a tube and its corresponding approximate balancing shunt had once been determined subsequent measurements could very quickly be made.

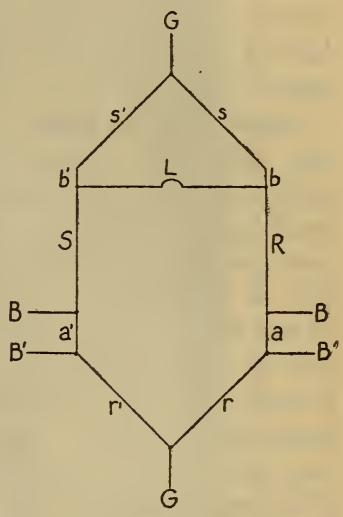


FIG. 16.—*Diagram of the Thomson bridge, with connecting resistances*

Preparation of the Tubes for Electrical Measurements.—Two tubes are made ready for electrical measurements at the same time. After cleaning and drying them in the usual way (this is always done before making a new filling) they are mounted on either side of a nickel-plated steel beam in carefully aligned V-grooved supports and clamped with suitable spring clips. The terminal bulbs are then seated, being adjusted so that the extensions are in a vertical plane. The bulbs are prevented from rotating by means of hard-rubber strips made to slip over the protecting tubes of the potential terminals and secured to uprights attached to the supporting beam. Under the center of each bulb is a vertical screw carrying horizontally a flat metal disk about 2 cm in diameter which is adjusted to support the bulb. A further support is provided between the bulb proper and the smaller bulb of the current terminal. This support consists of a rectangular piece of copper with a V-notch at the upper end

[16] This bulletin, 8, p. 585; 1912.

and with its lower end slotted so that the support may be adjusted to its proper height and held in place by a screw.

The mercury seals are next made secure by means of soft-rubber washers snugly fitting the ohm tubes and clamped against the face of the glass portion of the seal by screws through annular hard-rubber disks placed on the tubes and engaging slotted disks, also of hard rubber, slipped over the outlet of the bulbs just behind the enlargement forming part of the seal. The seals are filled with mercury and the proper seating of the tubes tested by partial evacuation of the tubes and bulbs. If satisfactory, the seals are capped and coated with celloidin to make them waterproof.

Preparatory to filling, a tube is connected to a Geryk pump and exhausted 15 minutes, the resulting observed pressure in most cases being less than 0.01 mm of mercury. It is filled by breaking the sealed and drawn-out end of the filling tube under pure mercury twice distilled in a Hulett still. The greater portion of the mercury is next removed from the upright portions of the filling and exhaust tubes and the final height of the mercury in each above the level of the ohm tubes recorded. Mercury is also introduced into the protecting tubes of the current and potential terminals. The filled tubes (see Fig. 17) are next removed to the ice bath and "iced up" preparatory to the electrical measurements.

The ice bath consists of a rectangular copper tank (Fig. 18) 128 cm long, 22 cm wide, and 16 cm deep, supported within a wooden tank with a clearance of about 6 cm all around. The inner tank is of double-walled, nickel-plated copper with intermediate air space of 0.6 cm. The outer tank is lined with nickel-plated copper sheeting and has the corners at the bottom rounded to facilitate its proper filling with ice. The outer tank is also provided with a drain pipe and a water gage.

Finely chipped, pure plate ice, made from distilled water, was used, care being taken to keep it free from contamination at all times. The ice was carefully packed about the tubes and terminal bulbs and extended to within 1 or 2 centimeters of the top of the terminal tubes. The outer bath was well packed with

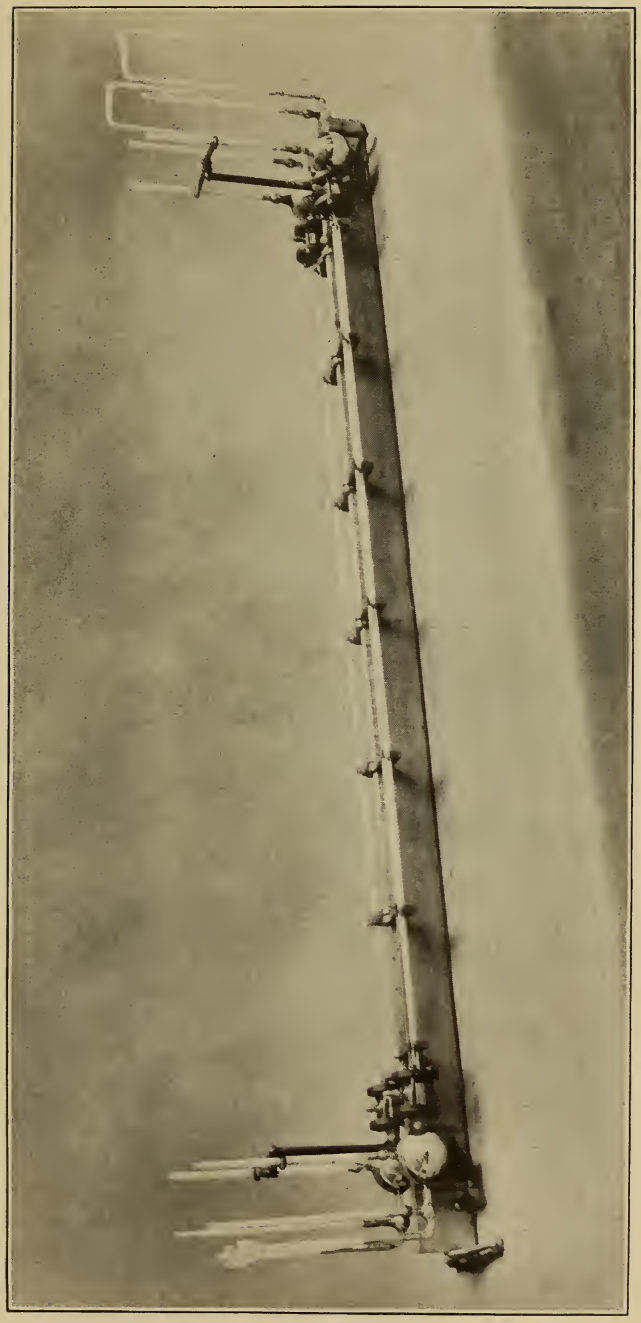


Fig. 17.—*View of two mercury units mounted and filled preparatory to placing in ice bath for electrical comparisons with wire standards*

coarser ice, and both tanks were filled to the top with distilled water. The resistance of the water in the inner bath was determined in nearly every case after the completion of the electrical measurements and averaged 3300 ohms. That of distilled water averaged 5500 ohms in the same apparatus. In one instance the measured resistance was abnormally low, 300 ohms, in which case the bath was cleaned and reiced and the electrical measurements repeated. With this one exception there was no cause for a rejection of the ice, the conductivity of the water being considerably less than that corresponding to a concentration of NaCl, which would affect the melting point by 0.001° C.

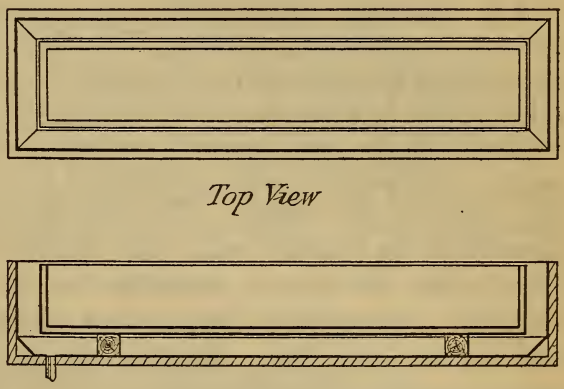


FIG. 18.—*Ice bath for mercury units under electrical comparison; also used in the determination of M_0 by Smith's method*

Insulation Resistance.—The insulation resistance of the inclosed column of mercury was measured in practically every case, and was found in most instances to exceed 100 megohms. The few exceptions were among the first fillings, where no attempt was made to protect the mercury seals from the water by coating them with celloidin.

The Connecting Leads.—Flexible stranded copper wire leads, cotton covered and 3 mm in diameter, were used for making electrical connections to the bridge. These leads terminate in well amalgamated No. 8 single copper wires, dipping into the mercury of the current and potential tubes. The height of the mercury in these tubes was so adjusted that with the leads in place the mercury was but slightly below the level of the ice. Heat conduction through the terminals into the end bulbs was thereby eliminated even when using the heavy conductors.

During the electrical measurements one of these terminals was several times heated in mercury at 90° C. and quickly placed in the potential terminal tube. The effect was principally thermoelectric, and the normal resistance was recovered within two or three minutes. No measurements were ever made after inserting the terminals until the galvanometer zero was steady, the usual time of waiting being from 5 to 10 minutes.

Effect of Test Current.—The effect of the test current of 0.035 ampere employed was found to be inappreciable. A current of 0.88 ampere steadily applied gave a resultant change in resistance of 238 millionths, corresponding to a change of approximately 300 parts per million per watt. The effect produced became constant within five minutes and the resistance returned to its original value within two minutes after opening the circuit. The current employed in the measurements corresponded to a load of 0.0012 watts, and therefore would correspond to a heating effect of but four parts in ten million. The experiments made also give a measure of the rapidity with which thermal equilibrium is established when the tube is placed in the ice bath. However, no electrical measurements were made without waiting at least half an hour after icing. Measurements occasionally repeated for various means confirm the above conclusions.

Effect of Pressure on Resistance.—The effect of the pressure on the conductivity of the mercury within the tube, due to the head in the filling and exhaust tubes, was necessarily very small, since the head in all cases was less than 6 cm. The change in resistance per unit resistance per atmosphere (internal and external pressure) of a column of mercury under pressure in ordinary glass as determined by A. De F. Palmer,[17] is approximately 33 parts per million. In tube No. 1 a decrease in resistance of but 3.7 parts per million was found for an increase in head of 15.6 cm of mercury, corresponding to a change of 1.4 parts per million for a head of 6 cm.

Effect of Curvature on Electrical Resistance.—A calculation was made to ascertain the influence of impressed curvature on the electrical resistance of an ohm tube on the assumption that the length of the axis remains unchanged and that the shape of the

[17] American Jr. Sci. (4), 4; p. 1; 1897.

cross section remains the same. The calculated resistance of the flexed tube was found to be

$$R_0\left\{1+\frac{1}{4}\left(\frac{a}{h}\right)^2+\frac{1}{8}\left(\frac{a}{h}\right)^4\right\}$$

in which R_0 represents the resistance of the unflexed tube, a the radius of the bore, and h the radius of curvature. It is apparent that there can be no appreciable effect due to this cause.

L. RESULTS OF ELECTRICAL COMPARISONS

In the electrical comparisons, readings were made, first on each of two tubes, then on the five wire coils with which they were compared, after which the readings on the tubes were repeated. The average difference between the first and last readings on the tubes (62 pairs in all) was less than 0.8 part per million.

The results of electrical comparisons of the ohm tubes with the wire standards in 1911 are shown in Table 28. Seven fillings were

TABLE 28

Results of Electrical Measurements, 1911

[The observed minus the calculated value in international ohms]

Filling	Tube No. 1		Tube No. 2		Tube No. 3		Tube No. 4	
	Difference in millionths	Δ	Difference in millionths	Δ	Difference in millionths	Δ	Difference in millionths	Δ
1	−27.8	+0.6	−27.0	+5.5	−17.7	+1.8	−14.4	+7.1
2	−28.0	+ .4	−34.8	−2.3	−19.2	+ .3	−16.4	+5.1
3	−28.2	+ .2	−34.0	−1.5	−18.1	+1.4	−16.6	+4.9
4	−31.4	−3.0	−28.7	+3.8	−20.0	− .5	−25.6	−4.1
5	−28.4	.0	−35.8	−3.3	−21.4	−1.9	−24.1	−2.6
6	−33.4	−5.0	−38.6	−6.1	−22.0	−2.5	−26.8	−5.3
7	−21.4	+7.0	−28.5	+4.0	−18.4	+1.1	−26.3	−4.8
Mean	−28.4	±2.3	−32.5	±3.8	−19.5	±1.4	−21.5	±4.8

Probable error of mean:

Tube No. 1 ±0.95 millionths
Tube No. 2 ±1.12 millionths
Tube No. 3 ±0.42 millionths
Tube No. 4 ±1.38 millionths

made of each tube, and the values tabulated are differences in international ohms of the observed and calculated values. The △s represent the deviation of the individual measurements from their mean. The average deviation from the mean and the probable error of the mean are also given. The calculated or theoretical resistances of the tubes have been given in Table 25, page 443.

The following table shows the mean results for the four tubes. The average deviation was 5 millionths, a very satisfactory agreement.

TABLE 29

Summary of Electrical Measurements, 1911

[Observed minus calculated resistance in international ohms]

Tube	Difference in millionths. Mean of seven fillings	△
1	−28.4	−2.9
2	−32.5	−7.0
3	−19.5	+6.0
4	−21.5	+4.0
Mean	−25.5	±5.0

Discussion of Results.—Table 29 shows that the international ohm, as defined by the four tubes, was found in 1911 to be 25.5 millionths smaller than the international ohm as represented by the manganin coils of the Bureau. These coils maintain the unit agreed upon in the 1910 international investigation, by intercomparison of the coils of England, France, Germany, and the United States.[18] The "1910 ohm" thus agreed upon represented the international ohm as closely as it was known at that time. The "1910 ohm" was so selected as to represent the mean of the English and German mercury ohm tubes, as no other countries had suitable ohm tubes at that time.

According to the comparisons made in the 1910 investigation, the international ohm, as represented by the German ohm tubes,

[18] Report to the International Committee on Electrical Units and Standards (issued by the Bureau of Standards, Jan. 1, 1912), pp. 82, 203.

was 10 millionths larger than that represented by the English ohm tubes. The first, therefore, differed by +5 millionths and the second by −5 millionths from the "1910 ohm." It follows that the international ohm as represented by the Bureau of Standards ohm tubes in 1911 was 30.5 millionths smaller than the unit defined by the German tubes and 20.5 millionths smaller than the unit defined by the English tubes.

PART III. SUPPLEMENTARY MEASUREMENTS

In July, 1912, two additional electrical comparisons of each of the four Bureau of Standards tubes were made. As the observed resistance of each tube minus its calculated resistance was greater than found in 1911 (see Tables 29 and 33), a third series of comparisons was made in November and December, 1912, which, in general, confirmed the results of the July series.

Redeterminations of the M_0 and L_0 of each tube were therefore made early in 1913 and the theoretical resistances recalculated. All measurements were made in the same way as those of 1910 to 1911, with the exception that in the redetermination of L_0 readings were made on but two positions of each tube with a smaller number of readings for each position, and hence with a correspondingly larger probable error. The work involved a recalibration of the weights employed and a recomparison of meters 39 and 22 at 0° C. The results are summarized in the following tables.

Tables 30, 31, and 32 show that the average change in L_0, M_0, and the calculated R_0 for all four tubes was 1.1, 9.4, and 8.2 parts per million, respectively.

Table 34 shows that the international ohm as defined by the four tubes in December, 1912, was 12.5 millionths smaller than the international ohm as represented by the manganin coils of the Bureau at that time. The ohm tubes and the manganin coils of the Bureau have thus come into better agreement since 1911, when the difference was 25.5 millionths. It would appear that the manganin coils changed in such a way that the unit which they define decreased by 13 millionths. That the change may actually be attributed to the manganin coils rather than to the

ohm tubes was indicated by comparisons made with the standards in other national laboratories. Coils were sent by the Bureau to England and Germany in 1911 and 1912 and measured in terms of the manganin standards of the National Physical Laboratory and the Reichsanstalt, respectively. According to the former the unit defined by the Bureau manganin coils decreased 9 millionths, and according to the latter 5 millionths during the interval.

TABLE 30

Redetermined L_0, 1913

Tube	L_0 in centimeters	L_0, 1913, minus L_0, 1911, in parts per million
1	95.082500	+2.00
2	96.260476	−1.17
3	96.996315	+0.89
4	100.693824	+0.53
Average change	.....	1.14

TABLE 31

Redetermined M_0, 1913

Tube	Fillings	Mean M_0 in grams	Average deviation from mean in parts per million	Probable error of results in parts per million	M_0, 1913, minus M_0, 1911, in parts per million
1	7	11.574870	±2.19	±0.91	+10.89
2	7	11.864101	±3.17	±0.95	+11.80
3	7	12.042745	±3.57	±1.07	+ 2.49
4	7	12.975108	±2.77	±0.90	+12.28
Mean	.....	.....	±2.92	±0.96	+ 9.36

TABLE 32

Recalculated R_0, 1913

Tube	R_0 in international ohms	Calculated R_0, 1913, minus R_0, 1911, in parts per million
1	0.9999467	− 6.88
2	0.9998674	−14.13
3	1.0001270	− 0.71
4	1.0004127	−11.20
Average change	.....	8.23

TABLE 33

Observed R_0—Calculated R_0 in International Ohms, Series 2, July, 1912

[The calculated R_0 was prorated on the assumption that the M_0 and the L_0 changed uniformally between the two determinations]

Tube	Fillings	Observed minus calculated R_0 in parts per million	△
1	2	−13.7	− 4.9
2	2	−11.8	− 3.0
3	2	−12.7	− 3.9
4	2	+ 3.0	+11.8
Mean	.....	− 8.8	± 5.9

TABLE 34

Observed R_0—Calculated R_0 in International Ohms, Series 3, November and December, 1912

[Calculated R_0 from 1913 determination of M_0 and L_0]

Tube	Fillings	Observed minus calculated R_0 in parts per million	△
1	4	−19.5	− 7.0
2	5	− 8.6	+ 3.9
3	4	−19.7	− 7.2
4	4	− 2.1	+10.4
Mean	.....	−12.5	± 7.1

The unit defined by the manganin coils of the Bureau, therefore, decreased by 7 millionths in terms of the mean of the English and German manganin standards, and decreased by 13 millionths in terms of the mercury tubes of the Bureau. This rather good agreement is partly accidental, since the mean change of 13 millionths is an average for the four tubes of changes ranging from 0 to 24 millionths, as may be seen from the following table:

TABLE 35

Observed R_0 — Calculated R_0, in Parts per Million

Tube	1911	December, 1912	Change	Δ
1	−28.4	−19.5	− 8.9	+ 4.1
2	−32.5	− 8.6	−23.9	−10.9
3	−19.5	−19.7	+ 0.2	+13.2
4	−21.5	− 2.1	−19.4	− 6.4
Mean	−25.5	−12.5	−13.0	± 8.6

The mean deviation of 8.6 millionths is a rough measure of the accuracy of reproduction of the mercury unit at different times, using the same tubes.

COMPARISON OF OHM TUBES OF SEVERAL COUNTRIES

A comparison of the mercury ohm tubes of the various national laboratories was made in 1913 by means of measurements upon a number of manganin coils. The English National Physical Laboratory sent coils to the Laboratoire Central d'Electricité of France, the Reichsanstalt of Germany, the Bureau of Measures and Weights of Russia, and to the Bureau of Standards. Each laboratory made measurements upon the coils and gave data by which the results could be expressed in terms of the mercury ohm tubes of the laboratory. The values given at that time by this Bureau were in terms of the mercury ohm measurements of July, 1912. In the following table these have been changed to correspond with the latest mercury ohm measurements, those of December, 1912. Mercury ohm tubes have also been set up in the Electrotechnical Laboratory at Tokyo, Japan,[19] and compared with tubes supplied

[19] Proc. Math.-Phys. Soc. Tokyo, 7, p. 346; 1914. Science Abstracts, 17, No. 1890; 1914.

by the National Physical Laboratory of England. The unit defined by the Japanese tubes was found to be 8 millionths smaller than the unit defined by these English tubes. Assuming (1) that these English tubes give the same mean unit as the tubes kept in the National Physical Laboratory, and (2) that the Japanese laboratory takes the mean of both sets of tubes to be the mercury resistance unit of Japan, it follows that the Japanese mercury unit is 4 millionths smaller than the English mercury unit. The first of these assumptions is probably not exact, but the result can not be far wrong.

TABLE 36

Comparison of Units Defined by Ohm Tubes in 1913

Country	Mercury unit of country minus mercury unit of United States
	Millionths
England	+10
Germany	+28
Japan	+6
Russia	+5
United States	0

New tubes have been set up in France, but the results have not yet become available.

As has been stated above, the unit defined by the ohm tubes of this Bureau in 1911 was 20.5 millionths smaller than the unit defined by the ohm tubes set up in England prior to 1910. The English mercury unit used in Table 36 is that defined by a set of new tubes. It would appear that the new English tubes gave a unit 10.5 millionths smaller than the old. This is corroborated by an estimate given by the National Physical Laboratory, viz, the new tubes define a unit 11 millionths smaller than the old.

Table 36 shows that the mercury unit of the Bureau of Standards was 28 millionths smaller than the German mercury unit in 1913. This is a good agreement with the difference of 30.5 millionths found in 1911.

In conclusion, the mercury ohm tubes of the various countries have been found to exhibit a very satisfactory agreement.

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